

Breast Cancer Wisconsin (Diagnostic)

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0.1 Setup

Load libraries

```
# library(GGally)      # for ggpairs
# library(ggfortify)   # for autoplot
# library(ggplot2)     # for ggplot
# library('MASS')      # for the glm model selection

#this is to check to see if package are installed and if not to install them or just load them
if(!require(pacman)) {install.packages(c("pacman", "remotes"))}

## Loading required package: pacman

if(!require(papaja)) {remotes::install_github("crsh/papaja")}
```

```

## Loading required package: papaja

pacman::p_load(GGally, ggfortify, ggplot2, MASS, here, kableExtra, papaja, glmnet)

```

1 Information about the dataset

[Link to dataset](#)

“Features are computed from a digitized image of a fine needle aspirate (FNA) of a breast mass. They describe characteristics of the cell nuclei present in the image.”

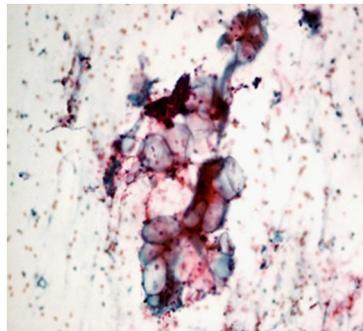


Figure 1: image of a fine needle aspirate (FNA) of a breast mass

Ten real-valued features are computed for each cell nucleus:

Name of the variables	type	Description
1) ‘radius’	num	distances from center to points on the perimeter
2) ‘texture’	num	standard deviation of gray-scale values
3) ‘perimeter’	num	perimeter of the nucleus
4) ‘area’	num	area of the nucleus
5) ‘smoothness’	num	local variation in radius lengths
6) ‘compactness’	num	$perimeter^2/area - 1.0$
7) ‘concavity’	num	severity of concave portions of the contour
8) ‘concave.points’	num	number of concave portions of the contour
9) ‘symmetry’	num	symmetry of the nucleus
10)‘fractal_dimension’	num	$coastlineapproximation - 1$

The mean, standard error and “worst” or largest (mean of the three largest values) of these features were computed for each image, resulting in 30 features. All feature values are recorded with four significant digits.

The aim is to **predict whether the cancer is benign or malignant**

diagnosis	radius	texture	perimeter	area	smoothness	compactness	concavity	concave.points	symmetry	fractal_dimension
M	17.99	10.38	122.80	1001.0	0.11840	0.27760	0.3001	0.14710	0.2419	0.07871
M	20.57	17.77	132.90	1326.0	0.08474	0.07864	0.0869	0.07017	0.1812	0.05667
M	19.69	21.25	130.00	1203.0	0.10960	0.15990	0.1974	0.12790	0.2069	0.05999
M	11.42	20.38	77.58	386.1	0.14250	0.28390	0.2414	0.10520	0.2597	0.09744
M	20.29	14.34	135.10	1297.0	0.10030	0.13280	0.1980	0.10430	0.1809	0.05883
M	12.45	15.70	82.57	477.1	0.12780	0.17000	0.1578	0.08089	0.2087	0.07613

2 Preprocessing

Load data

```
#get relative path
path = here("LUCILE")
setwd(path) #set working directory

# df<-read.csv('/Users/lucile/Library/Mobile Documents/com~apple~CloudDocs/STUDY/NEURO/LECTURE')

df<-read.csv('data.csv', stringsAsFactors = 1) # j'ai ajouté le dataset sur github
```

2.1 First look into the data

There are a lot of variables, we should pick the most relevant ones.

Let's delete the last variable and ID number because there are not relevant.

```
df<-df[,-33]
df<-df[,-1]
```

Let's create a new frame for the variables of type mean.

```
'data.frame': 569 obs. of 11 variables:
 $ diagnosis      : Factor w/ 2 levels "B","M": 2 2 2 2 2 2 2 2 2 2 ...
 $ radius         : num  18 20.6 19.7 11.4 20.3 ...
 $ texture        : num  10.4 17.8 21.2 20.4 14.3 ...
 $ perimeter      : num  122.8 132.9 130 77.6 135.1 ...
 $ area           : num  1001 1326 1203 386 1297 ...
 $ smoothness      : num  0.1184 0.0847 0.1096 0.1425 0.1003 ...
 $ compactness     : num  0.2776 0.0786 0.1599 0.2839 0.1328 ...
 $ concavity       : num  0.3001 0.0869 0.1974 0.2414 0.198 ...
 $ concave.points : num  0.1471 0.0702 0.1279 0.1052 0.1043 ...
 $ symmetry        : num  0.242 0.181 0.207 0.26 0.181 ...
 $ fractal_dimension: num  0.0787 0.0567 0.06 0.0974 0.0588 ...
```

diagnosis	radius	texture	perimeter	area
B:357	Min. : 6.981	Min. : 9.71	Min. : 43.79	Min. : 143.5

```

M:212      1st Qu.:11.700   1st Qu.:16.17    1st Qu.: 75.17   1st Qu.: 420.3
            Median :13.370   Median :18.84    Median : 86.24   Median : 551.1
            Mean   :14.127   Mean   :19.29    Mean   : 91.97   Mean   : 654.9
            3rd Qu.:15.780   3rd Qu.:21.80    3rd Qu.:104.10  3rd Qu.: 782.7
            Max.   :28.110   Max.   :39.28    Max.   :188.50   Max.   :2501.0
smoothness      compactness      concavity      concave.points
Min.   :0.05263   Min.   :0.01938   Min.   :0.00000   Min.   :0.00000
1st Qu.:0.08637   1st Qu.:0.06492   1st Qu.:0.02956   1st Qu.:0.02031
Median :0.09587   Median :0.09263   Median :0.06154   Median :0.03350
Mean   :0.09636   Mean   :0.10434   Mean   :0.08880   Mean   :0.04892
3rd Qu.:0.10530   3rd Qu.:0.13040   3rd Qu.:0.13070   3rd Qu.:0.07400
Max.   :0.16340   Max.   :0.34540   Max.   :0.42680   Max.   :0.20120
symmetry      fractal_dimension
Min.   :0.1060    Min.   :0.04996
1st Qu.:0.1619    1st Qu.:0.05770
Median :0.1792    Median :0.06154
Mean   :0.1812    Mean   :0.06280
3rd Qu.:0.1957    3rd Qu.:0.06612
Max.   :0.3040    Max.   :0.09744

```

Proportion of benign vs malignant cancer

```
prop.table(table(df_mean$diagnosis))
```

B	M
0.6274165	0.3725835

The two types of cancer are not represented in the same proportion, this can lead to a bias. Is this proportion representative of the reality ?

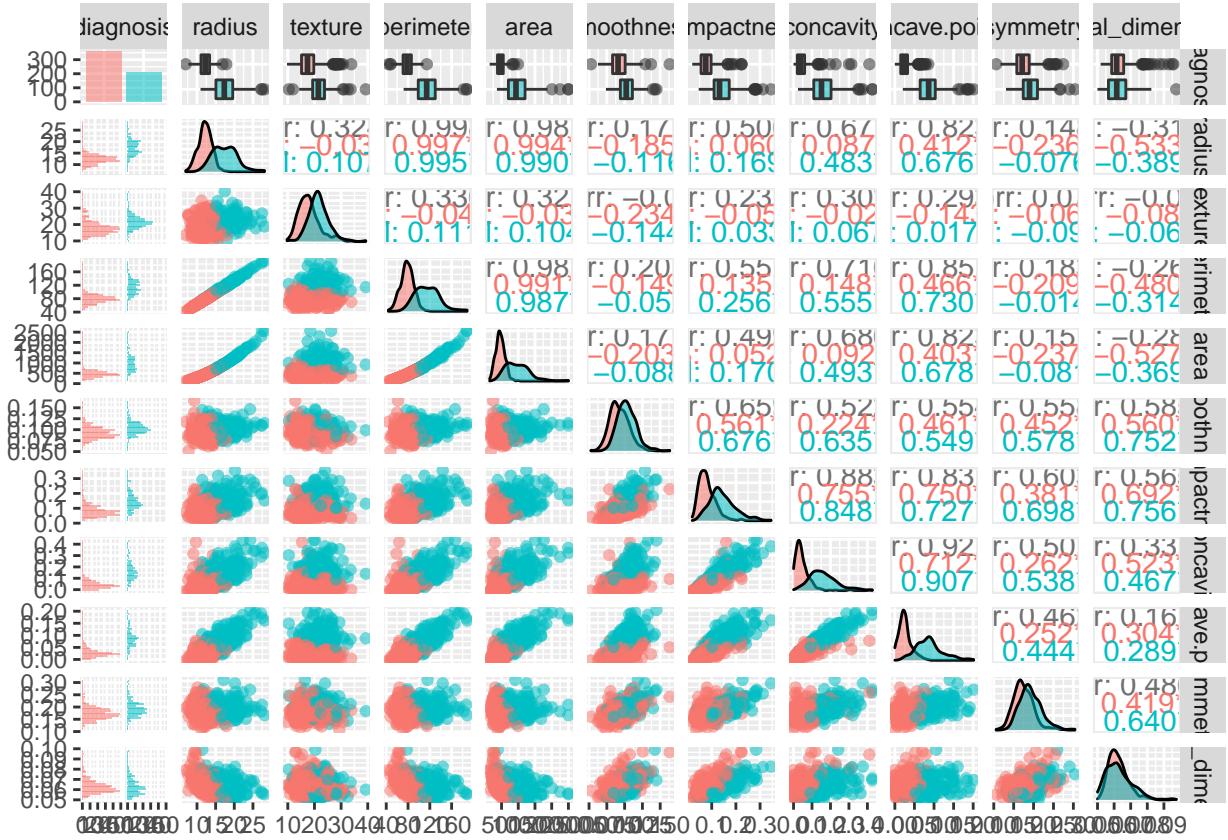
“The benign to malignant ratio (B:M ratio) among breast biopsies (number of benign breast lesions divided by number of breast cancers) is widely believed to be around 4:1 or 5:1” [2]

Blabla citation in parenthesis (James 1890) blbabla, or citation in text James (1890)

2.2 Selection of variables of interest

According to the description of the data, some variables are likely to be correlated. We will address this correlation with the mean group of variable.

Hypothesis about the correlation between variables : *radius and smoothness should be correlated* radius, perimeter, area and compactness should be perfectly correlated since it exists a formula between these variables * concavity and symmetry should be correlated * texture and fractal_dimension should not have any correlation



```
## [1] "B" "M"
```

By eye, the variables seem to be different according to the type of ‘diagnosis’ (first row of the plot), the variables coming from malignant cancer seem to be in general bigger than the data coming from benign cancer.

As expected, radius, perimeter and area are highly correlated; and texture and fractal_dimension don’t have strong correlation.

Surprisingly, radius and smoothness are not very correlated, and the compactness doesn’t show any strong correlation.

Concavity and compactness have a strong correlation. In annex we show that we have the same correlations with the standard deviation group and extreme group of variable.

The following function permit to see better the correlation :

```
ggcorr(df_mean, geom = "text", nbreaks = 5, hjust = 1, label = TRUE, label_alpha = 0.5)
```

```
## Warning in ggcorr(df_mean, geom = "text", nbreaks = 5, hjust = 1, label = ## TRUE, : data in column(s) 'diagnosis' are not numeric and were ignored
```

	fractal_dimension						
	symmetry 0.5						
	concave.points 0.5 0.2						
	concavity 0.9 0.5 0.3						
compactness	0.9	0.8	0.6	0.6			
smoothness	0.7	0.5	0.6	0.6	0.6		
area	0.2	0.5	0.7	0.8	0.2	-0.3	a (0.6,1]
perimeter	1	0.2	0.6	0.7	0.9	0.2	-0.3
texture	0.3	0.3	0	0.2	0.3	0.3	0.1 -0.1
radius	0.3	1	1	0.2	0.5	0.7	0.8 0.1 -0.3

Even if compactness is define as $\text{perimeter}^2/\text{area} - 1.0$ the correlation between this variable and area or perimeter is not 1 because the correlation show only the linear dependency. The correlation between radius perimeter and area is 1. We will only keep radius.

From here let's remove area, perimeter and compactness

```
m_perimeter <- lm(data = df_mean, perimeter ~ radius + texture + area + smoothness + compactness + concavity)
summary(m_perimeter)
```

```
##
## Call:
## lm(formula = perimeter ~ radius + texture + area + smoothness +
##     compactness + concavity + concave.points + symmetry + fractal_dimension,
##     data = df_mean)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.7781 -0.1859  0.0416  0.2280  3.8482 
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 2.627e+00 8.317e-01  3.159  0.00167 ***
## radius      6.126e+00 5.223e-02 117.301 < 2e-16 ***
## texture    -2.105e-03 5.886e-03 -0.358  0.72073  
## area        3.868e-03 4.676e-04  8.272 9.65e-16 ***
## smoothness -8.998e+00 2.816e+00 -3.196  0.00147 ** 
## compactness 3.407e+01 1.517e+00 22.459 < 2e-16 ***
```

```

## concavity      3.865e+00 9.842e-01  3.927 9.67e-05 ***
## concave.points 4.283e+00 2.785e+00  1.538 0.12456
## symmetry       -1.930e+00 1.127e+00 -1.712 0.08751 .
## fractal_dimension -4.119e+01 8.190e+00 -5.029 6.64e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5538 on 559 degrees of freedom
## Multiple R-squared:  0.9995, Adjusted R-squared:  0.9995
## F-statistic: 1.214e+05 on 9 and 559 DF,  p-value: < 2.2e-16

m_area <-lm(data = df_mean,area~ radius+texture+ perimeter+smoothness+compactness+concavity+con
summary(m_area)

```

```

##
## Call:
## lm(formula = area ~ radius + texture + perimeter + smoothness +
##     compactness + concavity + concave.points + symmetry + fractal_dimension,
##     data = df_mean)
##
## Residuals:
##    Min      1Q  Median      3Q      Max
## -119.45 -25.53   -8.03  18.13  395.17
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1033.1844    56.7713 -18.199 < 2e-16 ***
## radius        -81.3418   22.3034  -3.647 0.00029 ***
## texture        0.4089   0.5023   0.814 0.41588
## perimeter      28.1972   3.4086   8.272 9.65e-16 ***
## smoothness     92.0982  242.5499   0.380 0.70431
## compactness   -2169.2200  153.3091 -14.149 < 2e-16 ***
## concavity      221.0651   84.6672   2.611 0.00927 **
## concave.points 295.3508  237.9177   1.241 0.21498
## symmetry       92.1348   96.4335   0.955 0.33978
## fractal_dimension 6413.4180  661.4625   9.696 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 47.28 on 559 degrees of freedom
## Multiple R-squared:  0.9822, Adjusted R-squared:  0.9819
## F-statistic: 3434 on 9 and 559 DF,  p-value: < 2.2e-16

```

```

m_compactness <-lm(data = df_mean, compactness~ radius+texture+ perimeter+smoothness+area+conc
summary(m_compactness)

```

```
##
```

```

## Call:
## lm(formula = compactness ~ radius + texture + perimeter + smoothness +
##      area + concavity + concave.points + symmetry + fractal_dimension,
##      data = df_mean)
##
## Residuals:
##       Min        1Q     Median        3Q       Max
## -0.044976 -0.006177 -0.000675  0.005421  0.058426
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.053e-01  1.457e-02 -14.091 < 2e-16 ***
## radius       -7.702e-02  4.234e-03 -18.189 < 2e-16 ***
## texture       2.376e-04  1.185e-04   2.004  0.04555 *
## perimeter    1.392e-02  6.198e-04  22.459 < 2e-16 ***
## smoothness   1.759e-01  5.694e-02   3.089  0.00211 **
## area          -1.216e-04 8.592e-06 -14.149 < 2e-16 ***
## concavity     6.410e-02  1.998e-02   3.208  0.00141 **
## concave.points 1.077e-01  5.622e-02   1.915  0.05598 .
## symmetry      9.448e-02  2.250e-02   4.200  3.1e-05 ***
## fractal_dimension 2.348e+00  1.370e-01  17.136 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01119 on 559 degrees of freedom
## Multiple R-squared:  0.9558, Adjusted R-squared:  0.9551
## F-statistic:  1343 on 9 and 559 DF,  p-value: < 2.2e-16

```

The variables area, perimeter and compactness are well explained by the other variables (Adjusted R-squared very close to 1). So we discard them.

3 GLM

```
m <-glm(data = df_mean, diagnosis~ radius+texture+smoothness+concavity+concave.points+symmetry
summary(m)
```

```

## 
## Call:
## glm(formula = diagnosis ~ radius + texture + smoothness + concavity +
##      concave.points + symmetry + fractal_dimension, family = binomial,
##      data = df_mean)
##
## Deviance Residuals:
##       Min        1Q     Median        3Q       Max
## -2.35180 -0.13938 -0.03229  0.02046  3.15368

```

```

## 
## Coefficients:
##                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)           -28.38387   6.66946 -4.256 2.08e-05 ***
## radius                  0.88701   0.21852  4.059 4.93e-05 ***
## texture                 0.37262   0.06212  5.998 2.00e-09 ***
## smoothness                78.50170  32.64920  2.404  0.0162 *
## concavity                 15.52082   8.35462  1.858  0.0632 .
## concave.points            46.67203  26.16265  1.784  0.0744 .
## symmetry                  16.85783  10.75613  1.567  0.1170
## fractal_dimension -101.54448  61.26233 -1.658  0.0974 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## (Dispersion parameter for binomial family taken to be 1)
## 
## Null deviance: 751.44 on 568 degrees of freedom
## Residual deviance: 153.35 on 561 degrees of freedom
## AIC: 169.35
## 
## Number of Fisher Scoring iterations: 8

```

3.1 Validation

[751.44/153.35](#)

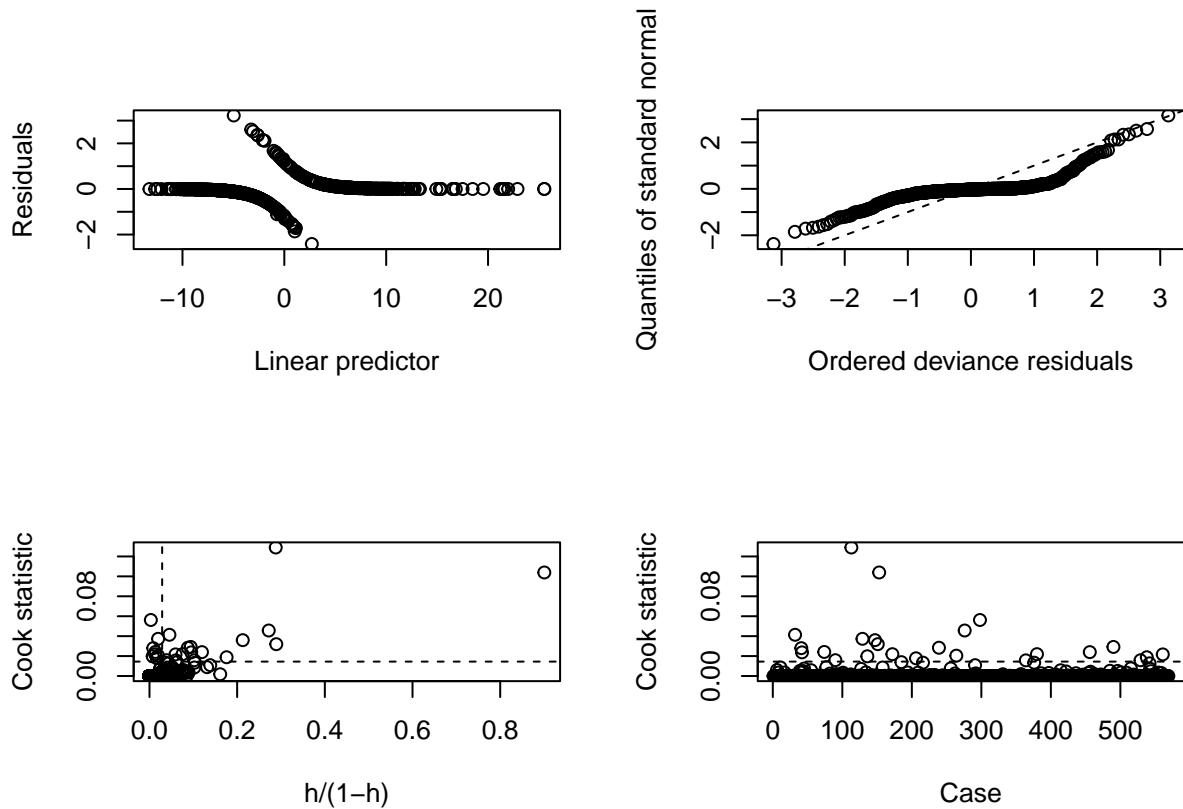
[## \[1\] 4.900163](#)

Overdistribution ?

```

library(boot)
diag <- glm.diag(m)
glm.diag.plots(m, diag)

```



Let's see with an anova test if we can remove symmetry,concavity,concave.points and fractal_dimension

```
m1 <- glm(data = df_mean, diagnosis ~ radius+texture+smoothness+concavity+concave.points+fractal_dimension, family = binomial, data = df_mean)
summary(m1)
```

```
##
## Call:
## glm(formula = diagnosis ~ radius + texture + smoothness + concavity +
##       concave.points + fractal_dimension, family = binomial, data = df_mean)
##
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max
## -2.33122 -0.15084 -0.03480  0.02274  3.04740
##
## Coefficients:
##                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)             -26.65706   6.59839 -4.040 5.35e-05 ***
## radius                  0.86784   0.22209  3.908 9.32e-05 ***
## texture                 0.36277   0.06098  5.949 2.70e-09 ***
## smoothness                90.53604  33.17393  2.729  0.00635 **
## concavity                 17.34487   8.23293  2.107  0.03514 *
## concave.points            45.65526  26.56395  1.719  0.08567 .
## fractal_dimension       -93.18039  59.52661 -1.565  0.11750
## ---
```

```

## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ',' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 751.44 on 568 degrees of freedom
## Residual deviance: 155.78 on 562 degrees of freedom
## AIC: 169.78
##
## Number of Fisher Scoring iterations: 8

```

```
anova(m,m1,test="Chisq")
```

Resid. Df	Resid. Dev	Df	Deviance	Pr(>Chi)
561	153.3485	NA	NA	NA
562	155.7755	-1	-2.426971	0.1192631

```
m2 <-glm(data = df_mean, diagnosis~ radius+texture+smoothness+concavity+concave.points,family=binomial)
summary(m2)
```

```

##
## Call:
## glm(formula = diagnosis ~ radius + texture + smoothness + concavity +
##       concave.points, family = binomial, data = df_mean)
##
## Deviance Residuals:
##      Min        1Q        Median        3Q        Max
## -2.28927 -0.15267 -0.03761  0.02390  3.03440
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -32.18048   5.69324 -5.652 1.58e-08 ***
## radius        0.99766   0.20762  4.805 1.55e-06 ***
## texture       0.36496   0.06131  5.953 2.64e-09 ***
## smoothness    72.72278  30.46830  2.387  0.0170 *
## concavity     10.21913   6.99120  1.462  0.1438
## concave.points 48.66262  26.54605  1.833  0.0668 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ',' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 751.44 on 568 degrees of freedom
## Residual deviance: 158.34 on 563 degrees of freedom
## AIC: 170.34
##
## Number of Fisher Scoring iterations: 8

```

```
anova(m1,m2,test="Chisq")
```

Resid. Df	Resid. Dev	Df	Deviance	Pr(>Chi)
562	155.7755	NA	NA	NA
563	158.3372	-1	-2.56172	0.1094794

```
m3 <-glm(data = df_mean, diagnosis~ radius+texture+smoothness+concave.points,family=binomial)
summary(m3)
```

```
##
## Call:
## glm(formula = diagnosis ~ radius + texture + smoothness + concave.points,
##      family = binomial, data = df_mean)
##
## Deviance Residuals:
##       Min        1Q        Median         3Q        Max
## -2.42132 -0.15010 -0.04247  0.02603  2.86598
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -28.57552   4.81406 -5.936 2.92e-09 ***
## radius        0.85081   0.17112  4.972 6.63e-07 ***
## texture       0.35845   0.05985  5.990 2.10e-09 ***
## smoothness    52.26403  26.08496  2.004  0.0451 *
## concave.points 78.73692  16.59332  4.745 2.08e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 751.44 on 568 degrees of freedom
## Residual deviance: 160.32 on 564 degrees of freedom
## AIC: 170.32
##
## Number of Fisher Scoring iterations: 8
```

```
anova(m2,m3,test="Chisq")
```

Resid. Df	Resid. Dev	Df	Deviance	Pr(>Chi)
563	158.3372	NA	NA	NA
564	160.3203	-1	-1.983059	0.1590686

3.2 Visualization

(to see) ## glm with glmnet Let's separate the dataset into train and test set

```

## 75% of the sample size
prop_train_test <- floor(0.75 * nrow(df_mean))

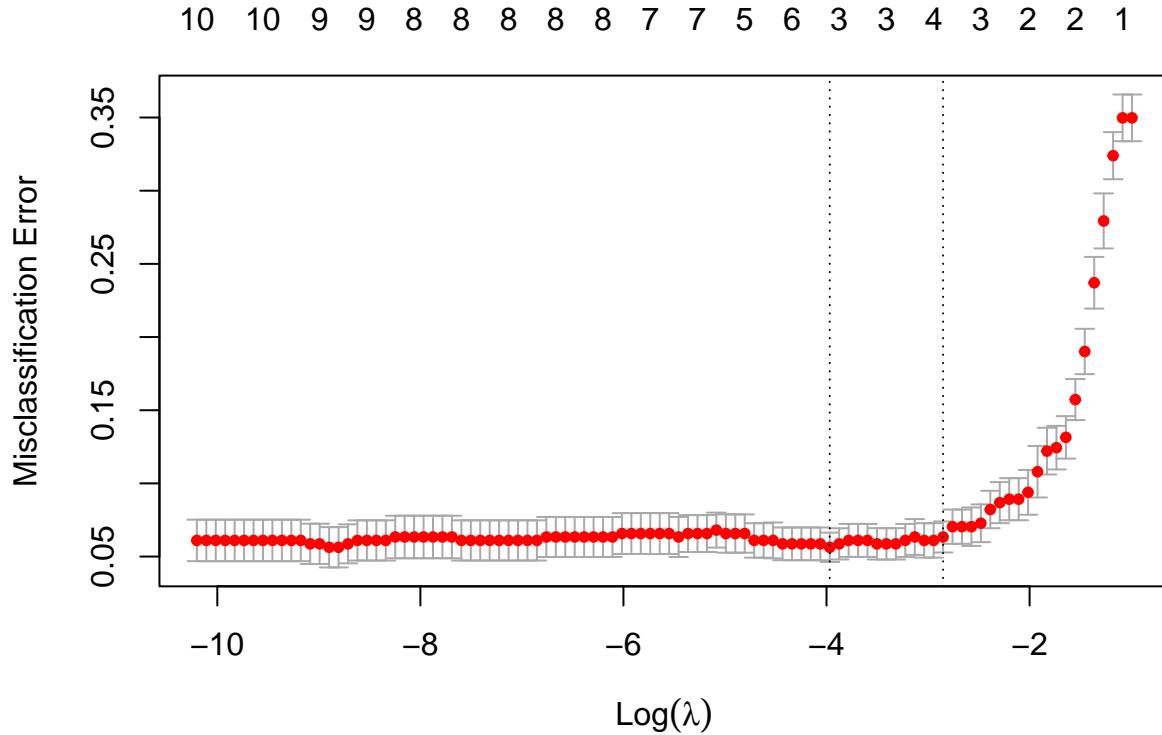
## set the seed to make your partition reproducible
set.seed(123)
train_ind <- sample(seq_len(nrow(df_mean)), size = prop_train_test)
#sample = sample.split(data$num, SplitRatio = .75)
#train = subset(data, sample == TRUE)
#test = subset(data, sample == FALSE)
train <- df_mean[train_ind, ]
test <- df_mean[-train_ind, ]

x_train <- train[,-1]
y_train <- train$diagnosis
x_test <- test[,-1]
y_test <- test$diagnosis

tol_length=length(levels(y_train))

# run glm : train and test with cross validation
cvfit<-cv.glmnet(as.matrix(x_train), y_train,family = "binomial", type.measure="class")
plot(cvfit)

```



```
cvfit$lambda.min
```

```
## [1] 0.01890567
```

```

assess<-assess.glmnet(cvfit,newx=as.matrix(x_test), newy=y_test, s='lambda.min')
confusion.glmnet(cvfit, newx =as.matrix(x_test), newy = y_test, s = 'lambda.min')

##          True
## Predicted B M Total
##      B    79 11   90
##      M     1 52   53
##      Total 80 63  143
##
##  Percent Correct:  0.9161

as.numeric(1-assess$class)

## [1] 0.9160839

coef(cvfit, s="lambda.min")

## 11 x 1 sparse Matrix of class "dgCMatrix"
##                               s1
## (Intercept)      -11.9549638
## radius           0.3363175
## texture          0.1750409
## perimeter        .
## area              .
## smoothness       .
## compactness      .
## concavity        .
## concave.points   63.6306102
## symmetry         .
## fractal_dimension .

```

Let's look with the first lambda:

```

cvfit$lambda.1se

## [1] 0.05773519

assess<-assess.glmnet(cvfit,newx=as.matrix(x_test), newy=y_test, s='lambda.1se')
confusion.glmnet(cvfit, newx =as.matrix(x_test), newy = y_test, s = 'lambda.1se')

##          True
## Predicted B M Total
##      B    80 13   93
##      M     0 50   50
##      Total 80 63  143
##
##  Percent Correct:  0.9091

```

```

as.numeric(1-assess$class)

## [1] 0.9090909

coef(cvfit, s="lambda.1se")

## 11 x 1 sparse Matrix of class "dgCMatrix"
##           s1
## (Intercept) -6.87636978
## radius       0.01235359
## texture      0.06882308
## perimeter    0.02960914
## area         .
## smoothness   .
## compactness  .
## concavity    .
## concave.points 40.52201326
## symmetry     .
## fractal_dimension .

```

4 PCA

(to do next week)

5 Annexe

5.1 More informations about the dataset

The 3-dimensional space is that described in: [3].

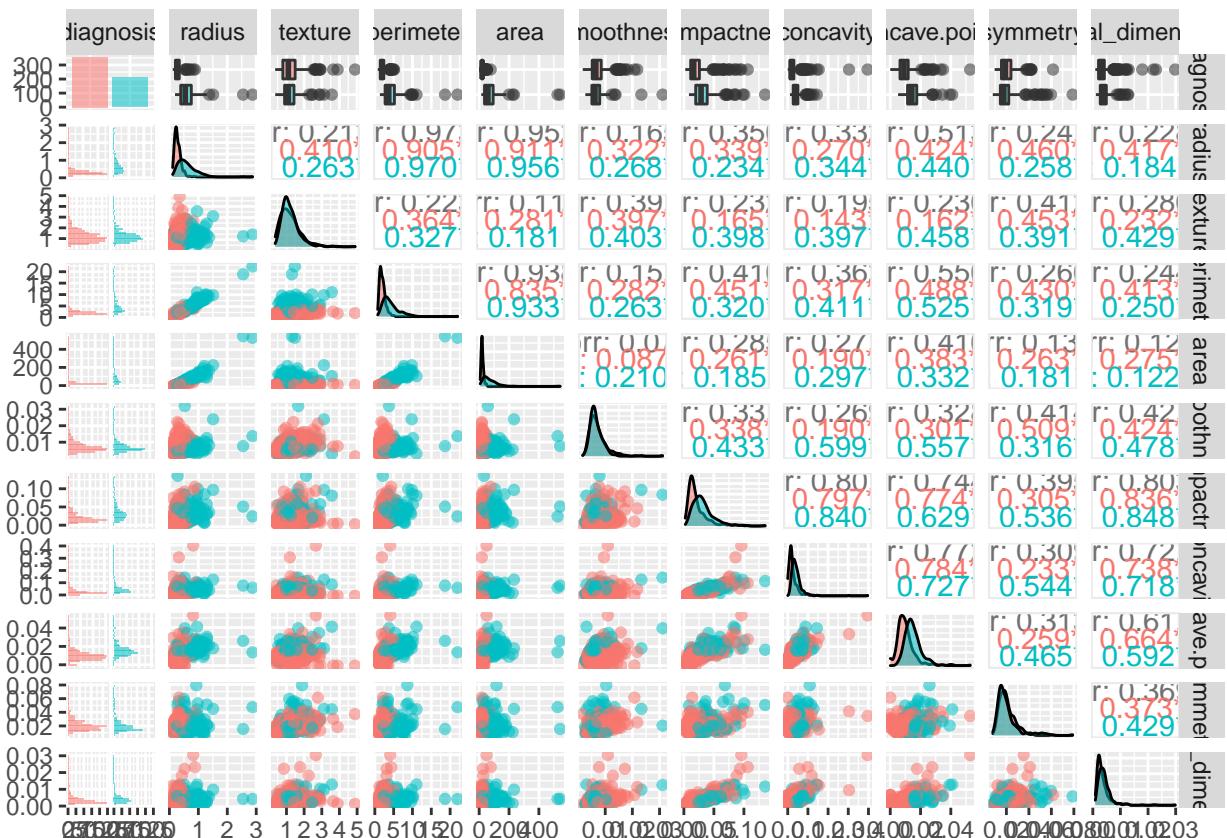
This database is also available through the UW CS ftp server: `ftp ftp.cs.wisc.edu cd math-prog/cpo-dataset/machine-learn/WDBC/`

5.2 Correlation in the ‘standard deviation’ and ‘worst’ group

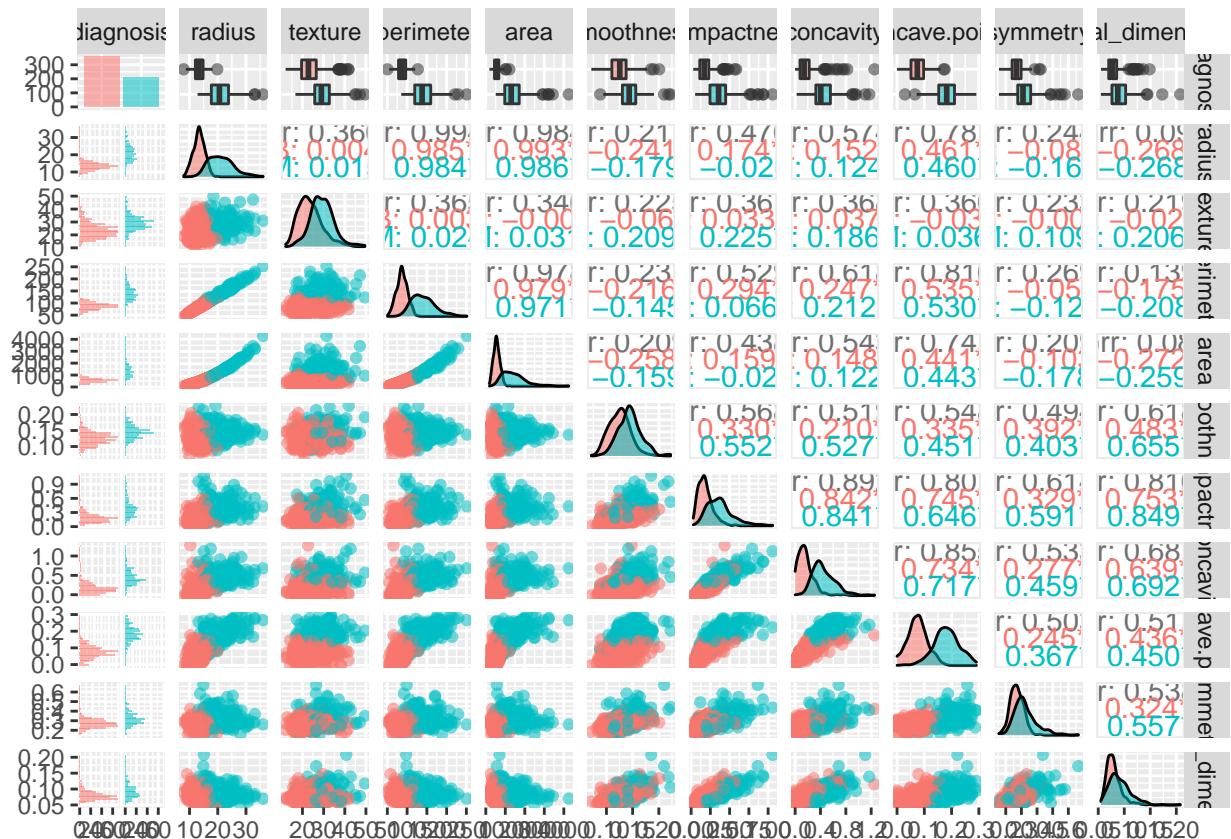
```

## 'data.frame': 569 obs. of 11 variables:
## $ diagnosis : Factor w/ 2 levels "B","M": 2 2 2 2 2 2 2 2 2 ...
## $ radius   : num  1.095 0.543 0.746 0.496 0.757 ...
## $ texture  : num  0.905 0.734 0.787 1.156 0.781 ...
## $ perimeter: num  8.59 3.4 4.58 3.44 5.44 ...
## $ area     : num  153.4 74.1 94 27.2 94.4 ...

```



```
## 'data.frame': 569 obs. of 11 variables:  
## $ diagnosis : Factor w/ 2 levels "B","M": 2 2 2 2 2 2 2 2 2 2 ...  
## $ radius    : num 25.4 25 23.6 14.9 22.5 ...  
## $ texture   : num 17.3 23.4 25.5 26.5 16.7 ...  
## $ perimeter : num 184.6 158.8 152.5 98.9 152.2 ...
```



We get the same results as for the mean group.

6 References

- [1] https://www.researchgate.net/figure/a-b-Fine-needle-aspiration-cytology-of-the-breast-lesion-showed-singly-lying_fig1_41548857
- [2] <https://pubmed.ncbi.nlm.nih.gov/7091922/>
- [3] K. P. Bennett and O. L. Mangasarian: “Robust Linear Programming Discrimination of Two Linearly Inseparable Sets,” Optimization Methods and Software 1, 1992, 23-34

7 Version of R used

```
## R version 4.1.1 (2021-08-10)
## Platform: x86_64-apple-darwin17.0 (64-bit)
## Running under: macOS Big Sur 10.16
##
## Matrix products: default
## BLAS:    /Library/Frameworks/R.framework/Versions/4.1/Resources/lib/libRblas.0.dylib
## LAPACK:  /Library/Frameworks/R.framework/Versions/4.1/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats      graphics   grDevices  utils      datasets   methods    base
##
## other attached packages:
## [1] boot_1.3-28      glmnet_4.1-3      Matrix_1.3-4      kableExtra_1.3.4
## [5] here_1.0.1       MASS_7.3-54       ggfortify_0.4.12  GGally_2.1.2
## [9] ggplot2_3.3.5    papaja_0.1.0.9997 pacman_0.5.1
##
## loaded via a namespace (and not attached):
## [1] Rcpp_1.0.7        svglite_2.0.0     lattice_0.20-44   tidyverse_1.1.4
## [5] rprojroot_2.0.2   digest_0.6.28     foreach_1.5.1     utf8_1.2.2
## [9] R6_2.5.1         plyr_1.8.6       evaluate_0.14    httr_1.4.2
## [13] highr_0.9        pillar_1.6.3     rlang_0.4.11     rstudioapi_0.13
## [17] rmarkdown_2.11    labeling_0.4.2    splines_4.1.1    webshot_0.5.2
## [21] stringr_1.4.0    munsell_0.5.0    compiler_4.1.1   xfun_0.26
## [25] pkgconfig_2.0.3   systemfonts_1.0.3 shape_1.4.6     htmltools_0.5.2
## [29] tidyselect_1.1.1  tibble_3.1.5     gridExtra_2.3    codetools_0.2-18
## [33] reshape_0.8.8    fansi_0.5.0      viridisLite_0.4.0 crayon_1.4.1
## [37] dplyr_1.0.7      withr_2.4.2     grid_4.1.1      gtable_0.3.0
## [41] lifecycle_1.0.1   magrittr_2.0.1    scales_1.1.1    stringi_1.7.5
## [45] farver_2.1.0     xml2_1.3.3      ellipsis_0.3.2  generics_0.1.0
## [49] vctrs_0.3.8      RColorBrewer_1.1-2 iterators_1.0.13 tools_4.1.1
## [53] glue_1.4.2       purrr_0.3.4     fastmap_1.1.0   survival_3.2-11
## [57] yaml_2.2.1       colorspace_2.0-2  rvest_1.0.2    knitr_1.36
```

8 References

James, William. 1890. "The Perception of Reality." *Principles of Psychology* 2: 283–324.