# 1 3.05, 10am-12pm

## 1.1 Exercise, normal distribution

MR, 26

Let $X \sim N(9, 25)$. Show that $P(X \leq 0) = 1 - \Phi(9/5)$, where $\Phi$ is the cdf of the standard normal variable, $N(0, 1)$.

*Hint:* If $X \sim (\mu, \sigma^2)$, and $Z = aX + b$, then $Z \sim N(a\mu + b, a^2\sigma^2)$.

## 1.2 Exercise, discrete, random vectors

The police department in Berlin is doing computer simulations for how to manage the football hooligans after the upcoming game between FC Union Berlin and Dynamo Dresden. Assume that there will be

A1: $h \geq 1$ hooligans transferring at Suedkreuz Bahnhof from Olympiastadion, where the game is, to their train home to Dresden

A2: $r \geq 1$ police officers to monitor them

A3: the simulation only cares about the relative positions of hooligans and police along a line parallel to the incoming train

A4: the ordering of hooligans and police officers along this line is uniformly random on $h + r$ elements.

For example, if there were $h = 5$ hooligans and $r = 3$ police officers, then the outcomes (elements of $\Omega$), where P represents a police officer, and H a hooligan

- PHHPPHHH, and
- HHHPHPHP

are equally probable.

For $k = 1, \ldots r - 1$, define the random variable $\xi_k$ as

$$\xi_k = \text{number of hooligans between the } k^{\text{th}} \text{ and } (k+1)^{\text{st}} \text{ police officers}$$

where we count from left to right, and define additionally

- $\xi_0$ as the number of hooligans to the left of the first police officer,
- $\xi_r$ as the number of hooligans to the right of the last police officer.

1. Determine the probability distribution function of the random vector $\Xi = (\xi_0, \ldots, \xi_r)$, i.e. $P(\xi_0 = k_0, \ldots, \xi_r = k_r)$ as a function of $k_0, \ldots, k_r$.
2. For a fixed $j \in \{0, \ldots, r\}$, determine the marginal probability distribution $P(\xi_j = k)$.
3. Are the component random variables $\xi_0, \ldots, \xi_r$ independent for all $r$ (and all $h$)?

## 1.3 Exercise, hat (or jacket check) problem

$n$ visitors to a museum in Berlin all check in black, Jack Wolfskin jackets upon arrival before 6pm. When they come to pick up their jackets later in the evening around 8pm, with uniform probability, the coat check clerk gives them a random black, Jack Wolfskin jacket.

Let $X$ be the random variable defined as the number of visitors who get the correct jacket back, so $X \in \{0, \ldots, n\}$.

On average, how many visitors do we expect to get back their own jacket?

*Hint:* Use indicator functions / variables, and expected value.

## 1.4 Exercise: A variant on the blue and red pills

(Adapted from Stirzaker, Probability and Random Variables, Chapter 5, ex. 11)

In the movie the Matrix, the character Morpheus offers another character Neo two pills. If Neo chooses the blue pill, he will return to his old life. If he chooses the red one, his eyes will be opened to some truth, and he will join Morpheus' band.

In this variant (also of Pólyeva's urn, see MR, vaja 4), Neo has to choose blindly from a bucket initially containing one red and one blue pill. If he picks red, he must swallow it and the game is over. If he picks blue, then he returns the blue pill and Morpheus adds an extra red pill.

Let $X$ be the number of draws. Calculate

1. The distribution $P(X > k)$
2. $\mathrm{E}(X)$ (hint: use the method of indicators)