ZS 1

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1. Computing with Matrices/Vectors

If-Statement: `if( abs(x) < numeric_limits<double>::epsilon)`

Absolute Error: $\varepsilon_{abs} := |x - \tilde{x}|$

Relative Error: $\varepsilon_{rel} := \frac{|x - \tilde{x}|}{|x|}$ $\tilde{x}$ has ℓ correct digits if $\varepsilon_{rel} \leq 10^{-\ell}$

EPS: $EPS := \max\limits_{x \in \mathbb{R}} \frac{|rd(x) - x|}{|x|}$

Rounding: $rd(x) : \begin{cases} \mathbb{R} \to \mathbb{M} \\ x \to \max(\operatorname{argmin}_{\tilde{x} \in \mathbb{M}} |x - \tilde{x}|) \end{cases}$

Trick: EPS is smallest number such that $1 \tilde{+} EPS \neq 1$ $(10 \tilde{+} EPS = 10)$

Assume you solve $Ax = b$ with x_{ex} being the exact value:

Forward error: $|x_{app} - x_{ex}|$ not computable

Backward error: $|b_{app} - b|$ computable

Small backward error $\not\Rightarrow$ Small forward error

S.p.d. Matrix: Matrix A is s.p.d. if

- $A^H = A$
- $\forall x \in \mathbb{K}^n : x^H A x \in \mathbb{R}$ $x^H A x > 0 \Leftrightarrow x \neq 0$

or

- All eigenvalues positiv

Complexity:

what	what	# of ·,/ operations	# of +,- operations	complexity
Dot product	$x^H y$	n	$n-1$	$O(n)$
Tensor product	$x y^H$	$n \cdot m$	0	$O(nm)$
Matrix product	$A B$	$n \cdot m \cdot k$	$m k (n-1)$	$O(nmk)$

Stability: An algorithm $\tilde{F}$ for finding y for a given b is numerically stable if:

$$\forall b \in \mathbb{M}^n : \frac{\|b - b_{app}\|}{\|b\|} = O(\operatorname{cost}(\tilde{F}) \cdot EPS)$$

How close is $\tilde{F}(x)$ $(= F(\tilde{x}))$ to $F(x)$? $\Rightarrow$ Condition Number

$$\operatorname{cond}(A) := \|A\| \cdot \|A^{-1}\| = \frac{\sigma_{max}}{\sigma_{min}} \quad \text{(singular values)}$$

Trick: Impact of roundoff errors within $\tilde{F}$ is not worse than impact of rounding the input for F