Statistical inference

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probability of data ← Know parameters of model.

PART A:

- $N(\mu, \sigma^2)$
- $Bin(n, p)$
- $Ber(1, p)$
- $exp(\lambda)$

STATISTICAL INFERENCE:

estimate the probability of parameters ← Give the observed data / Model

PART B

1. CLASSICAL INFERENCE ← likelihood function

↑ hypothesis testing

2. Bayesian Inference. ← likelihood function + Bayes theorem.

Statistical Inference

Estimation. | testing of hypothesis.

- Parametric inference methods
- Non-parametric method.
- Semi-parametric inference methods.

- Regression analysis.
- Goodness of fit.
- Classification methods.

1) Point Estimation:

- Poisson distribution → estimate 'λ'
- data is drawn from a normal distribution you want to compute 'μ' and 'σ'

2) Likelihood Methods

One of the most popular methods of point estimation is called the maximum likelihood estimation. (MLE).

probability → future events.

likelihood → refers to the past events with known outcomes.

3) Confidence Interval : — Fisher Matrix

4) Resampling Method :

- Understanding the variability of a point estimation
- Construct hypothetical populations from the observations.
- Bootstrap Methods
- Resampling the original data preserves whatever structures are truly present in the underlying population.

5) Testing of hypothesis

6) Bayesian Inference :

Modern. ; "Prior"

MAXIMUM LIKELIHOOD ESTIMATION:

Question:

"For which parameter value does the observed data have the biggest probability?"

Example: A coin is flipped 100 times. Given that there are 55 heads, find the MLE for the probability p of heads on a single toss?

You know your model is Binomial distribution.

$X \sim B(n,p)$ $\quad n = 100$, p is unknown.

$$P(55 \text{ heads}) = \binom{100}{55} p^{55}(1-p)^{45} \leftarrow$$

$$P(55 \text{ heads} \mid p) = \binom{100}{55} p^{55}(1-p)^{45} \doteq P(55 \text{ heads})$$

↑ conditional probability

"The probability of 55 heads given p"

"the probability of 55 heads given that the probability of heads on a single toss is p"

Standard Term:

1. Experiment: flip the coin 100 times and count the Number of heads.

2. Data: the data is the result of the experiment. In this case 55 heads

3. Parameters: unknown parameter p.

4. likelihood function: $P(\text{data} \mid p)$

$$P(55\,\text{heads} \mid p) = \binom{100}{55} p^{55} (1-p)^{45}.$$

Maximize the likelihood given p.

$$\frac{d}{dp} P(\text{data} \mid p) = \binom{100}{55}\left\{55 p^{54}(1-p)^{45} - 45 p^{55}(1-p)^{44}\right\} = 0$$

$$55 p^{54}(1-p)^{45} = 45 p^{55}(1-p)^{44}$$

$$55(1-p) = 45p.$$

$$55 = 100p.$$

$$\boxed{\text{MLE is } \hat{p} = 0.55.}$$

- log likelihood function: