On the Invalidity of the Riemann Hypothesis via Conditional Square Root Logic

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Author Note: This paper was written independently by a 14-year-old middle school student from South Korea, without formal academic affiliation and as a nonnative English speaker. I kindly ask for understanding in case of occasional linguistic imperfections. The core focus is on the logical and structural consequences of a minimal change to the square root operation.

Abstract

We show that adding the condition $x^2 \leq 0$ only when sqrt is invoked. This renders $\sqrt{-1}$ undefined outside specific contexts, collapsing the structure of complex numbers $\mathbb{C}$. As a consequence, the Riemann zeta function $\zeta(s)$, which is defined over $\mathbb{C}$, becomes undefined. Therefore, the Riemann hypothesis loses all meaning in this framework. No axioms are changed, and no new number system is introduced—only a localized constraint on $\sqrt{}$ removes the foundation on which RH stands.

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1 Introduction

The Riemann Hypothesis (RH) is one of the most studied and long-standing unsolved problems in mathematics. It posits that all nontrivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $Re(s) = 1/2$ in the complex plane C.

Most investigations of RH assume that the complex field C, and in particular the existence of $\sqrt{-1}$, is an unquestionable foundation. However, the necessity of C is not a formal requirement for discussing RH, since the hypothesis itself refers only to the real part of s.

In this paper, we question the role of complex numbers by applying a minimal change to the square root operation. We introduce a local constraint: the condition $x^2 \leq 0$ applies only when the square root is invoked. This causes $\sqrt{-1}$ to become undefined outside of explicit contexts $\sqrt{}$, rendering C structurally invalid.

As a result, the Riemann zeta function $\zeta(s)$ can no longer be defined, and thus the RH itself becomes logically meaningless within this framework.

This approach does not alter any axioms or introduce a new number system; it only places a context-dependent restriction on $\sqrt{}$.

In what follows, we define this modified operation, analyze its impact on C, and demonstrate the structural invalidation of RH.

2 Definition of the Conditional Square Root

We define a modification to the standard square root that introduces a conditional domain constraint without altering the underlying set of real numbers $\mathbb{R}$.

Let $x \in \mathbb{R}$. We define the conditional square root operator, denoted $\sqrt{\star}x$, as follows:

$$\sqrt{\star}x := \begin{cases} \sqrt{x}, & \text{if } x \geq 0 \\ \text{undefined}, & \text{if } x < 0 \text{ and } \textit{context} \neq \texttt{sqrt} \end{cases} \quad \text{where } x^2 \leq 0 \text{ is enforced only during root evaluation}$$

where $x^2 \leq 0$ is enforced only during root evaluation.

That is, condition $x^2 \leq 0$ is evaluated solely in the context of an active square root computation. We do not extend the real number system $\mathbb{R}$, nor redefine negative numbers. Instead, we restrict the use of square roots of negative values to a context-sensitive scope.

This restriction invalidates the use of $\sqrt{-1}$ as a universally defined object and thus renders the complex unit $i := \sqrt{-1}$ undefined outside of explicit $\sqrt{}$ operations. Consequently, the complex field $\mathbb{C}$, which relies on the structural existence of i, becomes formally undefined.

3 Collapse of the Complex Field $\mathbb{C}$

The standard definition of the complex field is given by:

$$\mathbb{C} := \{a + bi \mid a, b \in \mathbb{R},\ i = \sqrt{-1}\}$$

This construction assumes the existence of the imaginary unit i, which is globally defined as $i^2 = -1$, and thus $i := \sqrt{-1}$.

However, under the conditional square root operator $\sqrt{\star}$ defined in Section 1, the expression $\sqrt{-1}$ becomes undefined outside the specific context of an active square root operation. Since the condition $x^2 \leq 0$ is enforced only during $\sqrt{}$ evaluation, the symbolic identity $i := \sqrt{-1}$ loses its global meaning.

As a result:

- The field $\mathbb{C}$ can no longer be constructed, since it requires a universally defined i.
- All algebraic operations involving i (e.g., bi, $a+bi$, $|z|$, $\overline{z}$) become undefined or inconsistent in general computation.

- Any function or structure relying on the complex field — including the Riemann zeta function $\zeta(s)$ — loses its definitional basis.

Thus, the conditional constraint on $\sqrt{}$ alone suffices to dismantle the structural integrity of $\mathbb{C}$.

4 Logical Invalidity of the Riemann Hypothesis

The Riemann Hypothesis (RH) states that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\mathrm{RE}(s) = \frac{1}{2}$ in the complex plane $\mathbb{C}$. Formally:

$$\zeta(s) = 0 \Rightarrow \mathrm{RE}(s) = \frac{1}{2}, \quad \text{for all non-trivial zeros } s \in \mathbb{C}$$

This hypothesis presupposes the existence of:

- The complex field $\mathbb{C}$
- The analytic continuation of $\zeta(s)$ over $\mathbb{C}$
- A well-defined domain for s where non-trivial zeros may exist

However, as demonstrated in Section 2, the definition of $\mathbb{C}$ collapses under the conditional square root operator $\sqrt{\star}$ due to the contextual invalidity of $\sqrt{-1}$. Without a structurally sound $\mathbb{C}$, the function $\zeta(s)$ cannot be defined in the traditional analytic sense.

Therefore:

- The domain of $s \in \mathbb{C}$ is undefined
- The notion of "non-trivial zeros" loses mathematical coherence
- The logical structure of the Riemann Hypothesis fails at the level of definition

Conclusion: Under the conditional square root framework, the Riemann Hypothesis is neither true nor false-it is logically meaningless. It cannot be evaluated within a system where its foundational domain no longer exists.

5 Conclusion

We introduced a minimal yet impactful modification to the square root operator by adding the condition $x^2 \leq 0$ only during the invocation of $\sqrt{}$. This localized constraint leaves the real number system $\mathbb{R}$ intact but renders $\sqrt{-1}$ undefined in general computation.

As a result, the complex unit $i := \sqrt{-1}$ becomes undefined outside of the square root context, which collapses the foundation of the complex number field $\mathbb{C}$. Consequently, the Riemann zeta function $\zeta(s)$-which is defined over $\mathbb{C}$-cannot be analytically continued, and the Riemann Hypothesis, relying on $\zeta(s)$ and $\mathrm{Re}(s) = \frac{1}{2}$, becomes undefined.

This framework does not modify any axioms, nor does it introduce a new number system. It demonstrates that a single contextual condition on $\sqrt{}$ is sufficient to invalidate the logical structure supporting the Riemann Hypothesis.

Therefore, within this structure, the Riemann Hypothesis is not true, nor false, but logically inexpressible.

Final Remark

This work does not aim to resolve the Riemann Hypothesis within the traditional mathematical framework. Instead, it demonstrates that under minimal structural constraints applied to the square root operator, the hypothesis cannot even be stated coherently. In this sense, it is not a problem to be solved, but a question to be erased.

References

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