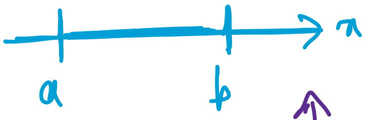
Distributions

Thursday, 10 February 2022 11:48

Continuous distribution function:

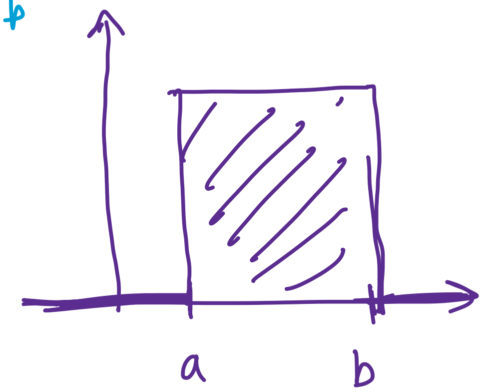
1) Uniform distribution;

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{otherwise} \end{cases}$$



$E(X) = (a+b)/2$

$Var(X) = \frac{1}{12}(b-a)^2$



$X \sim U(a,b)$

Simulations: Simulate a uniform random variable.
You can flip a coin repeatedly and define: $X_n = 1$ or $X_n = 0$

$$V = \sum_{n=1}^{\infty} \frac{X_n}{2} \sim U(0,1)$$ 'why 2'

If F is any c.d.f, The random variable

$$W = \inf\{x : F(x) \geq V\}$$

The smallest x such that $V \leq F(x)$, has distribution F.

2) Exponential Distribution:

$$F(x) = 1 - e^{-\lambda x} \qquad x > 0$$
$$= 0 \qquad x \leq 0$$

is called exponential with rate $\lambda > 0$. [

p.d.f f given by.

$$\boxed{f(x) = \lambda \exp(-\lambda x) \qquad x \geq}$$

A random variable with the exponential dist.

the called memoryless property.

$$P(X > t+s \mid X > s) = P(X > t) \text{ for}$$

Important in Modelling waiting times in Pr

Processes.

Mean: $E(X) = 1/\lambda$

$Var(X) = 1/\lambda^2$.

3) Gaussian Distribution (Normal dist.)

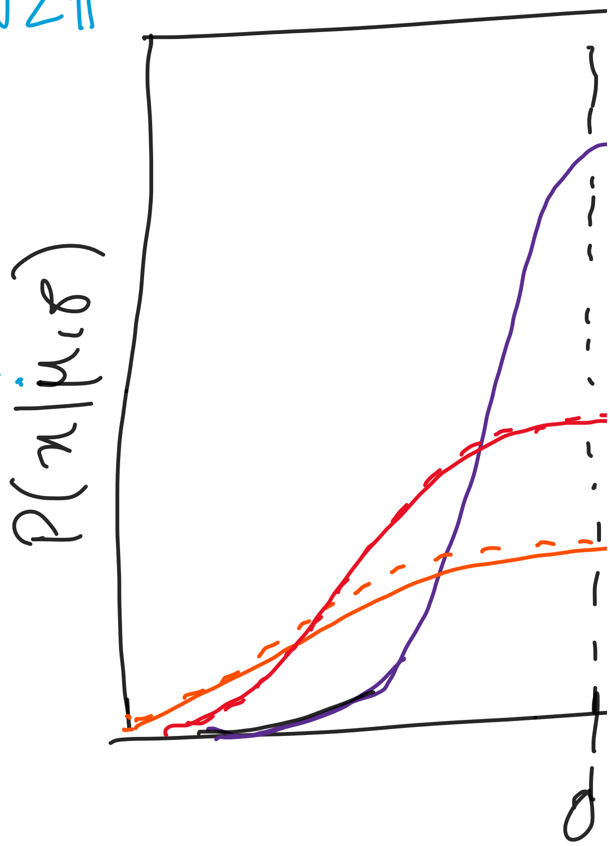
$$p(x \mid \mu, \sigma) = \frac{1}{\ } \exp\left(-\frac{(x-\mu)^2}{\ }\right.$$

$$P(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{\quad}{2\sigma^2} \right)$$

$N(\mu, \sigma^2)$

Mean : $E(x) = \mu$

Var : $Var(x) = \sigma^2$.



→ Symmetric.

- Skewness = 0
- Kurtosis = 0.

higher order Moments.

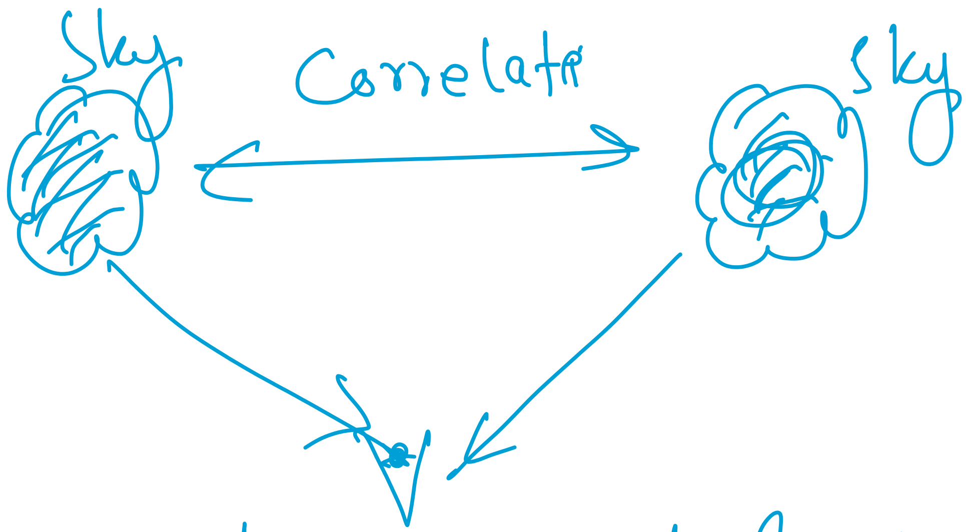
- all the statistical properties are Mean and Standard deviation.

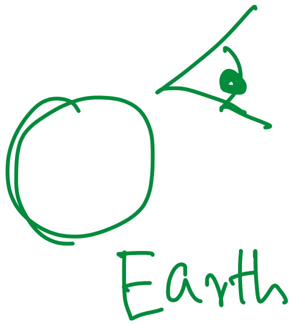
Cosmology : fluctuations in the temper

Cosmic Microwave Backgrou

→ Gaussian distributed.

– Two point function (power spectrum)





– higher point functions

Convolution

two functions f(x) and g(x

the convolution of these two functi

defined:

$$(f*g)(x) = \int_{-\infty}^{+\infty} f(x')g(x-$$

$$= \int^{+\infty} f(x-x')\, g$$

$-\infty$

The convolution of two gaussian f
a Gaussian function. $N(\mu_0, \sigma$

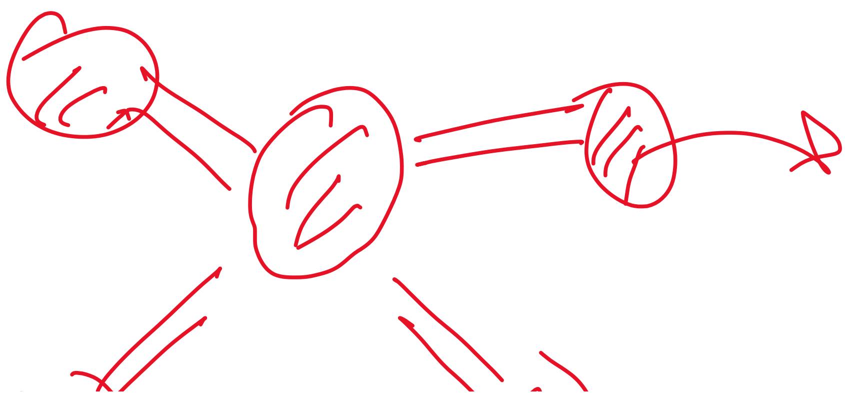
Cumulative distribution

$$P(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(}{}\right.$$

(Gauss error function)

$$\mathrm{err}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2$$

dark matter halo formation →
and



galaxies a

4) logNormal distribution:

For a random variable $X \approx N$

$Y = e^X$ has a lognormal

$$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left\{-\frac{(\ln(x}{2}\right.$$

$$E[Y] = e^{\mu + (1/2)\sigma^2}$$

$$Var[Y] = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}.$$

- Law of large Number
- Central limit theorem.

it tells us two things

(a) The average of many independen
(with high probability) close
of the underlying dist.

(b) density histogram of many
Samples (with high prob) is
graph of the density of the

$X_1, X_2, \ldots X_n$ indepe

$$\bar{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

Value & distribution of $\bar{X}_n$

LoLN: As n grows, the probe
is close to μ is 1.

CLT: As n grows, the distri[bution]
Converges to the normal
$N(\mu, \sigma^2/n)$.

$$S_n = X_1 + X_2 + \dots + X_n =$$

$$E(S_n) = n\mu. \quad var(S_n) = n\sigma$$

$$E(\bar{X}_n) = \mu \quad var(\bar{X}_n) = \sigma$$

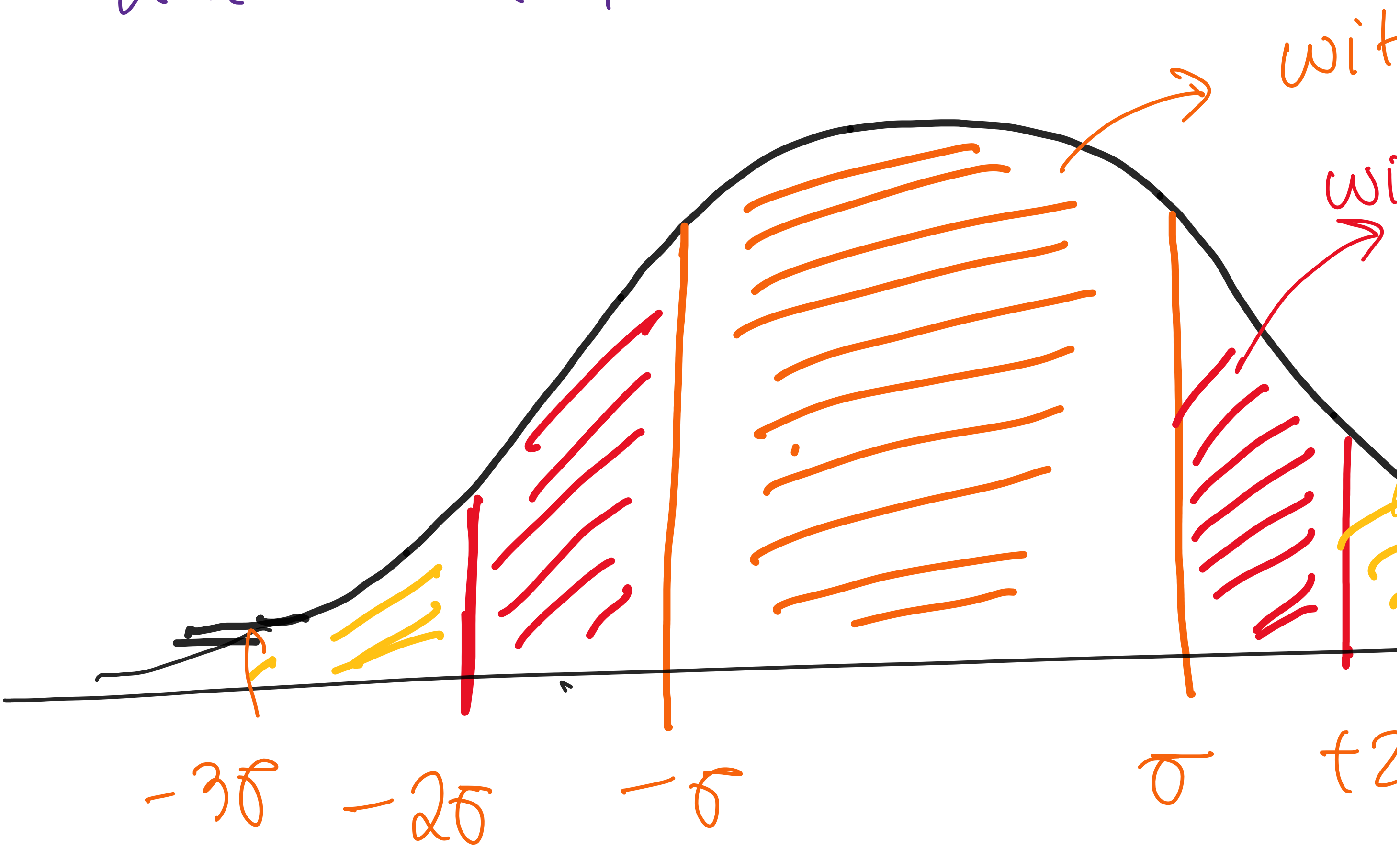
$$\boxed{\sigma_{\bar{X}_n} = \frac{\sigma}{\sqrt{n}}.}$$ → Popu

$$\bar{X}_n \sim N(\mu, \sigma^2/n)$$

$$S_n \sim N(n\mu, n\sigma^2)$$

Z

$Z \sim N(0,1)$



1. $P(|Z| < 1) = 0.68$
2. $P(|Z| < 2) = 0.95$
3. $P(|Z| < 3) = 0.997$

Claim:

(a) $P(Z < 1) = 0$

(b) $P(Z < 2) = 0$

(c) $P(Z < 3) = 0$

HW!

Next lecture:

- Some exam
- Quantile-((python).
- few examples Covariances