Espacio Proyectivo

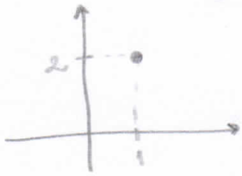
1) Transformar

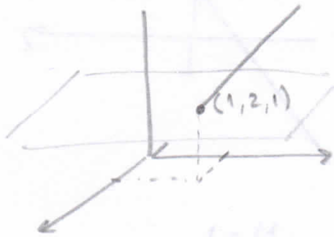
a) los puntos de $\mathbb{R}^2$ a P^2: $(1,2)$, $(-3,0)$ y $(0,8)$

$(1,2) \longrightarrow k(1,2,1) \in P^2$

$(-3,0) \longrightarrow k(-3,0,1)$

$(0,8) \longrightarrow k(0,8,1)$





b) los puntos de P^2 a $\mathbb{R}^2$: $(1,2,1)$
$(-3,0,2)$ y $(0,8,0)$

$(1,2,1) \longrightarrow (1,2)$

$(-3,0,2) \longrightarrow (-3/2\,;\,0)$

$(0,8,0) \longrightarrow$ punto en el infinito

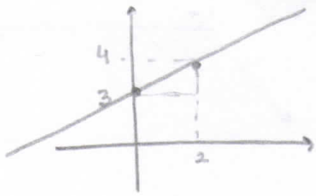
2) Escribir

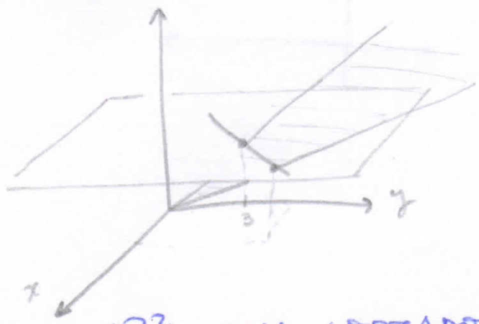
a) el vector homogéneo de P^2 que corresponde a la recta en $\mathbb{R}^2$

$y = 1/2\,x + 3 \longrightarrow -\frac{1}{2}x + y - 3 = 0 \Rightarrow l = \underbrace{(-1/2, 1, -3)}_{\text{vector normal al plano que representa la recta}}$

las coordenadas homogéneas

$(x,y,z) \in l \Leftrightarrow (x,y,z)\cdot(-1/2,1,-3) = 0$.





b) la ecuación de la recta en $\mathbb{R}^2$ que corresponde al vector homogéneo de P^2 $(2,4,8)$

$2x + 4y + 8 = 0 \Rightarrow 4y = -2x - 8$

$y = -\frac{1}{2}x - 2$

3) Utilizando la representación en vectores homogéneos de rectas y puntos en P^2 se pide:

a) Hallar la recta que pasa por los puntos $(1\ 1)$ y $(2,-1)$

$(1,1) \longrightarrow (1,1,1)$

$(2,-1) \longrightarrow (2,-1,1)$

$l = (1,1\,1) \times (2\ {-1}\ 1) = (2,1,-3)$

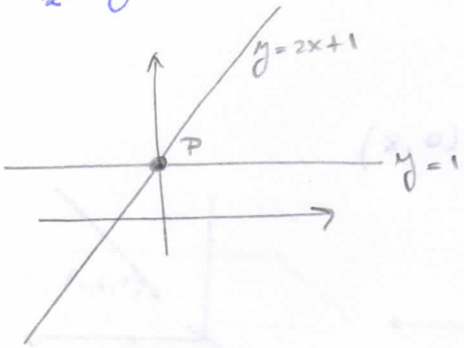
$l = (2,1,-3)$

$2x + y - 3 = 0$

$y = -2x + 3$

b) $l_1: y = 2x+1$ $\qquad l_1 = (-2, 1, -1)$ $\qquad x \in l_1 \iff l_1 . x = 0$

$l_2: y = 1$ $\qquad l_2 = (0, 1, -1)$ $\qquad x \in l_2 \iff l_2 . x = 0$



$\Rightarrow x \in l_1 \cap l_2 \iff \begin{matrix} l_1 . x = 0 \\ l_2 . x = 0 \end{matrix}$

$\Rightarrow x = (-2, 1, -1) \times (0, 1, -1)$
$= (0, -2, -2)$

$\boxed{x = (0, 1)}$

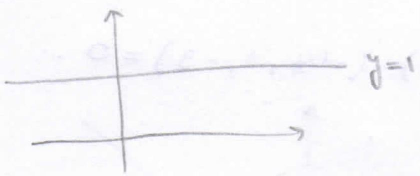
c) $y = 1$
$y = -1$ $\longrightarrow$ $l_1 = (0, 1, -1)$
Son paralelos $\qquad l_2 = (0, 1, 1)$

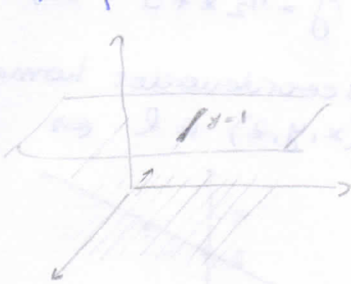
$x = (2, 0; 0) \longrightarrow$ punto en el infinito.

d) $y = 3x+1$ $\longrightarrow$ $l_1 = (-3, 1, -1)$
$y = 3x - 2$ $\qquad l_2 = (-3, 1, 2)$

$x = (3, 9, 0) \longrightarrow$ punto en el infinito

e) $y = 1$ y $l_\infty = (0, 0, 1)$ $\qquad x = (-1, 0, 0) \longrightarrow$ punto del infinito.
$\downarrow$
$l_1 = (0, 1, -1)$





Transformaciones Proyectivas

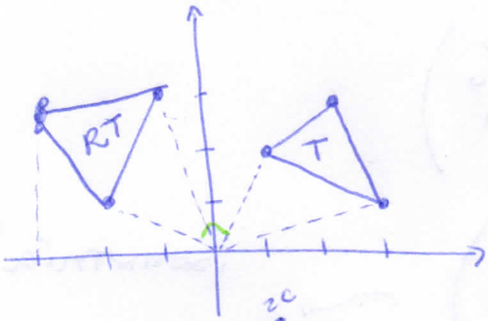
1) $A = (100, 200) \sim (100, 200, 1)$
$B = (300, 100) \sim (300, 100, 1)$
$C = (250, 300) \sim (250, 300, 1)$

a) $R = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ tomando $\theta = \pi/2$

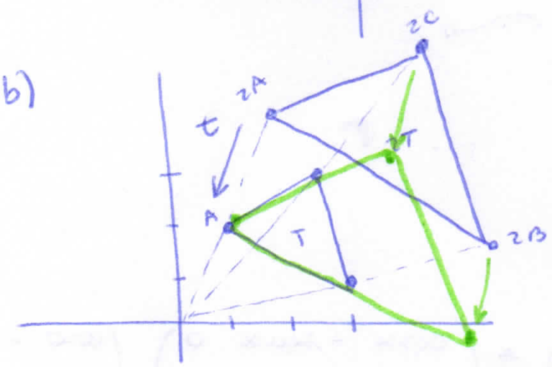
$R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Isometría

$R \cdot \begin{pmatrix} 100 \\ 200 \\ 1 \end{pmatrix} = \begin{pmatrix} -200 \\ 100 \\ 1 \end{pmatrix}$ $\quad R \cdot \begin{pmatrix} 250 \\ 300 \end{pmatrix} = \begin{pmatrix} -300 \\ 250 \\ 1 \end{pmatrix}$

$R \cdot \begin{pmatrix} 300 \\ 100 \\ 1 \end{pmatrix} = \begin{pmatrix} -100 \\ 300 \\ 1 \end{pmatrix}$



b)



$t = A - 2A$
$t = -A$

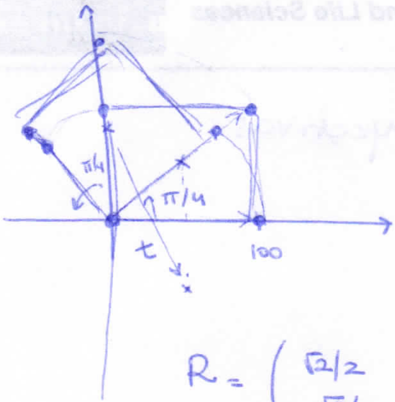
$M = \begin{pmatrix} 2 & 0 & -100 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{pmatrix}$

Similaridad

$\begin{pmatrix} 2 & 0 & -100 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 100 \\ 200 \\ 1 \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 1 \end{pmatrix}$ $\quad \begin{pmatrix} 2 & 0 & -100 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 250 \\ 300 \\ 1 \end{pmatrix} = \begin{pmatrix} 400 \\ 400 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 0 & -100 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 300 \\ 100 \\ 1 \end{pmatrix} = \begin{pmatrix} 500 \\ 0 \\ 1 \end{pmatrix}$

2)



$$T.R.\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$R = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R.\text{centro} = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 50 \\ 50 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} . 100 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2\sqrt{50} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 10\sqrt{2} \\ 1 \end{pmatrix}$$

$$t = \begin{pmatrix} 50 \\ -50 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 10\sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 50 \\ -50-10\sqrt{2} \\ 0 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 50 \\ 0 & 1 & -50-10\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underbrace{T.R.}_{M}\begin{pmatrix} 50 \\ 50 \\ 1 \end{pmatrix} = T\begin{pmatrix} 0 \\ 10\sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 50 \\ -50 \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 50 \\ \sqrt{2}/2 & \sqrt{2}/2 & -50-10\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix} \rightsquigarrow \text{isometría.}$$

$$M . \begin{pmatrix} 100 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 100\sqrt{2}/2 + 50 \\ 100\sqrt{2}/2 - 50 - 10\sqrt{2} \\ 1 \end{pmatrix}$$

3)

$$\begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha & 0 \\ \operatorname{sen}\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha & 0 \\ \operatorname{sen}\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta+\alpha) & -\operatorname{sen}(\theta+\alpha) & 0 \\ \operatorname{sen}(\theta+\alpha) & \cos(\theta+\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ya que

$$\cos(\theta+\alpha) = \cos(\theta).\cos(\alpha) - \operatorname{sen}\theta . \operatorname{sen}\alpha$$

$$\operatorname{sen}(\theta+\alpha) = \operatorname{sen}\theta . \cos\alpha + \cos\theta . \operatorname{sen}\alpha$$

b) $\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a+t_x \\ 0 & 1 & b+t_y \\ 0 & 0 & 1 \end{pmatrix} =$

$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x+a \\ 0 & 1 & t_y+b \\ 0 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha\beta & 0 & 0 \\ 0 & \alpha\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha\cos\theta & -\alpha\cdot\operatorname{sen}\theta & 0 \\ \alpha\operatorname{sen}\theta & \alpha\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} =$

4) $l_1 = (2,3,1)$

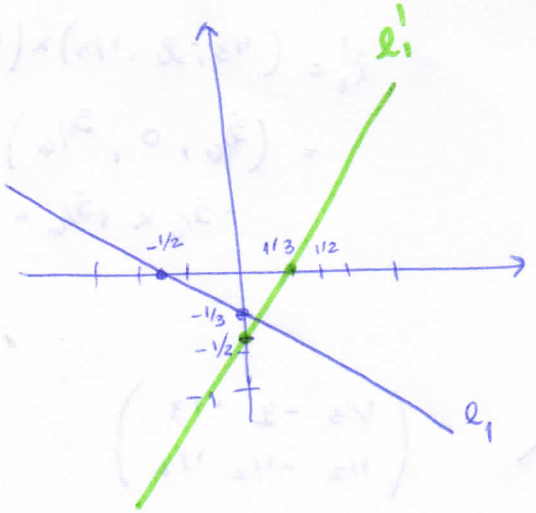
$l_2 = (4,6,5)$

$R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R^{-t} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

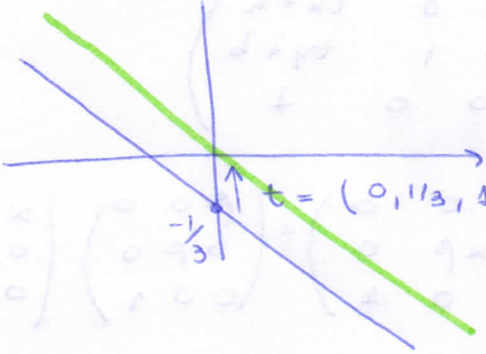
$R^{-t} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \longrightarrow \quad 2x+3y+1=0 \xrightarrow{R} -3x+2y+1=0$

$y = -\frac{2}{3}x + \frac{1}{3}$

$R^{t} \cdot \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 5 \end{pmatrix} \qquad 4x+6y+5=0 \xrightarrow{R} -6x+4y+5=0$



b) $$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{pmatrix}$$



$$T_1^{-t} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/3 & 1 \end{pmatrix}$$

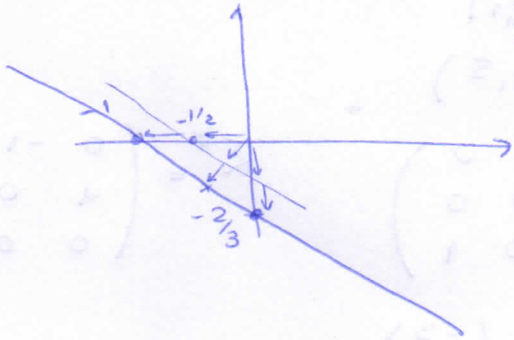
$$T_1^{-t} \cdot l_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$2x + 3y = 0$$

c) $$E_1 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_1^{-T} = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3/2 \\ 1 \end{pmatrix}$$

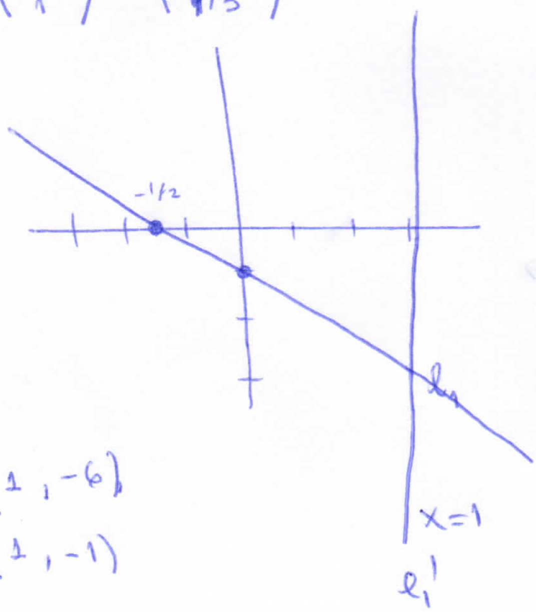
$$x + \frac{3}{2}y + 1 = 0.$$



d) $$M = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

$$M \cdot l_1 = l_1'$$

$$M \cdot \begin{pmatrix} 0 \\ -1/3 \\ 1 \end{pmatrix} = \begin{pmatrix} +1/3 \\ -2 \\ 1/3 \end{pmatrix} \qquad M \cdot \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$$



$(1, -6)$

$(1, -1)$

$$l_1' = (1/3, -2, 1/3) \times (1/2, -1/2, 1/2)$$
$$= (5/6, 0, 5/6)$$
$$-5/6\,x + 5/6 = 0.$$

$$\begin{pmatrix} 1/3 & -2 & 1/3 \\ 1/2 & -1/2 & 1/2 \end{pmatrix}$$

$$-1 + \frac{1}{6} = -5/6$$
$$-\frac{1}{6} + 1 =$$

5) $H_A(l_\infty) = l_\infty$ H_A afin

$l_\infty = (0,0,1)$

$$H_A(l_\infty) = H_A^{-t} \cdot l_\infty = \begin{pmatrix} A^{-t} & 0 \\ -tA^{-t} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = l_\infty$$

6) H_A es una tranformación afín $\Rightarrow$ existe H_A^{-t}

$x \in l \Rightarrow H_A(x) \in H_A(l)$

$x \in l \Leftrightarrow l^t . x = 0 \Rightarrow$ queremos ver que $H_A(x) \in H_A(l)$ es decir que $(H_A(l))^t . H_A(x) = 0$

$$(H_A^{-t} . l)^t . H_A . x = l^t . H_A^{-1} . H_A . x = l^t . x = 0$$

7) $l_1 // l_2$ q.v.q $H_A(l_1) // H_A(l_2)$ con H_A afín.

sabemos que si l_i es una recta $l_i' = H_A(l_i)$ también lo es

y además si $x \in l_i \Rightarrow H_A x \in H_A(l_i)$

con lo cual $x \in l_1 \cap l_2 \Rightarrow H_A(x) \in H_A(l_1) \cap H_A(l_2)$

y como $l_1 // l_2$ x es de la forma $(x_1, x_2, 0)$

luego $H_A(x) = \begin{pmatrix} A & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ 0 \end{pmatrix} \Rightarrow H_A(l_1) // H_A(l_2)$

8) H_A afín $\Rightarrow H_A = \begin{pmatrix} A & t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R(\theta) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R(\alpha) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I & t \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} R(\theta) . D . R(\alpha) & t \\ 0 & 1 \end{pmatrix}$$

$A = U . D . V^t = U . V^t . V . D . V^t$

$= R(\theta) . D . R(\alpha)$

con $D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

9) a) V $\longrightarrow$ $I = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & t_x \\ \operatorname{sen}\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$

b) F $\to$ tienen 3

$$I \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta x - \operatorname{sen}\theta y + t_x \\ \operatorname{sen}\theta x + \cos\theta y + t_y \\ 1 \end{pmatrix}$$

$$\sim (\cos\theta x - \operatorname{sen}\theta y + t_x ; \quad y + t_y)$$

$$I \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim (\cos\theta x' - \operatorname{sen}\theta y' + t_x , \operatorname{sen}\theta x' + \cos\theta y' + t_y$$

$$\operatorname{dist}(I\vec{x}, I\vec{x}') = \|(I\vec{x} - I\vec{x}')\| = \sqrt{}$$

$$\|(\cos\theta \cdot (x-x') - \operatorname{sen}\theta \cdot (y-y') ; \operatorname{sen}\theta \cdot (x-x') + \cos\theta \cdot (y-y'))\|$$

$$(\cos^2\theta \cdot (x-x')^2 - 2 \cdot \cos\theta \cdot \operatorname{sen}\theta \cdot (x-x') \cdot (y-y') + \operatorname{sen}^2\theta (y-y')^2 +$$

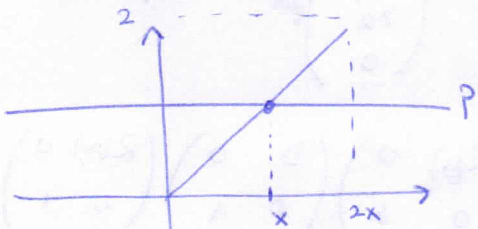
$$+ \operatorname{sen}^2\theta (x-x')^2 + 2 \cdot \cos\theta \cdot \operatorname{sen}\theta (x-x') \cdot (y-y') + \cos^2(\theta) \cdot (y-y')^2)^{1/2}$$

$$= ((x-x')^2 + (y-y')^2)^{1/2} = \operatorname{dist}(x, x')$$

c) F tienen 4

d) V

10) $h : P^1 \longrightarrow P^1 \implies h(\vec{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$



$$x_i = \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix}$$

$(x, 1) \sim x$

$(2x, 2) \sim \frac{2x}{2} \sim x$

veamos que $|H \cdot x_i , H x_j| = \det H \cdot |x_i , x_j|$

$$H \cdot x_i = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} = \begin{pmatrix} a x_{i1} + b x_{i2} \\ c x_{i1} + d x_{i2} \end{pmatrix} \qquad H \cdot x_j = \begin{pmatrix} a x_{j1} + b x_{j2} \\ c x_{j1} + d x_{j2} \end{pmatrix}$$

$$|H x_i , H x_j| = \det \begin{pmatrix} a x_{i1} + b x_{i2} & a x_{j1} + b x_{j2} \\ c x_{i1} + d x_{i2} & c x_{j1} + d x_{j2} \end{pmatrix}$$

$= \left(a\cdot x_{i_1} + b x_{i_2}\right)\cdot\left(c x_{j_1} + d x_{j_2}\right) - \left(a x_{j_1} + b x_{j_2}\right)\cdot\left(c x_{i_1} + d x_{i_2}\right)$

$= \cancel{ac\cdot x_{i_1} x_{j_1}} + ad\, x_{i_1}\cdot x_{j_2} + b\cdot c\, x_{i_2} x_{j_1} + \cancel{b\cdot d\, x_{i_2} x_{j_2}}$

$\cancel{-ca\, x_{j_1} x_{i_1}} - a\cdot d\, x_{j_1} x_{i_2} - b\cdot c\cdot x_{j_2} x_{i_1} - \cancel{b\cdot d\cdot x_{j_2}\cdot x_{i_2}}$

$= a\cdot d\cdot \underbrace{\left(x_{i_1}\cdot x_{j_2} - x_{j_1} x_{i_2}\right)}_{|x_i,\, x_j|} - b\cdot c\cdot\left(x_{i_1} x_{j_2} - x_{j_1} x_{i_2}\right)$

$= |x_i, x_j|\cdot\left(\underbrace{ad - b\cdot c}_{\det(H)}\right) = \det H\cdot |x_i, x_j|$

luego

$$\mathrm{Cross}\left(h(x_1), h(x_2), h(x_3), h(x_4)\right) = \frac{|h(x_1), h(x_2)|\cdot|h(x_3), h(x_4)|}{|h(x_1)\, h(x_3)|\;|h(x_2), h(x_4)|}$$

$$= \frac{\cancel{(\det H)^2}\cdot |x_1, x_2|\cdot|x_3, x_4|}{\cancel{(\det H)^2}\cdot |x_1, x_3|\cdot|x_2\, x_4|} = \mathrm{Cross}(x_1 x_2 x_3 x_4)$$

11) l_1 y l_2 son paralelos $\Rightarrow$ $l_1 = (a, b, c)$

$l_2 = (ka, kb, c')$

$l_\infty = (0, 0, 1)$

$x = l_1 \times l_2 = \left(b\cdot c' - c\cdot kb,\; -a\cdot c' + c\cdot k\cdot a,\; a k b - b\cdot k\cdot a\right)$

$= (a', b', 0) \Rightarrow$ punto en el infinito.

$l_1 \cap l_\infty \Rightarrow l_1 \times l_\infty = (b, -a, 0)$

$l_2 \times l_\infty = (kb, -k\cdot a, 0)$

representan el mismo punto.

$\left(b\cdot(c' - ck),\; -a\cdot(\underbrace{c' - ck}_{\text{cte}}),\; 0\right)$

$\Leftarrow)$

12) Si H_A es una afinidad $\Rightarrow$ $H_A(l_\infty) = l'_\infty$

$$H_A(l_\infty) = H_A^{-t} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} A^{-t} & 0 \\ -t^t \cdot A^{-t} & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = l'_\infty$$

$\Rightarrow)$ Sea $H_A = \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & v_3 \end{pmatrix}$

sup

$l'_\infty = H_A(l_\infty) = \cancel{H_A(1,0,0) \times H_A(0,1,0)}$

$l_\infty = (0,0,1)$

$x \in l_\infty \Leftrightarrow l_\infty^t x = 0$ tomo $x_1 = (1,0,0) \in l_\infty$

$x_2 = (0,1,0)$

luego $H_A(x_1)$ y $H_A(x_2) \in l'_\infty$

$\Rightarrow$ $l'_\infty = H_A(x_1) \times H_A(x_2)$

$$H_A(x_1) \cdot \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & v_3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ v_1 \end{pmatrix}$$

$$H_A(x_2) = \begin{pmatrix} a_{12} \\ a_{22} \\ v_2 \end{pmatrix}$$

$a_{11} \quad a_{21} \quad v_1$

$a_{22} \quad a_{22} \quad v_2$

$$l'_\infty = (\underbrace{a_{21} v_2 - a_{22} v_1}_{=0}, \underbrace{v_1 \cdot a_{21} - v_2 a_{11}}_{=0}; a_{11} a_{22} - a_{21} a_{12})$$

$(x,y,0) \in l_\infty \Rightarrow H_A \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \in l'_\infty$

$$H_A \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} x a_{11} + y \cdot a_{21} \\ x a_{21} + y \cdot a_{22} \\ x \cdot v_1 + y \cdot v_2 \end{pmatrix}$$

$x \cdot v_1 + y v_2 = 0 \quad \forall x, y$

$x = y = 1 \Rightarrow v_1 = -v_2$

$x = 1 \quad y = -1 \quad v_1 = v_2$

$\Rightarrow v_1 = v_2 = 0$

$\Rightarrow H_A = \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & v \end{pmatrix}$ afinidad!