1)

$Z = \begin{bmatrix}1\\3\\0\\2\end{bmatrix}$; $V_1 = \begin{bmatrix}3\\-1\\2\\-4\end{bmatrix}$; $V_2 = \begin{bmatrix}2\\-5\\-2\\-5\end{bmatrix}$

Use : $\frac{Z\cdot V_1}{V_1\cdot V_1}V_1 + \frac{Z\cdot V_2}{V_2\cdot V_2}V_2$

$Z\cdot V_1 = \begin{bmatrix}1\\3\\0\\2\end{bmatrix}\begin{bmatrix}3\\-1\\2\\-4\end{bmatrix} = 3+(-3)+0+(-8) = -8$

$V_1\cdot V_1 \begin{bmatrix}3\\-1\\2\\-4\end{bmatrix}\begin{bmatrix}3\\-1\\2\\-4\end{bmatrix} = 9+2+4+16 = 31$

$V_2\cdot V_2 \begin{bmatrix}2\\-5\\-2\\-5\end{bmatrix}\begin{bmatrix}2\\-5\\-2\\-5\end{bmatrix} = 4+25+4+25 = 58$

$Z\cdot V_2 \begin{bmatrix}1\\3\\0\\2\end{bmatrix}\begin{bmatrix}2\\-5\\-2\\-5\end{bmatrix} = 2+(-15)+0+(-10) = -23$

$\frac{-8}{31}\begin{bmatrix}3\\-1\\2\\-4\end{bmatrix} + \frac{-23}{58}\begin{bmatrix}2\\-5\\-2\\-5\end{bmatrix}$

$\begin{bmatrix}-24/31\\8/31\\-16/31\\-32/31\end{bmatrix} + \begin{bmatrix}-46/58\\115/58\\-46/58\\115/58\end{bmatrix}$

3)

$A = \begin{bmatrix}1 & 3\\1 & -1\\1 & 1\end{bmatrix}$; $b = \begin{bmatrix}5\\1\\0\end{bmatrix}$

3x2   2x3 = 2x2

$A^TA = \begin{bmatrix}1 & 1 & 1\\3 & -1 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\1 & -1\\1 & 1\end{bmatrix} = \begin{bmatrix}3 & 3\\3 & 9\end{bmatrix}$

$(A^TA)^{-1} = \begin{array}{cc} a & b \\ \end{array}$
$\begin{bmatrix}9 & -3\\-3 & 3\end{bmatrix}\frac{1}{36}$
$\begin{array}{cc} c & d \\ \end{array}$

$x^* = (A^TA)^{-1}A^T\vec{b} = \frac{1}{36}\begin{bmatrix}9 & -3\\-3 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\3 & -1 & 1\end{bmatrix}\begin{bmatrix}5\\1\\0\end{bmatrix}$

$= \frac{1}{36}\begin{bmatrix}9 & -3\\-3 & 3\end{bmatrix}\begin{bmatrix}6\\14\end{bmatrix} \to \frac{1}{36}\begin{bmatrix}12\\24\end{bmatrix}$

$x^* = \begin{bmatrix}12/36\\24/36\end{bmatrix}$;

9)

$Pv_i = tv_i \to X X^T v_i = av_i$

$C(Xv_i) \Rightarrow XX^T(Xv_i \Rightarrow X(av_i) = a(Xv_i)$

$\therefore$ Xv_i is an eigenvector of C

2)

given $\begin{bmatrix}1 & 0 & 4\\-2 & 3 & -2\\-2 & 0 & 6\end{bmatrix}$

Using gram scmidt

$Q = \begin{bmatrix}1/3 & \frac{2\sqrt{5}}{15} & \frac{2\sqrt{5}}{5}\\ -2/3 & \frac{\sqrt{5}}{3} & 0\\ -2/3 & \frac{-4\sqrt{5}}{15} & \frac{\sqrt{5}}{5}\end{bmatrix}$

$Q^TA = R$

$\begin{bmatrix}1/3 & -2/3 & -2/3\\ \frac{2\sqrt{5}}{15} & \frac{\sqrt{5}}{3} & \frac{-4\sqrt{5}}{15}\\ \frac{2\sqrt{5}}{5} & 0 & \frac{\sqrt{5}}{5}\end{bmatrix}\begin{bmatrix}1 & 0 & 4\\-2 & 3 & -2\\-2 & 0 & 6\end{bmatrix}$

$R = \begin{bmatrix}3 & -2 & -4/3\\ 0 & \sqrt{5} & -\frac{26\sqrt{5}}{15}\\ 0 & 0 & \frac{14\sqrt{5}}{5}\end{bmatrix}$

lsattr	# List file attributes on a Linux second extended file system
lsblk	# List block devices
ls	# List information about file(s)
lsof	# List open files
lspci	# List all PCI devices

8) $AA^T = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 13 & 6 \\ 6 & 4 \end{bmatrix}$

$\det(AA^T - \lambda I) = 0 \rightarrow \begin{bmatrix} 13 & 6 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 13-\lambda & 6 \\ 6 & 4-\lambda \end{bmatrix}$

$(13-\lambda)(4-\lambda) - 36 \rightarrow 52 - 13\lambda - 4\lambda + \lambda^2 - 36$

$\lambda^2 - 17\lambda + 16 \rightarrow (\lambda - 1)(\lambda - 16)$

$\lambda = 1, 16$

find eigenvectors

$\lambda_1 = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix} \begin{matrix} \cdot\frac{1}{6} \\ \cdot\frac{1}{3} \end{matrix} \sim \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{matrix} \\ -R1 \end{matrix} \sim \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda_2 = \begin{bmatrix} -3 & 6 \\ 6 & -12 \end{bmatrix} \begin{matrix} 1/3 \\ 1/6 \end{matrix} \sim \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{matrix} \cdot -1 \\ +R1 \end{matrix} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$

$\sigma_1 = 4$
$\sigma_2 = 1$

$\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

$V_1 = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} \frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \end{bmatrix}$

$U = \begin{bmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix}$

$V_2 = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -\frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \end{bmatrix}$

$V = \begin{bmatrix} \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} \end{bmatrix}$

(a) $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$; $\lambda = 1, 2, 3$

$\lambda = 1 \rightarrow A - I = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2/3 \\ 0 & 0 & 2/3 \end{bmatrix}$

Basis for EigenSpace $\lambda = 1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\lambda = 2 \rightarrow A - 2I = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{matrix} \\ \\ +R_1 \end{matrix} \begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{matrix} \\ +R_1 \\ \\ \end{matrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $x_3 = 0$, $x_2 = t$, $x_1 = -t/2$

Basis $E_2 = \left\{ \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}$

$\lambda = 3$ $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{matrix} \\ \\ +2R_1 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ +2R_1 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot -\frac{1}{2}$

$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ Basis for $E_3 = \left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$