Hypothesis Testing

Q1.What is a null hypothesis (H_0) and why is it important in hypothesis testing?

Ans– A null hypothesis (H_0) is a statement that assumes no change, no difference, or no effect in a situation.

It is called "null" because it says that nothing new or special is happening.

Before collecting or analyzing data, we start by believing H_0 is true.

Example:

- H_0: *The average marks of students are 60.*
- H_0: *The new medicine has no effect on blood pressure.*

The null hypothesis important

1. *Gives a starting point*
   It provides a clear baseline to test against.
2. *Helps in decision-making*
   We use sample data to decide whether to reject or not reject H_0.

3. *Reduces bias*
   Instead of assuming something new works, we demand strong evidence before rejecting H_0.
4. *Supports scientific testing*
   It ensures results are based on data and probability, not guesses.

Q2. What does the significance level (α) represent in hypothesis testing?

Ans– It tells us how much mistake we are okay with when testing something.

Think like this:

- We test something and say: “This result is true.”
- But sometimes it can be wrong.
- α is the allowed chance of being wrong.

Example:

If $\alpha = 0.05$ (5%):

- You are accepting a 5% chance of saying *“Yes, there is an effect”*
  when in reality there is no effect

Common α values:

- 0.05 → most commonly used (5% risk)
- 0.01 → very strict (1% risk)

- 0.10 → more relaxed (10% risk)

Decision rule:

- If p-value ≤ α → Reject the null hypothesis
- If p-value > α → Do not reject the null hypothesis

Q3.Differentiate between Type I and Type II errors.

Type I Error (False Positive)

- What it means: You reject a true null hypothesis.
- In simple words: You say *"Yes, there is an effect"* when actually there is no effect.

- Example:
 A medical test says a person has a disease, but in reality, the person is healthy.

- Symbol: α (alpha)
- Think of it as: *False alarm*

Type II Error (False Negative)

- What it means: You fail to reject a false null hypothesis.
- In simple words: You say *"No effect"* when actually there is an effect.

- Example:
  A medical test says a person is healthy, but the person actually has the disease.
- Symbol: β (beta)
- Think of it as: *Missed detection*

Easy Comparison Table

Type I Error	Type II Error
False positive	False negative
Reject true H_0	Accept false H_0
Error due to overreaction	Error due to ignoring evidence
Controlled by significance level (α)	Related to power of the test (1−β)

Q4. Explain the difference between a one-tailed and two-tailed test. Give an example of each.

Ans– One-Tailed Test

Meaning

A one-tailed test checks for an effect in only one direction — either greater than *or* less than, not both.

When it's used

When you are sure about the direction of the effect before testing.

Example

A teacher believes that a new teaching method increases students' scores.

- Null hypothesis (H_0): Average score ≤ 60
- Alternative hypothesis (H_1): Average score > 60

Here, we are only testing if the score is greater, so it's a one-tailed test.

Two-Tailed Test

Meaning

A two-tailed test checks for an effect in both directions — whether something is greater than or less than a value.

When it's used

When you want to detect any change, without assuming the direction.

Example

A company wants to know if a new machine produces items with a different average weight than 500 g.

- Null hypothesis (H_0): Average weight = 500 g
- Alternative hypothesis (H_1): Average weight ≠ 500 g

Here, the weight could be more or less, so it's a two-tailed test.

Quick Comparison

One-Tailed Test	Two-Tailed Test
Tests one direction	Tests both directions

More powerful in one direction	More conservative
Used when direction is known	Used when direction is unknown

Q5.A company claims that the average time to resolve a customer complaint is 10 minutes. A random sample of 9 complaints gives an average time of 12 minutes and a standard deviation of 3 minutes. At α = 0.05, test the claim.

Ans– Given information

- Claimed mean time (μ_0) = 10 minutes
- Sample mean ($\bar{x}$) = 12 minutes
- Sample standard deviation (s) = 3 minutes
- Sample size (n) = 9
- Significance level (α) = 0.05

Step 1: State the hypotheses

This is a two-tailed test (because we are checking whether the claim is true or not).

- Null hypothesis (H_0): $\mu = 10$
- Alternative hypothesis (H_1): $\mu \neq 10$

Step 2: Choose the test

- Sample size is small ($n < 30$)
- Population standard deviation is unknown

Step 3: Calculate the test statistic

Formula:

$$t = \bar{x} - \mu_0 \,/\, s / \sqrt{n}$$

Substitute values

$$t = 12 - 10 / 3 / \sqrt{9}$$
$$= 2 / 3/3$$
$$= 2 / 1$$
$$= 2$$

So,
$t = 2$

Step 4: Find critical value

- Degrees of freedom (df) = $n - 1 = 8$
- At $\alpha = 0.05$ (two-tailed),
  t critical $\approx \pm 2.306$

Step 5: Decision

- Calculated $t = 2$
- Critical $t = \pm 2.306$

Since

$|2| < 2.306$

We fail to reject the null hypothesis

Step 6: Conclusion

At the 5% significance level, there is not enough evidence to reject the company’s claim.

Q6. When should you use a Z-test instead of a t-test?

Ans– **Use a Z-test when:**

1. Sample size is large
    - Usually $n \geq 30$
    - Large samples follow the normal distribution well.
2. Population standard deviation (σ) is known
    - This is the main condition for a Z-test.
3. Data is approximately normally distributed
    - Or sample size is large enough to rely on the Central Limit Theorem.

Use a t-test when (for comparison):

- Sample size is small ($n < 30$)
- Population standard deviation is unknown
- You use sample standard deviation (s) instead of σ.

Key differences (easy table)

Condition	**Z-test**	**t-test**
Sample size	Large (≥30)	Small (<30)
Standard deviation	Known (σ)	Unknown (use s)
Distribution	Normal (Z)	t-distribution
Degrees of freedom	Not needed	Needed (n-1)

Q7 The productivity of 6 employees was measured before and after a training program. The data are given below:

Employee	**Before**	**After**
1	**50**	**55**

2	**60**	**65**
3	**58**	**59**
4	**55**	**58**
5	**62**	**63**
6	**56**	**59**

At α = 0.05, test if the training improved productivity

Ans– **Step 1: Set up hypotheses**

We want to check if productivity improved after training.

- Null hypothesis (H_0): Training has no effect
  μd=0
- Alternative hypothesis (H_1): Training improves productivity
  μd>0

(where d=After−Befored)

Step 2: Find the differences

Employee	Before	After	Difference (d)
1	50	55	5
2	60	65	5
3	58	59	1
4	55	58	3
5	62	63	1
6	56	59	3

Step 3: Calculate mean and standard deviation of differences

- Mean difference

$\bar{d}$= 5+5+1+3+1+3 / 6

= 18 / 6

= 3

- Standard deviation of differences

$S_d \approx 1.79$

Step 4 : Calculate t-value

$t = d / sd / \sqrt{n}$
$= 3 / 1.79 / \sqrt{6}$
≈ 4.10

Step 5: Critical value

- $\alpha = 0.05$
- Degrees of freedom = $n-1=5$
- t-critical (one-tailed) ≈ 2.015

Step 6: Decision

- Calculated t = 4.10
- Critical t = 2.015

4.10 > 2.015 → Reject H_0

Final Conclusion

At the 5% significance level, there is sufficient evidence to conclude that the training program improved employee productivity

Q8. A company wants to test if product preference is independent of gender

Gender	**Product A**	**Product B**	**Total**
Male	**30**	**20**	**50**
Female	**10**	**40**	**50**
Total	**40**	**60**	**100**

At α = 0.05, test independence

Ans– Step 1: State the Hypotheses

- Null Hypothesis (H_0):
  Product preference is independent of gender.

- Alternative Hypothesis (H_1):
  Product preference is dependent on gender.

Step 2: Find Expected Frequencies

Formula:

E=(Row Total)×(Column Total) **/** Grand Total

Expected Values Table

Gender	Product A	Product B
Male	(50×40)/100 = **20**	(50×60)/100 = **30**
Female	(50×40)/100 = **20**	(50×60)/100 = **30**

Step 3: Calculate Chi-Square Value

Formula:

X2 = ∑(O−E)2 / E

Cell	O	E	$(O-E)^2/E$
Male – A	30	20	$(10)^2/20$ = **5**
Male – B	20	30	$(-10)^2/30$ = **3.33**
Female – A	10	20	$(-10)^2/20$ = **5**
Female – B	40	30	$(10)^2/30$ = **3.33**

Total Chi-Square Value

$$\chi 2=5+3.33+5+3.33=16.66$$

Step 4: Degrees of Freedom

$$df=(r-1)(c-1)$$
$$df=(2-1)(2-1)=1$$

Step 5: Critical Value

From Chi-Square table:

- At $\alpha = 0.05$
- df = 1

X^2critical = 3.84

Step 6: Decision Rule

- If Calculated χ^2 > Critical χ^2, reject H_0

$16.66 > 3.84 \Rightarrow$ Reject H_0

Step 7: Conclusion

- There is sufficient evidence to conclude that:
- Product preference is dependent on gender.
- This means men and women prefer different products.