permutations /combinations

Thursday, 27 January 2022 11:50

Product of sets

The product of sets S & T is the set of ordered pairs.

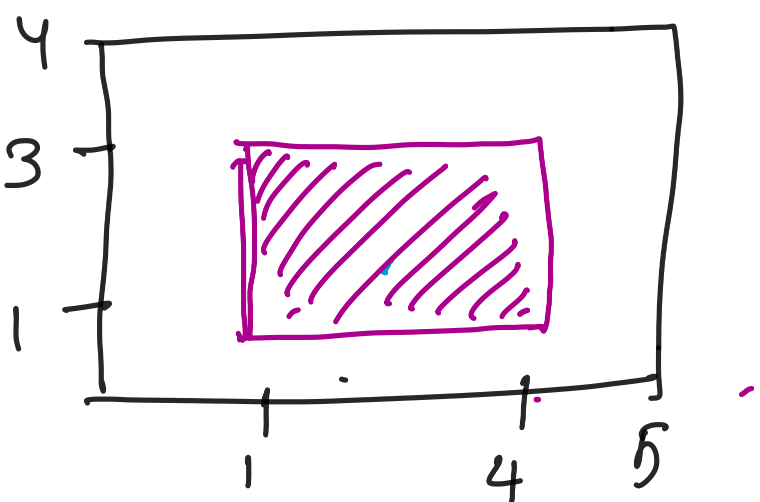
$$S \times T = \{(s,t) \mid s \in S, t \in T\}$$

Suppose we have two sets:

$$S = \{1,2,3\}$$
$$T = \{1,2,3,4\}$$

$S \times T$

×	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)



$$[1,4] \times [1,3] \subset [0,5] \times [0,4]$$

If $A \subset S$ and $B \subset T$ Then
$A \times B \subset S \times T$.

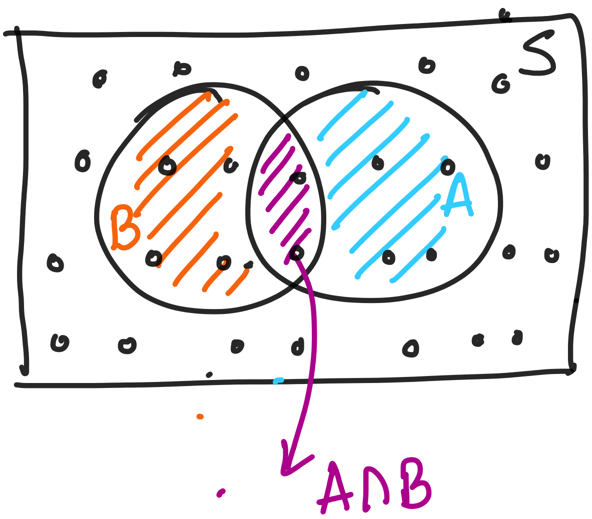
This means Subset.

Counting:

If S is finite we use $|S|$
or $\#S$ to denote the number of
elements in S.

Inclusion-Exclusion Principle:

$$|A \cup B| = |A| + |B| - |A \cap B|$$



$|A|$ is The number of dots in A
$|B|$ " " " " B

$|A| + |B|$ double-counting $|A \cap B|$

$|A \cup B| = |A| + |B| - |A \cap B|$.

Ex In a band of singers & guitarists 7 people sing, 4 play play the guitar, 2 do both. How big is the band?

S : Singers

G : Guitarists

Size of band $= |S \cup G| = |G| + |S| - |S \cap G| = 7 + 4 - 2 = 9$.

Rule of Product:

If there are n ways to perform action 1 and then m ways to perform action 2., then there are $n \cdot m$ ways to perform action 1 followed by action 2

1 followed by action 2.

"Rule of Multiplication"

Q: There are 5 competitors in the 100m final at the Olympics. In how many ways can the gold, silver and bronze medals be awarded?

$$5 \cdot 4 \cdot 3 = 60.$$

Permutations:

A permutations of a set is a particular ordering of its elements.

$\{a, b, c\}$: abc, acb, bac, bca, cab, cba.

$$3 \cdot 2 \cdot 1 = 6 = 3!$$

$$\boxed{k! = k \cdot (k-1) \ldots 3 \cdot 2 \cdot 1}$$

Permutations of 'k' things out of a set of 'n' things.

$\{a, b, c, d\}$

abc	acb	bac	cab	cba	bca
abd	adb	bad	bda	dab	dba
acd	adc	cad	cda	dac	dca
bcd	bdc	cbd	cdb	dbc	dcb.

There are 24 permutations.

$$4.3.2. = 24.$$

Combinations

Permutations are lists.

Combinations are set.

$\{a, b, c, d\}$

$\{a,b,c\}, \{a,b,d\}, \{a,c,d\}$
$\{b,c,d\}$.

Permutations: ${}_nP_k$

Combinations: ${}_nC_k = \binom{n}{k}$

$${}_nP_k = \frac{n!}{(n-k)!} = n(n-1)\dots(n-k+1)$$

$${}_nC_k = \frac{n!}{k!(n-k)!} = \frac{{}_nP_k}{k!}$$

Example:

(a) Number of ways to choose

6 ← 2 out 4.

12 ← (b) The Number of ways to list 2 out of 4 Things

120 ← (c) The number of ways to Choose 3 out of 10 Things:

(a) $\binom{4}{2} = \frac{4!}{2!2!} = 6$

(b) $4P_2 = \frac{4!}{2!} = 12$

(c) $\Rightarrow \binom{10}{3} = \frac{10!}{3!7!} = 120.$

Q: Cout The Number of ways to get (i) 3 heads in a sequence of 10 flips of a Coin.

(ii) if the coin is fair, what is the probability of exactly 3 heads in 10 flips.

(HHH)

(i) $\binom{10}{3} = \frac{10!}{3!7!} = \frac{10\cdot9\cdot8}{3\cdot2\cdot1} = 120$

(ii) $2^{10} = 1024$ sequences of 10 flips

Since the coin is fair each sequence is equally probable.

Probability of 3 Heads is

$$P(3\text{ heads}) = \frac{120}{1024} = 0.117$$