Competing for Donations: The Role of Tax Deductibility in the U.S. Charitable Sector.*

JOB MARKET PAPER

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Esteban Muñoz-Sobrado†

Toulouse School of Economics

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Abstract

Around the world, governments provide tax benefits to incentivize charitable giving. I argue that the current approaches to determining the optimal level of such tax benefits neglect a crucial ingredient. While higher tax benefits increase charitable giving, they also intensify potentially wasteful competition for funds among charities. I build a model where charities use informative advertising to attract individual donors. Competition leads to inefficient fundraising as charities incur excessive advertising costs, and the inefficiency increases as available funds increase. I then estimate the structural parameters of the model using data from the universe of Nonprofits in the U.S. paired with data from the country's most prominent charity assessment organization. I document that leakage, the proportion of charities' budget not spent on direct public good provision, goes up to 40 percent in my sample for 2014. Moreover, findings from counterfactual analyses suggest that fundraising accounts for significant endogenous leakage of gross donations into advertising. These findings suggest that estimates that ignore competition must be adjusted downwards to account for charities' endogenous responses to the tax code.

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†Email: esteban.munoz@tse-fr.eu. Website: https://www.munozsobrado.com . Affiliation: Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France.

1. Introduction

With total donations amounting to 2.1 percent of GDP yearly, the charitable sector constitutes an integral part of the U.S. economy[1]. Non-governmental organizations cater to donors to fund public goods and services. Consequently, governments typically establish tax deductions on donations made to charitable organizations, which means that tax policy is a standard instrument used to provide incentives for giving. This paper is interested in assessing the optimality of subsidies to giving.

Charities, NGOs, and Nonprofits are not passive players that merely wait for donations from well-intention samaritans[2]. In fact, there is one dimension in which the Nonprofit sector resembles a typical market: NGOs compete to capture funds from donors. The main novelty of this paper is that it provides a methodological tool to evaluate the optimality of subsidies to charitable giving that considers the strategic nature of NGOs. As Andreoni (2006) argues, it is fundamental to account that both sides of this market (donors and Nonprofits) are strategic players and will likely respond to changes in public policy, taxes, or other factors. Such interplay between the supply and demand for charitable goods and services has yet to be the subject of many theoretical or empirical analyses. Despite its immediate relevance, the extent to which such competition affects the optimal deductibility rates is still an open question, whose answer requires both a theoretical model of how NGOs may compete with each other and structural empirical estimates of such theoretical model.

In this paper, I proceed in two comprehensive steps. First, on the positive side, I provide a theoretical characterization of the impact of tax policy on the supply and demand for donations. I show how using the tax deductions for charitable giving as a tool for welfare maximization needs to account for a crucial statistic that summarizes the NGOs' conduct: what is the proportion of raised funds raised through costly advertising that gets allocated to fund the campaign as opposed to public good production? I dub this term the "endogenous leakage coefficient". Second, I use the model to perform normative analysis by estimating policy counterfactuals. The

[1] Giving USA reports the figure for 2021 at $484.85 billion.

[2] Although legal definitions may differ, in this paper, I will privilege the term "NGO" as an encompassing term that also includes "charity" and "Nonprofit."

counterfactual analysis allows us to answer critical policy questions. Does subsidizing donations necessarily increase welfare? Is competition in the sector desirable from a social welfare perspective?

This paper provides three main contributions. As a first contribution, I expand the optimal taxation formulae to account for competitive effects. Contributions to the literature on optimal taxation have solved the design of an optimal income tax system in an economy where agents value the public good, considering leakage into advertising as a constant parameter. The first contribution to consider leakage as a fixed parameter is found in the early work by Feldstein et al. (1980). This paper instead considers leakage to be an endogenous parameter determined by the strategic environment faced by NGOs, which decide on advertising intensities to fund their activities. Each NGO's decision over which share of resources to devote to advertising is a strategic choice that depends on the structure and characteristics of the charitable market and the total donations available. Since the Planner's design of the tax schedule determines total donations, advertising incentives also indirectly depend on tax policy.

As a second contribution, I build a model that considers charities as potentially pro-social entities that compete for donations through informative advertising and may differ in quality in a setting with atomistic donors that may value high-quality charities heterogeneously[3]. In the equilibrium of this model, advertising expenditures are excessive compared to a welfare-maximizing benchmark. Moreover, this inefficiency is increasing the amount of funds available in the market, which means that tax policies that increase donations may increase inefficient competition. More generally, two opposing effects arise when considering the strategic

[3]The model builds on specific stylized facts documented by Aldashev and Verdier (2010) regarding how charities compete for funds. First, NGO projects are horizontally differentiated. Bilodeau and Slivinski (1997) describe how Nonprofits can actively attempt to offer differentiated public goods to the public, for instance, through different types of in-kind assistance to indigents or support for different kinds of medical research. Second, NGOs compete for private donations through fundraising. Nonprofits exert effort to attract private donations through fundraising advertising, as accounted by De Waal and de Waal (1997) and Smillie (1995). In particular, De Waal and de Waal (1997) describes how the organizations with the most prominent media profiles often get the most funds from donors. Third, private donors have "spatial" preferences about NGOs and are sensitive to fundraising. Andreoni and Payne (2003) describes the latter by referring to donors having "latent demands for giving." Agents are often willing to give to nonprofit organizations but will not do so until asked for a contribution. Regarding the "spatial" dimension, Thornton (2006) establishes that differentiation in the nonprofit context may respond to factors such as ideology, methodology, or targeted beneficiaries.

responses of NGOs to tax policy. On one hand, NGOs react by competing more fiercely, generating a "business stealing" effect. Opposing this effect, increasing the availability of funds for donations may lead to competition between NGOs that seek to increase market coverage instead of increasing public good provision by NGOs.

In sum, ignoring such an interaction has likely led past studies to overestimate (resp. underestimate) the impact of marginal tax rates on donations when the detrimental (resp. beneficial) effects of advertising dominate. I aim to empirically assess this effect's magnitude and provide new policy estimates that account for competitive equilibrium outcomes.

The model yields three predictions: (i) increases in the deductibility rate of charitable donations should correlate positively with measures of intensity of competition between charities, (ii) equilibrium quality provision may be affected by the deductibility rate, depending on the extent to which donors value high-quality charities, and (iii) existing estimates of the optimal deductibility rate which do not take into account the effect of competition need to be adjusted downwards. I use data from the IRS, Kantar Media, and Charity Navigator to estimate a structural model of competition to assess (i) and (ii) and provide appropriate estimates on (iii) for the U.S.

I empirically asses (i)-(iii) as a third contribution. For this, I build a nested logit model (Berry, 1994), in which nests correspond to NGO classification as defined by the IRS filings, and markets are defined geographically using Nielsen's DMAs. I then estimate a structural model of vertical and horizontal differentiation in which NGOs compete inside their categories, deriving predictions over market shares and responses to changes in NGO characteristics.

Moreover, an event study allows us to complement our structural estimation: the Tax Reform Act of 1986 studied by Duquette (2016). The Tax Reform Act of 1986 (TRA86) was a significant legislation that sweepingly changed the U.S. federal income tax system. Among other things, it lowered tax rates and broadened the tax base by eliminating a number of tax loopholes and preferences. The impact of these changes was felt at the federal level and in state income tax systems across the country. Indeed, the effects of TRA86 on state tax systems were varied and complex, depending on each state's specific tax structure and policies. I simulate this tax reform for 2014 and document leakage in the entire same to be just below 40 percent pre-reform, with a substantial variation along the quality dimension as measured by the Charity Navigator Star System, which is higher for low-quality NGOs, as expected. Moreover, leakage elasticity, which measures how the leakage parameter varies when total donations are changed due to a tax change, is positive and also varies widely across Charity Navigator Scores.

The setting also allows testing relevant predictions obtained by Dewatripont et al. (2022) for environments with pro-socially motivated suppliers. My empirical results suggest that ethical NGOs (as measured by their Charity Navigator score) command higher market shares, suggesting that donors have preferences for high-quality NGOs, which means that, in the paragraph above, result (i) is dampened by the response of pro-social NGOs implied by result (ii).

Related literature. This paper contributes to several strands of literature concerning both Public Economics and the Industrial Organisation of Charitable Giving. First, it contributes to the literature that explores the optimal treatment of charitable donations. The two main contributions on this topic are the articles by Saez (2004) and Diamond (2006). They provide the solution to the optimal taxation problem that the government faces when agents derive some warm-glow utility of contributing to a public good. However, these two contributions do not consider the subsequent effect of these tax deductions on the fundraising market. This paper asks whether this result is robust to endogenous competition by NGOs. As they do not consider the impact of competition, their results likely overstate/understate the optimal deductibility rate given to charitable contributions.

This project also relates closely to investigations on the long-run equilibrium of the nonprofit sector and the optimal tax treatment of charitable donations. The earliest and most prominent contribution to this literature is found in Rose-Ackerman (1982), which builds a theoretical model in which charities are differentiated in one dimension described as "ideology," and donors are initially uninformed of charities. Fundraising serves as a way to inform donors about the charities that are closest to them. She finds that competition for contributions leads to excessive fundraising. The model presented in Section 3 of this paper relates more closely to Aldashev and Verdier (2010), which focuses on competition for funds in the market for development NGOs with horizontally differentiated projects, under the assumption that advertisement serves as a "cost reductor" and NGOs maximize public good provision instead of revenues. Crucially, their model yields a donation function that closely resembles a Tullock contest function (Tullock, 2013), which causes NGOs to decide on fundraising strategies independently of the amount of funds in the market, making NGO competition independent of tax policy.

This work also relates closely to that of Lapointe et al. (2018), who analyze implications of market size for market structure in the charity sector. Using data from six local markets in Canada, they find empirical evidence supporting a Cournot model where charities are concerned about providing public goods but may be biased towards their production. Their focus is, however, devoted to analyzing the question of market size and entry, which, in the context of the U.S., could be more relevant for the set of charities that react strategically to tax policy (Duquette, 2016). This work further complements recent theoretical work regarding charities' strategic decisions to cluster (Marini, 2020), delegate their decisions to motivated agents (Kopel and Marini, 2019), and react to publicly available contracts (Kopel and Marini, 2020). Schmitz (2019) conducted an experimental study in which he varies the set of similar real charities to which subjects can donate and finds weak substitution between charities when giving to more than one charity is possible, as the donated amounts to individual charities decrease with the size of the choice set. In another experimental study, Chatterjee et al. (2020) found that the average giving is unaffected by information provision and composition of the choice set of charities. However, subjects direct significantly more funds towards qualifying charities when information about the tax program is provided. Finally, it also relates to the recent literature that explores the strategic responses to charity ratings (Mayo, 2021) .

The paper also contributes to the extensive literature on philanthropy, which has been vastly studied both theoretically and empirically (see Andreoni (2006) and List (2011) for reviews on the matter) and advertising. Concerning this second literature, the model presented below builds directly on the model of informative advertising found in Grossman and Shapiro (1984). However, the model proposed here differs from theirs since charities typically do not charge competitive prices and often do not sell private goods. However, I take advantage of their convenient advertising technology without targeting. This adaptation yields an essential difference concerning the model of oligopolistic competition for private goods: the non-cooperative equilibrium level of advertising is independent of the market's competitiveness (which, in Hotelling models, is pinned down by the transport costs faced by consumers).

The structure of the paper proceeds as follows. Section 2 presents an optimal taxation model that considers endogenous leakage in a reduced manner. Section 3 presents the model of NGO competition and several theoretical results, and Section 4 and Section 5 present the data and empirical results, respectively. Section 6 describes the components of the welfare analyses. Finally, Section 7 concludes.

2. The optimal taxation problem with endogenous leakage

In this section, I study a "reduced" model of an economy in which public goods are funded partly or totally by charitable contributions. Consider an economy where the government and N charities provide one type of public good. Governmental provision is given by $G_0 \geq 0$, and the aggregate public good is denoted by $\bar{G} = \sum_{j=0}^{N} G_j$. I assume that donors have preferences over the aggregate vector of public goods but not the provision by individual NGOs.[4] Competitive forces will not be modeled yet but introduced indirectly as exogenous parameters; these will be endogenized in Section 3.

There is a continuum of donors indexed by i. Each donor derives utility from consumption $x_i \geq 0$, donations $d_i \geq 0$, and the vector of public goods $\boldsymbol{G}$. Donations are deducted at a rate $-\tau^d$, where a negative (resp. positive) rate $\tau^d < 0$ (resp. $\tau^d > 0$) constitutes a tax deduction (resp. increase). Income is taxed uniformly at a rate τ, which means that the budget constraint faced by each individual is given by $x_i + d_i(1 + \tau^d) \leq z_i(1 - \tau) + R$, where z_i denotes pre-tax income and R is a lump-sum transfer. Omitting indices, for a given vector of public goods $\mathbf{G}$, income z, and lump-sum transfer R, indirect utilities for agents i, v^i, are assumed to be given by $v^i\left(z, \mathbf{G}, R; \tau^d, \tau\right)$. When there is no ambiguity, I let sub-indices denote partial derivatives.

[4] More generally, we can consider an economy where the government and charities provide M types of public goods and $N_m \times M$ different NGOs provide each type of public good, which donors may have preferences over. This distinction does not affect the results of this section substantially, so this section presents a simplified model in which donors care about aggregate provision by category. In the empirical sections, this assumption is relaxed.

The planner's program. The government sets τ, τ^d, R and G_0 in order to maximize the utilitarian welfare function:

$$W(\tau, \tau^d, R, G_0) = \int \mu^i v^i \left(1-\tau, 1+\tau^d, R, \boldsymbol{G}\right) \, di \tag{1}$$

where μ^i is the weight associated to individual i, subject to the budget constraint:

$$\tau \bar{Z} + \tau^d \bar{D} \geq R + G_0 + E, \tag{2}$$

where E is exogenous government consumption per capita, $\bar{Z}$ is the aggregate income and $\bar{D}$ aggregate donations.

The leakage coefficient. As noted by Feldstein et al. (1980), when setting the price τ^d, the government must take into account the leakage that typically occurs, i.e., the portion of donated funds that cannot be allocated directly towards the production of public good, and are instead used to cover costs associated with raising charitable donations. To capture this, let each organization j transform donations, denoted D_j, into a public good G_j according to the following technology:

$$G_j = \rho_j D_j, \tag{3}$$

where $1 - \rho_j \in [0, 1]$ denotes the *leakage coefficient*. Leakage represents all the money raised by charity j that does not go directly into public good provision but is instead spent on administrative costs, advertising, and other activities. By contrast to the existing literature, I allow the leakage coefficient to be endogenous to tax policy. After gathering funds by advertising, a charity j transforms monetary donations D_j into a public good. However, since advertising intensity is considered endogenous, the leakage coefficient is not constant to total donations or tax liabilities.

I define by $1 - \eta_j$ as the *individual elasticity of leakage to an increase in total donations* $\bar{D}$

where:

$$\eta_j(\bar{D}) = \frac{\rho_j'(\bar{D})\bar{D}}{\rho_j(\bar{D})}, \tag{4}$$

and its counterpart at the aggregate level $1-\bar{\eta}$, i.e the *aggregate elasticity of leakage to an increase in total donations* $\bar{D}$ where:

$$\bar{\eta}(\bar{D}) = \frac{\bar{\rho}'(\bar{D})\bar{D}}{\bar{\rho}(\bar{D})}, \quad \text{for aggregate leakage: } \bar{\rho}(\bar{D}) = \frac{\bar{D}}{\bar{G}}, \tag{5}$$

these elasticities are the primary outcome of interest in this study. They are crucial to describing the model's implications for the optimal deductibility rate. The results in the sections below relate them to the competition model.

Optimal taxation. The first reason to incentivize charitable output is that since it is a public good, provision is typically inefficiently low in the absence of subsidies. To capture this, I proceed by defining the social marginal value of the public good in terms of public funds as:

$$e = \int \beta^i \frac{\partial v^i/\partial \bar{G}}{\partial v^i/\partial R} di,$$

where $\beta^i = \mu^i v_R^i/\lambda$ denotes the average social marginal value of consumption of agent i from a one-dollar lump-sum transfer from the government, for a planner with welfare weights μ^i and a multiplier $\lambda > 0$ of the budget constraint of the government in (2).

In order to simplify the problem and get solutions comparable to the baseline simulations found in Saez (2004), I impose three regularity conditions. First, I assume that there are no income effects on earnings, i.e., $z_R^i = 0$ for all i. Second, I posit independence between aggregate earnings and contributions, i.e., $\bar{Z}_{G_0} = 0$ and $\bar{Z}_{1+\tau^d} = 0$. Finally, the compensated supply of contributions does not depend on earnings. $\partial d^i/\partial(1-\tau) = 0$, which allows us to write and $\hat{D}_R = D_{1-\tau}/\overline{Z}$ as the average response weighted by earnings of contributions to a uniform one-dollar increase of the lump-sum. Finally, the elasticity of aggregate earnings to (one minus) the tax rate is given by $\epsilon_Z = (1-\tau)\partial\overline{Z}_{1-\tau}/\overline{Z}$. Finally, denote by $r = -\overline{G}_{1+\tau^d}/\overline{G}$ the size of the price response of contributions after a change in the deductibility rate. These assumptions are further detailed in the Appendix. The following proposition characterizes the solution to the planner's problem.

Proposition 1 *Suppose the government cannot supply the public good directly but can set the tax code and transfers optimally. In that case, the vector of policy parameters that maximizes welfare is described by the the solution* (τ, τ^d, R) *to the non-linear system:*

$$\tau^d = -e \cdot \bar{\rho}(1+\bar{\eta}) + \frac{1}{r}\left[1 - \int \beta^i d^i di / \overline{D}\right] \tag{6}$$

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon_Z}\left[1 - \int \beta^i z^i di/\overline{Z} - (\tau^d + e \cdot \bar{\rho}(1+\bar{\eta}))\hat{D}_R\right] \tag{7}$$

$$\int \beta^i di = 1 - (\tau^d + e \cdot \bar{\rho}(1+\bar{\eta}))\bar{D}_R. \tag{8}$$

If the government can supply the public good G_0 *and the solution implies positive provision* $G_0 > 0$. *The optimal vector* (τ, τ^d, R, G_0) *is characterized by the three equations above and additionally requires:*

$$e = 1 - (\tau^d + e \cdot \bar{\rho}(1+\bar{\eta}))\partial\bar{G}/\partial G_0. \tag{9}$$

See Appendix.

Proposition 1 establishes the non-linear system that solves the welfare-maximizing problem. The highlighted elements of the equations above describe the impact of endogenous leakage on the baseline optimality formulas found in Saez (2004). As seen from the first equation in Proposition 1, the endogenous leakage elasticity has a first-order effect that reduces the deductibility rate by a magnitude of the external effect e, pushing charitable deductions upwards. However, a change in the leakage elasticity also affects e. In order to assess the impact of varying the leakage elasticity on deductibility rates, Table 2 provides numerical computations that compare the tax rate for different values of η with those obtained in the benchmark case of Saez with no leakage elasticity.

The US income tax law authorizes some expenditures to be fully deductible (Saez, 2004) of income tax. The case of full deductibility is, hence, of immediate policy relevance. Full deductibility is modeled as considering an additional constraint $\tau^d = -\tau$, i.e., contributions are deducted at the income tax rate. The following proposition derives the optimal rates when the government faces the constraint $\tau = -\tau^d$.

Proposition 2 *If charitable donations are fully deductible from taxable income, i.e., the government is constrained to set $\tau^d = -\tau$, then the optimal tax rate on income τ is given by:*

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon_Y}\left[1 - \int \beta^i y^i di/\overline{Y} + e \cdot \bar{\rho}(1+\bar{\eta})\left(r\frac{\bar{G}}{\bar{Y}} - \hat{G}_R\frac{\bar{Z}}{\bar{Y}}\right)\right], \tag{10}$$

where $\bar{Y} = \overline{Z} - \overline{G}$ denotes aggregate taxable income, and $\epsilon_Y = (1-\tau)\overline{Y}_{1-\tau}/\overline{Y}$ is the aggregate taxable income elasticity. If the government can provide the public good, then:

$$e = 1 - (\tau^d + e \cdot \bar{\rho}(1+\bar{\eta}))\partial\bar{G}/\partial G_0 = \frac{1 - \tau^d \bar{D}_{G_0}}{1 + \bar{\rho}(1+\bar{\eta})\bar{D}_{G_0}} \tag{11}$$

Two effects to be discussed from Proposition 2 distinguish it from the baseline case. First, $\epsilon_Y > \epsilon_Z$, since contributions are more responsive than earnings, which drives the rate τ to be lower than in Proposition 1. Second, the tax rate on taxable income weakly increases contributions. The counterfactual analyses carried out in the empirical section revisit Proposition for a hypothetical tax change in 2014 are revisited in the counterfactual analyses carried out below.

3. A model of the competition between NGOs

I now present a model of competition that endogenizes the leakage coefficient and the leakage elasticity (see Equations (3) and (4)). The model comprises three actors: donors, NGOs, and the government. Each donor makes two decisions: he first pledges a donation amount after observing the tax code and subsequently selects his preferred NGOs among those in his choice set. Each NGO decides strategically on its advertisement intensity to maximize its social and private objectives. The government decides over tax policy, as argued previously. For this, I assume independence and focus on one of the M sectors to study competition between $N \geq 2$ NGOs inside the sector.

In this model, donors and NGOs are distributed along a Salop circle [5]. Donors derive positive utility from donating but passively wait for NGOs to inform them about their existence. As such, donors only donate to those NGOs whose existence is known to them, i.e., those whose ads have reached them.

3.1 Donors and NGOs

There is a continuum of donors of mass 1. The utility derived by donor i from donating to an NGO j is given by $U(\boldsymbol{\chi}_j, \boldsymbol{\theta}_i)$, where $\boldsymbol{\chi}_j$ is a vector that summarizes NGO characteristics, like geographical location r_j, service quality α_j, and horizontal position p_j. For the estimation, I distinguish $\boldsymbol{\chi}_j = (x_j, \xi_j)$, where x_j are observed NGO characteristics, and ξ_j are unobserved ones from $\boldsymbol{\theta}_i$, a vector of individual donor characteristics, like income $z_i \geq 0$, horizontal preference parameter $h_i \in [0, 1]$, and faced tax liability T_i.

I consider donors' choices as a two-step process. First, each donor pledges a constant fraction of his yearly expenses for donations (Bjornerstedt and Verboven, 2016). The amount of donations depends on tax policy but does not depend on the observed set of NGOs. Second, each donor observes NGO characteristics and donate the entire pledged amount to his preferred NGO inside his information set $\mathcal{I}_i \subset \mathcal{P}(\mathcal{J})$, where $\mathcal{J} = \{1, \ldots N\}$ is the set of all NGOs and $\mathcal{P}(\mathcal{J})$ power set over the NGO universe. The total supply of donations available in the market is $D\left(\tau, \tau^d\right)$, a measure of the market size for gross available donations. D is a function of the tax system faced by the donors and, as such, it is determined by the tax schedule τ^d and τ: $D(\tau^d, \tau)$, where τ^d is the tax rate paid on donations and τ the income tax rate.

After observing the ads and learning $\boldsymbol{\chi}_j$ for all $j \in \mathcal{I}_i$, each donor picks his preferred NGO in a discrete-choice fashion (Anderson et al., 1989). A given donor selects the NGO j^* that maximizes his indirect utility among all NGOs in his information set $\mathcal{I}_i$ according to a decision rule given by:

$$j^* \in \{j \in \mathcal{I}_i : u_{ij} > u_{ik}, \forall k \neq j\}. \tag{12}$$

[5] I consider an informative advertising setting that generalizes the model of Grossman and Shapiro (1984) to account for quality heterogeneity under fixed prices.

In the estimation, I will assume that the indirect utility follows a random coefficient specification (Berry, 1994). In more detail, this means that indirect utility from the donation to NGO j, u_{ij}, will be modeled as depending on NGO characteristics. In the model section, I will limit those to NGO quality and donors' horizontal taste and will allow for more general specifications in the empirical section 5.

An NGO, indexed by j, decides on fundraising intensity $\phi_j \in [0,1]$ and uses the rest of its proceeds to fund a public good $G_j \in \mathbb{R}_+$, for $j = 1, \ldots, N$. It solves the program:

$$\max_{(\phi_j, G_j)} \Pi_j(\phi_j; \boldsymbol{\phi_{-j}}) + \alpha_j \mathcal{W}^f \left(G_j; G_{-j}\right), \text{ subject to: } G_j = \Pi_j(\phi_j; \boldsymbol{\phi_{-j}}), \tag{13}$$

where the term $\Pi_j(\phi_j; \boldsymbol{\phi_{-j}})$ represents the total funds gathered by NGO j when advertising with intensity ϕ_j while the remaining ones choose intensities $\boldsymbol{\phi_{-j}} = (\phi_1 \ldots, \phi_{j-1}, \phi_{j+1}, \ldots \phi_N)$. As in Dewatripont et al. (2022), NGOs place a non-negative weight $\alpha_j \geq 0$ on philanthropic output $\mathcal{W}^f(\cdot) : \mathbb{R}_+^N \to \mathbb{R}$, as a function of its own public good G_j and the vector of public goods produced by other NGOs, denoted G_{-j}. This term captures NGOs' plausible concerns over social objectives, namely, the provision of public goods. NGOs are limited by a *non-distribution constraint*, which states that net funds Π_j must equal total public good provision G_j, i.e., $\Pi_j = G_j$. I assume that fund collection is described by:

$$\Pi_j(\phi_j; \boldsymbol{\phi_{-j}}) = A(\phi_j; \boldsymbol{\phi}_{-j}, D\left(\tau, \tau^d\right)) - K_j(\phi_j). \tag{14}$$

The first term represents the gross funds raised when advertising with intensity ϕ_j when the remaining NGOs advertise at the vector of intensities $\boldsymbol{\phi_{-j}}$. In the theoretical section, gross funds are taken as separable to ease the exposition: $A(\phi_j; \boldsymbol{\phi}_j, D) = D\left(\tau, \tau^d\right) a(\phi_j; \boldsymbol{\phi}_j)$.

The function K_j represents the cost of reach, which is taken as strictly increasing and weakly convex, i.e., $K_j : [0,1] \to \mathbb{R},\ K_j(0) = 0,\ K_j' > 0,\ K_j'' \geq 0$. In order to simplify the mathematical exposition, K_j is assumed to be given by the quadratic specification:

$$K_j(\phi_j) = \frac{1}{2} c_j \phi_j^2, \tag{15}$$

which implies that the marginal cost is linear and given by $K'(\phi_j) = c_j\phi_j$, where $c_j > 0$ is a cost shifter.

Philanthropic output, whose? The function $\mathcal{W}^f(G_j; G_{-j})$ in equation (13) captures the utility derived by NGO j from the impact of its activities over philanthropic output. One can consider NGOs as having narrow concerns over philanthropic output, privileging its provision over provision by competing suppliers, or instead consider them to be concerned with the overall output of its sector. With this important distinction in mind, I study two possible definitions of philanthropic output and study their implications for the equilibrium vector of intensities and public good provision [6]. Consider the following specification:

$$\mathcal{W}^f(G_j; G_{-j}) = G_j + \omega \sum_{k \neq j}^{N} G_k, \tag{16}$$

where $\omega \in \{0, 1\}$ parametrizes the type of philanthropic output NGOs are concerned with into two possible cases as defined below. The philanthropic outcome parameter ω is common knowledge.

I distinguish between two cases as determined by ω. When $\omega = 0$, I consider NGOs to be concerned with *narrow philanthropic output*: each NGO cares about the impact of its public good over philanthropic output, disregarding the activities of other competing suppliers. When $\omega = 1$, I say NGOs are motivated by *ethical philanthropic output*. An NGO that values philanthropic output considers its provision interchangeable with those carried out by different suppliers: it partially internalizes the negative externalities that intense advertising may induce on other competing NGOs within the sector in which it operates.

3.2 Equilibrium characterization

Given that the non-distribution constraint of each NGO binds and given the assumed functional form over philanthropic output concerns $\mathcal{W}^f(\cdot)$ the objective function of NGO j as a function

[6] There are other competing assumptions, among which the most prominent is to consider NGOs that internalize full welfare including misallocation costs. While this consideration is of theoretical interest, such an assumption would not change the qualitative result from the model.

of its fundraising profiles, and that of competing organizations, writes:

$$V_j(\phi_j, \boldsymbol{\phi}_{-j}) = \Pi_j(\phi_j, \boldsymbol{\phi}_{-j})(1+\alpha_j) + \alpha_j \omega \sum_{k \neq j} \Pi_k(\phi_k, \boldsymbol{\phi}_{-k}). \tag{17}$$

Assuming differentiability, the first-order necessary condition for NGO j to maximize its objective V_j is:

$$\frac{\partial V_j(\phi_j, \boldsymbol{\phi}_{-j})}{\partial \phi_j} = \frac{\partial \Pi_j(\phi_j, \boldsymbol{\phi}_{-j})}{\partial \phi_j}(1+\alpha_j) + \alpha_j \omega \sum_{k \neq j} \frac{\partial \Pi_k(\phi_k, \boldsymbol{\phi}_{-k})}{\partial \phi_j}. \tag{18}$$

Conditional on the N second-order conditions being also satisfied, an interior NGO fundraising equilibrium is a vector of advertising intensities that satisfies the above equation for all $j \in \{1, \dots N\}$.

3.3 The endogenous leakage coefficient

The equilibrium defined by NGO advertising decisions in eq. (18) and donors' discrete choices in eq. (12) allows us to express donations to an NGO j as a function of intensities:

$$D_j = D_j(\boldsymbol{\phi}) \tag{19}$$

Notice that this defines the *leakage coefficient* for NGO j as a function of equilibrium intensities, $1 - \rho_j^b(\boldsymbol{\phi}, \boldsymbol{\alpha})$, in a market in which donors have a taste for quality $b > 0$ and the vector of NGO quality is given by $\boldsymbol{\alpha}$, so I will write the NGO-level and market-level elasticities, respectively, as: $\eta_j^b(\boldsymbol{\phi}, \boldsymbol{\alpha})$, and $\bar{\eta}^b(\boldsymbol{\phi}, \boldsymbol{\alpha})$. Henceforth, I omit the reach vector as an argument whenever there is no ambiguity[7].

[7]Notice that equation (3) defines the leakage coefficient at the sector level instead.

3.4 Theoretical results

I now specialize the model to derive a handful of theoretical results that will guide the empirical analysis and the interpretation of the empirical results. More precisely, I reduce NGO characteristics to be NGOs horizontal position $p_j \in [0,1]$ and quality α_j, which means that $\boldsymbol{\chi}_j = (p_j, \alpha_j)$. I let each donor be characterized by two parameters: his position $h_i \in [0,1]$ and a common taste $b \geq 0$ for NGO quality, captured by the parameter α_j. Additionally, a donor derives utility from income z_i, his donation d_i, the aggregate public good provision G, which means that $\boldsymbol{\theta}_i = (h_i, b, z_i, d_i, G)$. A donor first decides on the fraction of his income that will be devoted to charitable giving by taking into account the deductibility rate $\tau^d \in \mathbb{R}$ and a linear income tax $\tau \in \mathbb{R}$ [8], according to a generic function:

$$d^*(1+\tau^d, z(1-\tau)),$$

with $\partial d^*/\partial \tau^d < 0$ and $\partial d^*/\partial z(1-\tau) > 0$, meaning that $1+\tau_d$ is the effective price of giving, and donations are a normal good. After pledging his donation, each donor receives and observes the ads of those charities that reach him. Each charity j discloses two elements in its advertisement: its location p_j and its concern for philanthropic output α_j. The selected NGO j^* is such that:

$$(\alpha_{j^*}, p_{j^*}) = \underset{(\alpha_j, p_j) \in \mathcal{I}_i}{\arg\max}\, b\alpha_j - \Delta(h_i, p_j), \tag{20}$$

where $\Delta(h_i, p_j)$ is the smallest arc distance between NGO j, located at p_j and donor located at h_i, and the term b captures the donor's tastes over NGO quality. NGOs are distributed along a unitary Salop Circle with generic position $p_j = j/N$, as shown in Figure 1.

The taste parameter h_i is distributed independently of income z_i, which means that gross aggregate donations $D(\tau, \tau^d)$ are given by:

$$D(\tau, \tau^d) = \int_{\underline{z}}^{\overline{z}} d^*(1+\tau^d; z(1-\tau)) dF(z). \tag{21}$$

[8] In general, τ may be a non-linear function $\tau(z) : \mathbb{R} \to \mathbb{R}$. In the baseline model, I limit τ to be a linear function, following the optimal taxation literature, e.g., Diamond (2006), and Saez (2004).

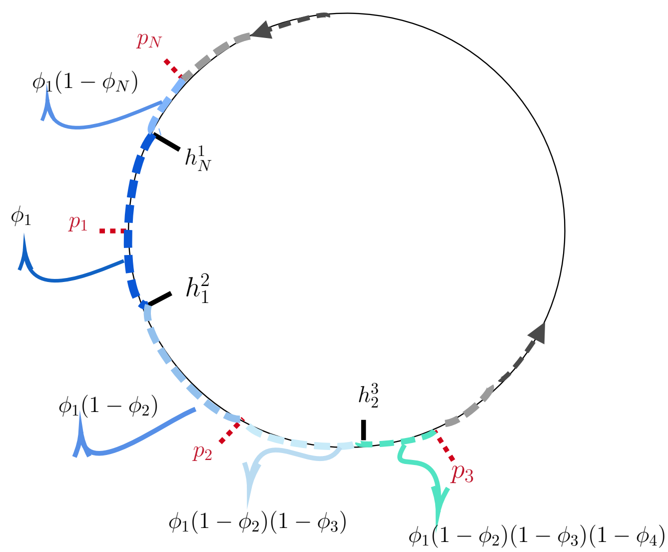

Figure 1: Salop circle depicting the NGO market. NGO j is located at p_j, and the arc distance between two neighboring NGOs is $1/N$. The value h_l^j denotes the indifferent donor between NGO j and l.

Equilibrium results are benchmarked to the public-good maximizing profile of intensities $\boldsymbol{\phi}^*$ defined as:

$$\boldsymbol{\phi}^* = (\phi_1^*, \dots, \phi_N^*) \in \arg\max_{\boldsymbol{\phi}} G(\boldsymbol{\phi}), \tag{22}$$

where:

$$G(\boldsymbol{\phi}) \triangleq \sum_{k=1}^{N} G_k(\boldsymbol{\phi}) \tag{23}$$

is the aggregate public good supplied by the N charities of the sector. This measure allows us to capture whether decentralized public good provision leads to inefficient provision due to competitive forces.

The 3-NGO Benchmark with symmetric costs. In order to ease the exposition, I consider the case in which there are three NGOs in the market [9], and I fix $c_j = c > 0$ for all j.

The market that NGO j faces in this environment is such that A in equation (14) writes:

$$\begin{aligned} A(\phi_j;\boldsymbol{\phi}_j) =&\phi_j\left[(1-\phi_{j+1})(1-\phi_{j-1})X_j^j+(1-\phi_{j-1})\phi_{j+1}X_j^{j+1}\right] \\ &+\phi_j\left[(1-\phi_{j+1})\phi_{j-1}X_j^{j-1}+\phi_{j-1}\phi_{j+1}X_j^{j-1,j+1}\right]D(\tau^d,\tau), \end{aligned} \tag{24}$$

where X_j^{j+1} (resp. X_j^{j-1}) describes the mass of consumers that donates to NGO j when after having also received an ad from NGO $j+1$, which happens with probability $\phi_j(1-\phi_{j-1})\phi_{j+1}$ (resp. $j-1$, with probability $\phi_j(1-\phi_{j+1})\phi_{j-1}$), and $X_j^{j-1,j+1}$ is the mass of donors that donates to j after received ads of both $j+1$ and $j-1$, which occurs with probability $\phi_j\phi_{j-1}\phi_{j+1}$. X_j^j denotes the mass of consumers that give to j when only NGO j is in their information set. Since donors impose no minimum quality, it follows that $X_j^j = 1$. Importantly, as shown in Figure 1, indices are $\mod 3$ (i.e, if $j=1$, then $j-1=3$ and $j+1=2$).

Using the equation (24), we obtain a characterization of necessary and given the concavity of the objective function implied by (14) sufficient first-order conditions. After some manipulation (details included in appendix) we obtain the following system of equations:

$$\begin{aligned} \frac{K'(\phi_1)}{D\left(\tau,\tau^d\right)} &= (1-\phi_2)(1-\phi_3)+(1-\phi_3)\phi_2\left[\frac{X_1^2}{1+\omega\alpha_1}\right]+(1-\phi_2)\phi_3\left[\frac{X_1^3}{1+\omega\alpha_1}\right]+\phi_3\phi_2\left[\frac{X_1^{23}}{1+\omega\alpha_1}\right] \\ \frac{K'(\phi_2)}{D\left(\tau,\tau^d\right)} &= (1-\phi_1)(1-\phi_3)+(1-\phi_3)\phi_1\left[\frac{1-X_1^2}{1+\omega\alpha_2}\right]+(1-\phi_1)\phi_3\left[\frac{X_2^3}{1+\omega\alpha_2}\right]+\phi_3\phi_1\left[\frac{X_2^{13}}{1+\omega\alpha_2}\right] \\ \frac{K'(\phi_3)}{D\left(\tau,\tau^d\right)} &= (1-\phi_2)(1-\phi_1)+(1-\phi_1)\phi_2\left[\frac{1-X_2^3}{1+\omega\alpha_3}\right]+(1-\phi_2)\phi_1\left[\frac{1-X_1^3}{1+\omega\alpha_3}\right]+\phi_1\phi_2\left[\frac{1-X_2^{13}-X_1^{23}}{1+\omega\alpha_3}\right]. \end{aligned}$$

This system characterizes equilibrium strategy profiles as functions of the type of welfare concerns faced by NGOs (either $\omega = 1$ or $\omega = 0$), the strength of welfare concerns by the α-terms, and the generic market shares X. Significantly, these last ones will depend on donors' taste for quality b, and the quality parameters α_1, α_2, α_3. I first provide a result for when $b = 0$ and then examine the case in which $b > 0$.

Proposition 3 *[Benchmark with insensitive donors] Let donors be insensitive to NGO quality, i.e.,*

[9] Derivations for larger markets with $N > 3$ result substantially more complicated, since the $N \times N$ system equivalent to the $N = 3$ case becomes increasingly hard to solve analytically for larger polynomials. Certain benchmark results still hold, namely Proposition 1. Appendix 8.3.4 includes these results.

$b = 0$. *Then:*

1. *For any* $\omega \in 0, 1$ *and any* $\boldsymbol{\alpha} \in [0, 1]^3$*, the leakage coefficient is increasing in total donations* D *:* $\bar{\eta}^0(\boldsymbol{\alpha}) > 0$*, and public good provision is lower than in the benchmark* $G(\boldsymbol{\phi}) < G(\boldsymbol{\phi}^*)$*;*

2. *If* $\omega\alpha_j = 0$ *for all* j*, there exists a unique equilibrium. This equilibrium is symmetric, i.e., all NGOs exert the same fundraising effort* $\phi_j = \phi^{sym} \in (0, 1)$*. In equilibrium, the following properties hold:*

   (a) *Reach is excessive with respect to the public-good maximizing level of reach* ϕ^**:* $\phi^{sym} > \phi^*$*. Total public good provision is lower than in the public-good maximizing Benchmark:* $G(\phi^{sym}) < G(\phi^*)$*.*

   (b) $\eta^0(\boldsymbol{\alpha}) > \eta^0(0, 0, 0)$ *for all* $\boldsymbol{\alpha} \in \mathbb{R}/\{0, 0, 0\}$

3. *If* $\omega\alpha_j = \alpha > 0$ *for all* $j \in \{1, 2, 3\}$*, there exists a unique symmetric equilibrium, ie. where all NGOs exert the same level of fundraising effort* $\phi_j = \phi^{sym}(\alpha) \in (0, 1)$*, such that* $\phi^{sym}(\alpha) > \phi^{sym}(0)$ *for any* $\alpha > 0$*.*

4. *If* $\omega = 1$ *and instead the philanthropic output weights are heterogeneous and such that* $\alpha_1 < \alpha_2 < \alpha_3$*, then the market shares obtained by each NGO are such that* $s_1 > s_2 > s_3$*, and where individual leakage is increasing in* α_j*:* $\eta_1^0(\boldsymbol{\alpha}) < \eta_2^0(\boldsymbol{\alpha}) < \eta_3^0(\boldsymbol{\alpha})$*.*

See Appendix.

Proposition 3 establishes that in a reach equilibrium, the public good will be under-provided with respect to the public-good maximizing benchmark defined in (22). The leakage coefficient is positive, and the elasticity of leakage to donations is positive. The calibration Section 6 combines these results with Propositions 1 and 2 to examine the implications of such effects over optimal tax formulas.

When $\alpha_j = 0$ for all j, each NGO is only concerned with the funds it captures from donors. It does not internalize the negative externalities that its advertising imposes on the other NGOs competing against it. This situation leads to excessive advertising in the market, exacerbated by increased available funds in the charitable market.

When NGOs have symmetric concerns over broad philanthropic output, such symmetric equilibria still exist, and in it, advertising efforts are still excessive with respect to public good maximization. However, advertising efforts are reduced as α increases.

Finally, when the weights on overall welfare differ, and NGOs care about social welfare, we have that low-α NGOs will command larger market shares in equilibrium. These results predict the sign of the correlation between the measured taste and the market shares observed from NGOs in our data. I now let $b > 0$ to consider the more involved case in which the NGO heterogeneity matters for donors. First, one can consider the benchmark case where NGOs have narrow philanthropic output.

Proposition 4 *[Narrow philanthropic output]*

Let $b > 0$, $\omega = 0$, and $\alpha_1 < \alpha_2 < \alpha_3$. Then, in equilibrium $\phi_1 < \phi_2 < \phi_3$ and $s_1 < s_2 < s_3$, and individual leakage is decreasing in α_j: $\eta_1^b > \eta_2^b > \eta_3^b$.

With narrow philanthropic output, NGOs with higher philanthropic output concerns become more aggressive due to their narrow mandate, leading them to adopt more potent fundraising strategies and command higher market shares. Moreover, there is a second effect: even for equal advertising efforts, the NGO with the highest α gets the highest market share when b>0.

Proposition 5 *[Broad Philanthropic Output] Let $b > 0$, $\alpha_1 \leq \alpha_2 \leq \alpha_3$ and $\omega = 1$. Then, the following properties hold in equilibrium:*

1. *if $b < \frac{1}{2(\alpha_2 - \alpha_1)}$ the equilibrium system of market shares is such that $s_1 > s_2 > s_3$ and $\eta_1^b(\boldsymbol{\alpha}) < \eta_2^b(\boldsymbol{\alpha}) < \eta_3^b(\boldsymbol{\alpha})$;*
2. *if $b > \frac{1}{2(\alpha_2 - \alpha_1)}$ the equilibrium system of market shares is such that $s_1 < s_2 < s_3$, moreover $\partial |s_k - s_j| / \partial D(\tau, \tau^d) > 0$ and $\eta_1^b(\boldsymbol{\alpha}) > \eta_2^b(\boldsymbol{\alpha}) > \eta_3^b(\boldsymbol{\alpha})$.*

See Appendix.

When NGOs are concerned with social welfare, they must balance two forces. First, intensive advertising imposes an externality over their competitors, valued with intensity α_j by NGOs. Second, advertising more intensely "reallocates" resources away from inferior-quality

NGOs. Notably, the second effect is proportional to the donors' valuation for quality provision, b. Consequently, when donors' preferences for NGO quality are sufficiently strong, the theoretical model predicts that the NGOs with high perceived quality will command higher market shares in equilibrium. The converse is true when the donors' taste for quality is low.

This result tells us that we should expect more ethical firms to command higher market shares in equilibrium as long as the value for quality exceeds the threshold value of b, which is inversely proportional to the difference between the best and the worst NGO. This result is crucial as it is empirically testable with the rankings data from Charity Navigator. Moreover, if donors' taste for quality is significant, increases in gross donations should result in good NGOs commanding larger market shares and advertising more intensely.

4. Data

I work with a panel of tax filings from IRS Form 990 that contains observations at the NGO level, with 106 variables for each charity, including fundraising expenditures, tax-exemption status, year of creation, total revenues and total assets, and geolocalization. I define geographical markets by using Nielsen's DMA regions (Figure 2).

Data from the first-dollar tax cost faced by donors to asses the Tax Reform Act of 1986 comes from TAXSIM and completing tax liabilities from estimates reported by Duquette (2016) obtained using the IRS Public Use File, which uses a nationally representative sample of tax returns at the individual level to estimate the marginal tax subsidy for the first dollar given for each state in the U.S.

The IRS Form 990 provides donation data at the Nonprofit level, a financial disclosure form that most tax-exempt nonprofits must file annually. Additionally, for the reduced-form comparison with the previous literature, some specifications use the IRS Statistics of Income Division (SOI) data sample all organizations with over 10 million USD in assets for 1982, 1983, and 1985 to the present. The SOI data also try to follow the same organizations each year.

Data about organizations' quality scores are obtained from the Charity Navigator website. Charity Navigator is, by large, the most used source of ratings for Nonprofits. The Charity Nav-

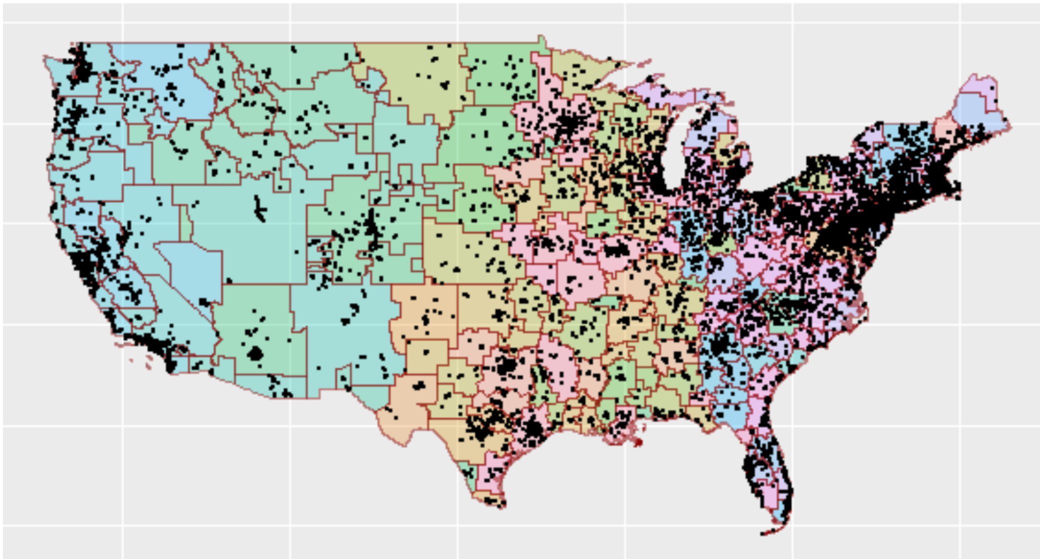

Figure 2: DMAs and NGO geolocalization for 2014.

igator website ranks organizations in several dimensions: finances, transparency, governance, and others. Charity Navigator rates a subset of registered 501(c)(3) public charities in the U.S. based on guidelines such as allocating at least 1 percent of expenses to fundraising and administrative expenses for three consecutive years. The rating system has evolved, and highly rated charities are awarded a star rating based on an underlying score. The ratings are published roughly once a year, with a one-year lag between the release of Form 990 data and the publication of the rating. The ratings and underlying metrics are made available through the Charity Navigator API.

As an instrument of our advertising technology, I rely on the dataset provided by Spenkuch and Toniatti (2018), which measures the intensity of political advertising during presidential campaigns at the level Nielsen DMA level.

I also run robustness checks using data from Kantar Media, which tracks advertising expenditures by specific media providers for the period of interest, and Charity Navigator. Charity Navigator allows us to identify quality and advertising targeting measures.

5. Empirical Specification and Estimation

I bring the general model described in section 3 to the data in this section. To do this, I proceed in three broad steps. First, I estimate the structural preference parameters that guide donation decisions. I follow a nested logit specification (Berry, 1994) of the discrete choice presented in Equation (12). Second, I use these estimates and the NGO model to obtain estimates of the marginal cost of unconditional reach of each NGO at equilibrium. Third, I use my estimates to perform counterfactual analyses of interest.

5.1 Donation supply and market shares

I consider a setting with N_l NGOs in each market $l \in \{0, \ldots, L\}$, where each market corresponds to one of Nielsen's geographical DMAs. Henceforth, I stick as closely as possible to the notation of Berry (1994). NGOs are nested into 5 exhaustive and mutually exclusive NTMAJ5 sets, $m = 0, 1, \ldots 5$ [10]. Denote the set of NGOs in group m as $\mathcal{J}_m$, and the outside good, $j = 0$, be the only member of group 0. For NGO $j \in \mathcal{J}_m$, let the random coefficient specification of utility (20) for a donor i that donates to NGO j be:

$$u_{ij} = \delta_j + \varsigma_{im} + \beta_T f(z_i, T_l) + (1 - \sigma)\epsilon_{ij},$$

where ϵ_{ij} is iid extreme value and the mean utility term δ_j is given by:

$$\delta_j = x_j' \beta + \beta_\phi \phi_j + \beta_q q_j + \xi_j, \tag{25}$$

where x_j' is a vector of observed NGO characteristics, and ϕ_j is the advertising intensity of NGO j, q_j is the measure of NGO, as given by the CN Score (i.e., the measure of α_j in the theoretical model), z denotes individual income, T_l is the mean tax liability faced by donors in market l, and where the function $f(z, T_l)$, as in Bjornerstedt and Verboven (2016), is taken to be the fixed

[10] There are five major subsectors as categorized by the National Center for Charitable Statistics, each represented by a two-letter code. These codes are AR for Arts, culture, and humanities; ED for Education; HE for Health; HU for Human Services; and OT for Other.

expenditure demand specification [11]. The idiosyncratic group preference, ς_{im}, follows the unique distribution such that $\varsigma_{im} + (1-\sigma)\epsilon_{ij}$ is also an extreme value random variable. The parameter σ, with $0 \leq \sigma < 1$, characterizes the correlation of utilities that a donor experiences among the NGOs in the same group. As standard, I normalize the mean utility of the outside good to zero $\delta_0 = 0$.

I allow mean utility in Equation (25) to depend on fundraising intensity ϕ_j. I interpret this specification as allowing advertising to increase the supply of donations under a persuasive motive, an approach often adopted by marketing studies (Shapiro, 2018). In terms of the theoretical model from Section 3.4, this is equivalent to allowing for indifferent donors at a given information set to be influenced by the equilibrium profile of intensities of NGOs within those NGOs that have reached them. Letting advertising influence mean utilities allows the supply of donations and the characterization of NGO equilibrium behavior to describe a setting where advertising informs and persuades donors[12].

Aggregate and Inverted Aggregated Donations. Aggregate donations for NGO j are given by the probability that a consumer buys that product, multiplied by the donation amount, $d_j(z_i)$, aggregated over all donors and according to income distribution F_z :

$$\begin{aligned} \mathcal{D}_j &= \int s_j(\boldsymbol{\delta}, \sigma) d_j(z) dF_z(z) \\ &= s_j(\boldsymbol{\delta}, \sigma) \int d_j(z) dF_z(z). \end{aligned} \tag{26}$$

The last equality follows from the fact that the choice probability $s_j(\delta, \sigma)$ is independent of income. We can solve the remaining integral using (26). The constant expenditure specification of donations is such that, for a $\gamma \in [0, 1]$, $f(z_i, T_l) = \gamma^{-1} \ln z_i - \ln T_l$, so donations are given by:

[11] More generally, the mean tax liability T_l is a function of aggregate income and the interaction between the federal and state-level tax policies, this may lead us to consider more general specifications in which the tax liability varies at the individual level, $T_{i,r}$. However, due to the lack of variation in tax liabilities within markets for the period after 2010, the present model cannot distinguish between this specification and the one in (25)

[12] Notably, a distinction between the role of advertising as informative as opposed to persuasive allows us to decide on whether or not to include it in welfare estimation. When assessing welfare changes of the counterfactual tax change, I further consider implications of this distinction.

$d_j(z_i) = \gamma \frac{z_i}{T_l}$. . Using this last equation in the expression for the choice probabilities, we obtain:

$$\frac{T_l \mathcal{D}_j}{\gamma Z} = s_j(\boldsymbol{\delta}, \sigma)$$

where Z is the total income of all donors. We can now recur to the standard approach and invert choice probabilities to solve for mean utilities. Following the constant expenditure specification of Bjornerstedt and Verboven (2016), in the estimation of (28), we let the random utility component be given by the logarithmic specification:

$$\delta_j = x_j'\beta + \beta_\phi \phi_j + \beta_T \gamma^{-1}(\log Z_i - \log T_l) + \beta_q q_j + \xi_j. \tag{27}$$

The estimation equation is then given by:

$$\log s_j - \log s_0 = x_j\beta + \beta_\phi \phi_j + \sigma \log \bar{s}_{j/m} + \beta_T T_l + \beta_q q_j + \xi_j, \tag{28}$$

where s_j is the market share of NGO j, s_0 that of the outside option, $s_{j/m}$ is the share of NGO j within it's nest, and δ_j is defined as in (25). Additionally, market shares are introduced in value terms instead of linearly. At last, the potential market is assumed to be a fixed Fraction of GDP, γZ. As standard, γ is not estimated but imposed according to a range of reasonable values.

I estimate (28) by using an instrumental variable regression of market shares on NGO characteristics, tax liabilities, fundraising intensities, and nest market shares. Here, fundraising intensities and nest shares are endogenous variables. I use the standard approach of recurring to the number of other NGOs present in the market of NGO j as an instrument for the inside-nest parameter. The fundraising intensities are instrumented using data on political advertising as gathered by Spenkuch and Toniatti (2018), which is a shifter of the advertising effectiveness at the DMA-level.

	Dependent variable:	
	$\log(s_j) - \log(s_0)$	
	(1)	(2)
Reach (β_ϕ)	0.921***	0.964***
	(0.004)	(0.004)
Tax Liability (β_T)	0.0003	0.0002
	(0.003)	(0.003)
Nesting parameter (σ)	0.219***	0.218***
	(0.003)	(0.003)
CN Score (β_q)	0.009***	0.009***
	(0.0005)	(0.0005)
Fixed effects	Yes	No
Observations	101,750	101,750
R^2	0.569	0.569
Adjusted R^2	0.569	0.569
Residual Std. Error	1.187 (df = 101711)	1.187 (df = 101715)
F Statistic	3,532.527*** (df = 38; 101711)	3,945.579*** (df = 34; 101715)

Note: *p<0.1; **p<0.05; ***p<0.01

Estimation results are presented in Table 5.1, which displays a substantial positive effect of advertising on log market shares for specifications with and without state-year and market fixed effects. I interpret this as evidence that advertising decisions are essential determinants of donation supply. Moreover, results suggest low substitutability of donations between nests and a positive and significant quality effect (as measured by the Charity Navigator Score) on predicted market shares. This last result and the theoretical model of Section 3 provide evidence of the predicted relationship between competition and market shares in a setting with organizations motivated by more than self-interest.

5.2 Counterfactual

The Tax Reform Act of 1986 (TRA86) was a significant legislation that sweepingly changed the US federal income tax system. Among other things, it lowered tax rates and broadened the tax base by eliminating a number of tax loopholes and preferences. The impact of these changes was felt at the federal level and in state income tax systems across the country. Indeed, the effects of TRA86 on state tax systems were varied and complex, depending on each state's specific tax structure and policies. As an considerably large source of variation in the tax code, TRA86 has been used widely by the empirical public finance literature, most notably for this study is Duquette (2016).

I take advantage of the estimation results to implement a counterfactual of interest. Since TRA86 constitutes the most considerable policy change in immediate history, I simulate its effects on the economy for 2014. Figure 1 summarizes the results. The estimates obtained using this approach are documented by Duquette (2016) and put forward an elasticty of roughly 4 percent to the price of giving, which contrasts the one obtained by the literature that relies on surveys (Peloza and Steel, 2005), which reports an estimate of around one percent. For completeness, I compute the counterfactual tax change for both possible elasticities.

To compute the equilibrium effects of a change in the tax liability, I proceed in two broad steps: first, I estimate the equilibrium value of the vector of marginal cost to reach c_j. Second, I solve for the new equilibrium using the system of first-order conditions characterized by the non-linear system in (74). Implementation details are included in Appendix 8.5.

Notice that leakage in the full same is estimated at just below 40 percent pre-reform, with a substantial variation along the quality dimension as measured by the Charity Navigator Star system, which is higher for low-quality NGOs, as expected. Moreover, leakage elasticity is positive and also varies widely across quality. Moreover, leakage elasticity is substantially higher when a larger elasticity to the deductibility rate is imposed, as documented by columns 2 and 3 of Table 1.

The hypothetical tax reform increases donor surplus. The effect is indeed more than proportional when the large income elasticity of 4 percent is imposed. This effect could induce us to consider changes that lower the cost of giving as beneficial, but a caveat applies. These effects are mainly driven by estimated responses to advertising that enter the utility function. If advertising is potentially wasteful, such considerable positive surplus change need not apply.

	Case 1 ($\beta_T \propto 1$)		Case 2 ($\beta_T \propto 4.1$)	
	Estimate	SE	Estimate	SE
Leakage (l)				
Full sample	0.39	0.23	-	-
One star	0.558	0.251	-	-
Two stars	0.406	0.231	-	-
Three stars	0.390	0.226	-	-
Four stars	0.385	0.220	-	-
Leakage elas. (η^l_D)				
Full sample	0.008	0.0004	0.015	0.066
One star	0.001	0.002	0.100	0.222
Two stars	0.004	0.004	0.00155	0.00244
Three stars	0.012	0.005	0.0148	0.0520
Four stars	0.007	0.014	0.00620	0.0147
Consumer Surplus (ΔCS)				
Full sample	186.6	0.16925	691.77	6.7
One star	103.5	0.1235	641.3	1.39
Two stars	150.35	0.3265	691.3	1.42
Three stars	124.2756	0.18351	697.89	1.38
Four stars	191.45	0.1565	183.74	1.28

Table 1: Counterfactual results. Simulating an equivalent tax liability change in 1986.

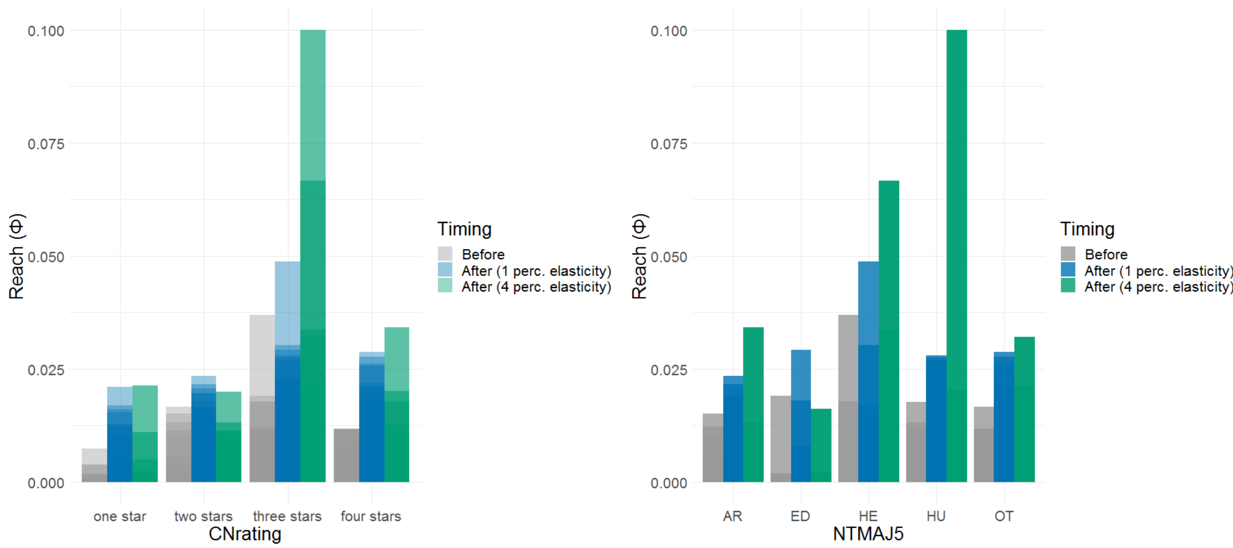

Figure 3: Counterfactual.

Robustness shows that the significant differences documented in Table 1 are broadly mitigated by excluding advertising from mean-utilities.

The counterfactual results evidence a large degree of response heterogeneity regarding fundraising and leakage elasticities at the CN-rating level and the NTMAJ5 one (see Figure 3). Fundraising elasticities display an inverse-U relationship with respect to NGO quality as measured by Charity Navigator Stars. This result may be due to ratings being perceived in a binary fashion by donors as either positive or negative or to bunching in some categories (Mayo, 2021). In light of the model from Section 3.4, we can think about this result as capturing NGOs that are motivated by narrow philanthropic output competing with somewhat a broader mandate.

6. Calibrations and welfare analysis

This section leverages the previous estimation to perform welfare assessments that consider the endogenous competition between NGOs when obtaining normative estimates for the welfare-maximizing deductibility rate from our model section, accounting for the fact that competition between NGOs induces endogenous leakage into advertising due to competitive effects. We bring the estimated leakage elasticity coefficients to the welfare analysis of the two propositions of the welfare analysis of the model section in 2. In the first case, the government solves policy parameters to maximize welfare, considering endogenous leakage. The second case constrains the government to deduct contributions at the income tax rate.

Welfare characterizations require making two technical assumptions (further detailed in the appendix). First, a separation between discrete choices and marginal utilities from public good provision must be assumed to disentangle donations decisions from overall public good provision satisfaction. Second, preferences for public good provision are assumed to replicate preferences from discrete choices. This is the equivalent of requiring warm-glow giving to reflect overall public good provision preferences in the aggregate. I also require welfare to not consider advertising persuasiveness, meaning that I evaluate mean utilities at $\phi_j = 0$.

General deductibility		Full deductibility		Parameters			
τ^d	τ^d_{Saez}	τ	τ_{Saez}	$\bar{\eta}$	$1-\rho$	ϵ_Z	ϵ_G
-0.17	-0.40	1.01	0.60	0.008	0.39	0.25	1.00
-0.30	-0.52	1.01	0.59	0.008	0.39	0.25	1.50
0.10	-0.05	1.02	0.60	0.008	0.39	0.25	0.50
0.01	-0.31	1.03	0.48	0.008	0.39	0.50	1.00
-0.09	-0.45	1.02	0.47	0.008	0.39	0.50	1.50
0.34	0.14	1.04	0.48	0.008	0.39	0.50	0.50
-0.08	-0.40	0.97	0.60	0.015	0.39	0.25	1.00
-0.21	-0.52	0.97	0.59	0.015	0.39	0.25	1.50
0.1	-0.05	0.95	0.60	0.015	0.39	0.25	0.50
-0.04	-0.31	0.95	0.48	0.015	0.39	0.50	1.00
-0.14	-0.45	0.95	0.47	0.015	0.39	0.50	1.50
0.31	0.14	0.91	0.48	0.015	0.39	0.50	0.50

Table 2: Solution to Propositions 1 and 2 for given parameters.

The full description of the derivations and functional forms used in the simulations is given in Appendix 9. The table above summarizes the solution for the deductibility rate and income tax as described in Propositions 1 and 2. For it, I fix the leakage parameter to be $\rho = 0.39$, and vary the aggregate elasticity $\bar{\eta}$ to match the two different values obtained in the counterfactual analysis. Notice that when the leakage elasticity is positive, meaning that leakage increases with donations, the estimates for τ^d in Proposition 2 imply a higher deductibility rate than the one found by Saez in a model with no competitive effects; the variation is a substantial range of parameter values. On the other hand, when the leakage elasticity is positive, the deductibility rate is higher in absolute terms than that proposed by the baseline model with no competitive effects. Competitive forces push the Saez estimates downward in absolute terms.

7. Results and discussion

Many governments worldwide offer tax benefits to encourage charitable donations. However, the current methods for determining the ideal level of these benefits overlook a significant factor. Higher tax benefits do increase charitable giving, but they also contribute to wasteful competition among charities for funding.

This paper presents a model in which NGOs compete for donations endogenously to tax policy. It uses data from the U.S. to estimate the model's structural parameters and then exploits these estimates to perform positive and normative analyses. It provides evidence for a low substitute between categories of charitable giving and a high sensibility of giving to fundraising expenditures. A counterfactual study shows evidence of considerable sensitivity of fundraising to changes in deductibility rates. Ongoing welfare analyses suggest that such estimates indicate previous normative estimates found in the literature to be overestimating the social impact of charitable giving and, therefore implying deductibility rates that are too high with respect to a baseline scenario with no competition. Finally, the counterfactual exercise presented allows to compute a measure of donor surplus of giving.

The research shows that leakage, the proportion of charities' budget not spent on direct public good provision, reached up to 40 percent in the 2014 sample. In addition, the findings suggest that fundraising plays a significant role in the endogenous leakage of gross donations into advertising. Therefore, any estimates that do not account for the effects of competition on charities must be adjusted downwards to accurately reflect the impact of the tax code.

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8. Appendix

8.1 Indirect utility and discrete choice

Consider an economy with $J+1$ goods and statistically identical and independent donors, all endowed with income z. Good 0 is a perfectly divisible outside good. The other J goods are the indivisible variants of a differentiated product. These N NGOs can be classified into G exhaustive and mutually exclusive groups with J_g variants in the gth group, such that $\sum_{g=1}^{G} N_g = N$. The log-price of giving is $T_{kh} \leqslant z$. A donor's conditional indirect utility of good k in group h is

$$u_{kh} = \gamma^{-1} \log z - T_{kh} + b_{kh} + \epsilon_{kh},$$

where b_{kh} is the quality of good k in group h and ϵ_{kh} is the random part of utility. Note that price and income enter linearly in (1). For the nested logit model, the $\epsilon_{11}, \ldots, \epsilon_{J_G G}$, follow the multivariate cumulative distribution function:

$$F(\epsilon_{11}, \ldots, \epsilon_{J_G G}) = \exp\left[-\sum_{g=1}^{G} \left(\sum_{j=1}^{N_g} \mathrm{e}^{-\epsilon_{g'}\mu_g} \right)^{\mu_g'^{\mu}} \right],$$

where $0 \leqslant \mu_g \leqslant \mu$.[4] A consumer chooses the good with the highest utility. The probability $\boldsymbol{P}_{kh}$ that a consumer buys good k from group h then equals the probability that $\tilde{U}_{jg}$ is maximized at good k from group h. Demand for NGO k from group h is $\boldsymbol{P}_{kh}$.

For the nested logit distribution function (2), it is well known that $\boldsymbol{P}_{kh}$ equals

$$\boldsymbol{P}_{kh} = \frac{\exp((b_{kh} - T_{kh})/\mu_h)}{\sum_{j=1}^{J_h} \exp((b_{jh} - T_{jh})/\mu_h)} \cdot \frac{\exp(I_h/\mu)}{\sum_{g=1}^{G} \exp(I_g/\mu)},$$

where

$$I_g = \mu_g \ln \sum_{j=1}^{J_g} \exp((b_{jg} - T_{jg})/\mu_g)$$

is called the inclusive value of group g. It can be shown that I_g is the expected value of the maximum of the utilities of the goods within a group g, and that $\mu \ln \sum_{g=1}^{G} \exp(I_g/\mu)$ is the expected value of the maximum of the utilities of all goods. Some calculations transform (3) into

$$\ln\left[\left(\frac{\boldsymbol{P}_{kh}}{\sum_{j=1}^{N_h}\boldsymbol{P}_{jh}}\right)^{\mu_h}\cdot\left(\sum_{j=1}^{N_h}\boldsymbol{P}_{jh}\right)^{\mu}\right]=-\mu\cdot\ln\left(\sum_{g=1}^{G}\exp(I_g/\mu)\right)+b_{kh}-T_{kh},$$

where $\boldsymbol{P}_{kh}/\sum_{j=1}^{J_h}\boldsymbol{P}_{jh}\equiv\boldsymbol{P}_{k|h}$ is the probability that a consumer donates to k, given that he/she buys from group h, and where $\sum_{j=1}^{J_h}\boldsymbol{P}_{jh}\equiv\boldsymbol{P}_h$ is the probability that a donor donates to group h^5.

Now consider an alternative economy with $J+1$ perfectly divisible goods and one representative consumer, endowed with income Z. Good 0 is the outside good sold at price $T_0=1$. The other J goods are the variants of a differentiated product. A good j in group g is sold at price p_{jg}. The representative consumer's budget constraint is

$$\sum_{g=1}^{G}\sum_{j=1}^{N_g}T_{jg}X_{jg}+X_0\leqslant Z$$

where X_{jg} is the donation amount to j from group g and X_0 is the quantity of the outside good. Then:

Proposition 1. A representative consumer's direct utility function consistent with the nested logit demand system (3) or its transformation (5) is

$$U=v_0(G)+\sum_{g=1}^{G}\left[\sum_{i=1}^{N_g}\left[b_{ig}\log\left(\frac{X_{ig}}{\sum_{j=1}^{J_g}X_{jg}}\right)^{\mu_g}\left(\frac{\sum_{j=1}^{J_g}X_{jg}}{}\right)^{\mu}\right]X_{ig}\right]+X_0$$

Proof. The Lagrangian for the consumer's maximization problem is written as

$$L=\sum_{g=1}^{G}\left[\sum_{i=1}^{J_g}\left[b_{ig}-\ln\left(\frac{X_{ig}}{\sum_{j=1}^{J_g}X_{jg}}\right)^{\mu_g}\left(\frac{\sum_{j=1}^{J_g}X_{jg}}{N}\right)^{\mu}\right]2_{ig}\right]+X_0$$
$$+\lambda\left(Z-X_0-\sum_{g=1}^{G}\sum_{i=1}^{J_X}T_{ig}X_{ig}\right),$$

where λ is the traditional budget constraint multiplier. The first-order condition for X_0 yields $\lambda = 1$. The first-order condition for an X_{kh} yields, after some rearrangements,

$$\ln\left(\left(\frac{X_{kh}}{\sum_{j=1}^{J_h} X_{jh}}\right)^{\mu_h} \cdot \left(\frac{\sum_{j=1}^{J_h} X_{jh}}{N}\right)^{\mu}\right) = (-\mu) + b_{kh} - T_{kh}.$$

Reinterpreting the market shares $X_{jg}/1$ of the representative consumer model as the probabilities $\boldsymbol{P}_{jg}$ of the discrete choice model, (8) and (5) become strikingly similar. They coincide if and only if

$$(-\mu) = -\mu \ln\left(\sum_{g=1}^{G} \exp(I_g/\mu)\right).$$

To show that this is indeed the case, rewrite (8) as

$$\frac{X_{kh}}{\sum_{j=1}^{J_h} X_{jh}} \cdot \left(\frac{\sum_{j=1}^{J_h} X_{jh}}{N}\right)^{\mu/\mu_h} = \exp\left(\frac{-\mu + b_{kh} - T_{kh}}{\mu_h}\right)$$

and add for $j = 1, \ldots, N_h$:

$$\left(\frac{\sum_{j=1}^{J_h} X_{jh}}{1}\right)^{\mu/\mu_h} = \exp\left(\frac{-\mu + I_h}{\mu_h}\right).$$

Rewrite this as

$$\frac{\sum_{j=1}^{J_h} X_{jh}}{1} = \exp\left(\frac{\mu + I_h}{\mu}\right)$$

and add for $g = 1, \ldots, G$. This gives (9). To verify that the solution to the first-order conditions does indeed maximize U, calculate the second-order condition for an X_{kh} :

$$\frac{\mu_h}{X_{kh}} + \frac{\mu - \mu_h}{\sum_{j=1}^{J_h} X_{jh}}.$$

This is negative if $\mu \geqslant \mu_h \geqslant 0$.

8.2 Proofs of taxation problem

The planner solves the problem of $\max_{\tau,\tau^d,R,G_0} W$ subject to equation (2) and (**??**). Denote by λ the multiplier of the government's budget constraint, then first-order conditions to this problem are given by:

$$-\int \mu^i \left[v^i_{1-\tau} + v^i_G \bar{G}_{1-\tau}\right] dv(i) + \lambda \left[\bar{Z} - \tau \bar{Z}_{1-\tau} - \tau^d \bar{G}_{1-\tau}\right] = 0, \tag{29}$$

$$\int \mu^i \left[v^i_{1+\tau^d} + v^i_G \bar{G}_{1+\tau^d}\right] dv(i) + \lambda \left[\bar{D} + \tau \bar{Z}_{1+\tau^d} + \tau^d \bar{G}_{1+\tau^d}\right] = 0, \tag{30}$$

$$\int \mu^i \left[v^i_R + v^i_G \bar{G}_R\right] dv(i) + \lambda \left[-1 + \tau \bar{Z}_R + \tau^d \bar{G}_R\right] = 0. \tag{31}$$

Given the leakage equation in (**??**), we have that the derivatives of the average public good with respect to taxes and the lump-sum, namely $\bar{G}_{1-\tau}$, $\bar{G}_{1-\tau^d}$, and $\bar{G}_R$, are given by the following three equations:

$$\bar{G}_{1-\tau} = \rho(D) D_{1-\tau}(1 + \eta^\rho_D) = D_{1-\tau}(1 - l(D)(1 + \eta^l_D)), \tag{32}$$

$$\bar{G}_{1+\tau^d} = \rho(D) D_{1+\tau^d}(1 + \eta^\rho_D) = D_{1+\tau^d}(1 - l(D)(1 + \eta^l_D)), \tag{33}$$

$$\bar{G}_R = \rho(D) D_R(1 + \eta^\rho_D) = D_R(1 - l(D)(1 + \eta^l_D)). \tag{34}$$

Moreover, if the government can contribute to the public good the first-order condition with respect to G_0 writes:

$$\int \mu^i \left[v^i_G + v^i_G \bar{G}_{G_0}\right] dv(i) + \lambda[-1 + \tau \bar{Z}_{G_0} + \tau^d \partial \bar{G}/\partial G_0] = 0.$$

Where, in an analogous fashion as above, we have that:

$$\bar{G}_{G_0} = \rho(D) D_{G_0}(1 + \eta^\rho_D) = D_{G_0}(1 - l(D)(1 + \eta^l_D))$$

We can therefore re-express the previous system of equations as:

$$\left[1 - \frac{\int \beta^i z^i di}{\bar{Z}}\right] \bar{Z} = \tau \bar{Z}_{1-\tau} + (\tau^d + e \cdot \rho(1+\eta_D^\rho))\bar{D}_{1-\tau}, \tag{35}$$

$$\left[1 - \frac{\int \beta^i d^i di}{\bar{D}}\right] \bar{D} = -\tau \bar{Z}_{1+\tau^d} - (\tau^d + e \cdot \rho(1+\eta_D^\rho))\bar{D}_{1+\tau^d}, \tag{36}$$

$$1 - \int \beta^i di = \tau \bar{Z}_R + (\tau^d + e \cdot \rho(1+\eta_D^\rho))\bar{D}_R. \tag{37}$$

and, finally, we have that if the government can contribute to the public good:

$$e = 1 - \tau \bar{Z}_{G_0} - (\tau^d + e \cdot \rho(1+\eta_D^\rho))\partial \bar{G}/\partial G_0 \tag{38}$$

Three assumptions are made in order to simplify the system determined by the four equations above (see Saez (2004) for further discussion).

Assumption T1. There are no income effects on earning, i.e: $z_R^i = 0$ for all i.

Assumption T2. Independence between aggregate earnings and contributions , i.e: $\bar{Z}_{G_0} = 0$ and $\bar{Z}_{1+\tau^d} = 0$.

Assumption T3. Compensated supply of contributions does not depend on earnings. $\partial d^i/\partial(1-\tau) = 0$. This implies that:

$$\bar{D}_{1-\tau} = \bar{Z}\hat{D}_R \tag{39}$$

where $\hat{D}_R$ corresponds to the average response to a uniform one dollar increase in the lumpsum R, weighted by earnings. We can use Assumptions 1-3 to simplify our system in the following way:

$$\tau^d = -e \cdot (1 - l(D)) + \frac{1}{r}\left[1 - \frac{\int \beta^i d^i di}{\overline{D}} - \eta(D)\right]$$
$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon_Z}\left[1 - \frac{\int \beta^i z^i di}{\overline{Z}} - (\tau^d + e \cdot (1 - l(D)))\hat{D}_R - \epsilon_Z^D\right]$$
$$\int \beta^i di = 1 - (\tau^d + e \cdot (1 - l(D)))\bar{D}_R - \eta(D) \cdot \bar{D}.$$

If the government can choose G_0 optimally:

$$e = 1 - (\tau^d + e(1 - l(D)))\partial\bar{G}/\partial G_0 - \eta(D) \cdot \bar{D} = \frac{1 - \tau^d \bar{D}_{G_0} - \eta(D) \cdot \bar{D}}{1 + (1 - l(D))\bar{D}_{G_0}} \tag{40}$$

$$\tau^d = -(1 - \eta(D))(1 - l(D)) + \frac{1}{r}(1 + (1 - l(D)) \cdot \partial\bar{G}/\partial G_0)\left[(1 - \frac{\int \beta^i d^i di}{\overline{G}} - \eta(D)\right]$$

When the government is constrained to set $\tau^d = -\tau$, the first-order condition with respect to income becomes:

$$\begin{aligned}\left[1 - \frac{\int \beta^i z^i di}{\overline{Z}}\right]\bar{Z} - \left[1 - \frac{\int \beta^i d^i di}{\overline{D}}\right]\bar{D} &= \tau\bar{Z}_{1-\tau} + (-\tau + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1-\tau} \\ &+ \tau\bar{Z}_{1+\tau^d} + (-\tau + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1+\tau}.\end{aligned}$$

The formula for the optimal income tax follows from this.

8.3 Proofs of section 3

8.3.1 Preliminaries

Consider 3 charities with positions p_1,p_2,p_3 and qualities α_1,α_2,α_3. The power set that describes all possible information sets is given by $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

Among the donors with information set $\{j, r\}$ the indifferent donor is defined by:

$$\Delta(h, p_j) + b\,\alpha_j = \Delta(h, p_r) + b\,\alpha_r$$

For $j, r = 1, 2, 3$ and $j < r$ and $p_j < p_r$ and information sets composed of at most two NGOs the indifferent donor is given by:

$$h_j^r = \frac{p_j + p_r}{2} + \frac{b}{2}(\alpha_j - \alpha_r)$$
$$h_r^j = h_j^r - b(\alpha_j - \alpha_r) + \frac{1}{2} = \frac{1}{2} + \frac{p_j + p_r}{2} + \frac{b}{2}(\alpha_r - \alpha_j)$$

as illustrated in figure 1, where h_j^r denotes the mid point in the arc between NGO j and NGO r starting at position p_j and moving anti-clockwise, (resp. h_r^j for the complementary case, starting at point p_r). The associated mass of donors who give to NGO j for each one of these arc segments is given by:

$$\int_{p_j}^{h_j^r} i\,di = \frac{p_r - p_j}{2} + \frac{b}{2}(\alpha_j - \alpha_r)$$
$$\int_{h_r^j}^{p_j+1} i\,di = \frac{p_j - p_r}{2} + \frac{b}{2}(\alpha_j - \alpha_r) + \frac{1}{2}$$

Among those donors with informations sets given by $\{j, r\}$ NGO j thus raises:

$$X_j^r = min\left\{1, max\left\{\frac{1}{2} + b(\alpha_j - \alpha_r), 0\right\}\right\},$$

while NGO r raises the amount $1 - X_j^r$. Consider now the information set described by $\mathcal{J} = \{1, 2, 3\}$. Here I assume that competition is stronger among the two immediate neighbors, the donor who is indifferent between NGOs j and r is located on the shortest arc segment between these two NGOs. Hence I study the shares given by:

$$\int_{p_j}^{h_j^{j+1}} i\,di = \frac{p_{j+1} - p_j}{2} + \frac{b}{2}(\alpha_j - \alpha_{j+1})$$

$$\int_{h^j_{j-1}}^{p_{j+1}} i\; di = \frac{p_j - p_{j-1}}{2} + \frac{b}{2}(\alpha_j - \alpha_{j-1}) + \frac{1}{2}$$

And hence:

$$X_j^{j+1\; j-1} = \frac{1}{2} + \frac{b}{2}(2\alpha_j - \alpha_{j+1} - a_{j-1}) + \frac{p_{j+1} - p_{j-1}}{2}$$

In sum:

$$\begin{aligned}
X_1^{23} &= \frac{1}{2} + \frac{b}{2}(2\alpha_1 - \alpha_2 - a_3) - \frac{p_3 - p_2}{2} \\
X_3^{12} &= \frac{1}{2} + \frac{b}{2}(2\alpha_3 - \alpha_2 - a_1) - \frac{p_2 - p_1}{2} \\
X_2^{13} &= 1 - X_1^{23} - X_3^{12} \\
X_1^1 &= X_2^2 = X_3^3 = 1
\end{aligned}$$

We can write the objective of NGO 1 as:

$$V_1 = \Pi_1 + \alpha_1(\Pi_1 + \omega(\Pi_2 + \Pi_3)) = \Pi_1(1 + \alpha_1) + \alpha_1\omega(\Pi_2 + \Pi_3),$$

FOCs write:

$$\frac{\partial\, V_1}{\partial\, \phi_1} = \frac{\partial\, \Pi_1}{\partial\, \phi_1}(1 + \alpha_1) + \alpha_1\omega\left(\frac{\partial\, \Pi_2}{\partial\, \phi_1} + \frac{\partial\, \Pi_3}{\partial\, \phi_1}\right) = 0$$

Where:

$$\Pi_1 = D\left(\tau, \tau^d\right)\; \phi_1\left[(1-\phi_2)(1-\phi_3)X_1^1 + (1-\phi_3)\phi_2 X_1^2 + (1-\phi_2)\phi_3 X_1^3 + \phi_3\phi_2 X_1^{23}\right]$$

$$\frac{\partial \Pi_1}{\partial\, \phi_1} = D\left(\tau, \tau^d\right)\; \left[(1-\phi_2)(1-\phi_3)X_1^1 + (1-\phi_3)\phi_2 X_1^2 + (1-\phi_2)\phi_3 X_1^3 + \phi_3\phi_2 X_1^{23}\right]$$

We can write revenues of NGOs 2 and 3 as:

$$\Pi_2 = D\left(\tau, \tau^d\right)\; \phi_2\left[(1-\phi_1)(1-\phi_3)X_2^2 + (1-\phi_3)\phi_1 X_2^1 + (1-\phi_1)\phi_3 X_2^3 + \phi_3\phi_1 X_2^{13}\right]$$

$$\Pi_3 = D\left(\tau, \tau^d\right)\; \phi_3\left[(1-\phi_2)(1-\phi_1)X_3^3 + (1-\phi_1)\phi_2 X_3^2 + (1-\phi_2)\phi_1 X_3^1 + \phi_1\phi_2 X_3^{21}\right]$$

And hence:

$$\frac{\partial \Pi_2}{\partial\, \phi_1} = D\left(\tau, \tau^d\right)\ \phi_2 \left[-(1-\phi_3)X_2^2 + (1-\phi_3)X_2^1 - \phi_3 X_2^3 + \phi_3 X_2^{13}\right]$$
$$= -D\left(\tau, \tau^d\right)\ \phi_2 \left[X_2^2 - X_2^1 + \phi_3\left[X_2^3 + X_2^1 - X_2^{13} - X_2^2\right]\right]$$

$$\frac{\partial \Pi_3}{\partial\, \phi_1} = D\left(\tau, \tau^d\right)\ \phi_3 \left[-(1-\phi_2)X_3^3 - \phi_2 X_3^2 + (1-\phi_2)X_3^1 + \phi_2 X_3^{21}\right]$$
$$= -D\left(\tau, \tau^d\right)\ \phi_3 \left[X_3^3 - X_3^1 + \phi_2\left[X_3^2 + X_3^1 - X_3^{21} - X_3^3\right]\right]$$

Repeating these operations for NGO 2 and 3 we obtain the system:

$$\frac{c_1\phi_1}{D(\tau,\tau^d)} = (1-\phi_2)(1-\phi_3) + (1-\phi_3)\phi_2\left[\frac{X_1^2}{1+\omega\alpha_1}\right] + (1-\phi_2)\phi_3\left[\frac{X_1^3}{1+\omega\alpha_1}\right] + \phi_3\phi_2\left[\frac{X_1^{23}}{1+\omega\alpha_1}\right] \tag{41}$$

$$\frac{c_2\phi_2}{D(\tau,\tau^d)} = (1-\phi_1)(1-\phi_3) + (1-\phi_3)\phi_1\left[\frac{1-X_1^2}{1+\omega\alpha_2}\right] + (1-\phi_1)\phi_3\left[\frac{X_2^3}{1+\omega\alpha_2}\right] + \phi_3\phi_1\left[\frac{X_2^{13}}{1+\omega\alpha_2}\right] \tag{42}$$

$$\frac{c_3\phi_3}{D(\tau,\tau^d)} = (1-\phi_2)(1-\phi_1) + (1-\phi_1)\phi_2\left[\frac{1-X_2^3}{1+\omega\alpha_3}\right] + (1-\phi_2)\phi_1\left[\frac{1-X_1^3}{1+\omega\alpha_3}\right] + \phi_1\phi_2\left[\frac{1-X_2^{13}-X_1^{23}}{1+\omega\alpha_3}\right] \tag{43}$$

8.3.2 Proof for Proposition 3

Let $\omega\alpha_j = 0$ for all j in the system (41)-(43). Then the proof becomes equivalent to the general proof for arbitrary N is provided in Appendix 8.3.4.

8.3.3 Proofs for Propositions 4 and 5

With aims of recurring to the Inverse Function Theorem, define the continuously differentiable function $\mathbf{F} : [0,1]^3 \to \mathbf{R}^3$ as $\mathbf{F} = (F_1(\boldsymbol{\phi}), F_2(\boldsymbol{\phi}), F_3(\boldsymbol{\phi}))$ by rewriting system **??** as:

$$
\begin{aligned}
F_j(\boldsymbol{\phi}) =& \frac{c_j \phi_j}{D\left(\tau, \tau^d\right)} - (1-\phi_{j+1})(1-\phi_{j-1}) - (1-\phi_{j+1})\phi_j \left[\frac{X_j^{j+1}}{1+\omega\alpha_j}\right] \\
& - (1-\phi_{j+1})\phi_{j-1}\left[\frac{X_j^{j-1}}{1+\omega\alpha_j}\right] - \phi_{j+1}\phi_{j+1}\left[\frac{X_j^{j-1j+1}}{1+\omega\alpha_j}\right] = 0
\end{aligned}
$$

For all $j = 1, 2, 3$, we have:

$$
\frac{\partial F_j(\boldsymbol{\phi})}{\partial \phi_j} = \frac{c_j}{D\left(\tau, \tau^d\right)}
$$

Also:

$$
\begin{aligned}
\frac{\partial F_1(\boldsymbol{\phi})}{\partial \phi_2} &= (1-\phi_3) - (1-\phi_3)\left[\frac{X_1^2}{1+\omega\alpha_1}\right] + \phi_3\left[\frac{X_1^3}{1+\omega\alpha_1}\right] - \phi_3\left[\frac{X_1^{23}}{1+\omega\alpha_1}\right] = (1-\phi_3)\left[1-\frac{X_1^2}{1+\omega\alpha_1}\right] + \phi_3\left[\frac{-X_1^{23}+X_1^3}{1+\omega\alpha_1}\right] \\
&= \left[1-\frac{X_1^2}{1+\omega\alpha_1}\right] + \phi_3\left[\frac{-X_1^{23}+X_1^3+X_1^2}{1+\omega\alpha_1}-1\right] \\
&= \left[1-\frac{X_1^2}{1+\omega\alpha_1}\right] - \phi_3\left[1-\frac{X_1^{23}-X_1^3-X_1^2}{1+\omega\alpha_1}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F_1(\boldsymbol{\phi})}{\partial \phi_3} &= (1-\phi_2) + \phi_2\left[\frac{X_1^2}{1+\omega\alpha_1}\right] - (1-\phi_2)\left[\frac{X_1^3}{1+\omega\alpha_1}\right] - \phi_2\left[\frac{X_1^{23}}{1+\omega\alpha_1}\right] = (1-\phi_2)\left[1-\frac{X_1^3}{1+\omega\alpha_1}\right] + \phi_2\left[\frac{-X_1^{23}+X_1^2}{1+\omega\alpha_1}\right] \\
&= \left[1-\frac{X_1^3}{1+\omega\alpha_1}\right] - \phi_2\left[1-\frac{X_1^{23}-X_1^2-X_1^3}{1+\omega\alpha_1}\right]
\end{aligned}
$$

We can then write:

$$
\begin{aligned}
\frac{\partial F_1(\boldsymbol{\phi})}{\partial \phi_2} &= \left[1-\frac{X_1^2}{1+\omega\alpha_1}\right] - \phi_3\left[\frac{\left(X_1^{23}+\frac{1}{3}\right)}{1+\omega\alpha_1}+1\right] \\
\frac{\partial F_1(\boldsymbol{\phi})}{\partial \phi_3} &= \left[1-\frac{X_1^3}{1+\omega\alpha_1}\right] - \phi_2\left[\frac{\left(X_1^{23}+\frac{1}{3}\right)}{1+\omega\alpha_1}+1\right]
\end{aligned}
$$

And due to symmetry we have that:

$$\frac{\partial F_2(\boldsymbol{\phi})}{\partial \phi_1} = \left[1 - \frac{X_2^1}{1+\omega\alpha_2}\right] - \phi_3 \left[\frac{\left(X_2^{13} + \frac{1}{3}\right)}{1+\omega\alpha_2} + 1\right]$$
$$\frac{\partial F_2(\boldsymbol{\phi})}{\partial \phi_3} = \left[1 - \frac{X_2^3}{1+\omega\alpha_2}\right] - \phi_1 \left[\frac{\left(X_2^{13} + \frac{1}{3}\right)}{1+\omega\alpha_2} + 1\right]$$

And for the third NGO:

$$\frac{\partial F_3(\boldsymbol{\phi})}{\partial \phi_1} = \left[1 - \frac{X_3^1}{1+\omega\alpha_3}\right] - \phi_2 \left[\frac{\left(X_3^{12} + \frac{1}{3}\right)}{1+\omega\alpha_3} + 1\right]$$
$$\frac{\partial F_3(\boldsymbol{\phi})}{\partial \phi_2} = \left[1 - \frac{X_3^2}{1+\omega\alpha_3}\right] - \phi_1 \left[\frac{\left(X_3^{12} + \frac{1}{3}\right)}{1+\omega\alpha_3} + 1\right]$$

Consider the Jacobian Matrix:

$$\mathcal{J} = \begin{bmatrix} \frac{\partial F_1}{\partial \phi_1} & \frac{\partial F_1}{\partial \phi_2} & \frac{\partial F_1}{\partial \phi_3} \\ \frac{\partial F_2}{\partial \phi_1} & \frac{\partial F_2}{\partial \phi_2} & \frac{\partial F_2}{\partial \phi_3} \\ \frac{\partial F_3}{\partial \phi_1} & \frac{\partial F_3}{\partial \phi_2} & \frac{\partial F_3}{\partial \phi_3} \end{bmatrix}.$$

The determinant of the above matrix is given by:

$$\begin{aligned} \det \mathcal{J} &= \frac{\partial F_1}{\partial \phi_1}\left[\frac{\partial F_2}{\partial \phi_2}\frac{\partial F_3}{\partial \phi_3} - \frac{\partial F_2}{\partial \phi_3}\frac{\partial F_3}{\partial \phi_2}\right] - \frac{\partial F_1}{\partial \phi_2}\left[\frac{\partial F_2}{\partial \phi_1}\frac{\partial F_3}{\partial \phi_3} - \frac{\partial F_2}{\partial \phi_3}\frac{\partial F_3}{\partial \phi_1}\right] + \frac{\partial F_1}{\partial \phi_3}\left[\frac{\partial F_2}{\partial \phi_1}\frac{\partial F_3}{\partial \phi_2} - \frac{\partial F_2}{\partial \phi_2}\frac{\partial F_3}{\partial \phi_1}\right] \\ &= \frac{c_1}{D}\frac{c_2}{D}\frac{c_3}{D} + \frac{\partial F_1}{\partial \phi_2}\frac{\partial F_2}{\partial \phi_3}\frac{\partial F_3}{\partial \phi_1} + \frac{\partial F_1}{\partial \phi_3}\frac{\partial F_2}{\partial \phi_1}\frac{\partial F_3}{\partial \phi_2} - \frac{c_1}{D}\frac{\partial F_2}{\partial \phi_3}\frac{\partial F_3}{\partial \phi_2} - \frac{\partial F_1}{\partial \phi_2}\frac{\partial F_2}{\partial \phi_1}\frac{c_3}{D} - \frac{\partial F_1}{\partial \phi_3}\frac{\partial F_3}{\partial \phi_1}\frac{c_2}{D}. \end{aligned}$$

It is verified numerically that $\det \mathcal{J}(\boldsymbol{\phi}) \neq 0$ for any $\boldsymbol{\phi} \in [0,1]^3$. Which implies that the solutions obtaineed below numerically are unique.

For the comparative statics results we can recur to the Implicit Function Theorem. For this consider the partial derivatives of the system with respect to a generic variable v. We have that:

$$M \begin{bmatrix} \frac{\partial \ \phi_1^*}{\partial \ v} \\ \frac{\partial \ \phi_2^*}{\partial \ v} \\ \frac{\partial \ \phi_3^*}{\partial \ v} \end{bmatrix} = \mathbf{w}(v)$$

where $M = [M_1, M_2, M_3]^T$ is an invariable 3x3 matrix of marginal effects given by:

$$M = \begin{bmatrix} \frac{c_1}{D\left(\tau,\tau^d\right)} & (1-\phi_3)\left[1-\frac{X_1^2}{1+\omega\alpha_1}\right]+\phi_3\left[\frac{X_1^3-X_1^{23}}{1+\omega\alpha_1}\right] & (1-\phi_2)\left[1-\frac{X_1^3}{1+\omega\alpha_1}\right]+\phi_2\left[\frac{X_1^2-X_1^{23}}{1+\omega\alpha_1}\right] \\ (1-\phi_3)\left[1-\frac{X_2^1}{1+\omega\alpha_2}\right]+\phi_3\left[\frac{X_2^3-X_2^{13}}{1+\omega\alpha_2}\right] & \frac{c_2}{D\left(\tau,\tau^d\right)} & (1-\phi_1)\left[1-\frac{X_2^3}{1+\omega\alpha_2}\right]+\phi_1\left[\frac{X_2^1-X_2^{13}}{1+\omega\alpha_2}\right] \\ (1-\phi_3)\left[1-\frac{X_3^1}{1+\omega\alpha_3}\right]+\phi_3\left[\frac{X_3^2-X_3^{12}}{1+\omega\alpha_3}\right] & (1-\phi_1)\left[1-\frac{X_3^2}{1+\omega\alpha_3}\right]+\phi_1\left[\frac{X_3^1-X_3^{12}}{1+\omega\alpha_3}\right] & \frac{c_3}{D\left(\tau,\tau^d\right)}, \end{bmatrix}$$

and $\boldsymbol{w}(v) = [w_1(v), w_2(v), w_3(v)]^T$ is a vector of marginal effects specific to each variable. We can hence use Cramer's Rule to study partial derivatives. We can then solve for our $\partial\phi_j/\partial v$ for $j = 1, 2, 3$ using Cramer's Rule:

$$\frac{\partial \ \phi_j^*}{\partial \ v} = \frac{\det M_j}{\det M}, \tag{44}$$

where:

$$M_1 = \begin{bmatrix} w_1 & (1-\phi_3)\left[1-\frac{X_1^2}{1+\omega\alpha_1}\right]+\phi_3\left[\frac{X_1^3-X_1^{23}}{1+\omega\alpha_1}\right] & (1-\phi_2)\left[1-\frac{X_1^3}{1+\omega\alpha_1}\right]+\phi_2\left[\frac{X_1^2-X_1^{23}}{1+\omega\alpha_1}\right] \\ w_2 & \frac{c_2}{D\left(\tau,\tau^d\right)} & (1-\phi_1)\left[1-\frac{X_2^3}{1+\omega\alpha_2}\right]+\phi_1\left[\frac{X_2^1-X_2^{13}}{1+\omega\alpha_2}\right] \\ w_3 & (1-\phi_1)\left[1-\frac{X_3^2}{1+\omega\alpha_3}\right]+\phi_1\left[\frac{X_3^1-X_3^{12}}{1+\omega\alpha_3}\right] & \frac{c_3}{D\left(\tau,\tau^d\right)} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} \frac{c_1}{D\left(\tau,\tau^d\right)} & w_1 & (1-\phi_2)\left[1-\frac{X_1^3}{1+\omega\alpha_1}\right]+\phi_2\left[\frac{X_1^2-X_1^{23}}{1+\omega\alpha_1}\right] \\ (1-\phi_3)\left[1-\frac{X_2^1}{1+\omega\alpha_2}\right]+\phi_3\left[\frac{X_2^3-X_2^{13}}{1+\omega\alpha_2}\right] & w_2 & (1-\phi_1)\left[1-\frac{X_2^3}{1+\omega\alpha_2}\right]+\phi_1\left[\frac{X_2^1-X_2^{13}}{1+\omega\alpha_2}\right] \\ (1-\phi_3)\left[1-\frac{X_3^1}{1+\omega\alpha_3}\right]+\phi_3\left[\frac{X_3^2-X_3^{12}}{1+\omega\alpha_3}\right] & w_3 & \frac{c_3}{D\left(\tau,\tau^d\right)} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} \frac{c_1}{D\left(\tau,\tau^d\right)} & (1-\phi_3)\left[1-\frac{X_1^2}{1+\omega\alpha_1}\right]+\phi_3\left[\frac{X_1^3-X_1^{23}}{1+\omega\alpha_1}\right] & w_1 \\ (1-\phi_3)\left[1-\frac{X_2^1}{1+\omega\alpha_2}\right]+\phi_3\left[\frac{X_2^3-X_2^{13}}{1+\omega\alpha_2}\right] & \frac{c_2}{D\left(\tau,\tau^d\right)} & w_2 \\ (1-\phi_3)\left[1-\frac{X_3^1}{1+\omega\alpha_3}\right]+\phi_3\left[\frac{X_3^2-X_3^{12}}{1+\omega\alpha_3}\right] & (1-\phi_1)\left[1-\frac{X_3^2}{1+\omega\alpha_3}\right]+\phi_1\left[\frac{X_3^1-X_3^{12}}{1+\omega\alpha_3}\right] & w_3 \end{bmatrix}$$

Comparative statics are then obtained by differentiating $\boldsymbol{F}$ with respect to each variable of interest, obtaining $\boldsymbol{w}$ and computing (44). Indeed we have:

$$\boldsymbol{w}(b) = \begin{bmatrix} \phi_2\left[\frac{(\alpha_1-\alpha_2)}{1+\omega\alpha_1}\right]-\phi_3\left[\frac{(\alpha_1-\alpha_3)}{1+\omega\alpha_1}\right]+\phi_3\phi_2\left[\frac{(2\alpha_1-\alpha_2-\alpha_3)/2}{1+\omega\alpha_1}\right] \\ \phi_1\left[\frac{(\alpha_2-\alpha_1)}{1+\omega\alpha_2}\right]-\phi_3\left[\frac{(\alpha_2-\alpha_3)}{1+\omega\alpha_2}\right]+\phi_3\phi_1\left[\frac{(2\alpha_2-\alpha_1-\alpha_3)/2}{1+\omega\alpha_2}\right] \\ \phi_1\left[\frac{(\alpha_3-\alpha_1)}{1+\omega\alpha_3}\right]-\phi_2\left[\frac{(\alpha_3-\alpha_2)}{1+\omega\alpha_2}\right]+\phi_2\phi_1\left[\frac{(2\alpha_3-\alpha_1-\alpha_2)/2}{1+\omega\alpha_2}\right] \end{bmatrix}, \quad \boldsymbol{w}(D(\tau^d,\tau)) = \begin{bmatrix} \frac{c_1\phi_1}{D\left(\tau,\tau^d\right)^2} \\ \frac{c_2\phi_2}{D\left(\tau,\tau^d\right)^2} \\ \frac{c_3\phi_3}{D\left(\tau,\tau^d\right)^2} \end{bmatrix} \tag{45}$$

Comparative statics are them obtained by noting that $\det M > 0$, which means that the numerator of expression (44) determines the sign in question and substituting (45) accordingly.

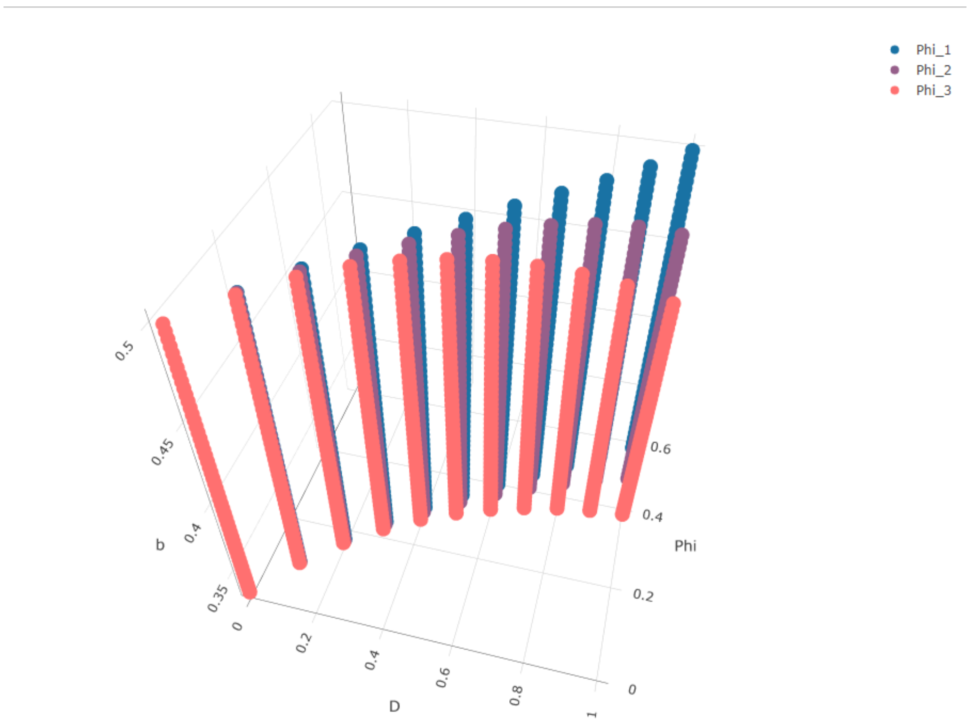

Figure 4: Equilibrium reach for $N = 3$, $\alpha = 0.5$, $\omega = 1$

8.3.4 Proofs for a large N

Under symmetry and $b = 0$, the objective function of an NGO that advertises at intensity ϕ while the remaining NGOs advertise $\bar{\phi}$ is:

$$\Pi_j(\phi; \bar{\phi}) = \phi \cdot D \cdot \left(1 + (1-\bar{\phi}) + (1-\bar{\phi})^2 + \cdots + (1-\bar{\phi})^{N-1}\right) - K(\phi) \tag{46}$$

$$= \phi \cdot \frac{D}{N} \cdot \frac{1-(1-\bar{\phi})^N}{\bar{\phi}} - K(\phi). \tag{47}$$

Welfare at a symmetric level of reach is then given by:

$$W(\phi) = N \cdot \Pi_j(\phi; \bar{\phi}) = D\left[1-(1-\bar{\phi})^N\right] - N \cdot K(\phi). \tag{48}$$

The first-order conditions that pin-down ϕ^* and ϕ^{sym} are, respectively:

$$D(1-\phi^*)^{N-1} - K'(\phi^*) = 0, \tag{49}$$

$$\frac{D}{N} \cdot \frac{1-(1-\phi^{sym})^N}{\phi^{sym}} - K'(\phi^{sym}) = 0. \tag{50}$$

By assumption $K'(\phi) > 0$ and $K''(\phi) > 0$, while the functions $D(1-\phi)^{N-1}$ and $\frac{D}{N} \cdot \frac{1-(1-\phi)^N}{\phi}$ are both strictly decreasing and convex in $\phi \in [0,1]$, they hence cross $K'(\phi)$ at most once. Moreover, since for any $\phi \in (0,1]$ and $N > 1$ $(1-\phi)^{N-1}(N+1-\phi) > 1$, the follow inequality holds true:

$$D(1-\phi)^{N-1} > \frac{D}{N} \cdot \frac{1-(1-\phi)^N}{\phi}. \tag{51}$$

Together, the first-order conditions, equation (51), and the fact that the cost function $K(\phi)$ in increasing an covex imply that $\phi^{sym} > \phi^*$. At last, second-order conditions are met since both objectives are globally concave for all $\phi \in [0,1]$:

$$\frac{\partial^2 \Pi_j}{\partial \phi^2} = -K''(\phi) < 0, \text{for all } j, \text{ and } \frac{\partial^2 W}{\partial \phi^2} = -N(N-1)(1-\phi)^{N-2} - NK''(\phi) < 0. \tag{52}$$

To compare these solutions for a large N, let $K(\phi) = 0.5\phi^2$ and study the solutions for the functions:

$$f(x) \triangleq x - D(1-x)^{N-1} = 0,$$
$$g(x) \triangleq x^2 - \frac{D}{N}\left[1 - (1-x)^N\right] = 0.$$

Define x_f and x_g as solutions to the above equations. This means that:

$$f(x_f) = 0, \text{ and } f(x_g) = 0.$$

To analyse x_f consider the change of variable $M = N - 1$ and $x_f = \frac{z_f}{M}$. We then study $\frac{z_f}{M} = a\left(1 - \frac{z_f}{M}\right)^M$. As M gets large then the exponential approximation implies:

$$\left(1 - \frac{z_f}{M}\right)^M \approx \exp(-z_f)$$

For a large M then $z_f \approx MD\exp(-z_f) \Leftrightarrow z_f \exp(z_f) \approx MD$. We can then express z_f approximately using the Lambert W function as $z_f \approx W(MD) \approx \log MD - \log\log MD + o(1)$ which gives

$$x_f \approx \frac{W(MD)}{M} \approx \frac{\log MD - \log\log MD + o(1)}{M}.$$

So x_f grows roughly like $\frac{\log DM}{M}$. For x_g use the change of variables $x_g = z_g\sqrt{\frac{D}{N}}$. And study:

$$z_g^2 = \left(1 - \left(1 - z_g\sqrt{\frac{D}{N}}\right)^N\right).$$

The exponential approximation yields $\left(1 - z_g\sqrt{\frac{D}{N}}\right)^N \approx \exp\left(-z_g\sqrt{DN}\right)$. The approximation allows to obtain a lower bound; we have $\left(1 - z_g\sqrt{\frac{D}{N}}\right)^N \leq \exp\left(-z_g\sqrt{DN}\right)$ which implies:

$$z_g^2 \geq 1 - \exp\left(-z_g\sqrt{DN}\right).$$

Since $\exp(-x) = \frac{1}{\exp(x)} \leq \frac{1}{1+x}$ we can then obtain:

$$z_g^2 \geq 1 - \frac{1}{1 + z_g\sqrt{DN}}.$$

Now, notice that we have $g(0) < 0$ and $g(1) = 1$ so x_g is the unique real root between 0 and 1 and it lies between a sign change from negative to positive; this tells us that if $g(x) \leq 0$ then $x \leq x_g$. The bounds above applied to $g(x)$ give

$$\begin{aligned} g(x) &\leq x^2 - \frac{a}{N}\left(1 - \exp\left(-Nx\right)\right) \\ &\leq x^2 - \frac{D}{N}\left(1 - \frac{1}{1 + Nx}\right) \end{aligned}$$

and substituting in $x = \frac{\sqrt{D}}{N}$ gives that

$$g\left(\frac{D}{N}\right) \leq \frac{D}{N^2} - \frac{D}{N}\frac{\sqrt{D}}{1 + \sqrt{D}}$$

which is ≤ 0 as long as $N \geq 1 + \frac{1}{\sqrt{D}}$. So, assuming this from now on, we conclude that $x_g \geq \frac{\sqrt{D}}{N}$ and hence that $z_g \geq \frac{1}{\sqrt{N}}$. This gives

$$z_g^2 \geq 1 - \frac{1}{1 + z_g\sqrt{DN}} \geq 1 - \frac{1}{1 + \sqrt{D}} = \frac{\sqrt{D}}{1 + \sqrt{D}}$$

which gives

$$x_g \geq \frac{\sqrt{D}}{(1 + \sqrt{D})\sqrt{N}}.$$

We can now bootstrap a second time to get

$$z_g^2 \geq 1 - \exp\left(-z_g\sqrt{DN}\right) \geq 1 - \exp\left(-\frac{a}{1 + \sqrt{D}}\sqrt{N}\right).$$

This means that z_g is in fact exponentially close to 1 when N is large. We have established that for N sufficiently large, x_f is bounded from above by $\frac{\log DN}{N}$ while x_g is bounded from below by $\sqrt{\frac{D}{N}}$.

We can therefore establish that an approximation to the ratio $x_f/x_g \approx (\sqrt{N}\log D(N-1))/((N-1)\sqrt{D})$. Coming back too our problem of interest, this means that:

$$\frac{\phi^*}{\phi^{sym}} \approx \frac{\sqrt{N}\log D(N-1)}{N-1)\sqrt{D}}. \tag{53}$$

And it follows that $\frac{\phi^*}{\phi^{sym}} < 1$ and $\frac{\phi^*}{\phi^{sym}} \to 0$ as $N \to \infty$. Moreover, $\frac{\phi^*}{\phi^{sym}}$ decreases in D, which implies that increases in market size imply a larger absolute difference between ϕ^{sym} and ϕ^{OP}.

8.4 Proofs for optimal taxation problem

The planner solves the problem of $\max_{\tau,\tau^d,R,G_0} W$ subject to equation (2) and (**??**). Denote by λ the multiplier of the government's budget constraint, then first-order conditions to this problem are given by:

$$-\int \mu^i \left[v^i_{1-\tau} + v^i_G \bar{G}_{1-\tau}\right] dv(i) + \lambda\left[\bar{Z} - \tau\bar{Z}_{1-\tau} - \tau^d \bar{G}_{1-\tau}\right] = 0, \tag{54}$$

$$\int \mu^i \left[v^i_{1+\tau^d} + v^i_G \bar{G}_{1+\tau^d}\right] dv(i) + \lambda\left[\bar{D} + \tau\bar{Z}_{1+\tau^d} + \tau^d \bar{G}_{1+\tau^d}\right] = 0, \tag{55}$$

$$\int \mu^i \left[v^i_R + v^i_G \bar{G}_R\right] dv(i) + \lambda\left[-1 + \tau\bar{Z}_R + \tau^d \bar{G}_R\right] = 0. \tag{56}$$

Given the leakage equation in (**??**), we have that the derivatives of the average public good with respect to taxes and the lump-sum, namely $\bar{G}_{1-\tau}$, $\bar{G}_{1-\tau^d}$, and $\bar{G}_R$, are given by the following three equations:

$$\bar{G}_{1-\tau} = \rho(D)D_{1-\tau}(1+\eta^\rho_D) = D_{1-\tau}(1 - l(D)(1+\eta^l_D)), \tag{57}$$

$$\bar{G}_{1+\tau^d} = \rho(D)D_{1+\tau^d}(1+\eta^\rho_D) = D_{1+\tau^d}(1 - l(D)(1+\eta^l_D)), \tag{58}$$

$$\bar{G}_R = \rho(D)D_R(1+\eta^\rho_D) = D_R(1 - l(D)(1+\eta^l_D)). \tag{59}$$

Moreover, if the government can contribute to the public good the first-order condition with respect to G_0 writes:

$$\int \mu^i \left[v_G^i + v_G^i \bar{G}_{G_0}\right] dv(i) + \lambda[-1 + \tau \bar{Z}_{G_0} + \tau^d \partial \bar{G}/\partial G_0] = 0.$$

Where, in an analogous fashion as above, we have that:

$$\bar{G}_{G_0} = \rho(D) D_{G_0}(1 + \eta_D^\rho) = D_{G_0}(1 - l(D)(1 + \eta_D^l))$$

We can therefore re-express the previous system of equations as:

$$\left[1 - \frac{\int \beta^i z^i di}{\bar{Z}}\right] \bar{Z} = \tau \bar{Z}_{1-\tau} + (\tau^d + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1-\tau}, \tag{60}$$

$$\left[1 - \frac{\int \beta^i d^i di}{\bar{D}}\right] \bar{D} = -\tau \bar{Z}_{1+\tau^d} - (\tau^d + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1+\tau^d}, \tag{61}$$

$$1 - \int \beta^i di = \tau \bar{Z}_R + (\tau^d + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_R. \tag{62}$$

and, finally, we have that if the government can contribute to the public good:

$$e = 1 - \tau \bar{Z}_{G_0} - (\tau^d + e \cdot \rho(1 + \eta_D^\rho))\partial \bar{G}/\partial G_0 \tag{63}$$

Three assumptions are made in order to simplify the system determined by the four equations above (see Saez (2004) for further discussion).

Assumption T1. There are no income effects on earning, i.e: $z_R^i = 0$ for all i.

Assumption T2. Independence between aggregate earnings and contributions , i.e: $\bar{Z}_{G_0} = 0$ and $\bar{Z}_{1+\tau^d} = 0$.

Assumption T3. Compensated supply of contributions does not depend on earnings. $\partial d^i/\partial(1-\tau) = 0$. This implies that:

$$\bar{D}_{1-\tau} = \bar{Z}\hat{D}_R \tag{64}$$

where $\hat{D}_R$ corresponds to the average response to a uniform one dollar increase in the lumpsum R, weighted by earnings. We can use Assumptions 1-3 to simplify our system in the following way:

$$\tau^d = -e \cdot (1 - l(D)) + \frac{1}{r}\left[1 - \frac{\int \beta^i d^i di}{\bar{D}} - \eta(D)\right]$$
$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon_Z}\left[1 - \frac{\int \beta^i z^i di}{\overline{Z}} - (\tau^d + e \cdot (1 - l(D)))\hat{D}_R - \epsilon_Z^D\right]$$
$$\int \beta^i di = 1 - (\tau^d + e \cdot (1 - l(D)))\bar{D}_R - \eta(D) \cdot \bar{D}.$$

If the government can choose G_0 optimally:

$$e = 1 - (\tau^d + e(1 - l(D)))\partial\bar{G}/\partial G_0 - \eta(D) \cdot \bar{D} = \frac{1 - \tau^d \bar{D}_{G_0} - \eta(D) \cdot \bar{D}}{1 + (1 - l(D))\bar{D}_{G_0}} \tag{65}$$

$$\tau^d = -(1 - \eta(D))(1 - l(D)) + \frac{1}{r}(1 + (1 - l(D)) \cdot \partial\bar{G}/\partial G_0)\left[(1 - \frac{\int \beta^i d^i di}{\overline{G}} - \eta(D)\right]$$

When the government is constrained to set $\tau^d = -\tau$, the first-order condition with respect to income becomes:

$$\left[1 - \frac{\int \beta^i z^i di}{\overline{Z}}\right]\bar{Z} - \left[1 - \frac{\int \beta^i d^i di}{\overline{D}}\right]\bar{D} = \tau\bar{Z}_{1-\tau} + (-\tau + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1-\tau}$$
$$+ \tau\bar{Z}_{1+\tau^d} + (-\tau + e \cdot \rho(1 + \eta_D^\rho))\bar{D}_{1+\tau}.$$

The formula for the optimal income tax follows from this.

8.5 Estimation Details

8.5.1 Linking NGO decisions to donation supply estimates

Having estimated the donation supply at (28), I use the system (**??**) to obtain marginal costs of reach at equilibrium. First, write aggregate donations $\mathcal{D}(\boldsymbol{\phi}_j)$ as a function of reach:

$$\mathcal{D}_j(\boldsymbol{\phi}) = \mathcal{D}(\boldsymbol{\delta}(\boldsymbol{\phi})) = \gamma T_r s_j(\boldsymbol{\delta}(\boldsymbol{\phi}), \sigma),$$

which is the empirical equivalent to equation (19) from the model section. The net fundraising function for NGO j, in turn writes:

$$\Pi(\phi_j; \boldsymbol{\phi}_{-j}) = -K_j(\phi_j) + \phi_j A(\phi_j; \boldsymbol{\phi}_{-j}, \mathcal{D}_j), \tag{66}$$

where the fund-collection function $A(\phi_j; \boldsymbol{\phi}_-j)$ is given by:

$$A(\phi_j; \boldsymbol{\phi}_{-j}, \mathcal{D}_j) = \prod_{k \in \mathcal{J}_g/j} (1-\phi_k)\mathcal{D}_j\left(\phi_j, \boldsymbol{\phi}^c_{-j}\right) + \sum_{S \subset \mathcal{J}_g/j}^{N_g} \prod_{\substack{m \in S \\ k \in \overline{S}/\mathcal{J}_g}} \phi_m(1-\phi_k)\mathcal{D}_j\left(\phi_j, \boldsymbol{\phi}_m, \boldsymbol{\phi}^c_k\right) \tag{67}$$

Here, $\mathcal{D}_j(\phi_j, \boldsymbol{\phi}^c_{-j})$ represents the gross donations perceived by NGO j when advertising at intensity ϕ_j, while other NGOs advertise with intensities summarized by the vector of dimension $N_g - 1$ that represents the probability that no other NGOs reach a donation segment: $\boldsymbol{\phi}^c_{-j} = \mathbf{1} - \boldsymbol{\phi}_{-j}$. Similarly, $\mathcal{D}_j\left(\phi_j, \boldsymbol{\phi}_m, \boldsymbol{\phi}^c_k\right)$ corresponds to gross donations perceived by NGO j when $|S|$ NGOs indexed by m are in the same segment while the remaining, indexed by k are not: $\boldsymbol{\phi}_m$ is a vector with entries ϕ_m for $m \in S$, and $\boldsymbol{\phi}^c_k$ is a vector with entries $\boldsymbol{\phi}^c_k = 1 - \boldsymbol{\phi}_k$ for $k \in \overline{S}/j$.

8.5.2 The effects of a change in the price of giving

Similarly, I compute the equilibrium effects of a change in tax liabilities differentiating (74) with respect to the tax liability, which yields:

$$\phi_j c_j = (1+\alpha_j)\frac{\partial A(\phi_j;\boldsymbol{\phi}_{-j},\mathcal{D}_j)}{\partial \mathcal{D}_j}\frac{\partial \mathcal{D}_j}{\partial T_r} + \alpha_j \omega \sum_{k\neq j}^{N} \frac{\partial A_k}{\partial \mathcal{D}_k}\frac{\mathcal{D}_K}{\partial T_r}, \tag{68}$$

which becomes:

$$\phi_j c_j = (1+\alpha_j)A(\cdot)\frac{\beta_T}{1-\sigma}s_j(1-\sigma\bar{s}_{j|g}(1-\sigma)s_j) + \alpha_j\omega \sum_{k\neq j}^{N} A(\cdot)\frac{\beta_T}{1-\sigma}s_k(1-\sigma\bar{s}_{k|g}-(1-\sigma)s_k), \tag{69}$$

8.5.3 Estimation algorithm

Estimation proceeds in the following way. First, define the elements $\bar{j}$ and $\underline{j} \in S$ as:

$$\bar{j} = argmin_{k\in S/j}\ (k \bmod N) - j$$
$$\underline{j} = argmin_{k\in S/j}\ j - (k \bmod N)$$

For instance, if $j=1$, and $S=\{1,\dots N\}$ then:

$$\bar{j} = 2$$
$$\underline{j} = N$$

For instance, if $j=1$, and $S=\{4,\dots N-1\}$ then:

$$\bar{j} = 4$$
$$\underline{j} = N-1$$

Define the cardinality of S by $|S|$. We then have a general formula:

$$X_j^S = \begin{cases} \frac{1}{2} + b(\alpha_j - \alpha_r) & S = \{r\},\ r \neq j \\ 1 & S = \{j\} \\ \frac{b}{2}(2\alpha_j - \alpha_q - \alpha_r) + \frac{p_q - p_r}{2} & S = \{q, r\},\ q > r,\ q, r \neq j, j\ \neq\ 1,\ N \\ \frac{b}{2}(2\alpha_j - \alpha_{\overline{j}} - \alpha_{\underline{j}}) + \frac{p_{\overline{j}} - p_{\underline{j}}}{2} & |S| > 2,\ j \notin S\ j\ \neq\ 1,\ N \\ \frac{1}{2} + \frac{b}{2}(2\alpha_j - \alpha_q - \alpha_r) - \frac{p_q - p_r}{2} & S = \{q, r\},\ q > r,\ q, j = 1 \text{ or } j = N \\ \frac{1}{2} + \frac{b}{2}(2\alpha_j - \alpha_{\overline{j}} - \alpha_{\underline{j}}) - \frac{p_{\overline{j}} - p_{\underline{j}}}{2} & |S| > 2,\ j \notin S\ ,\ j = 1 \text{ or } j = N \end{cases} \tag{70}$$

I do not observe p_j directly, so I will assume that $p_j = j/N$. I also do not observe the ordering j; the ordering matters for our computations.

Notice that $j = 1, N$ are special; there is a 1/2 and a change of sign (this is because they represent the end of the circle). We estimate in the code:

$$\hat{X}_j^S = \max\left\{\min\left\{X_j^S, 1\right\}, 0\right\}$$

For this, first, pick a random line j=1, and then compute

$$\frac{1}{2} + \frac{b}{2}(2\alpha_1 - \alpha_r - \alpha_q)$$

for all the possible combinations of $r, q \in \{2, 3, \ldots N\}$. Then select the minimal (maximal) value and define the positions $\overline{r} = N$ and $\underline{q} = 2$. Now compute again

$$\frac{1}{2} + \frac{b}{2}(2\alpha_1 - \alpha_r - \alpha_q)$$

For all possible combinations of $r, q \in \{3, \ldots N - 1\}$. Then select the minimal (maximal) value and define the positions $\overline{r} = N - 2$ and $\underline{q} = 3$. Repeat until all the observations have assigned positions.

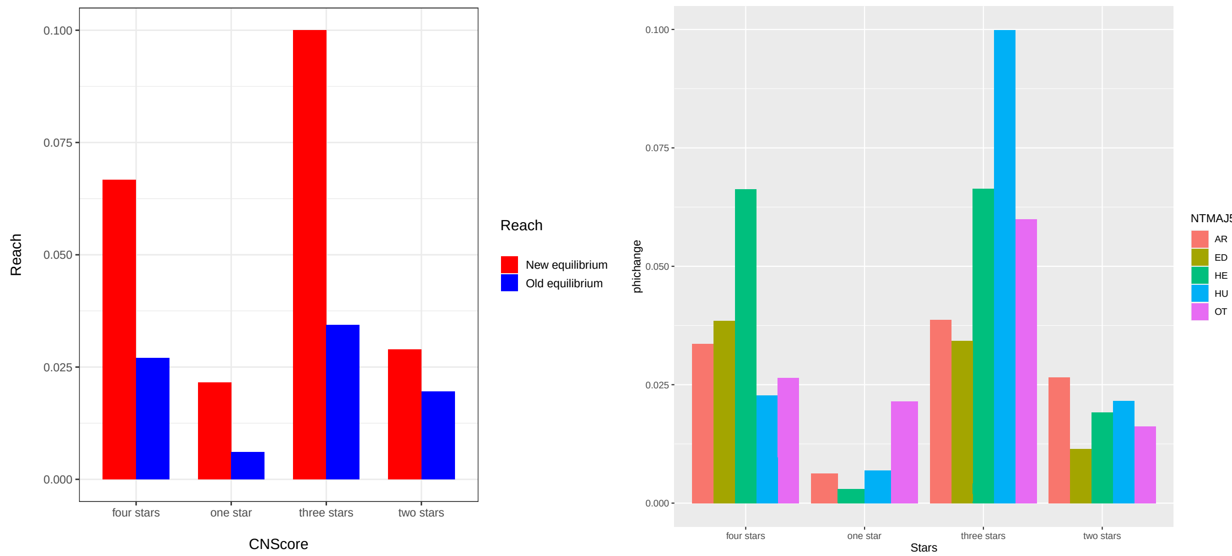

Figure 5: A subsample of estimated Best-Responses, where the total donations D are normalized to one.

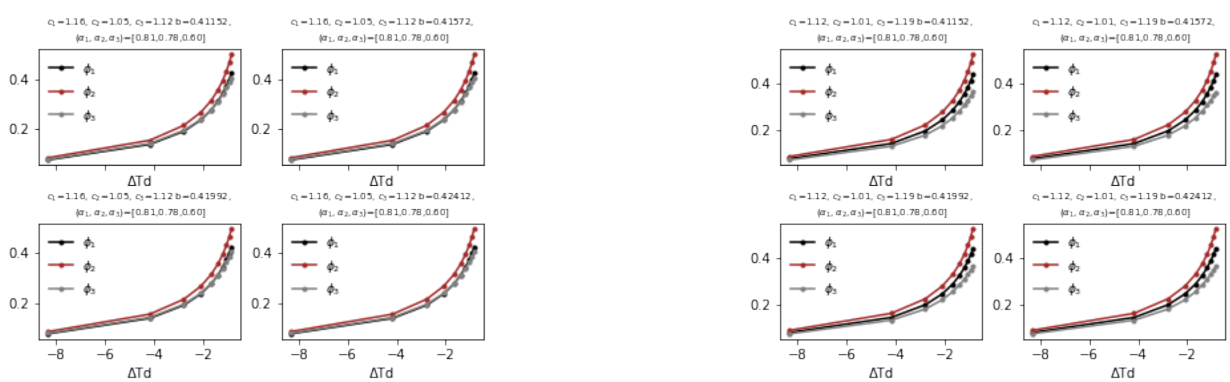

Figure 6: A subsample of estimated best responses as a function of a change in the deductibility rate, assuming the donation supply elasticity of Peloza and Steel (2005) of -1.2.

8.6 Estimation tables

9. Appendix: numerical analysis

In order to compare to the baseline simulations presented by Saez (2004), the numerical analysis adopts the functional forms present in that paper, together with the majority of parameter values.

Government consumption per capita, E is fixed at $\$6000$. Aggregate earnings are given by:

$$\bar{Z} = \bar{Z}_0 \left(\frac{1-\tau}{1-\tau_0} \right)^{\epsilon_Z},$$

where the earnings elasticity ϵ_Z is assumed to be constant, τ_0 is the current average marginal income tax rate taken as equal to 30%, and $\bar{Z}_0$ corresponds to the baseline aggregate earnings.

Aggregate donations $\bar{D}$ are given by:

$$\bar{D} = \bar{D}_0 \frac{e^{-\rho(1+t)}}{e^{-\rho(1+t_0)}} \left[\frac{\bar{Z}(1-\tau)+R}{\bar{Z}_0 \left(1-\tau_0\right)+R_0} \right]^{\epsilon_R} - \alpha G^0,$$

where $r = -\bar{D}_{1+/}/\bar{D}$ is a constant parameter that measures the price response of contributions,$\bar{D}_0$ are the baseline aggregate donation, ϵ_R corresponds to the income elasticity of donations (assumed to be constant), and α is a crowding out parameter.

I assume that$\frac{v_D^h}{v_R^h} = B \cdot \left(s \cdot \bar{G} + G^0\right)^{-l}$, for constant parameter B and l. This implies that the external effect e given by:

$$e = B \cdot \left(s \cdot \vec{G} + G^0\right)^{-1} \beta(R). \tag{71}$$

Individual earnings are given by,

$$z^h = z_0^i \left(\frac{1-\tau}{1-\tau_0} \right)^{\epsilon_Z}.$$

Where τ^0 is the average marginal tax rate, and z_0^i is the baseline earnings level for individual i. The elasticity is taken as constant and uniform across individuals (recall that only linear taxation is considered, which makes this assumption fairly harmless).

The marginal welfare weights β^i depend on disposable income only and thus are specified as, $\beta^i = 1/\left(z^i(1-\tau)+R\right)^v$, where v is a measures the redistributive preferences of the government.

9.1 Estimation details

If NGO j is in group g, i.e., $j \in \mathcal{J}_g$, then the selection probability of product j conditional on group g being selected equals:

$$\overline{s}_{j|g} = \frac{\exp\left(\frac{\delta_j}{1-\sigma}\right)}{D_g},$$

where the denominator D_g is described by:

$$D_g = \sum_{k\in\mathcal{J}_g} \exp\left(\frac{\delta_k}{1-\sigma}\right). \tag{72}$$

In the same manner, the probability of choosing one of the g NGO groups is given by:

$$\overline{s}_g(\boldsymbol{\delta},\sigma) = \frac{D_g^{1-\sigma}}{\sum_g D_g^{1-\sigma}},$$

and hence market shares are given by:

$$s_j(\boldsymbol{\delta},\sigma) = \overline{s}_{j|g}(\boldsymbol{\delta},\sigma)\overline{s}_g(\boldsymbol{\delta},\sigma) = \frac{\exp\left(\frac{\delta_k}{1-\sigma}\right)}{D_g^{\sigma}\sum_g D_g^{1-\sigma}}. \tag{73}$$

We can now use the expression for the NGOs objective function in (17) together with (66) to obtain the first-order conditions of the estimated model. Assuming a quadratic cost specification $K_j(\phi_j) = 0.5c_j\phi_j^2$ we have that this system is given by:

$$\phi_j\left(c_j - \frac{\partial A_j}{\partial \phi_j}\right) = A(\phi_j;\boldsymbol{\phi}_{-j},\mathcal{D}_j)(1+\alpha_j) + \alpha_j\omega \sum_{k\neq j}^{N} \frac{\partial A_k}{\partial \phi_j}, \tag{74}$$

where the derivative on the right-hand-side corresponds to the aggregate elasticity given by:

$$\frac{\partial A_j}{\partial \phi_j} = \gamma T_r \prod_{k \neq j} (1 - \phi_k) \frac{\partial s_j \left(\phi_j, \boldsymbol{\phi}_{-j}^c\right)}{\partial \phi_j} + \gamma T_r \sum_{S \subset \mathcal{J}_g / j}^{N_g} \prod_{\substack{m \in S \\ k \in \overline{S} / \mathcal{J}}} \phi_m (1 - \phi_k) \frac{\partial s_j \left(\phi_j, \boldsymbol{\phi}_m, \boldsymbol{\phi}_k^c\right)}{\partial \phi_j}, \tag{75}$$

and the derivatives of the choice probabilities above are computed with the standard formulas:

$$\frac{\partial \ \overline{s}_j}{\partial \ \phi_j} = \beta_\phi \frac{\partial \ \overline{s}_j}{\partial \ \delta_j} = \frac{\beta_\phi}{1 - \sigma} s_j (1 - \sigma \overline{s}_{j|g} - (1 - \sigma) s_j), \tag{76}$$

evaluated at their respective intensity profiles as in (77). Similarly, the derivative at the right-hand-side of (77) is given by:

$$\frac{\partial A_k}{\partial \phi_j} = \gamma T_r \sum_{S \subset \mathcal{J}_g / j}^{N_g} \prod_{\substack{m \in S \\ k \in \overline{S} / \mathcal{J}}} \phi_r (1 - \phi_m) \frac{\partial s_k \left(\phi_j, \boldsymbol{\phi}_m, \boldsymbol{\phi}_k^c\right)}{\partial \phi_j}, \tag{77}$$

and again the cross-derivatives of the choice probabilities above are computed as:

$$\frac{\partial s_k}{\partial \ \phi_j} = \beta_\phi s_j s_k. \tag{78}$$

Together, equations (74) to (78) allow us to estimate the marginal cost of reach at the observed equilibrium for each year. These estimates are summarized in Table 3. Details on estimation are included in Appendix 8.5.

	market	Costs	Elasticity
1	Atlanta	6.08	0.27
2	Baltimore	0.67	0.02
3	Bangor	1.78	0.09
4	Billings	15.22	0.41
5	Binghamton	4.58	1.73
6	Boise	15.06	0.73
7	Boston	0.93	0.01
8	Buffalo		0.02
9	Charleston, SC	16.12	2.47
10	Charlotte	7.28	0.85
11	Chicago		0.01
12	Cincinnati	2.77	0.06
13	Cleveland	2.24	0.06
14	Columbus, OH		0.06
15	Dayton	1.78	0.08
16	Denver	13.55	1.96
17	Detroit		0.03
18	Erie		0.02
19	Evansville	2.77	0.07
20	Houston	9.59	0.48
21	Indianapolis	1.59	0.06
22	Kansas City		0.25
23	Lansing		0.04
24	Los Angeles	2.10	0.08
25	Louisville	1.92	0.10
26	Madison	1.42	0.04
27	Marquette	0.61	0.03
28	Memphis	3.38	0.11
29	Milwaukee	1.59	0.05
30	Nashville	3.07	0.08
31	New York		0.01
32	Oklahoma City	15.37	0.50
33	Omaha	2.61	0.08
34	Philadelphia		0.02
35	Pittsburgh	1.85	0.04
36	Portland, OR	7.51	0.44
37	Rochester, NY		0.02
38	Rockford	0.50	0.01
39	Salisbury	1.45	0.32
40	Salt Lake City	15.02	0.41
41	San Antonio	6.47	0.36
42	San Diego		0.06
43	Spokane	15.40	0.38
44	Syracuse	0.80	0.02
45	Toledo	3.03	0.06
46	Youngstown	0.79	0.06
47	Zanesville	1.45	0.04

	Cost parameter		
NTMAJ5	$\omega = 0$	$\omega = 1$	Adver. Elasticity
AR	2.73	1.95	0.05
ED	2.65	1.87	0.10
HE	4.26	1.65	0.13
HU	5.75	2.92	0.06
OT	5.72	3.40	0.12

Table 3: Mean estimates by nest.