

# 1 СЕМИНАРЫН БОДЛОГО 1

## 1.1 ДАСГАЛ, БОДЛОГО

1. Цувааны хэсгийн нийлбэрийг ол.

1.  $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots + \frac{2}{3^{n-1}} + \dots$
2.  $\frac{9}{100} + \frac{9}{100^2} + \frac{9}{100^3} + \dots + \frac{9}{100^n} + \dots$
3.  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + (-1)^{n-1} \frac{2}{2^{n-1}} + \dots$
4.  $1 - 2 + 4 - 8 + \dots + (-1)^{n-1} 2^{n-1} + \dots$

2. Дараах цуваанууд нийлэх, сарних эсэхийг тогтоо. Нийлдэг цуваануудын хувьд цувааны нийлбэрийг ол.

- |  |  |
|--|--|
| 1. $\sum_{k=0}^{\infty} 3 \left(\frac{1}{5}\right)^k$            | 11. $\sum_{k=1}^{\infty} \frac{2}{k}$                                |
| 2. $\sum_{k=0}^{\infty} \frac{1}{3} (5)^k$                       | 12. $\sum_{k=1}^{\infty} \frac{4}{k+1}$                              |
| 3. $\sum_{k=0}^{\infty} \frac{1}{2} \left(-\frac{1}{3}\right)^k$ | 13. $\sum_{k=1}^{\infty} \frac{2k+1}{k^2(k+1)^2}$                    |
| 4. $\sum_{k=0}^{\infty} 4 \left(\frac{1}{2}\right)^k$            | 14. $\sum_{k=1}^{\infty} \frac{4}{k(k+1)(k+2)(k+3)}$                 |
| 5. $\sum_{k=3}^{\infty} (-1)^k \frac{3}{2^k}$                    | 15. $\sum_{k=1}^{\infty} 2e^{-k}$                                    |
| 6. $\sum_{k=0}^{\infty} \frac{1}{2} (3)^k$                       | 16. $\sum_{k=1}^{\infty} \sqrt[k]{3}$                                |
| 7. $\sum_{k=1}^{\infty} \frac{4}{k(k+2)}$                        | 17. $\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{k+1}\right)$ |
| 8. $\sum_{k=1}^{\infty} \frac{4k}{k+2}$                          | 18. $\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{3^k}\right)$ |
| 9. $\sum_{k=1}^{\infty} \frac{3k}{k+4}$                          | 19. $\sum_{k=1}^{\infty} \left(\frac{2}{3^k} + \frac{1}{2^k}\right)$ |
| 10. $\sum_{k=1}^{\infty} \frac{9}{k(k+3)}$                       | 20. $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{4^k}\right)$   |

3. Дараах тоонуудыг хоёр бүхэл тооны ноогдвор хэлбэрт бич.

- |                                       |   |
|---------------------------------------|---|
| 1. $0.\overline{23} = 0.232323\dots$  | 5. $0.\overline{7} = 0.7777\dots$         |
| 2. $0.\overline{99} = 0.999999\dots$  | 6. $1.24\overline{123} = 1.24123123\dots$ |
| 3. $0.\overline{234} = 0.234234\dots$ | 7. $0.\overline{d} = 0.ddd\dots$          |
| 4. $1.\overline{414} = 1.414414\dots$ | 8. $3.\overline{142857} = 3.142857\dots$  |

4. Хэрэв  $\sum_{n=1}^{\infty} a_n$  цуваа нийлдэг бол  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  цуваа сарнидаг болохыг батал.

5. Сарнилтын шинжүүр хэрэглэн дараах цувааг сарних эсэхийг тогтоо.

- |  |  |
|--|--|
| 1. $\sum_{n=1}^{\infty} \frac{n}{n+10}$            | 3. $\sum_{n=0}^{\infty} \frac{1}{n+4}$   |
| 2. $\sum_{n=0}^{\infty} \frac{n(n+1)}{(n+2)(n+3)}$ | 4. $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$ |

$$5. \sum_{n=1}^{\infty} \ln \frac{1}{n}$$

$$7. \sum_{n=1}^{\infty} \cos \frac{1}{n}$$

$$6. \sum_{n=0}^{\infty} \frac{e^n}{e^n + n}$$

$$8. \sum_{n=0}^{\infty} \cos \pi n$$

## 1.2 ДАСГАЛ, БОДЛОГО

1. Даламберийн шинж хэрэглэн цувааны нийлэх эсэхийг тогтоо.

$$1. \sum_{n=1}^{\infty} \frac{2^n}{n!}$$

$$4. \sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$$

$$2. \sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$$

$$5. \sum_{n=1}^{\infty} \frac{3^{n+2}}{\ln n}$$

$$3. \sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!3^{2n}}$$

$$6. \sum_{n=1}^{\infty} \frac{n5^n}{(2n+3)\ln(n+1)}$$

2. Кошийн шинж хэрэглэн цувааны нийлэх эсэхийг тогтоо.

$$1. \sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$$

$$5. \sum_{n=1}^{\infty} \left( \ln \left( e^2 + \frac{1}{n} \right) \right)^{n+1}$$

$$2. \sum_{n=1}^{\infty} \left( \frac{4n+3}{3n-5} \right)^n$$

$$6. \sum_{n=1}^{\infty} \sin^n \left( \frac{1}{\sqrt{n}} \right)$$

$$3. \sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$$

$$7. \sum_{n=1}^{\infty} \frac{1}{n^{1+n}}$$

$$4. \sum_{n=1}^{\infty} \left( 1 - \frac{1}{n} \right)^{n^2}$$

$$8. \sum_{n=1}^{\infty} n^2 e^{-n}$$

3. Кошийн интеграл шинж хэрэглэн цувааны нийлэх эсэхийг тогтоо.

$$1. \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$5. \sum_{n=1}^{\infty} \frac{n}{n^2+4}$$

$$2. \sum_{n=1}^{\infty} \frac{1}{n^{0.2}}$$

$$6. \sum_{n=1}^{\infty} \frac{\ln(n^2)}{n}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$$

$$7. \sum_{n=1}^{\infty} \frac{n-4}{n^2-2n+1}$$

$$4. \sum_{n=1}^{\infty} e^{-2n}$$

$$8. \sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$$

4. Жиших шинж хэрэглэн цувааны нийлэх эсэхийг тогтоо.

$$1. \sum_{n=1}^{\infty} \frac{1}{n^2+30}$$

$$4. \sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$$

$$2. \sum_{n=1}^{\infty} \frac{n-1}{n^4+2}$$

$$5. \sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{\frac{3}{2}}}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$$

$$6. \sum_{n=1}^{\infty} \frac{\frac{1}{n}}{\ln n \sqrt{\ln^2 n - 1}}$$

$$7. \sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{\sqrt{n^2+3}}$$

$$8. \sum_{n=1}^{\infty} \frac{1}{\ln n}$$

5. Цувааны нийлбэрийг ашпроксимацилах өгөгдсөн хэсгийн нийлбэрийн алдааны үнэлгээг хий.

$$1. S_{100}, \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$4. S_{80}, \sum_{n=1}^{\infty} \frac{2}{n^2+1}$$

$$2. S_{100}, \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$5. S_{40}, \sum_{n=1}^{\infty} n e^{-n^2}$$

$$3. S_{50}, \sum_{n=1}^{\infty} \frac{6}{n^8}$$

$$6. S_{200}, \sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$$

6. Цувааны нийлбэрийн дөхөлтийн алдаа  $10^{-6}$ -аас хэтрэхгүй байхын тулд эхний хэдэн гишүүний нийлбэрийг авах шаардлагатай вэ?

$$1. \sum_{n=1}^{\infty} \frac{3}{n^4}$$

$$3. \sum_{n=1}^{\infty} \frac{2}{n^2}$$

$$2. \sum_{n=1}^{\infty} n e^{-n^2}$$

$$4. \sum_{n=1}^{\infty} \frac{4}{n^5}$$

### 1.3 ДАСГАЛ, БОДЛОГО

1. Тэмдэг ээлжлэх цуваа нийлэх эсэхийг тогтоо.

$$1. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{3}{k}$$

$$11. \sum_{k=3}^{\infty} (-1)^k \frac{3}{\sqrt{k+1}}$$

$$2. \sum_{k=1}^{\infty} (-1)^k \frac{2}{k^2}$$

$$12. \sum_{k=4}^{\infty} (-1)^k \frac{k+1}{k^3}$$

$$3. \sum_{k=1}^{\infty} (-1)^k \frac{4}{\sqrt{k}}$$

$$13. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{k!}$$

$$4. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{k+1}$$

$$14. \sum_{k=3}^{\infty} (-1)^{k+1} \frac{k!}{3^k}$$

$$5. \sum_{k=2}^{\infty} (-1)^k \frac{k}{k^2+2}$$

$$15. \sum_{k=5}^{\infty} (-1)^{k+1} 2e^{-k}$$

$$6. \sum_{k=7}^{\infty} (-1)^k \frac{2k-1}{k^3}$$

$$16. \sum_{k=6}^{\infty} (-1)^{k+1} 3e^{\frac{1}{k}}$$

$$7. \sum_{k=5}^{\infty} (-1)^{k+1} \frac{k}{2^k}$$

$$17. \sum_{k=2}^{\infty} (-1)^k \ln k$$

$$8. \sum_{k=4}^{\infty} (-1)^{k+1} \frac{3^k}{k}$$

$$18. \sum_{k=2}^{\infty} (-1)^k \frac{1}{\ln k}$$

$$9. \sum_{k=1}^{\infty} (-1)^k \frac{4^k}{k^2}$$

$$19. \sum_{k=0}^{\infty} (-1)^{k+1} \frac{1}{2^k}$$

$$10. \sum_{k=1}^{\infty} (-1)^k \frac{k+2}{4^k}$$

$$20. \sum_{k=0}^{\infty} (-1)^{k+1} 2^k$$

2. Цувааны нийлбэрийн дөхөлт 0.01-ээс хэтрэхгүй алдаатай байхаар нийлбэрийг үнэл.

$$1. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{4}{k^3}$$

$$5. \sum_{k=0}^{\infty} (-1)^k \frac{3}{k!}$$

$$2. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{k^3}$$

$$6. \sum_{k=0}^{\infty} (-1)^{k+1} \frac{2}{k!}$$

$$3. \sum_{k=3}^{\infty} (-1)^k \frac{k}{2^k}$$

$$7. \sum_{k=2}^{\infty} (-1)^{k+1} \frac{4}{k^4}$$

$$4. \sum_{k=4}^{\infty} (-1)^k \frac{k^2}{10^k}$$

$$8. \sum_{k=3}^{\infty} (-1)^{k+1} \frac{3}{k^5}$$

3. Цувааны нийлбэрийн дөхөлт 0.0001-ээс бага алдаатай байхын тулд хэдэн гишүүний нийлбэрээр үнэлгээ хийх вэ?

$$1. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{k}$$

$$2. \sum_{k=0}^{\infty} (-1)^{k+1} \frac{2^k}{k!}$$

$$3. \sum_{k=0}^{\infty} (-1)^k \frac{10^k}{k!}$$

$$4. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k!}{k^k}$$

4. Абсолют, нөхцөлт нийлэх эсвэл сарних эсэхийг тогтоо.

$$1. \sum_{k=0}^{\infty} (-1)^k \frac{3}{k!}$$

$$2. \sum_{k=0}^{\infty} (-1)^k \frac{6}{k!}$$

$$3. \sum_{k=0}^{\infty} (-1)^k 2^k$$

$$4. \sum_{k=0}^{\infty} (-1)^k \frac{2}{3^k}$$

$$5. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k^2+1}$$

$$6. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2+1}{k}$$

$$7. \sum_{k=3}^{\infty} (-1)^k \frac{3^k}{k!}$$

$$8. \sum_{k=4}^{\infty} (-1)^k \frac{10^k}{k!}$$

$$9. \sum_{k=2}^{\infty} (-1)^{k+1} \frac{k}{2k+1}$$

$$10. \sum_{k=3}^{\infty} (-1)^{k+1} \frac{4}{2k+1}$$

$$11. \sum_{k=6}^{\infty} (-1)^k \frac{k 2^k}{3^k}$$

$$12. \sum_{k=1}^{\infty} (-1)^k \frac{k^2 3^k}{2^k}$$

$$13. \sum_{k=1}^{\infty} \left( \frac{4k}{5k+1} \right)^k$$

$$14. \sum_{k=1}^{\infty} \left( \frac{1-3k}{4k} \right)^k$$

$$15. \sum_{k=0}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{k+1}$$

$$16. \sum_{k=2}^{\infty} (-1)^{k+1} \frac{k}{k^3+1}$$

$$17. \sum_{k=1}^{\infty} \frac{\sin k}{k^2}$$

$$18. \sum_{k=1}^{\infty} \frac{\cos k}{k^3}$$

$$19. \sum_{k=1}^{\infty} \frac{\cos \pi k}{k}$$

$$20. \sum_{k=1}^{\infty} \frac{\arctan k}{k}$$

$$21. \sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$$

$$22. \sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k}$$

$$23. \sum_{k=1}^{\infty} \frac{(-1)^k}{k \sqrt{k}}$$

$$24. \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$$

$$25. \sum_{k=6}^{\infty} (-1)^{k+1} \frac{k!}{4^k}$$

$$26. \sum_{k=4}^{\infty} (-1)^{k+1} \frac{k^2 4^k}{k!}$$

$$27. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^{10}}{(2k)!}$$

$$28. \sum_{k=0}^{\infty} (-1)^k \frac{4^k}{(2k+1)!}$$