Counting

Monday, 24 January 2022 12:40

Example:

A coin is fair if it comes up heads or tails with equal prob.

You flip a coin 3 times.
What is the probability of getting exactly one of the flips results in a head?

Ω : Sample Space. All possible outcomes.

There are 8 possible outcomes.

$\{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\} = \Omega$

$E = \{TTH, THT, HTT\}$

$$P(1 \text{ head in } 3 \text{ flips}) = \frac{\text{Number of outcomes with 1 head}}{\text{total number of outcomes.}} = \frac{3}{8}$$

A deck of 52 Cards has 13 ranks (2, 3, 4, ..., 10, J, Q, K, A) and

4 Suits (♡, ♠, ◇, ♣). A poker hand consist of 5 cards. A one-pair hand consists of 2 card having one rank and 3 card having other rank.

{2♡, 2♠, 5♡, 8♣, K◇}

same rank

One-pair hand.

Is it
(a) less than 5%
(b) b/w 5% — 10%
(c) b/w 10% — 20%
(d) b/w (20% — 40%
(e) greater than 40%

$P(\text{one-pair}) = \dfrac{\text{number of one-pair hand}}{}$

total number of hand.

$$\frac{\binom{13}{1}\cdot\binom{12}{3}\binom{4}{2}\binom{4}{1}^3}{{}^{52}C_5 = \binom{52}{5}} = 0.49$$

Def:

- A set 'S' is a collection of elements.

Element: $x \in S$

Subset: The set A is a subset of S if all of its elements are in S. $A \subset S$

Complement:

The set of elements of S

that are not in A.

$A^c = S - A$

Union: $A \cup B$. (A or B)

Intersection: A and B.

$A \cap B$.

Empty: $\emptyset$

Disjoint: A & B are disjoint if they have no common elements

$A \cap B = \emptyset$

Difference:

The set of elements in A that

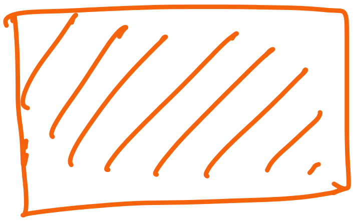
$A - B$.

DeMorgan's law.

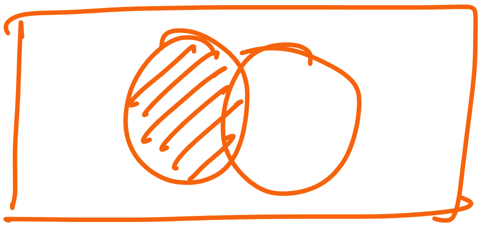
(1) $(A \cup B)^c = A^c \cap B^c$

(2) $(A \cap B)^c = A^c \cup B^c$.

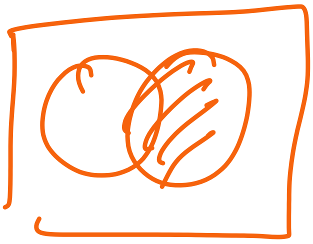
Venn Diagrams



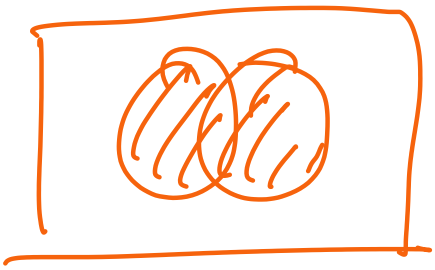
S



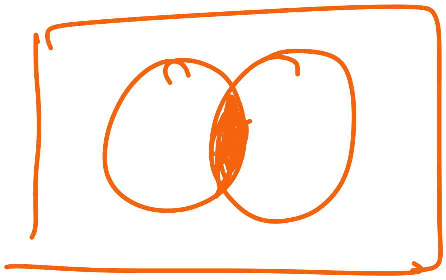
L



R

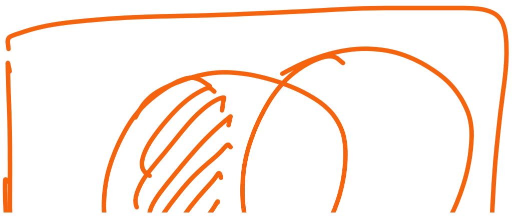


$L \cup R$



$L \cap R$

L^c



L - R