Problem Set 4 – Maximum Likelihood Estimation

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Likelihood Function

We estimate the following model:

$$\ln(w_{i,t}) = \beta_0 + \beta_1\, Educi,t + \beta_2\, Agei,t + \beta_3\, Age_{i,t}^2 + \beta_4\, Blacki,t + \beta_5\, OtherRacei,t + \epsilon_{i,t},$$

where $\epsilon_{i,t} \sim N(0, \sigma^2)$.

The log-likelihood function for this model is:

$$\ell(\beta, \sigma^2 \mid y, X) = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(y - X\beta)^\top (y - X\beta),$$

with y the vector of log wages, X the matrix of regressors (intercept, education, age, age^2, Black, Other), and $\beta = (\beta_0, \beta_1, \ldots, \beta_5)$.

MLE Results

The table below shows the maximum likelihood estimates for each year:

Year	1971	1980	1990	2000
Education coefficient	0.0665	0.0660	0.0955	0.1102

Interpretation

The coefficient on education measures the percentage change in wages associated with an additional year of schooling, holding age and race constant.

- In 1971, one more year of education is associated with about a 6.7% increase in wages.
- In 1980, the effect is very similar, about 6.6%.
- By 1990, the return rises to roughly 9.6%.
- By 2000, the return increases further to about 11.0%.

Conclusion: The return to education increased steadily between 1971 and 2000, indicating that education became increasingly valuable in the labor market.

Screenshots

```
C:\Users\David Munson\Desktop\git\CompEcon_Fall25\Problemsets>pytest
================================================ test session starts ================================================
platform win32 -- Python 3.13.7, pytest-8.4.2, pluggy-1.6.0
rootdir: C:\Users\David Munson\Desktop\git\CompEcon_Fall25\Problemsets
collected 3 items

Test_ProblemSet4_MLE.py .                                                                                      [ 33%]
Test_ProblemSet4_Munson.py ..                                                                                  [100%]

================================================= 3 passed in 0.98s =================================================
```

Figure 1: Successful test results from `pytest`.

MLE Results:

Year	Success	LogLik	Intercept	Educ	Age	Age^2	Black	Other	Sigma^2
1971	True	-713.706788	0.586333	0.066500	0.064904	-0.000617	-0.164137	0.017512	0.164718
1980	True	-1142.420586	1.002273	0.066002	0.045569	-0.000399	-0.103028	0.012315	0.200524
1990	True	-1385.080151	0.277244	0.095499	0.057864	-0.000540	-0.167987	-0.052001	0.231832
2000	True	-2043.016695	-0.295246	0.110212	0.084495	-0.000888	-0.259771	-0.062168	0.285320

Figure 2: MLE results table.