$$\frac{\partial^2}{\partial t^2} s(x,t) = c^2 \frac{\partial^2}{\partial x^2} s(x,t)$$

$$s(0,t) = s(L,t) = 0 \quad t > 0$$

$$s(x,0) = f(x) \longleftrightarrow F(u,0) = F(u)$$

$$\frac{\partial}{\partial t} s(x,0) = 0$$

→ Aplicando transformada de Fourier

$$\frac{\partial}{\partial t^2} F(u,t) = c^2 (i2\pi u)^2 F(u,t)$$

$$F''(u,t) + c^2 4\pi u^2 \cdot F(u,t) = 0$$

$$r^2 + c^2 4\pi u^2 = 0$$

$$r = \pm\sqrt{c^2 4\pi u^2}\, i = \pm c 2u\sqrt{\pi}\, i$$

$$X(u,t) = e^{\pm c2u\sqrt{\pi} i} = A\cos(c2u\sqrt{\pi}t) + B\operatorname{sen}(c2u\sqrt{\pi}t)$$

$$F(u,0) = A$$

Derivando y aplicando condiciones iniciales

$$B = 0.$$

Luego, la solución viene dada por:

$$X(u,t) = F(u)\cos(2cu\sqrt{\pi}t)$$

En este caso: $f(x) = a\Lambda\left(\frac{x-\frac{L}{2}}{L/2}\right)$

Luego $F(u) = a\frac{L}{2}\operatorname{sinc}^2\left(\frac{L}{2}u\right)e^{-\frac{i2\pi uL}{2}}$

$$\boxed{X(u,t) = \frac{aL}{2}\operatorname{sinc}^2\left(\frac{L}{2}u\right)e^{-\frac{i2\pi uL}{2}}\cdot\cos(2c\cdot u\sqrt{\pi}t)}$$