Fiscal capacity and moral preferences

This paper introduces semi-Kantian Homo Moralis preferences (Alger and Weibull 2013, 2016) as a new framework to model how citizens' preferences influence the long-term fiscal capacity of states. This approach explains notable correlations between government trust and tax compliance, as shown in various surveys. It serves as an alternative microfoundation to Besley's (2020) reciprocity-based model. In both models, the fair distribution of tax proceeds by the Elite enhances citizens' tax compliance. However, this paper's framework extends beyond Besley's by linking the equilibrium of higher taxation and the emergence of strong civic cultures to individual moral considerations, offering insights into the relationship between fiscal policies and intrinsic individual moral values.

1. Introduction

Scholars have developed two distinct theories about the origins of states. The first theory is based on the notion of a "social contract" between the citizens and the State (Rousseau, 1762) . It suggests that community members voluntarily give a select group the power to rule, with this governing entity tasked with delivering crucial public services. The second strand of theories, developed after the work of Thomas Hobbes, instead focus on the exploitative nature of the government as cohersive institution. These extractive theories of government argue instead that a powerful elite group forms the state mainly to exploit resources through taxation and similar methods.

In this paper, I consider Kantian moral motivations as determinants of citizens' sense of civic duty, consistent with the social contract view of the State. In the model, agents consider the role of the government as a provider of public goods and transfers to the citizens when undertaking their compliance decisions. Particularly, they ask themselves about the hypothetical public good provision and transfers that would arise if other members of the society made the same compliance decision as them. This universalization logic resembles Kant's (1785) categorical imperative, which posits the question: What if a fraction of the population were to act in the same way that I am acting?.

As shown by Figure 1, evidence from the World Values Survey demonstrates a positive correlation between trust in the government and tax compliance. This finding is consistent with the social contract theories, suggesting that trust in the government and institutions is fundamental for fostering well-functioning States.

So far, most of the economic literature has focused on views that resemblle the Hobbesian nature of the relationship between citizens and a ruler. The literature that has focused on the social contract view has instead relied on the concept of w'reciprocity" between the citizens and the State (Levi, 1989, Besley, 2020). In this paper, instead, I propose an alternative model based on individual moral preferences for universalization.

Instead of relying on the concept of reciprocity, it presents a novel approach to understanding the dynamics of fiscal capacity development in states; this approach consists of considering the role of semi-Kantian Homo Moralis preferences, as developed by

TABLE I

DETERMINANTS OF ATTITUDES TOWARDS TAX COMPLIANCE[a]

Low confidence in government	–	−0.177(0.029)	–
Low trust in people	–	–	0.009 (0.022)
Male	−0.212 (0.018)	−0.210 (0.018)	−0.212 (0.018)
Age 30–49	0.222 (0.021)	0.220 (0.021)	0.222 (0.021)
Age 50+	0.571 (0.043)	0.564 (0.043)	0.572 (0.043)
Education: Middle	0.042 (0.026)	0.046 (0.026)	0.042 (0.026)
Education: Upper	0.097 (0.032)	0.101 (0.033)	0.098 (0.031)
Income 2	−0.017 (0.055)	−0.014 (0.029)	−0.017 (0.055)
Income 3	−0.034 (0.054)	−0.032 (0.054)	−0.034 (0.053)
Income 4	−0.087 (0.056)	−0.087 (0.057)	−0.087 (0.056)
Income 5	−0.043 (0.056)	−0.041 (0.057)	−0.043 (0.056)
Income 6	−0.141 (0.056)	−0.141 (0.057)	−0.141 (0.056)
Income 7	−0.161 (0.057)	−0.160 (0.057)	−0.161 (0.057)
Income 8	−0.158 (0.069)	−0.157 (0.069)	−0.158 (0.069)
Income 9	−0.194 (0.063)	−0.191 (0.062)	−0.193 (0.063)
Income 10	−0.234 (0.081)	−0.236 (0.080)	−0.233 (0.081)
Number of observations	303,229	303,229	303,229

[a]The dependent variable is based on question "Is it justifiable to cheat on your taxes if you have a chance?" from the World Values Survey and European Values Survey (various waves) with the scale reversed so that the highest score is associated with cheating not being justified. All specifications include wave and country dummies with standard errors clustered at the country level. The data cover 104 countries. Standard errors adjusted for clustering at the country level. For income: Here is a scale of incomes. We would like to know in what group your household is, counting all wages, salaries, pensions and other incomes that come in. Just give the letter of the group your household falls into, before taxes and other deductions. For confidence: I am going to name a number of organizations. For each one, could you tell me how much confidence you have in them: is it a great deal of confidence, quite a lot of confidence, not very much confidence or none at all? Use answers on "government in capital". Coded 1 if answer is "Not very much" or "None at all". Generalized Trust: Generally speaking, would you say that most people can be trusted or that you can't be too careful in dealing with people? Coded 1 if "You cannot be too careful"

FIGURE 1. World Values Survey: determinants of tax compliance, from Besley (2020)

Alger and Weibull (2013) and Alger and Weibull (2016). In more detail, the work extends the current theoretical framework to include the influence of citizens' moral preferences on their interactions with the state, providing an alternative set of microfoundations for modeling the evolution of state fiscal capacity.

While Besley's model primarily examines the role of the Elite's tax strategies in shaping civic culture and tax compliance through the lens of civic-minded citizens who make their compliance decisions based on the expenditure patterns of a ruling Elite, this paper derives predictions based on moral dimensions that motivate citizens' compliance behavior as captured by citizens' degree of Kantian morality. It explores how a fair distribution of tax proceeds by the Elite not only fosters voluntary compliance but is also dependent with intrinsic moral values of the citizens. This paper hypothesizes that individual moral considerations significantly impact the long-term fiscal strategies of states, influencing both the equilibrium of higher taxation and the emergence of robust civic cultures.

With the intention of comparing the conclusions implied by this new model of preferences, I present a model that stays as closely as possible to that of Besley but instead lets citizens be equipped with Homo moralis preferences. In the model, agents can be laymen or part of the ruling Elite. The Elite chooses the income tax rate the rest of the population pays. Tax proceeds have three uses: to finance a pure public good to redistribute among the Elite and the non-elite. The non-elite pays the tax, but they can try to hide part of their income. Civic-motivated citizens are more likely to pay taxes if the redistribution to the Elite is sufficiently moderate, that is if the Elite is 'fair.' In a coordination game, the Elite finds it optimal not to appropriate the resources as long as the collected taxes are 'large enough,' which depends on the fiscal capacity (the degree of tax enforcement), compliance, and the degree of Kantian morality in the population. However, the degree of compliance depends on whether the elite appropriates the resources. Ultimately, the equilibrium depends on fiscal capacity and individual morality.

2. BASELINE MODEL

I consider a model in which citizens can belong to two different groups: (i) a ruling elite that makes decisions about transfers and public goods and (ii) tax-paying citizens. To simplify mathematical expressions, I fix the population size to 2 and let all the citizens have income $w \geq 0$.

The material utility function of all citizens is linear in public and private goods, that is, $U(G, y) = \alpha G + y$, where G is expenditure on a public good financed through taxes, $\alpha > 0$ is the marginal utility from the consumption of the public good and y is private consumption. We may interpret α as capturing the intensity of threat of war (Besley and Persson, 2011). Therefore, larger values of α are associated with a larger necessary investment in defensive capabilities. Citizens can hide a fraction $n \in [0, 1]$ of their income from the tax authorities (n stands for non-compliance).

The paper is organized as follows: Sections 2 to 4 lay out the baseline model and derive its main predictions. Section 5 discusses the results and proposes avenues for future research.

2.1 *Policy and institutions*

The Elite decides on policy, which is comprised of four elements:

1. t: tax rate on income w,
2. G: expenditure of the public good,
3. B: transfers to the Elite[1],
4. b: transfers to the taxpayers.

As in the baseline model by Besley and Persson (2011), to capture the strength of institutions, I assume that for every unit that the elite transfers to itself, it must give $\sigma \in (0, 1)$ units to the taxpayers: $b = \sigma B$. An increase in σ implies that institutions are more cohesive and, in turn, motivates the state to spend on the public good G. Substituting $b = \sigma B$, the government budget constraint can be written as:

$$B = \theta(\sigma)[T - G],$$

where T stands for taxation per capita and $\theta(\sigma) = [1 + \sigma]^{-1} \in [1/2, 1]$ is the effective "price" of the public good to the Elite, taking into account that the transfers spent to provide them. It is convenient to then decompose the tax revenues as the proportion $1 - \rho \in [0, 1]$ used for transfers and the one ρ used for the public good. That is, $G = \rho T$, which means that we can write total transfer to the Elite and citizens as:

$$B + b = (1 - \rho)T. \tag{1}$$

[1] An alternative interpretation of B is money lost due to loopholes that are exploited by the members of the Elite.

2.2 *Compliance by moral agents*

Citizens decide how much of their income they should misreport to the authorities by maximizing their Homo-Moralis utilities.

DEFINITION 1. Assume that every citizen has a **degree of morality** $\kappa \in [0,1)$. Let G denote the global public good. *Homo moralis* preferences over the provision of public good for a citizen that conceals a fraction of their income $n \in [0,1]$ are given by:

$$(1-\kappa)U\left(G, y(w(1-n), b)\right) + \kappa U\left(G^{\mathcal{M}}(n), y\left(w(1-n), b^{\mathcal{M}}(n)\right)\right), \tag{2}$$

where $G^{\mathcal{M}}(n)$ and $b^{\mathcal{M}}(n)$ stand for the universalized levels of public good and transfers to the citizens implied by the compliance decision of each agent. They are given by:

$$G^{\mathcal{M}}(n) = \rho T(t, w(1-n)), \quad \text{and } b^{\mathcal{M}}(n) = (1-\rho)\frac{\sigma}{1+\sigma}T(t, w(1-n)), \tag{3}$$

where $T(t, w(1-n)) = tw(1-n)$ stands for total government revenues raised when imposing a flat income tax rate t and when all the citizens report income $w(1-n)$.

Following Definition 1, each citizen solves for the level of concealment n that maximizes her utility function in (2). Given the linear utility specification we can write the problem solved by each citizen as:

$$\max_{n}(1-\kappa)\left(\alpha G + b\right) + \kappa\left(\alpha G^{\mathcal{M}}(n) + b^{\mathcal{M}}(n)\right) + w\left(1 - t(1-n) - c\cdot C(n)\right)$$

where $c \cdot C(n)$ is the expected cost of non-compliance, and $c > 0$ is a parameter that captures detection effort and the rightmost component of the utility function stands for the expected net savings from concealment after paying income tax. For simplicity, assume that $C'(n) = n^2/2$. The solution to the taxpayers' problem is given by:

$$\hat{n}(\kappa;\rho,\alpha,c) = \min\left\{\max\left\{\frac{t\left(1-\kappa\left(\rho\alpha + (1-\rho)\frac{\sigma}{1+\sigma}\right)\right)}{c}, 0\right\}, 1\right\},$$

which means that an agent with a degree of morality κ decides on concealment proportional to the income tax levied by the Elite, t, but inversely proportionally to its degree of morality κ weighted by the government's effective provision as given by the convex combination $\rho\alpha + (1-\rho)\frac{\sigma}{1+\sigma}$. Homo moralis agents are more likely to be compliant when their tax revenues are being used efficiently for the citizenry. This can either mean funding a relatively useful public good when α is high by setting a high public share ρ, or instead focusing on providing transfers given the distribution constraint implied by institutions, captured by σ.

3. Fiscal Capacity and the Laffer Curve

Fiscal capacity is defined as the maximum tax revenue a government can raise given the degree of morality κ and the coercive power of government given by c. Tax revenue per capita, given a tax rate of t and an expenditure mix duplet of transfers and public good (b, G), is given by

$$T(t, \rho, \kappa, c, \sigma) = tw\left[1 - \hat{n}(\kappa; \rho, \alpha, c)\right] = \frac{tw}{c}\left[c - t\left(1 - \kappa\left(\rho\alpha + (1-\rho)\frac{\sigma}{1+\sigma}\right)\right)\right]. \tag{4}$$

As shown Figure 2, equation (4) leads to the emergence of a variation of the Laffer Curve. As we increase the degree of morality κ, the Elite has the ability to raise taxes further without finding itself on the downward-sloping region to the right of the revenue-maximizing tax rate. This becomes apparent when observing the upwards shift in $T(t)$ when we allow citizens to have a higher degree of morality κ.

The revenue-maximizing tax rate is then found by solving for the optimum of $T(t, \rho, \kappa, c, \sigma)$. We can find it by maximizing expression (4) with respect to the income tax rate t. It is given by:

$$\hat{t}(\rho, \kappa, c, \sigma) = \arg\max_{t \geq 0}\{T(t, \rho, \kappa, c)\} = \frac{c/2}{1 - \kappa\left(\rho\alpha + (1-\rho)\frac{\sigma}{1+\sigma}\right)}. \tag{5}$$

Substituting equation (5) into (4) we can express total tax revenues as a function of the public good share ρ, the morality parameter κ, the marginal utility of the public good α, and c. Indeed:

$$T(\hat{t}(\rho, c, \sigma), \rho, \kappa, c) = \frac{wc}{1 - \kappa\left(\rho\alpha + (1-\rho)\frac{\sigma}{1+\sigma}\right)}\frac{1}{4} \tag{6}$$

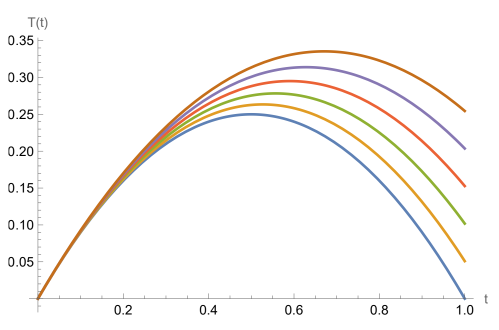


FIGURE 2. Laffer Curves for different κ: $T(t)$ for $\alpha = 0.5$, $\rho = 0.4$, $\sigma = 0.1$, $w = c = 1$.

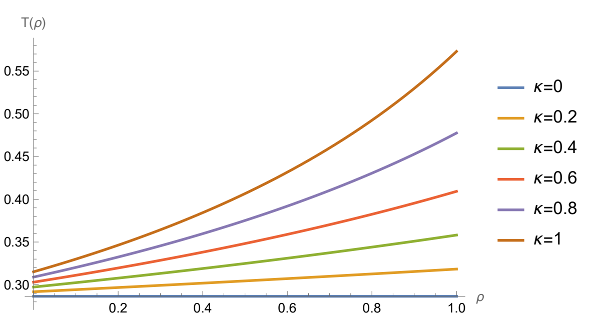


FIGURE 3. Tax proceeds as a function of the public good share $T(\rho)$ for $\alpha = 0.5$, $\sigma = 0.1$, $w = c = 1$.

4. THE ELITE'S PROBLEM

The Elite chooses the policy mix of public good, transfers, and taxes, $(G(\rho), B(\rho), \hat{t}(\rho,\kappa,c))$ that maximizes its utility subject to the resource constraint given the institutions of the economy and morality parameter κ of the agents in the population:

$$\max_{\rho} \alpha G(\rho) + B(\rho) \tag{7}$$

$$\text{s.t: } [T(\hat{t}(\rho,c,\sigma),\rho,\kappa,c) - G]\theta(\sigma) = B(\rho) \tag{8}$$

$$\tag{9}$$

The Elite will choose the revenue-maximizing tax rate for any given expenditure mix. Moreover, the constraint set is convex, so the Elite will choose a corner solution where either G or B is zero.

PROPOSITION 1. The solution to the Elite's problem in the program 7 in terms of the public good proportion $\rho^* \in [0,1]$ is such that:

1. if $\alpha > \frac{1}{1+\sigma}$ the Elite spends all the tax proceeds in the public good and sets $\rho^* = 1$.
2. if $\alpha \leq \frac{1}{1+\sigma}$ then there exists a threshold level of the degree of morality $\bar{\kappa}(\alpha,\sigma)$ such that:

$$\rho^* = \begin{cases} 1, & \text{if } \kappa \geq \bar{\kappa}(\alpha,\sigma) \\ 0, & \text{if } \kappa < \bar{\kappa}(\alpha,\sigma) \end{cases}.$$

Moreover, the threshold level of the degree of morality is decreasing in α unconditionally and on σ provided $\alpha > 1/2$. Formally,

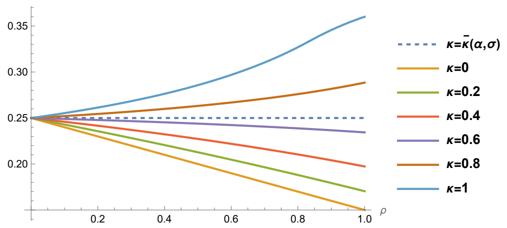
$$\frac{\partial\bar{\kappa}(\alpha,\sigma)}{\partial\alpha} < 0; \; \frac{\partial\bar{\kappa}(\alpha,\sigma)}{\partial\sigma} = \frac{-2+\alpha^{-1}}{(1-\sigma)^2} > 0 \text{ iff } \alpha < 1/2.$$

The Elite will decide to devote all of the collected revenue whenever the marginal utility from doing so is sufficiently large. More interestingly, even for cases in which the public good is less desirable, a sufficiently large degree of morality $\kappa \geq \bar{\kappa}(\alpha,\sigma)$ induces the elite to spend its resources in public provision. This result is somewhat striking: while the Elite's marginal utility of appropriating the money of the citizens, given by $\frac{1}{1+\sigma}$ is larger than the marginal value of provision given by α, the conditional compliance of Homo moralis agents leads the ruler's to spend instead on the public good G by setting $\rho^* = 1$.

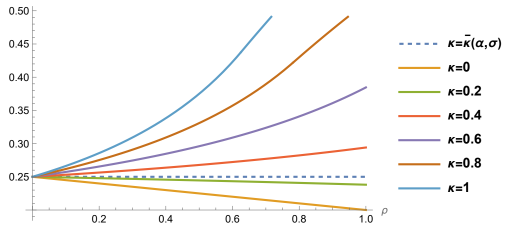
COROLLARY 1. In the solution to the Elite's program above, tax revenues are described by:

$$T^*(\rho,\kappa,c,w,\sigma) = \begin{cases} \frac{wc}{1-\kappa\alpha}\frac{1}{4}, & \text{if } \rho^* = 1 \\ \frac{wc}{1-\frac{\kappa}{1+\sigma}}\frac{1}{4}, & \text{if } \rho^* = 0 \end{cases} \tag{10}$$

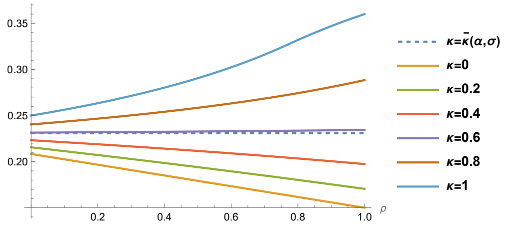
This says that taxation is proportional to c, resulting from the quadratic compliance cost specification. More substantively, the result says that when κ increases, civic-minded citizens increase compliance when the Elite is committed to public expenditure.



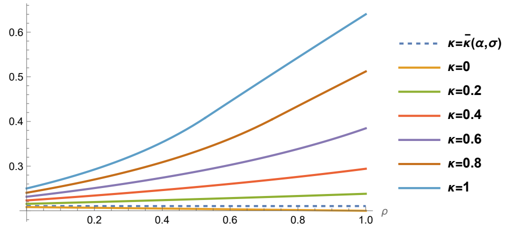
(a) Low productivity and weak institutions $\alpha = 0.6$, $\sigma = 0$.



(b) High productivity and weak institutions $\alpha = 0.8$, $\sigma = 0$.



(c) Low productivity and strong institutions $\alpha = 0.6$, $\sigma = 0.2$



(d) High productivity and strong institutions $\alpha = 0.6$, $\sigma = 0.2$

FIGURE 4. Elite's objective function as a function of public good share ρ for different levels of degree of morality κ, $c = 1$, $w = 1$.

Proposition 1 its Corollary are illustrated by Figure 4. In it, the plots of the Elite's objective function for different levels of the population's degree of morality (κ), productivity levels (α), and institutions' cohesiveness (σ). The apparent first conclusion to observe is the effect of the degree of morality on the provision optimality of public provision by the state: Holding institutions and public good productivity constant, societies that are more moral are more likely to incentivize the Elite to pursue the funding of public goods and set $\rho = 1$. We may interpret this as incentivizing the emergence of common interest states (Besley and Persson, 2011).

Second, notice that moral agents condition their cooperative behavior upon public good productivity, this means that any increase in the productivity of the public good α also incentivizes the emergence of common interest states, as moral agents entirely react to the hypothetical changes that their provision would have on the aggregate economy.

Third, Kantian moral agents also react to institution strength as measured by σ. This means that they incorporate the hypothetical impact of their transfers in redistribution through transfers; this effect is evident when comparing panels (a) and (b) with (c) and (d) vertically in Figure 4.

5. DISCUSSION AND AVENUES FOR FUTURE RESEARCH

As put forward by the results of the previous section, semi-Kamtian preferences offer a potential micro foundation for the emergence of fiscal capacity. This microfoundation has similar implications to those explored Besley (2020), under which civic-minded citizens link their compliance decisions to the Elite's pattern of expenditure. In this section, I present a comparison of the two approaches and contrast their different implications.

First just as with agents that are motivated by reciprocity, under Homo moralis preferences, compliance decisions are also linked to the expenditure patterns of the Elite, as capture by the public good share ρ. Indeed, agents equipped with semi-Kantian preferences universalize the impact of their compliance decision, considering the magnitude of their decisions as determined by the optimal mix ρ.

Contrastingly, Homo moralis agents care about the perceived usefulness of their contributions, as captured by the preference parameter α. This means that they do not simply adjust their compliance decisions to the expenditure mix of the government. However, they do so conditionally based on the government funding public goods that are considered useful to the public. This is not the case for civic-minded citizens, as modeled by Besley, who simply react to the divergence between taxes raised and public expenditures in the public good G.

A similar effect is captured by the way Homo moralis agents react to σ, the parameter that guides institution cohesiveness. Kantian moral agents are fully aware that a more cohesive redistributive institution implied by a higher σ, makes them better off for each possible expenditure mix (ρ, G); this leads them to be more compliant and leads to the emergence of common interest states in which $\rho = 1$ trough a purely preference-based channel. This conclusion strikingly distinguishes the Homo moralis model from the reciprocity model proposed by Besley: Kantian agents internalize the full expenditure mix of the government and lead to the emergence of equilibrium public good provision that is not only driven by government parameters but mediated by pure preference parameters, in this case, the degree of morality κ.

Another important distinction is the required assumptions over α. While with reciprocity-motivated agents, one needs to require that $\alpha \geq 1$ so that citizens always prefer public production to pure transfers, as put forward by Proposition 1 part 2, societies in which morality is high enough may manage to incentivize public provision even for lower levels of α, as long as the marginal value of transfer *net of the Elite's appropiation*, i.e $\sigma/(1+\sigma)$ is high enough to lead to conditional compliance.

This last distinction drives an important prediction that opens a lot of avenues for future research. In particular, the model of state capacity as based in Homo moralis agents offers a sharp prediction: we should observe societies in which individual measures of Kantian morality, more precisely κ, are large, to be more prone to the emergence of common interest states in which the relative levels of taxation s large and fiscal capacity is relatively large. Empirical and experimental work in the near future may focus on establishing whether these predictions hold. For that, it is necessary to have adequate estimates of the degree of morality, κ. Techniques pioneered by Van Leeuwen and Alger (2019) may be exploited for this purpose.

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Appendix A: Appendix: proofs

PROOF. We can manipulate the objective function of the Elite to obtain:

$$\alpha G + \theta(\sigma)(1-\rho)\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4}$$

subject to

$$\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4} - G = (1-\rho)\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4}$$

We can rearrange the constraint above and write:

$$wG = \rho\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4}$$

Now substitute this in the objective function:

$$\alpha\rho\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4} + \theta(\sigma)(1-\rho)\frac{wc}{1-\kappa\left(\rho\alpha+\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4}$$
$$= \frac{wc}{1-\kappa\left(\rho\alpha+\sigma\dfrac{1-\rho}{1+\sigma}\right)}\frac{1}{4}(\alpha\rho+\theta(\sigma)(1-\rho))$$

Using the definition of $\theta(\sigma) = (1+\sigma)^{-1}$:

$$= \frac{wc}{4} \frac{\frac{1}{1+\sigma} + \rho\left(\alpha - \frac{1}{1+\sigma}\right)}{1 - \kappa\left(\frac{\sigma}{1+\sigma} + \rho\left(\alpha - \frac{\sigma}{1+\sigma}\right)\right)}$$

$$= \frac{wc}{4} \frac{\frac{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1}}{1+\sigma} + \rho}{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} + \rho\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)} \text{ if } \alpha \neq \frac{1}{1+\sigma}$$

The first order condition to this problem with respect to ρ writes:

$$= \frac{wc}{4} \frac{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} + \rho\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right) + \kappa\left(\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)\left(\frac{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1}}{1+\sigma} + \rho\right)}{\left(\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} + \rho\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)\right)^2}$$

$$= \frac{wc}{4} \frac{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1}\right) + \frac{\kappa}{1+\sigma}\frac{\alpha - \frac{\sigma}{1+\sigma}}{\left(\alpha - \frac{1}{1+\sigma}\right)^2}}{\left(\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} + \rho\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)\right)^2}$$

$$= \frac{wc}{4} \frac{\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\frac{1}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1}\left(\sigma - \frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)}{\left(\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} - \kappa\left(\frac{\sigma}{1+\sigma}\left(\alpha - \frac{1}{1+\sigma}\right)^{-1} + \rho\frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right)\right)^2}$$

If $\alpha > \frac{1}{1+\sigma}$ the derivative is positive iff:

$$1 - \kappa\frac{1}{1+\sigma}\left(\sigma - \frac{\alpha - \frac{\sigma}{1+\sigma}}{\alpha - \frac{1}{1+\sigma}}\right) > 0$$

$$1 - \kappa\frac{1}{1+\sigma}\left(\frac{\alpha(\sigma - 1)}{\alpha - \frac{1}{1+\sigma}}\right) > 0$$

$$1+\kappa\frac{\alpha}{1+\sigma}\left(\frac{1-\sigma}{\alpha-\dfrac{1}{1+\sigma}}\right)>0$$

$$\implies \kappa>0>-\frac{\alpha-\dfrac{1}{1+\sigma}}{1-\sigma}\frac{1+\sigma}{\alpha}$$

Which always holds.

If $\alpha<\frac{1}{1+\sigma}$ the derivative is positive iff:

$$1-\kappa\frac{1}{1+\sigma}\left(\sigma-\frac{\alpha-\dfrac{\sigma}{1+\sigma}}{\alpha-\dfrac{1}{1+\sigma}}\right)<0$$

$$1-\kappa\frac{\alpha}{1+\sigma}\left(\frac{1-\sigma}{-\alpha+\dfrac{1}{1+\sigma}}\right)<0$$

$$\kappa>\frac{1+\sigma}{\alpha}\frac{\dfrac{1}{1+\sigma}-\alpha}{1-\sigma}==\frac{\alpha^{-1}-(1+\sigma)}{1-\sigma}$$

$$\implies \kappa>\overline{\kappa}(\alpha,\sigma)=\frac{\alpha^{-1}-(1+\sigma)}{1-\sigma}$$

Notice that:

$$\frac{\partial\overline{\kappa}(\alpha,\sigma)}{\partial\alpha}<0;\ \frac{\partial\overline{\kappa}(\alpha,\sigma)}{\partial\sigma}=\frac{-(1-\sigma)+\alpha^{-1}-(1+\sigma)}{(1-\sigma)^2}=\frac{-2+\alpha^{-1}}{(1-\sigma)^2}>0 \text{ iff } \alpha<1/2$$

□