Useful Concepts for Decline-Curve Forecasting, Reserve Estimation, and Analysis

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Summary

The purpose of this paper is to give engineers responsible for making forecasts and determining reserves for numerous operated or nonoperated wells some guidelines and fundamental concepts to allow them to make these forecasts and determinations more quickly and accurately.

Introduction

Decline-curve analysis, based on the Arps[1] equations, has always been considered to be purely empirical with no basis on physical laws governing the flow of oil and gas through the formation. The works of Fetkovich[2–11] and others have attempted to place decline-curve analysis on a sound, fundamental basis using the constant wellbore pressure analytical solution and simple combinations of material-balance equations and pseudosteady-state rate equations to derive rate/time decline equations for oil and gas wells. (See additional references listed in Ref. 14.) The derivations illustrate under what circumstances specific values of the hyperbolic decline exponent, b, should result. It is from these derivations that the variables in Arps equations can be expressed in terms of reservoir variables and reservoir engineering concepts. Decline-curve analysis, or more specifically rate/time analysis, is not simply an art based on applying a purely empirical equation to be analyzed with statistical approaches. Reliance on a statistical analysis, void of reservoir engineering concepts, often leads to unrealistic and unreliable forecasts and reserve estimates.

Oil and Gas Decline Equations

Arps Equations. Conventional decline-curve analysis is based on the empirical equations of Arps.[1] In his work, Arps proposed a set of equations described by an exponent, b, having a range between 0 and 1. The resulting rate time and cumulative production equations are shown in **Table 1** (see also Ref. 14). He used his equations analyzing some 149 oil fields' production data, assembled by Cutler–1924, in 1945, for determining the distribution of b. He found that the range of exponent, b, was between 0 and 0.7 with 90% having b less than 0.5. No harmonic declines were found and less than 15% had a b value less than 0.1. The rapid development and virtual wide-open production of many of these oil fields resulted in the ideal situation for constant wellbore pressure production over their entire life.

Although the original Arps exponential and hyperbolic equations were developed empirically on the basis of oil production data, Arps forms of equations can be rigorously derived for volumetric dry gas systems. This permits a physical basis to be placed on each of the variables, q_i, $q(t)$, b, and D_i.

Exponential Decline (Oil) (Derived) (Above Bubblepoint). The single-phase liquid, exponential decline equation, valid for undersaturated oil or water flow, has been derived in Refs. 2 and 3. The derivation is based on a simple material-balance equation and pseudosteady-state or stabilized backpressure curve equation. Both equations are depicted graphically in **Fig. 1.** The use of this approach has also been used in Ref. 3 to derive the Arps form of equations for single-phase gas and solution-gas-drive volumetric systems.

The complete form of the derived exponential equation for constant wellbore pressure production is

$$q(t) = q_i / e^{\left\{D_i/\left[1-\left(p_{wf}/\overline{p}_R\right)\right]\right\}t}, \quad (1)$$

where
$$D_i(p_{wf}) = q_i/N_{puo}\Big/\left[1-\left(p_{wf}/\overline{p}_R\right)\right]. \quad (2)$$

Both q_i and N_{puo} are functions of the constant pressure, p_{wf}. For $p_{wf}=0$,

$$D_i = q_{i\,\max}/N_{puo,o}, \quad (3)$$

where $N_{puo,o}$ is the ultimate recoverable to $q(t)=0$ and $p_{wf}=0$.

Note that different levels of constant wellbore pressures, p_{wf}, always result in an exponential decline and D_i is the same for all levels of backpressure. (This is not the case for derived forms of the hyperbolic equation.) The goal of any prudent operator to maximize production and recoverable reserves and to minimize drainage of reserves to offset wells is to maintain the flowing pressure, p_{wf}, as close to 0 as is economically possible. Therefore, $p_{wf}\approx 0$ was the basis for deriving hyperbolic forms of decline equations which turn out to be very useful for understanding some important concepts of decline-curve analysis.

Solution-Gas Decline Equations (Derived) (Below Bubblepoint). As presented and discussed in Ref. 3, the material-balance and rate equations used to derive rate/time equations for solution-gas-drive oil wells are

$$\overline{p}_R^2 = -\left(\overline{p}_{Ri}^2/N_{puo}\right)N_p + \overline{p}_{Ri}^2, \quad (4)$$

and
$$q_o = J'_{oi}\left(\overline{p}_R/\overline{p}_{Ri}\right)\left(\overline{p}_R^2 - p_{wf}^2\right)^n. \quad (5)$$

The ratio $(\overline{p}_R/\overline{p}_{Ri})$ is used in the rate equation to approximate the decrease in relative permeability, k_{ro}, with pressure depletion. The reader is referred to Ref. 6, Table 9 and Fig. 25 where the works of Levine and Pratts and Vogel were used to justify the pressure ratio approximation to the k_{ro} change in Eq. 5.

The derived rate/time equation for an oil well producing against a constant pressure, p_{wf}, is for all backpressure curve exponents $n > 0.5$, hyperbolic.

$$q(t) = \frac{q_i}{\left\{1+\left[(2n-1)/2\right]\left(q_i/N_{puo}\right)t\right\}^{(2n+1)/(2n-1)}} \quad (6)$$

and exponential for $n=0.5$

$$q(t) = q_i\Big/e^{-\left(q_i/N_{puo}\right)t}. \quad (7)$$

High-capacity, flowing oil wells that are "tubing limited," friction pressure drop dominated, will tend to exhibit exponential decline, as is the case for tubing limited gas wells.

With the derivations based on $p_{wf}=0$, $q_i=q_{i\,\max}$. At backpressures other than $p_{wf}=0$, q_i is a rate from the stabilized backpressure curve at the specified flowing pressure, p_{wf}. N_{puo} is equal to the original oil-in-place times a recovery factor (RF)

$$N_{puo} = N \times \mathrm{RF}, \quad (8)$$

where the recovery factor may be obtained from correlations, historical performance, or Tarner type material-balance forecasts at some selected gas/oil relative permeability curve.

D_i, in terms of reservoir engineering terms, is for both the hyperbolic and exponential forms

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TABLE 1—SUMMARY OF PRODUCTION DECLINE EQUATIONS

Decline Type	Hyperbolic	Exponential	Harmonic
Rate–Time	$q(t) = q_i \Big/ (1 + bD_i t)^{1/b}$	$q(t) = q_i / e^{D_i t}$	$q(t) = q_i \Big/ (1 + D_i t)$
Time to $q(t)$	$t = \left\{ [q_i/q(t)]^b - 1 \right\} \Big/ bD_i$	$t = \ln[q_i - q(t)]/D_i$	$t = \{[q_i/q(t)] - 1\}/D_i$
Cumulative–Time	$Q_p = [q_i/(1-b)D_i]\left[1 - (1 + bD_i t)^{(b-1/b)}\right]$	$Q_p = (q_i/D_i)\left(1 - e^{-D_i t}\right)$	$Q_p = (q_i/D_i)\left[\ln(1 + D_i t)\right]$
Rate–Cumulative	$Q_p = [q_i b/(1-b)D_i]\left[q_i^{(1-b)} - q(t)^{(1-b)}\right]$	$Q_p = [q_i - q(t)]/Q_{D_i}$	$Q_p = (q_i/D_i)\ln[q_i/q(t)]$
From Rate–Cum. D_i at $q(t) = 0$	$D_i = [1/(1-b)](q_i/Q_{puo})$	$D_i = q_i/Q_{puo}$	D_i is not definable; (Q_{puo} is infinite).
D_i(oil)	$D_i = [(2n+1)/2](q_i/N_{puo})$ $N_{puo} = N \times$ (RF) where RF $= f(k_g/k_o)$	$n = 0.5$; $D_i = (q_i/N_{puo})$	Not derivable
D_i(gas)	$D_i = 2n(q_i/G)$ $G = G_i \times$ (RF) where RF $= \left[1 - (p_{wf}/\bar{p}_R)\right]$	$n = 0.5$; $D_i = (q_i/G)$	Not derivable
b(oil) where $p_{wf} \approx 0$	$b = (2n-1)/(2n+1)$ where n is between 0.5 and 1		
b(gas) where $p_{wf} \approx 0$	$b = (2n-1)/2n$ where n is between 0.5 and 1		

$$D_i = [(2n+1)/2](q_i/N_{puo}). \quad \text{(9)}$$

The Arps decline exponent, b, can be expressed by the backpressure curve exponent, n, for wells producing at very low flowing pressures

$$b = (2n-1)/(2n+1). \quad \text{(10)}$$

For $n = 1$, the value that one would assume in the absence of an actual multipoint test having been run on any well in the field, b would be equal to 0.33. A slope $n = 1$ should be considered as being typical for most solution-gas-drive reservoirs and also would be equivalent to assuming the Vogel IPR relationship. With $n = 1$,

$$D_i = 3/2\,(q_i/N_{puo}). \quad \text{(11)}$$

In the absence of a clearly defined decline exponent from field data, a value of $b = 0.3$ should be assumed for a solution-gas-drive reservoir; i.e., $n = 1$ in Eq. 10. However, indication of an unfavorable relative permeability, k_g/k_o, relationship would dictate a decline exponent approaching exponential decline, $b = 0$, because of the anticipated lower recovery factor (see **Fig. 2**).

Gas Well Decline Equations (Derived). Most of the fundamental reservoir engineering concepts and definitions of the terms in the Arps equations, q_i, $q(t)$, b, and D_i, can be made from the derivation of the single-phase gas equations.

Derivation of the gas well decline equations is completely analogous to that previously used to derive the single-phase liquid exponential decline. The rate decline $q(t)$ with time for a single-phase, single-layer, gas flow system, as in the single-phase liquid solution, is based entirely on reservoir pressure depletion of a closed finite system. For single-phase gas, we use the pseudosteady state or stabilized backpressure equation and material-balance equation, as depicted graphically in Fig. 1. The derived rate/time equation for a gas well producing against a constant pressure, p_{wf}, is for all backpressure curve exponents $n > 0.5$ a hyperbolic

$$q(t) = q_i \Big/ \left[1 + (2n-1)(q_i/G)\,t\right]^{2n/(2n-1)}, \quad \text{(12)}$$

and an exponential for the backpressure curve exponent $n = 0.5$.

$$q(t) = q_i / e^{(q_i/G)t}. \quad \text{(13)}$$

With the derivations based on $p_{wf} = 0$, $q_i = q_{i\max}$, and $G = G_i$, where $q_{i\max}$ is the stabilized absolute open-flow potential and G_i is the original gas-in-place. At backpressures other than $p_{wf} = 0$, q_i is a rate from the stabilized backpressure curve at the specified flowing pressure,

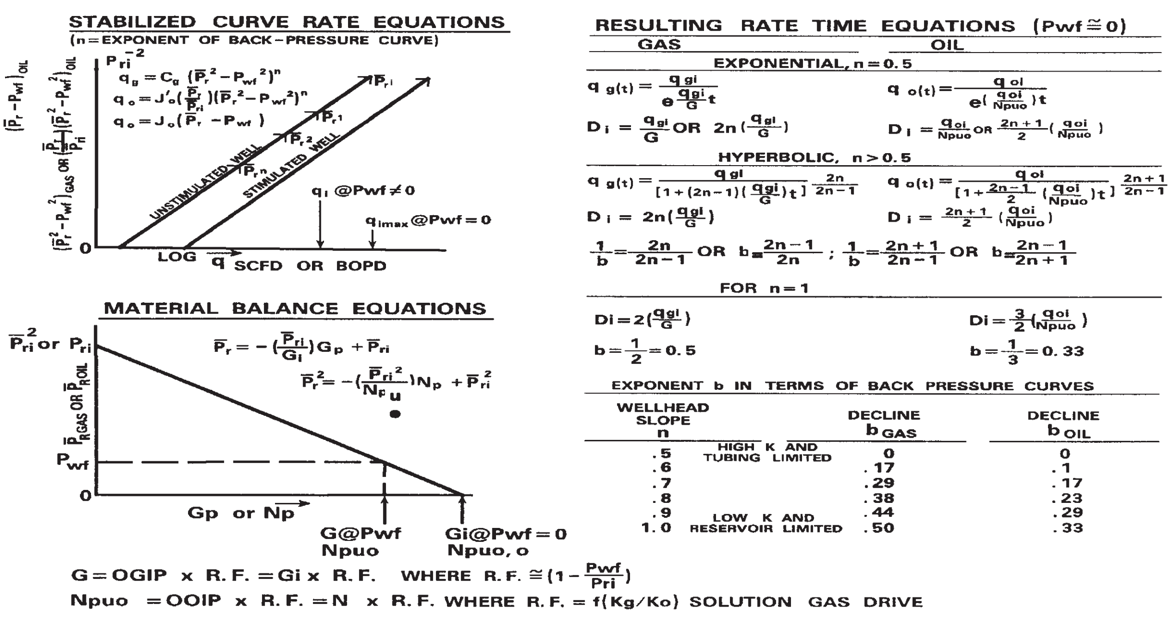

Fig. 1—Rate and material-balance equations used to derive rate/time decline equations for gas and oil wells.

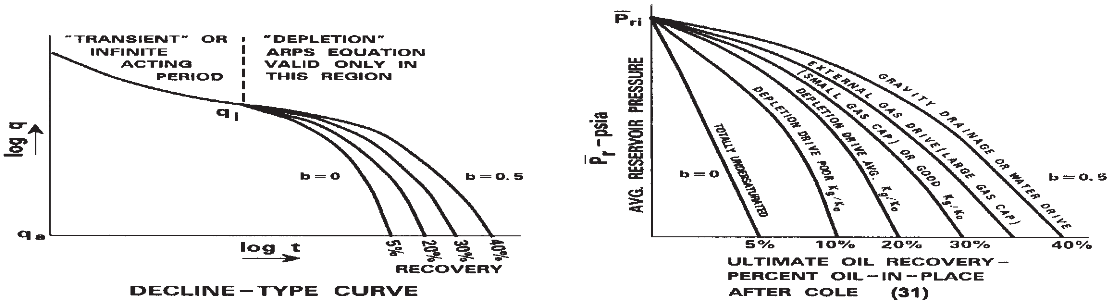

Fig. 2—Decline exponent *b* as a reflection of recovery efficiency or drive mechanism.

p_{wf}. G is the recoverable gas to an abandonment pressure equal to p_{wf}, where G is equal to the original gas-in-place times a recovery factor.

$$G = G_i(\text{RF}), \quad (14)$$

$$\text{where RF} \approx \left[1 - \left(p_{wf}/\bar{p}_R\right)\right]. \quad (15)$$

These same type definitions involving in-place volume and recovery factor were also used for single-phase and multi-phase oil systems—i.e., q_i and N_{puo}.

Note that the basic form of the gas well decline equations, Eqs. 12 and 13, are identical to that of the Arps form (see Table 1). They exactly reproduce or overlay the Arps type curve over its entire length. D_i expressed in more familiar and readily available reservoir engineering terms is for both the hyperbolic and exponential forms

$$D_i = 2n\left(q_i/G\right). \quad (16)$$

All the terms in Eq. 16 can be calculated or estimated from initial multipoint, pressure transient analysis results and geological data before any rate decline data is even available. Or in the case of a replacement well or offset location to be drilled, values from offset wells can be used.

With regard to q_i, q_i is a rate from the stabilized wellhead or the bottomhole backpressure curve. If tubular friction is not significant, the bottomhole and wellhead curves will be essentially the same except for a hydrostatic head term. q_i is not simply a producing rate at early time, it is very specifically a pseudosteady-state rate at the surface. It can be substantially less than actual early time transient flow rates as would be produced from low-permeability wells with large negative skins.

The Arps decline exponent, b, can be expressed in terms of the backpressure curve exponent, n, for wells producing at very low flowing pressure

$$b = (2n - 1)/2n. \quad (17)$$

The exponent n from a gas well backpressure performance curve can therefore be used to calculate or estimate b and D_i. Using Eq. 17, we can determine the physical limits of b, which is between 0 and 0.5, over the accepted theoretical range of the backpressure curve exponent, n, which is between 0.5 and 1 for a single-layer homogeneous system (see **Table 2**).

The harmonic decline exponent, $b=1$, cannot be obtained. In fact, no other investigators have been able to derive an exponent b greater than 0.5 for any reasonable single-layer, homogeneous reservoir system or drive mechanism. Arps study of the range of the exponent b for 149 oil fields also tends to support the 0.5 value as a physical upper limit of b. He found that 90% of the oil fields studied had b values less than 0.5.

As will be discussed later, a layered no-crossflow reservoir system, or its equivalent, can result in decline exponents that cover the range of b between 0.5 and 1—values of b greater than 0.5 can be used to identify layered no-crossflow reservoirs. In Arps distribution study he found no value of b greater than 0.7. It should also be pointed out that attempting to fit all or some of the "transient" production rate data of a well with the Arps pseudosteady-state equation will result in an "apparent" b value higher than it really is. In some cases, it will even be greater than 1.

TABLE 2—DECLINE EXPONENT *b* AS A FUNCTION OF BACKPRESSURE EXPONENT *n*

Decline Exponent, b	Backpressure Exponent, n
0	.50 (High k)
.1	.56
.2	.62
.3	.71
.4	.83
.5	1.0 (Low k)

Cumulative Production Equations. The cumulative production/time equation is for $n>0.5$, the hyperbolic form

$$G_p/G = 1 - \left[1 + (2n-1)\left(q_i/G\right)t\right]^{1/(1-2n)}, \quad (18)$$

and for $n=0.5$, the exponential form

$$G_p/G = 1 - e^{-\left(q_i/G\right)t}, \quad (19)$$

both of which can be readily reduced to the Arps cumulative production-time equations and rate/cumulative production equations shown in Table 1 (see also Ref. 14, Eqs. 6 through 11).

Rate/Cumulative Production Equations (Derived). The rate/cumulative production equations for single-phase liquid and gas can be directly derived from the backpressure curve and material-balance equations (see Fig. 1). From the backpressure curve equation at $\bar{p}_{Ri}$ and $\bar{p}_{R(t)}$ where

$$C_{gi} = C_g(t) = q/\left(\bar{p}_R^2\right)^n, \quad (20)$$

$$\text{then } q(t)/q_i = \left[\bar{p}_{R(t)}/\bar{p}_{Ri}\right]^{2n}. \quad (21)$$

From the material-balance equation slope

$$\bar{p}_{R(t)}/\bar{p}_{Ri} = \left[G_i - G_{p(t)}\right]/G_i. \quad (22)$$

Substituting Eq. 22 into 21 and simplifying, we get

$$q(t)/q_i = \left[1 - G_{p(t)}/G_i\right]^{2n}. \quad (23)$$

For $n=0.5$ or exponential decline, $b=0$,

$$q(t) = q_i - (q_i/G_i)G_{p(t)}. \quad (24)$$

The Arps hyperbolic rate/cumulative production equation converted to our nomenclature with $q(t)=0$ is

$$G_p = \left\{q_i^b/\left[(1-b)D_i\right]\right\}\left[q_i^{(1-b)}\right]. \quad (25)$$

Using the gas relationship $b=(2n-1)/2n$ and $D_i=2n(q_i/G)$, the hyperbolic rate/cumulative equation given in Table 1 reduces to

$$q(t)/q_i = \left[1 - G_{p(t)}/G\right]^{2n}, \quad (26)$$

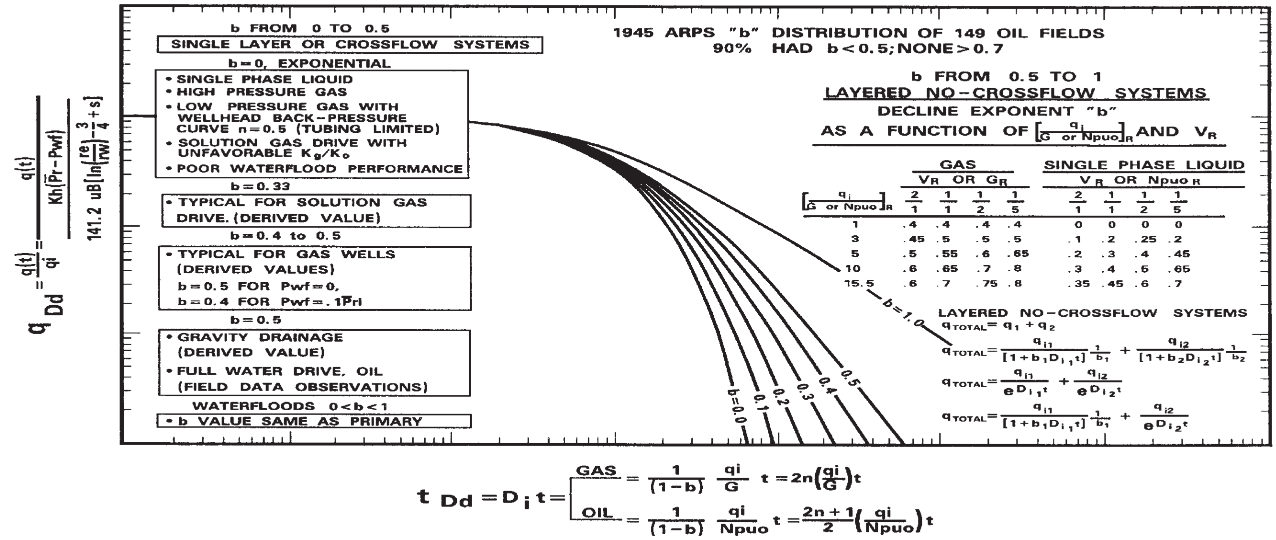

Fig. 3—Decline exponents *b* for various reservoir drive mechanisms.

which is identical to Eq. 23. A similar approach was used for the single-phase liquid and solution-gas-drive decline equations with similar results.

Because both the rate/time and rate/cumulative equations are derived using the same backpressure and material-balance equations, both methods of analyzing production data should get identical results.

Recovery or Drive Mechanisms and *b*. In many instances, rate/time data existing in the depletion period is of such poor quality or limited extent that a unique value of b cannot be determined. Reliance on a statistical analysis, void of reservoir engineering concepts, to determine the decline exponent often leads to unrealistic or illogical values of b and unreliable or indefensible forecasts and reserves estimates.

From basic reservoir engineering principles, several of the expected values of b have been derived for different reservoir drive or recovery mechanisms. These values should be used when production data is poor, insufficient, or totally lacking. They can also be used to support or confirm clearly defined values of b determined from good quality production data.

Following is a tabulation of the values of b that should be expected for homogeneous single-layer or layered crossflow systems (see **Fig. 3**). The range of the expected b values for these systems is from 0 to 0.5.

b* = *undeterminable. Constant-rate or increasing-rate production period. Flow rates are all in the transient or infinite-acting period with no supplemental engineering or geological information.

b* = *0, exponential. Single-phase liquid (highly undersaturated oil wells). High-pressure gas. Low-pressure gas with wellhead backpressure curve exponent $n \approx 0.5$ (tubing limited wells, both gas and oil flowing wells). Depletion or solution-gas-drive with unfavorable k_g/k_o. Poor waterflood performance. Wells with a high backpressure, $p_{wf}/\bar{p}_R \rightarrow 1$. Gas wells undergoing liquid loading. Gravity drainage with no free surface (derived value).

b* = *0.3. Typical for solution-gas-drive (a derived value).

b* = *0.4 to 0.5. Typical for gas wells (derived values). $b = 0.5$ for $p_{wf} \approx 0$; $b = 0.4$ for $p_{wf} = 0.1\,\bar{p}_{Ri}$.

b* = *0.5. Gravity drainage with a free surface (derived value). Full waterdrive in oil reservoirs (field data observations).

An unpublished study conducted several years ago on West Texas fields being waterflooded found values of b ranging from exponential, $b = 0$, to $b = 0.9$ or nearly harmonic. The decline exponent, b, was essentially the same for different leases within a given "field" but different fields had different values of b.

Fig. 2 is an attempt to illustrate the concept that b is a reflection of recovery efficiency or drive mechanism. The pressure-percent recovery figure, after Cole,[12] depicts typical values of percentage recoveries (recovery efficiency) for various reservoir drive mechanisms from the least efficient, totally undersaturated reservoir with a $b = 0$, to a much more efficient gravity drainage or waterdrive recovery mechanism with a $b = 0.5$. Different drive mechanisms and typical recoveries are also depicted in between.

Also shown on this figure is a corresponding rate/time decline type curve consisting of a transient period followed by depletion stems ranging from $b = 0$ to 0.5. Note that as b increases, the percent recovery increases. (These same recovery values are also depicted on the Cole pressure-% recovery plot.) With the transient or infinite-acting production period fixed, it requires a larger value of b, or better recovery efficiency mechanism, to get a larger area under the rate/time curve to achieve higher fraction recoveries. Before the outermost boundary is encountered, the transient period would be the same regardless of what later drive mechanism is established once a depletion process begins.

Relationship Between Backpressure Curve Exponent *n* and Decline-Curve Exponent *b*. Because gas and oil are sold at the wellhead, we must consider the affect that the tubing string has on production for flowing gas and oil wells. It is the exponent, n, of the wellhead backpressure curve that affects the decline exponent, b. Examination of field performance backpressure curves[7] indicates that low-permeability gas wells yield bottomhole backpressure curves with n values more nearly approaching 1.0 while high-permeability gas wells yield n values approaching 0.5.

The Forchheimer form of the backpressure equation is

$$\bar{p}_R^2 - p_{wf}^2 = Aq + Bq^2, \quad \text{(27)}$$

where A is a laminar flow pressure drop term and B is a turbulent flow pressure drop term. When kh is large, the Aq term becomes small and we have

$$q \approx \left(1/\sqrt{B}\right)\,(\bar{p}_R^2 - p_{wf}^2)^{0.5}. \quad \text{(28)}$$

Similarly, when kh is small Aq becomes large with the Bq^2 term becoming negligible when compared with the laminar pressure drop term. We would then have

$$q \approx (1/A)(\bar{p}_R^2 - p_{wf}^2)^{1.0}. \quad \text{(29)}$$

Expressing the flow equation at a surface or wellhead datum and including a pressure drop term for the tubing or flow string, we have

$$p_c^2 - p_t^2 = (A_{WH})q + (B_{WH} + T_{WH})q^2, \quad \text{(30)}$$

where T_{WH} represents a friction pressure drop term for flow through the tubing and is indistinguishable from the reservoir turbulent pressure drop component (B_{WH}) at the surface datum. Eq. 30 is more

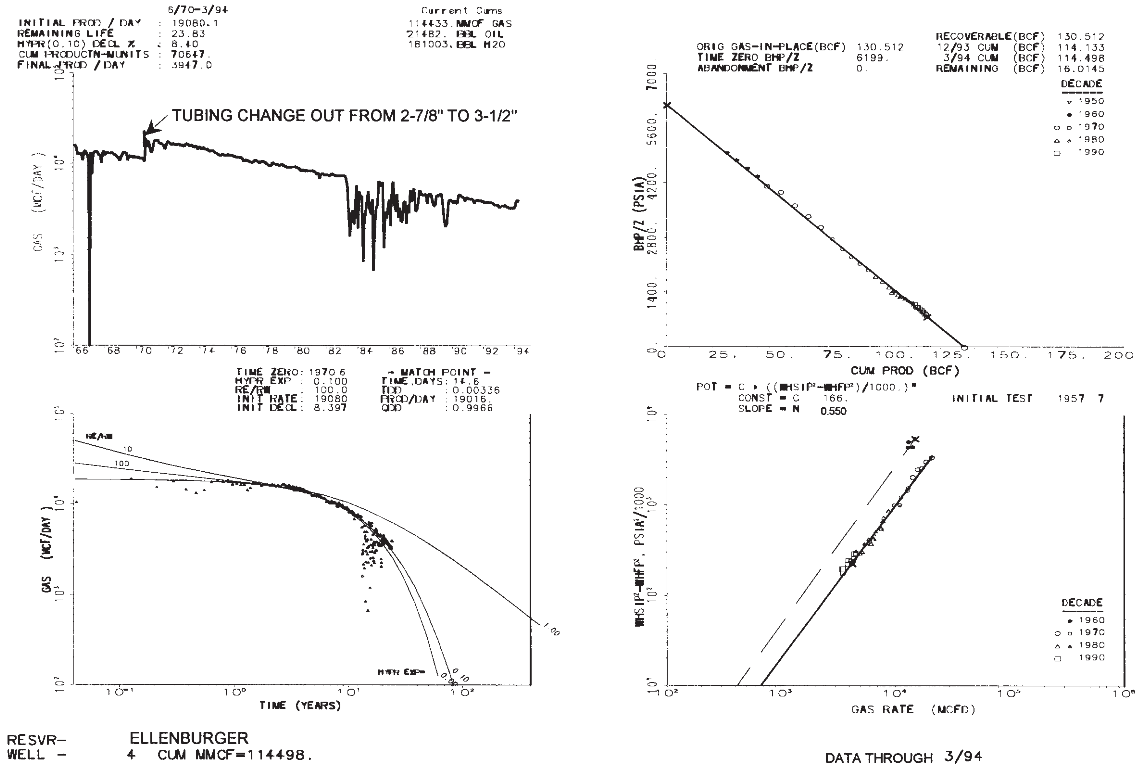

Fig. 4—Well performance plots of material-balance, backpressure curve, and rate/time semilog and log-log graphs.

useful for performance monitoring when expressed as the wellhead backpressure curve

$$q \approx C_{WH}\ (p_c^2 - p_t^2)^n, \quad \ldots \quad (31)$$

where the range of n is between 0.5 and 1.

It should be recognized that a storm choke, flowline, and other surface equipment can be included in Eqs. 30 and 31, even further driving the value of n to 0.5 in high-capacity flowing gas and oil wells. Note that as a limiting condition in Eq. 30 if T_{WH} is large compared with A_{WH} and B_{WH}, a very large bottomhole deliverability well with a relatively small diameter tubing string, Eq. 30 reduces to

$$q \approx C_{TWH}(p_c^2 - p_t^2)^{0.5}. \quad \ldots \quad (32)$$

When the exponent (n) of the wellhead backpressure curve approaches 0.5, the well is considered to be "tubing limited" and will always exhibit exponential rate/time decline. Once tubing limited "always tubing limited"—the backpressure performance curve does not shift to the right as reservoir pressure, $\bar{p}_R$ declines. The wellhead backpressure curve based on 25+years of performance data presented in **Fig. 4** has an exponent $n=0.55$. The effect of a tubing changeout, 2⅞ to 3½ in., is shown both on the backpressure curve and the semilog production plot. A 50% increase in production was achieved by simply increasing the tubing diameter. Yet the well is still "tubing limited". (Well depth and casing programs often dictate a maximum tubing size.) Note that the log-log production plot initialized after the tubing changeout to a larger diameter fits a decline exponent, $b=0.1$, stem of the type curve. The decline exponent is exactly what would be predicted by using Eq. 17 and simply knowing that the backpressure curve exponent is 0.55.

This example illustrates that the wellhead deliverability curve and exponent n could, in some instances, be totally described by the pressure drop through the tubing string. We could use Eq. 32 to establish for a given tubing size a maximum position for a wellhead curve, its wellhead potential $q_{i\max}$ or q_i, exponent $n \approx 0.5$ and it will exhibit exponential decline, $b=0$. For any other diameter flow string, one need only ratio $(D^{2.612}\text{new}/D^{2.612}\text{present}) \times C_{TWH}$ to draw in its new wellhead deliverability curve. Using 3.476 and 2.992 in. ID for the tubing sizes in Fig. 4, we get a curve shift 1.5 $\times C_{TWH}$ that reproduces the actual results obtained (i.e., a 50% increase in production).

When a well is tubing limited, $n_{\rightarrow}0.5$, it should be recognized that a significant deterioration in the bottomhole performance curve will not be reflected in any change in the wellhead backpressure performance curve. Conversely, any improvement in the bottomhole performance curve, such as a stimulation or restimulation, will not result in any increase in surface production; i.e., there will be no change in the wellhead deliverability curve and no increase in production. Clearly, one will not be able to calculate reservoir variables from rate/time analysis on tubing limited gas or oil wells since most of the pressure drop in the well is tubing friction pressure drop.

Backpressure Effects on *b*. The level of backpressure, p_{wf}, does not affect b for single-phase liquid flow, it's always exponential—i.e., $b=0$. The effect of backpressure on a gas well is demonstrated for a backpressure curve exponent $n=1$ in Fig. 9 given in Ref. 3. Backpressure is expressed as a ratio of $p_{wf}/\bar{p}_{Ri}$ where p_{wf} is the constant pressure against which the rate declines with time and can also be considered as an abandonment pressure, p_{wfa}. The rate/time type curve for a backpressure ratio, $p_{wf}/\bar{p}_{Ri}=0$ has a decline exponent b value of 0.5 and would yield a recovery factor $[1-p_{wf}/\bar{p}_{Ri}]$ of 100%. At a backpressure ratio of $p_{wf}/\bar{p}_{Ri}=0.9$, where $p_{wf\rightarrow}\bar{p}_{Ri}$ the rate/time type curve exhibits exponential decline, $b=0$, and would yield a recovery factor $[1-p_{wf}/\bar{p}_{Ri}]$ of only 10%. Backpressure ratios shown on the figure yielding 20% and 50% recoveries will not trace any of the Arps type curve stems over its entirety—they cut across several of the b stems. From a practical standpoint, no operator would produce a well that is on decline at such high backpressure to abandonment. A more realistic ultimate backpressure ratio limit, $p_{wf}/\bar{p}_{Ri}$, for volumetric gas wells in particular, would be 0.1. This would yield a maximum recovery factor of 90% and result in a decline exponent $b=0.4$. The b value of 0.4 should be considered as a good limiting value for gas wells when not clearly defined by actual production data.

Some Basic Concepts. Fig. 1 depicts in a graphical form the $(\bar{p}_R - p_{wf})$, pseudosteady-state rate equation, and the $\bar{p}_R - N_p$, material-balance equation, from which the exponential decline equation was derived. The drawdown backpressure curve is also a depletion

TABLE 3—DECLINE EXPONENT b AS A FUNCTION OF $(q_i/G)_R$ or $(q_i/N_{puo})_R$ AND VOLUME RATIO, V_R

Single-Phase Gas					Single-Phase Liquid				
	V_R or G_R					V_R or $(N_{puo})_R$			
$(q_i/G)_R$	2/1	1/1	1/2	1/5	$(q_i/N_{puo})_R$	2/1	1/1	1/2	1/5
1	.4	.4	.4	.4	1	0	0	0	0
3	.45	.5	.5	.5	3	.1	.2	.25	.2
5	.5	.55	.65	.65	5	.2	.3	.4	.45
10	.6	.65	.7	.8	10	.3	.4	.5	.65
15.5	.6	.7	.75	.8	15.5	.35	.45	.6	.7

$q_{total} = q_1 + q_2$; individual layer $b = 0.4$ for gas and $b = 0$ for single-phase liquid

$$\text{Gas}: \; q_{total} = \frac{q_{iT}}{\left\{1 + b_T\left[1/(1-b_T)\right](q_{iT}/G_T)t\right\}^{1/b_T}} = \frac{q_{i1}}{\left\{1 + 0.4\left[1/(1-0.4)\right](q_{i1}/G_1)t\right\}^{1/0.4}} + \frac{q_{i2}}{\left\{1 + 0.4\left[1/(1-0.4)\right](q_{i2}/G_2)t\right\}^{1/0.4}}$$

$$\text{Liquid}: \; q_{total} = \frac{q_{iT}}{\left\{1 + b_T\left[1/(1-b_T)\right]\left(q_{iT}/N_{puoT}\right)t\right\}^{1/b_T}} = \frac{q_{i1}}{e^{\left(q_{i1}/N_{puo1}\right)t}} + \frac{q_{i2}}{e^{\left(q_{i2}/N_{puo2}\right)t}}$$

curve in that all flow rates $q(t)$, including q_i, will trace down the curve as the reservoir pressure, $\overline{p}_R$, declines as a result of production. The backpressure curve is fixed, J_o is constant, for all stages of pressure depletion providing there are no skin changes or the drainage radius, r_e, does not change. In equation form, the pseudosteady-state backpressure equation is

$$q(t) = \frac{kh \;\; (\overline{p}_{R(t)} - p_{wf})}{141.2\mu B\left[\ln\left(r_e/r_w\right) - \frac{3}{4} + s\right]}, \quad \ldots \ldots \ldots \ldots \ldots \ldots \; (33)$$

or in its simplest form

$$q(t) = J_o\left[\overline{p}_{R(t)} - p_{wf}\right]^{1.0}. \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \; (34)$$

For a total field or lease backpressure curve, we would have

$$q_T(t) = WJ_{o\,avg}(\overline{p}_R - p_{wf}), \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \; (35)$$

where W is the total number of wells and J_{oavg} represents our average well PI. Or we could sum the individual wells to obtain a total field or lease composite backpressure curve.

$$q_T(t) = \sum_{j=1}^{W} J_{oj}(\overline{p}_R - p_{wf}). \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \; (36)$$

The material-balance equation, graphically depicted in Fig. 1, relates the cumulative production, N_p, to the reservoir pressure, $\overline{p}_R$, is

$$\overline{p}_R = -\left(\overline{p}_{Ri}/N_{puo}\right)N_p + \overline{p}_{Ri}. \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \; (37)$$

A basic assumption again is that r_e does not change; i.e., the well (or lease) is neither draining nor being drained by offset wells. For a total field, the reservoir volume, or r_e, always remains fixed.

An important reservoir engineering concept useful for interpreting decline performance and understanding and applying decline-curve analysis is that each well in a common reservoir undergoing pseudosteady–state depletion drains a volume in proportion to its producing rate. From this we can point out a couple of important concepts that result.

1. The decline rate, D_i or q_i/N_{puo}, theoretically should be the same for all wells in the common reservoir. Wells that have a noticeably different D_i are not in communication.

2. When a new well(s) is added (drilled) or an old well(s) is restimulated in a common reservoir, the decline rate, D_i, will increase for each well. The depletion of the total field will be essentially accelerated. All other wells will lose some of their remaining reserves.

Layered, No-Crossflow Decline Behavior

Most reservoirs consist of several layers with reservoir properties varying between layers. The fact that no-crossflow reservoirs are perhaps the most prevalent and important, reservoir heterogeneity is of considerable significance in long-term forecasting and reserve estimates. If crossflow exists in a layered reservoir, adjacent layers can simply be combined into a single equivalent layer using the average reservoir properties of the crossflowing layers. It will then perform as an equivalent homogeneous single-layer system. For a homogeneous single-layer system, the maximum value of b is 0.5. Decline-curve exponents, b, ranging between 0.5 and 1 are a predicted response for layered, no-crossflow reservoirs[10] and can therefore be used to identify them. We have found that layered, no-crossflow reservoirs have the greatest potential for increasing current production and recoverable reserves.

Low-permeability, stimulated wells' production performance can appear similar to layered, no-crossflow reservoir responses on a semilog production curve. However, a log-log type curve plot can be used to distinguish between the two. Further confirmation of no-crossflow can be made by measuring layer pressures and having some idea of the well's permeability level.

For two or more layers producing against a common flowing pressure, p_{wf}, the commingled production rate, q_T, is simply the sum of separate producing rates or forecasts from each of the individual layers. Producing at a constant wellbore pressure, production from each layer is independent of all the other layers present.

$$q_T = q_1 + q_2 + ,\ldots, + q_n. \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \; (38)$$

For a two-layered system, or its equivalent (most multilayered systems can be reduced to an equivalent of two layers by combining layers with similar (q/G) or (q/N_{puo}) values, we can write

$$q_T = \frac{q_{i1}}{\left(1 + b_1 D_{i1} t\right)^{1/b_1}} + \frac{q_{i2}}{\left(1 + b_2 D_{i2} t\right)^{1/b_2}}, \quad \ldots \ldots \ldots \; (39)$$

$$\text{or } q_T = \left(q_{i1}/e^{D_{i1}t}\right) + \left(q_{i2}/e^{D_{i2}t}\right), \quad \ldots \ldots \ldots \ldots \ldots \ldots \; (40)$$

$$\text{or even } q_T = \frac{q_{i1}}{\left(1 + b_1 D_{i1} t\right)^{1/b_1}} + \frac{q_{i2}}{e^{D_{i2}t}}, \quad \ldots \ldots \ldots \ldots \; (41)$$

where D_i is defined in Table 1 and Layer 1 normally represents the higher-deliverability layer.

The sum of two or more separate forecasts usually results in a forecast with the decline exponent, b, greater than that of each individual layer. We have, for a commingled system using single-phase gas variables, for example

$$q_T = \frac{q_{iT}}{\left\{1 + \left[b_T/(1-b_T)\right](q_{iT}/G_T)t\right\}^{1/b_T}}, \quad \ldots \ldots \ldots \ldots \; (42)$$

where q_{iT} is the sum of each layer's flow rates, q_i, and G_T is the total ultimate recoverable gas in each layer. The exponent b_T is the single

TABLE 4—LAYER SKIN VALUES

s_1	s_2
−7	0
0	+72
−7.6	−4
−2.2	+50

value of b that results in the best match, or fit, of the commingled production rates, excluding late-time commingled production rates when necessary. Very-late-time production rates will ultimately result in the b value of Layer 2 producing by itself. Use of the early to middle time production data permits reasonable calculated values of q_{iT} and G_T from actual production performance data.

Table 3 contains predicted values of the decline exponent, b, for a two-layer system as a function of (q_i/G) ratio vs. volume ratio, V_R or G_R for single-phase gas and (q_i/N_{puo}) ratio vs. volume ratio, V_R or $[N_{puo}]_R$, for single-phase liquid. The high-deliverability layer is normally the numerator in both ratios. The q_i/G or q_i/N_{puo} ratio can be described as a rate-of-take or rate-of-depletion indicator. The layer that has the higher q_i/G or q_i/N_{puo} value is depleting its reserves faster. It is important to recognize that the q_i for the high-deliverability layer is not just a reflection of layer permeability, k, but also of layer skin, s, and layer shut-in pressure, $\bar{p}_R$. With regard to layer volume, V or (G or N_{puo}), the PV ratio, V_R, will essentially remain constant for all producing times, whereas the gas-in-place ratio, G_R or $[N_{puo}]_R$, changes with time due to differential pressure depletion. This becomes significant when we consider a constant rate production period followed by a declining production rate period; i.e., the volume ratio, G_R, will be different at the beginning and end of the constant-rate period.

From Table 3, we see that the highest b values listed are for $(q_i/G$ or $q_i/N_{puo})$ ratios between 10 and 15.5 (see Ref. 10 for the results of larger ratios). Further, for any given $(q_i/G$ or $q_i/N_{puo})$ ratio, b increases as the volume ratio decreases. In either case, this occurs because the reserves in the high-deliverability layer are being depleted faster with respect to the low-deliverability layer. As a result, depletion of the high-deliverability layer occurs over a shorter time interval with respect to the low-deliverability layer. The composite or combined surface flow rate will have b values that are driven further towards a value of b equal to 1 as either the q_i/G or q_i/N_{puo} ratio increases, the V_R decreases, or both. The $(q_i/G)_R$ ratio is written as

$$(q_i/G)_R = (q_{i1}/G_1)/(q_{i2}/G_2), \qquad (43)$$

where Layer 1 is generally the higher-deliverability layer. Assuming, for simplicity, that $p_{i1}=p_{i2}$ and (μc_t) are equal for both layers and substituting reservoir variables, we have

$$(q_i/G)_R = \frac{k_1/\phi_1}{k_2/\phi_2}\frac{\left[\ln(0.472\,r_{e2}/r_w) + s_2\right]}{\left[\ln(0.472\,r_{e1}/r_w) + s_1\right]}\frac{r_{e2}^2}{r_{e1}^2}. \qquad (44)$$

With a (q_i/G) ratio of 10, one normally assumes that this would represent a layered, no-crossflow system with a 10 to 1 contrast in permeability. From Eq. 44, this would be true if $\phi_1=\phi_2$, $r_{e1}=r_{e2}$, and $s_1=s_2$. Note that thickness, h, has cancelled from the equation. However, if layer k, ϕ, and r_e are all equal, we would still have a (q_i/G) ratio of 10 for the following skin conditions shown in **Table 4** on each layer.

Some small contrast in permeability would lead to even more reasonable values of skin on each layer to still maintain the 10 ratio and the resulting high decline exponents, b, reflected in the table. For all reservoir variables of each layer being equal, including skins, s, an $r_{e1}=630$ ft and $r_{e2}=2{,}000$ ft would also result in a ratio ≈ 10. One need not have a high contrast in permeability to have significant layered, no-crossflow reservoir production performance behavior that exhibits high values of the decline exponent, b.

In a layered, no-crossflow system, the maximum recoverable reserves from the low-deliverability layer will occur as $(q_i/G)_R$ approaches 1. We, therefore, must have as large a negative skin, s_2, on the low-permeability layer(s) as possible and as early in the production life as possible. On commingled wellbore systems, even with attempts at diversion, the stimulation seems to preferentially occur in the higher permeability layer(s). This preferential stimulation of the high-permeability layer(s) will most likely occur in openhole completions that have been acidized commingled. By restimulating wells with the objective of targeting the low-permeability layers, substantial increases in production and recoverable reserves can be achieved. This potential can normally be identified through high decline exponents, b. Note from Table 3 that the higher the b value, the more volume in the low-permeability layer and therefore the more potential for increased production and recoverable reserves. With a single-layered system exhibiting a low b value, restimulation generally results in accelerated production. With a layered, no-crossflow system exhibiting a high b value, restimulation can result in substantial increased recoverable reserves, depending on the volume of the low-permeability layer(s). The (hydraulically) fractured well example given in Ref. 3 had a b of between 0.6 and 1 to fit both the before and after stimulation data. The rather successful treatment reflected in both the increase in production and additional recoverable reserves is an example of what one could expect from a proper restimulation of a layered, no-crossflow well.

Following is a list of some of the more important characteristics of layered, no-crossflow behavior:

1. High value of the decline exponent, b; $b>0.5$. This is reflected as an early rapid decline in rate followed by an extended period of a low percentage decline.
2. Reservoir has an indicated unusually long producing life.
3. Unusually rapid decline in reservoir shut-in pressure. Later, a nearly constant measured shut-in pressure with increasing cumulative production. [This is because the pressures usually represent the most permeable layer, but production is coming mainly from the low-permeability layer(s).]
4. Large discrepancy between pressure performance oil or gas-in-place and volumetrics. (Indicated low recovery factor.)
5. Thick reservoirs have a very high likelihood of exhibiting layered, no-crossflow behavior. (There appears to be a strong correlation of increasing b with thickness.)
6. Thin reservoirs don't automatically mean they will behave as a single layer. (We have measured differential depletion in a 9-ft sand.)
7. Naturally fractured reservoirs permit a contrast in $(q_i/G)_R$ required for layered, no-crossflow behavior (i.e., high b values as seen in the Monterey formation).
8. Layer pressures indicating differential depletion.
9. Production logs indicating wellbore backflow during shut-ins or rate reductions. Lack of backflow does not mean lack of layered, no-crossflow behavior.

The reader is referred to Ref. 10 for additional material on layered, no-crossflow reservoir behavior.

We recognize the similarity of rate/time and pressure cumulative production and a few of the other listed characteristics are similar to that of a composite reservoir model; however, we find it to be an unrealistic model on an individual well basis. It is a possible model on a fault-block or a field-wide basis. The question of which it may be is readily resolved by the taking of layer pressures and comparing pressure data on an areal basis.

Production Data Treatments

Production data from a commercial database is usually available on a monthly production basis, with the production sometimes being erratic and difficult to analyze. We have found that 6-month or 12-month averaging, with the average rate $q_{avg}(t)$ plotted at the midpoint of the time period, works very satisfactorily (see Ref. 4, Figs. 20 and 21 for examples). Material is neither created nor destroyed this way, as can be done with purely mathematical smoothing approaches. Further, this averaging is nearly equivalent to cumulative production/time plots that are often made to make the same erratic rate/time data "look" better. A derivative at the midpoint of the year would result then in rate/time data for a rate/time plot. It is much easier to relate rate/time data to one's experience, particularly transient behavior, than it is cumulative production time plots.

Transient production data, however, result in the opposite recommendation. Average monthly production on low-permeability, highly stimulated wells or fields will not reveal some useful data (see Ref. 4, Figs. 32 and 33). We have had extremely good success using daily production from meter charts on individual wells in the Carthage Cotton Valley field.

Normalization. Another important technique we use in rate/time analysis for low-permeability stimulated wells is rate normalization—i.e., $q(t)/[p_i - p_{wf}(t)]$. Often, during transient production, both the rate q(t) and flowing pressure, $p_{wf}(t)$ are both declining smoothly and monotonically. Normalizing the rate by dividing by $[p_i - p_{wf}(t)]$ is identical to rigorous superposition and will permit the normalized data to overlay the $q_D - t_D$ type curves. Extrapolation of the fit into the future by tracing along the type curve results in a rate/time forecast as if the well were being produced at the last value of p_{wf}. Production rates at a different backpressure, p_{wf}, would then require a superposition calculation.

Abrupt changes in flowing pressure, p_{wf}, normally a reduction in p_{wf} to increase production and recoverable reserves, is adequately discussed in Ref. 3 (Table 2 and Fig. 14), Ref. 5 (Tables 9 and 9A and Fig. 9), and Ref. 13 (Pages 438–41). The principle of superposition also applies to an increase in p_{wf2} as would be applicable in attempting to handle the gas bubble curtailment period of the mid-1980's. See Figs. 34 and 35 of Ref. 4 for the negligible effect of numerous field shut-ins on the overall total field production.

One additional normalization procedure that we have used is well count normalization when the well count is increasing smoothly and monotonically as a result of continuous drilling in a field or lease. One of the more noticeable effects we have seen was in the Sooner Trend field between 1970 and 1982. The total field production data matched a decline exponent $b = 1$ during this period of continuous drilling. Normalized by well count to an average well basis, the decline exponent was found to be $b = 0.23$ instead of $b = 1$ unnormalized.

Reinitialization of Data. There are several obvious situations where rate/time data must be reinitialized because the drive or production mechanism has changed. The most obvious situation is production above and below the bubblepoint pressure. Above the bubblepoint production will follow exponential decline, $b = 0$. Further, the calculation of reservoir variables, kh, s, and original oil-in-place, N, will be most accurate when calculated during the undersaturated period. Below the bubblepoint pressure, the values of the decline exponent, b, and N_{puo}, would change and the value of b will be greater than exponential decline depending on the k_g/k_o relationship. Pressure-cumulative production and gas/oil-ratio performance often help indicate when the reinitialization should be done. Some other conditions where the production data might be reinitialized are:

1. Primary, secondary, and tertiary recovery periods would all be reinitialized. The decline exponent, b, however should be similar for each period.
2. An abrupt change in the number of wells on a lease or in a field. An infill well on a one-well section, for example, will result in a change in q_{iT}, $(q_{i1} + q_{i2})/G$ compared with q_{i1}/G before the infill well.
3. Restimulation of an original well would be similar, $q_{i\,\text{new}}/G$, as compared to $q_{i\,\text{old}}/G$.
4. Also similar would be a tubing changeout to a larger diameter where the well was tubing limited. Not only would q_i change afterwards, but the decline exponent, b, could also change due to the increase in wellhead backpressure exponent, n (see Fig. 4 and Eq. 17).

Finally, anytime during the depletion period, one can reinitialize the problem and get the same resulting forecast. However, there becomes a point where there is not sufficient rate/time data to uniquely match or define the decline exponent, b. Using a different value of b would result in a different forecast and reserve estimate. One of the problems with reinitializing is obtaining a new $\bar{p}_R$ to use in attempting to calculate reservoir variables.

Stimulation and Restimulation (Depletion). The effects of a stimulation or a restimulation, providing a well is not tubing or equipment limited, result in a change in deliverability, q_i, and possibly the remaining recoverable gas or oil, G or N_{puo}. The decline exponent, b, normally can be assumed to remain the same.

For an individual layer, a convenient "rule-of-thumb" equation[13] to approximate an increase in rate due to a stimulation or restimulation is

$$q_{i\,\text{new}} = q(t)_{\text{old}}\left[(7 + s_{\text{old}})/(7 + s_{\text{new}})\right], \quad \ldots\ldots\ldots\ldots\ldots (45)$$

where $q(t)_{\text{old}}$ is the producing rate just prior to stimulation, and $q_{i\,\text{new}}$ is the new q_i for the reinitialized forecast. We would then have

$$q(t)_{\text{new}} = q_{i\,\text{new}}/\left(1 + bD_{i\,\text{new}}t\right)^{1/b}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

where b is assumed to be the same before and after stimulation and

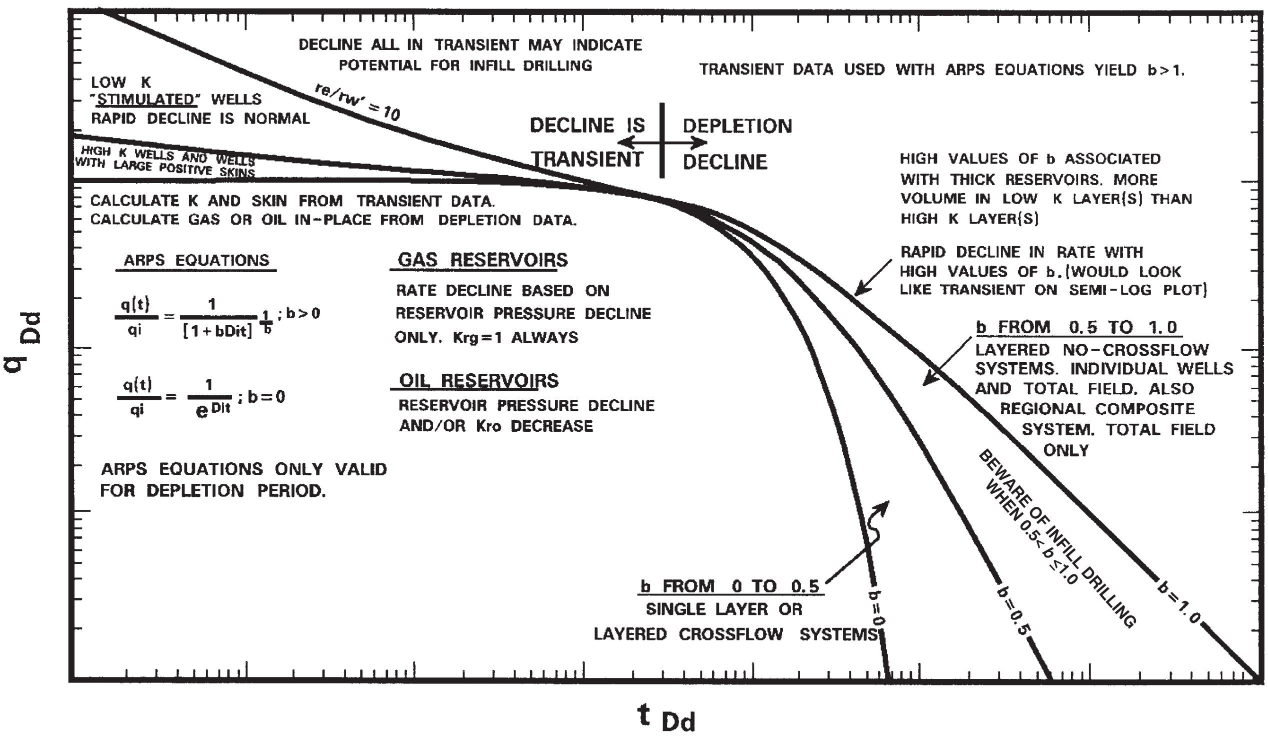

Fig. 5—Composite of analytical and empirical type curves with comments.

$$D_{i\,\text{new}} = \frac{1}{(1-b)}\left(\frac{q_{i\,\text{new}}}{G \text{ or } N_{puo\,\text{remaining}}}\right). \quad (47)$$

G or N_{puo}, if assumed to be the same as that obtained prior to stimulation, would be simply acceleration, which would be the case if all offsets were stimulated or restimulated at the same time. If offsets were not restimulated, then the remaining reserves would increase in direct proportion to the increase in producing rate as a result of a radius of drainage readjustment; i.e., drainage of offset wells would occur.

For layered, no-crossflow systems, we would assume that the low-deliverability layer was not stimulated originally and calculate its increase due to restimulation to q_{i2} with the high-deliverability layer, $q_i(t)$, remaining the same. In this case, the increase in reserves would be a result of reducing the amount of differential depletion between layers.

Calculating Reservoir Variables

There are two periods of rate decline, transient decline followed by depletion decline. These two periods, and their production response are illustrated on the log-log type curve of **Fig. 5.**

Transient Decline. Production data existing during the transient decline period can be matched to a constant wellbore pressure type curve to calculate both kh and skin, s, from the following equations

$$kh = 141.2\mu B\left\{[q/(p_i - p_{wf})]/q_D\right\}_{\text{match}} \quad (48)$$

and $$r_{wa} = \sqrt{\left[0.00634\ kh/(\mu c_t)\phi h\right](t/t_D)_{\text{match}}}, \quad (49)$$

where $$r_{wa} = r_w e^{-s}, \quad (50)$$

or in terms of skin

$$s = -\ln(r_{wa}/r_w). \quad (51)$$

Eq. 48 is written to emphasize the need to normalize the rates q by using flowing pressure, p_{wf}, when necessary. To calculate kh and s from transient data, we also prefer to use the more general radial flow constant wellbore pressure solution type curve in terms of $q_D - t_D$, Fig. 2-A of Ref. 3, with the effective wellbore radius, r_{wa}. This type curve covers a much broader range of $q_D - t_D(r_{wa})$ that applies to most of the very low-permeability, successfully stimulated wells. Wells with large, positive skins of any permeability level lose uniqueness in that the rate/time data would be in the region where $q(t)$ appears to be nearly constant because of large $t_D(r_{wa})$ values. Eq. 49 is written in terms of kh and ϕh to emphasize the point that when we pick h we are assigning an oil or gas-in-place—not simply determining k. The significance is now apparent in that we are making production forecasts. It's one thing to reduce the k by a half by assigning a range of thickness, h, and quite different to cut the oil or gas-in-place by half. Generally, people who do pressure transient analysis seldom apply the results to production forecasting. See Refs. 3 through 5 for some good field examples.

Depletion Decline. Rapidly declining transient production data in tabular form, or even on a semilog plot, often is misinterpreted to be depletion. The primary use of the composite type curve (Fig. 5) is to be able to distinguish between transient and depletion production decline by making a log-log plot of the production data, matching it and then forecasting future production by extending the match down the appropriate depletion stem. If production data exist only in depletion, we can calculate oil or gas-in-place and a combined kh and r_{wa} or skin and PI, J.

In its simplest form, using the Arps equation and match points from a type curve match on the depletion stem, b, calculate

$$q_i = [q(t)/q_{Dd}]_{\text{match}} \quad (52)$$

and $$D_i = (t_{Dd}/t)_{\text{match}}, \quad (53)$$

where $$\frac{kh}{\left[\ln(0.473 r_e/r_{wa})\right]} = \frac{141.2\mu B}{(\bar{p}_R - p_{wf})} q_i. \quad (54)$$

By assuming some value of skin, or r_{wa}, we can calculate a value of kh.

Calculate the pseudosteady-state PI for the rate equation

$$J = q_i/\left[\bar{p}_R - p_{wf}\right]. \quad (55)$$

Using the hyperbolic expression for D_i from Table 1, we can then calculate the recoverable reserves

$$G \text{ or } N_{puo} = \left[1/(1-b)\right](q_i/D_i). \quad (56)$$

Then, gas or oil-in-place at the start of decline is

$$G_i \text{ or } N = (G \text{ or } N_{puo})/\text{RF}. \quad (57)$$

Conclusions

Rate/time analysis (decline-curve analysis) has been placed on a sound, fundamental basis using the constant wellbore pressure analytical solutions. Derivations combining the material-balance and pseudosteady-state rate equations result in rate/time decline equations for oil and gas wells. It is from these derivations that the variables in Arps equations can be expressed in terms of reservoir variables. Using these solutions with sound reservoir engineering concepts, it becomes possible to provide more accurate production forecasts and better oil and gas reserves for both wells and/or individual fields.

Nomenclature

b = decline exponent
C_g = gas-well PI (backpressure curve coefficient) Mscf/D/(psi)^{2n}
D_i = initial decline rate, t^{-1}
G = gas-in-place, surface-measured
G_p = cumulative gas produced, surface-measured
J_o = PI, STB/D/psi
J_o' = PI, (backpressure curve coefficient) STB/D/(psi)^{2n}
n = exponent of backpressure curve
N_p = cumulative production, STB
N_{puo} = ultimate recoverable oil at $q(t)=0$, STB
$N_{puo,o}$ = ultimate recoverable oil at $q(t)=0$ and $p_{wf}=0$, STB
N = oil-in-place, STB
p_c = wellhead shut-in pressure, psia
$\bar{p}_R$ = average reservoir pressure, psia
p_t = flowing wellhead tubing pressure, psia
p_{wf} = bottomhole flowing pressure, psia
q_i = initial surface rate of flow from stabilized curve
$q_{i(\max)}$ = initial surface rate of flow from stabilized curve at p_{wf} or $p_t=0$
$q(t)$ = surface rate of flow at time t
q_D = dimensionless rate
q_{Dd} = decline-curve dimensionless rate
Q_p = oil or gas cumulative production, N_p or G_p, surface measure
r_{wa} = effective wellbore radius, ft.
t_D = dimensionless time
t_{Dd} = decline-curve dimensionless time
V_R = layer volume ratio, PVs
V_p = reservoir PV, ft^3 or BBl

Subscripts

a = at abandonment
g = gas
i = initial
o = oil
R = ratio
T = total of all layers' productions
u = ultimate

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References

1. Arps, J.J.: "Analysis of Decline Curves," *Trans.*, AIME (1945) 160, 228–47.
2. Fetkovich, M.J.: "A Simplified Approach to Water Influx Calculations—Finite Aquifer Systems," *JPT* (July 1971) 814–828.
3. Fetkovich, M.J.: "Decline Curve Analysis Using Type Curves," *JPT* (June 1980) 1065–77.
4. Fetkovich, M.J. *et al.*: "Decline Curve Analysis Using Type Curves – Case Histories," *SPEFE* (Dec. 1987) 637–656.
5. Fetkovich, M.J. *et al.*: "Case Study of a Low-Permeability Volatile Oil Field Using Individual-Well Advanced Decline Curve Analysis," paper SPE 14237 presented at the SPE 60th Annual Technical Conference and Exhibition, Las Vegas, NV (Sept. 22–25, 1985).
6. Fetkovich, M.J.: "The Isochronal Testing of Oil Wells," paper SPE 4529 presented at the 48th Annual Fall Meeting, Las Vegas, NV (Sept. 30–Oct. 3, 1973).
7. Fetkovich, M.J.: "Multipoint Testing of Gas Wells," SPE Mid-Continent Section, Continuing Education Course, Tulsa, (March 17, 1975).
8. Fetkovich, M.D. *et al.*: "Oil and Gas Relative Permeabilities Determined From rate/time Performance Data," paper SPE 15431 presented at the 61st Annual Technical Conference and Exhibition of SPE, New Orleans, LA (Oct. 5–8, 1986).
9. Fetkovich, M.J.: "Advances in Well Deliverability and Production Forecasting," PhD dissertation, U. of Trondheim (1988).
10. Fetkovich, M.J. *et al.*: "Depletion Performance of Layered Reservoirs Without Crossflow," *SPEFE* (Sept. 1990).
11. Fetkovich, M.J., Needham, R.B., and McCoy, T.F.: "Analysis of Kansas Hugoton Infill Drilling, Part II: Twelve Year Performance History of Five Replacement Wells," paper SPE 20779 presented at the 65th SPE Annual Technical Conference and Exhibition, New Orleans, LA (Sept. 23–26, 1990).
12. Cole, F.W.: *Reservoir Engineering Manual*, Gulf Publishing Co., Houston, TX (1969) 48.
13. Golan, M. and Whitson, C.H.: *Well Performance*, Prentice Hall, Englewood Cliffs, NJ, Second Edition (1991), 669.
14. Fetkovich, M.J., Fetkovich, E.J., and Fetkovich, M.D.: "Useful Concepts for Decline Curve Forecasting, Reserve Estimation, and Analysis," paper SPE 28628 presented at the 69th Annual Technical Conference and Exhibition, New Orleans, LA (Sept. 25–28, 1994).

SI Metric Conversion Factors

ft	$\times 3.048^*$	$E-01 = m$
in.	$\times 2.54^*$	$E+00 = cm$

*Conversion factor is exact.

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