Probability Fundamentals

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Probability: Mathematical formalisation describing uncertain event.

Statistics: the practice of Collecting and analysing data.

Central Questions:

- why do we need this, is it useful?
    - Make inference about uncertain events.
    - Form the basis of information theory.
    - Test the strength of Statistical evidence.
- How is it possible to Say something about uncertain (or stochastic) events?
- How Can we measure uncertainty (or information)?

Example:
(a) Three differents Labs have measured the Speed of light with

measured its speed ... slightly different results. What is the true speed likely to be?

(b) Two drugs are compared. 5 out of 10 patients responded to treatment with drug A. whereas 7 out 10 responded to drug B. What do we conclude?

Probability Theory is the Calculus of uncertaint events

It enables one to infer probabilities of interest based on assumptions and observations

Example
probability of getting 2 heads when we toss a pair of coins is 1/4.

Frequentist Approach

Bayesian Approach

The probability of an event is defined as its long run frequency in a repeated experiment.

Example if you roll a fair die what is the probability of getting a 6. Ans: 1/6

ifinite time Approach

$P(6) \rightarrow 1/6$

Example: "There is a 50% chances that the arctic polar ice-cap will have melted by 2100"

it is not possible to define a repeated experiment.

Interpretation of prob. Subjective to degree of belief is possible here. This type of interpretation is called Bayesian interpretation

What is Randomness?

* event is random when it is uncertain whether it is going to happen or not.

Inherent Uncertainty → A

whether a radioactive atom may decay within some time interval.

lack of Knowledge

I may be uncertain about the number of legs my pet Centipede has

Random Sample

An opinion poll was based on telephone interviews of a representative Sample of a 994 voters.

The Foundations of Probability

3 Axioms

i) The probability of an Event E is a non-negative real number

$$P(E) \geq 0 \quad \forall\, E \subseteq \Omega$$

for all. events belongs to sample Space.

2) The certain event has unit probability.

$$P(\Omega) = 1.$$

3) (Countable) additivity:

for disjoint event: $E_1, E_2, E_3 \ldots$

$$P(E_1 \cup E_2 \cup E_3 \ldots \cup E_n) = \sum_{i=1}^{n} P(E_i).$$

Complementary Rule: $P(\Omega - E) = P(\bar{E}) = 1 - P(E)$

Impossible Events: $P(\emptyset) = 0$

If $E_1 \subseteq E_2$ then $P(E_1) \leq P(E_2)$

$\{2\} \subseteq \{2, 4, 6\}$ $\Omega = \{1, 2, 3, 4, 5, 6\}$

subset.

$$P(E_2) = \frac{3}{6} = 1/2$$

$$P(E_1) = \frac{1}{6}$$

$$1/6 \leq 1/2.$$

General Additive Rule:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$P(E_1 \cap E_2) = P(E_1, E_2)$$

joint probability!