



## DiSPy Examples

### Oxygen Diffusion on Graphene

Oxygen diffusion across a carbon ring on graphene can be used as a simple example to demonstrate the distortion symmetry method. Details regarding this example can be found in Ref. [4]. Linearly interpolating between the end states shown generates an initial path with a distortion symmetry group of  $m^*m2^*$ . Images for this path can be found in the “init\_path” folder.

Running DiSPy with the sample input file will identify the distortion group, print out its irreducible representations, and perturb the images in the path according to distortion symmetry-adapted modes constructed using the  $\Gamma_2$  irreducible representation. This can be found as no. 1259 in the listing by Stokes *et al.* [3], and produces a new initial path with a distortion group of  $m^*$ .

### Ferroelectric Switching in $\text{Ca}_3\text{Ti}_2\text{O}_7$

Currently, there is a strong interest in ferroelectric materials that have a coupling between their polarization and another order parameter. Recently, a class of materials called hybrid improper ferroelectrics was predicted theoretically in  $n = 2$  Ruddlesden-Popper (RP) materials, and shown experimentally in  $\text{Ca}_3\text{Ti}_2\text{O}_7$  (CTO). This class of materials is characterized by their ferroelectricity being induced by octahedral rotations.

An initial ferroelectric switching pathway in  $\text{Ca}_3\text{Ti}_2\text{O}_7$  can be made by linearly interpolating between initial and final states of opposite polarization shown in Ref. [2]. This path has a distortion symmetry group of  $Cmcm^*$ , and can be found in the “init\_path” folder.

Running DiSPy with the sample input file will identify the distortion group, print out its irreducible representations, and perturb the images in the path according to distortion symmetry-adapted modes constructed using the  $Y_{4+}$  irreducible representation. This can be found as no. 2857 in the listing by Stokes *et al.* [3], and produces a new initial path with a distortion group of  $Pbcn^*$ .

## Ferroelectric Switching in $\text{LiNbO}_3$

$\text{LiNbO}_3$  is a commonly used ferroelectric material. The origin of its ferroelectricity can be traced to a polar mode associated with the displacement of the Li cations along the three-fold rhombohedral axis. Reversing the polarization of the structure then involves the motion of these cations along this axis [5].

Images for an initial ferroelectric switching pathway can be found in “init\_path”. These was constructed using a linear interpolation between the initial and final supercell images ( $2 \times 2 \times 2$  rhombohedral unit cells) of opposite polarization which can be found in Ref.[5]s.

Running DiSPy with the sample input file will identify the distortion group, print out its irreducible representations, and perturb the images in the path according to distortion symmetry-adapted perturbations constructed using the  $F_{1+}$  irreducible representation. This can be found as no. 7611 in the listing by Stokes *et al.* [3]

The  $F_{1+}$  irreducible representation is three-dimensional, and is associated with the  $k = (1/2, 1/2, 0)$  point. This means that coefficients for the order parameter vector need to be set in order to choose the appropriate isotropy subgroup, and that there will be a loss in translational symmetry after perturbation. This loss is characterized by all of the  $k$ -vectors in the star of the wave-vector  $\{k = (1/2, 1/2, 0), (0, 1/2, 1/2), (1/2, 0, 1/2)\}$ , and is appropriately accommodated by the  $2 \times 2 \times 2$  supercell.

Since we would like to break the most amount symmetry in this case, the isotropy subgroup associated with the kernel of the irreducible representation is desired. This corresponds to  $P\bar{1}^*$ , and can be obtained by choosing three distinct entries for the order parameter vector:  $\eta = (0.8, 1.0, 1.4)$ . This information can be found using the ISOSUBGROUP listing by Stokes *et al.* [3]

## Rigid Sliding in $\text{MoTe}_2$

The rigid sliding between layers in 2D materials is of interest to characterize atomic contributions to nanoscale friction in the material. This example concerns the rigid sliding path for  $\text{MoTe}_2$  shown in Ref. [1].

An initial path can be made by linearly interpolating between the initial and final states shown in Ref. [1]. This path has a distortion symmetry group of  $Cmcm$ , which does not contain distortion reversal symmetry (starred elements). Nonetheless, the same procedure may be applied.

Running DiSPy with the sample input file will identify the distortion group, print out its irreducible representations, and perturb the images in the path according to distortion symmetry-adapted perturbations constructed using the  $\Gamma_{3+}$  irreducible representation.

This can be found as no. 2836 in the listing by Stokes *et al.* [3], and produces a new initial path with a distortion symmetry group of  $C2/m$ .

## References

- [1] A. Cammarata and T. Polcar. Overcoming nanoscale friction barriers in transition metal dichalcogenides. *Physical Review B*, 96(8):085406, Aug. 2017.
- [2] E. a. Nowadnick and C. J. Fennie. Domains and ferroelectric switching pathways in  $\text{Ca}_3\text{Ti}_2\text{O}_7$  from first principles. *Physical Review B*, 94(10):104105, Sept. 2016.
- [3] H. T. Stokes, B. J. Campbell, and R. Cordes. Tabulation of irreducible representations of the crystallographic space groups and their superspace extensions. *Acta Crystallographica Section A Foundations of Crystallography*, 69(4):388–395, July 2013.
- [4] B. K. VanLeeuwen and V. Gopalan. The antisymmetry of distortions. *Nature Communications*, 6:8818, 2015.
- [5] M. Ye and D. Vanderbilt. Ferroelectricity in corundum derivatives. *Physical Review B*, 93(13):134303, Apr. 2016.