CV HW#4

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1.1 Camera Models

(a) for only one pair (x_i, y_i, z_i) and (u_i, v_i),

$$K\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} k_{11}x_i + k_{12}y_i + k_{13}z_i \\ k_{22}y_i + k_{23}z_i \\ k_{33}z_i \end{pmatrix} \equiv \begin{pmatrix} u_i \\ v_i \\ 1 \end{pmatrix}$$

$$\therefore k_{11}x_i + k_{12}y_i + k_{13}z_i = k_{33}u_i z_i$$
$$k_{22}y_i + k_{23}z_i = k_{33}v_i z_i$$

$$\begin{bmatrix} x_i & y_i & z_i & 0 & 0 & -u_i z_i \\ 0 & 0 & 0 & y_i & z_i & -v_i z_i \end{bmatrix}\begin{bmatrix} k_{11} \\ k_{12} \\ k_{13} \\ k_{22} \\ k_{23} \\ k_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for multiple pairs, (n pairs)

$$\underbrace{\begin{bmatrix} x_1 & y_1 & z_1 & 0 & 0 & -u_1 z_1 \\ 0 & 0 & 0 & y_1 & z_1 & -v_1 z_1 \\ & & & \vdots & & \\ & & & \vdots & & \\ x_n & y_n & z_n & 0 & 0 & -u_n z_n \\ 0 & 0 & 0 & y_n & z_n & -v_n z_n \end{bmatrix}}_{2n\times 6\ (A)}\underbrace{\begin{bmatrix} k_{11} \\ k_{12} \\ k_{13} \\ k_{22} \\ k_{23} \\ k_{33} \end{bmatrix}}_{6\times 1\ (X)} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{6\times 1}$$

it's form of $AX = 0$

(b)
$$\begin{bmatrix} x_1 & y_1 & z_1 & 0 & 0 \\ 0 & 0 & 0 & y_1 & z_1 \\ & & \vdots & & \\ x_n & y_n & z_n & 0 & 0 \\ 0 & 0 & 0 & y_n & z_n \end{bmatrix} \begin{bmatrix} k_{11} \\ k_{12} \\ k_{13} \\ k_{22} \\ k_{23} \end{bmatrix} = \begin{bmatrix} u_1 z_1 \\ v_1 z_1 \\ \vdots \\ u_n z_n \\ v_n z_n \end{bmatrix}$$

$2n \times 5$ $\quad$ 5×1 $\quad$ $2n \times 1$

set this as A $\quad$ b

from least squares,

$$\begin{bmatrix} k_{11} \\ k_{12} \\ k_{13} \\ k_{22} \\ k_{23} \end{bmatrix} = (A^T A)^{-1} A^T b$$

1.2 Epipolar Geometry

(a) $M = K \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \left(K \mid 0 \right)$

$M' = K'\left(R \mid t\right) = \left(K'R \mid K't\right)$

(b) $e = M \begin{pmatrix} -R^T t \\ 1 \end{pmatrix} = -KR^T t$

$e' = M' \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = K't$

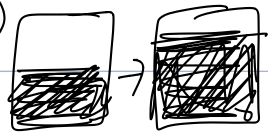
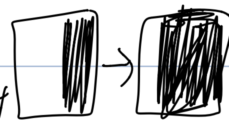
(c) $e' \times x' = [e']_\times x' = [e']_\times \left(M' \begin{pmatrix} X \\ 1 \end{pmatrix} \right) = [e']_\times \left(M' \begin{pmatrix} K^{-1}x \\ 1 \end{pmatrix} \right)$

$= [e']_\times \left(K'RK^{-1}x + \underbrace{\cancel{K't}}_{e'} \right) 0$

$= [e']_\times K'RK^{-1}x$

(d) vector perpendicular to epipolar plane.

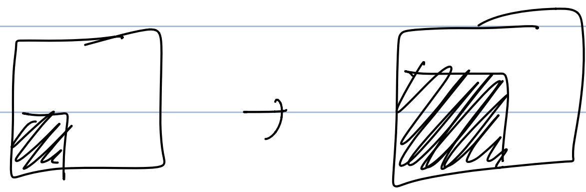
1.3 Optical flow

(a)  can't recover horizontal component  can't recover vertical component

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

2 unknowns $u, v \Rightarrow$ need 2 indep equations to get u, v.

if $\sum I_x^2 \sum I_y^2 - (\sum I_x I_y)^2 = 0$, cannot recover u or v.

(b) 

Can recover both u and v in this case

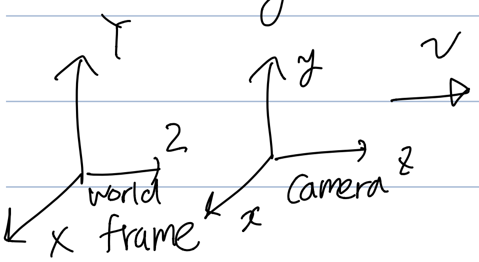
$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$ is not singular because $\sum I_x^2 \sum I_y^2 - (\sum I_x I_y)^2 \neq 0$.

$\Rightarrow$ 2 indep equations for 2 unknowns u, v

(c) Optical flow equation still holds.

It is relevent to movement of pixel, not real motion.

To figure out which scene point the object is moving toward,

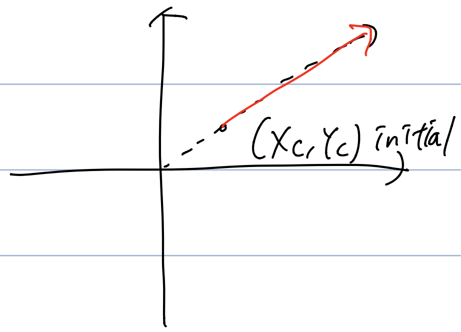


- (X_w, Y_w, Z_w) in world coordinate

$(X_c, Y_c, Z_w - vt)$ in camera coordinate $= (X_c, Y_c, Z_c)$

$(X_c, Y_c, Z_w - vt)$

$$\equiv \left(\frac{fX_c}{Z_w - vt}, \frac{fY_c}{Z_w - vt}, f \right)$$



getting away from the origin

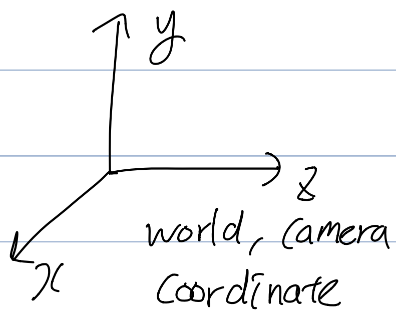
(camera coordinate)

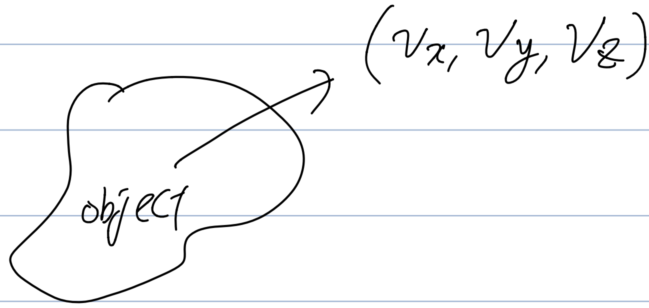
velocity: $\left(\frac{fvX_c}{(Z_w - vt)^2}, \frac{fvY_c}{(Z_w - vt)^2} \right)$

for image coordinate (u, v)

origin corresponds to $K \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, where K is intrinsic matrix.

(d) like (c)





let initial position of some point of the object

is (x, y, z), $\equiv (fx/z,\ fy/z,\ f) := (x_1, y_1, f)$

at time t, its position is $(x + v_x t, y + v_y t, z + v_z t)$

$$\equiv \left(\frac{(x + v_x t)f}{z + v_z t}, \frac{(y + v_y t)f}{z + v_z t}, f \right) = (x_c, y_c, f)$$

$$\frac{dx_c}{dt} = \frac{v_x f(z + v_z t) - (x + v_x t)v_z f}{(z + v_z t)^2} = \frac{(v_x z - v_z x)f}{(z + v_z t)^2}$$

$$\frac{dy_c}{dt} = \frac{(v_y z - v_z y)f}{(z+v_z t)^2}$$

Show that $\begin{pmatrix} f v_x / v_z \\ f v_y / v_z \\ f \end{pmatrix} := \begin{pmatrix} x_2, \\ y_2, \\ f \end{pmatrix}$ in camera coordinate is the focus of expansion

$$\frac{dx_c}{dt} = \frac{(x_2 - x_1) v_z z}{(z+v_z t)^2} \qquad \frac{dy_c}{dt} = \frac{(y_2 - y_1) v_z z}{(z+v_z t)^2}$$

$$\frac{dx_c}{dt} : \frac{dy_c}{dt} = x_2 - x_1 : y_2 - y_1$$

if $x_2 = x_1$ and $y_2 = y_1$, its optical flow is 0.

in image plane,

the focus of expansion corresponds to

$K \begin{pmatrix} x_2 \\ y_2 \\ f \end{pmatrix}$, where K is intrinsic matrix

2_1 dominant Motion Estimation

Lukas-Kanade Method

$$\Delta p = H^{-1} \sum_{x} \left[\nabla I \frac{\partial W}{\partial p} \right]^T \left[T(x) - I(W(x;p)) \right]$$

where

hessian $H = \sum_{x} \left[\nabla I \frac{\partial W}{\partial p} \right] \left[\nabla I \frac{\partial W}{\partial p} \right]$

for element in sigma, only consider

this part of the image, where cars never appear.

(want Δp of the background, not cars)

and when calculating dI/dx dI/dy,

we should deride Gx, Gy by 128,

be cause when applying 5x5 sobel filter

to

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$$

we get 128, not 1

(it should be 1)

2.2 Moving Object Detection

we update global p by

$$p(t+\Delta t) = p(t) + \Delta p$$

and check absolute value of pixel value difference between img2 and warped img1 (warped by p).

thresholds:

high: $0.2 * 256$

low: $0.15 * 256$