Joint distribution

Monday, 21 February 2022 10:52

- joint distribution
- Covariances & Correlations.
- Bivariate or Multi-variate Gaussian Distribution. ← (Covariances)

Discretes Case:

Suppose we have X & Y

$X = \{x_1, x_2, \dots x_n\}$

$Y = \{y_1, y_2, \dots y_m\}$

Ordered Pair: $\{(x_1, y_1), (x_2, y_2), \dots (x_n, y_m)\}$.

The joint probability function : $P(x_i, x_j)$

$X = x_i \; , \; Y = y_j$

X/Y	y_1	y_2	...	y_j	...	y_m
x_1	$P(x_1 y_1)$	$P(x_1 y_2)$	...	$P(x_1 y_j)$	...	$P(x_1 y_m)$
x_2	$P(x_2 y_1)$	$P(x_2 , y_2)$	...	$P(x_2 y_j)$	...	$P(x_2 y_m)$
$\vdots$						
x_i	$P(x_i , y_1)$	$P(x_i , y_2)$	...	$P(x_i , y_j$	...	$P(x_i , y_m)$
$\vdots$						
x_n	$P(x_n , y_1)$	$P(x_n , y_2)$		$P(x_n , y_j)$		$P(x_n , y_n$

$\text{Dim}(X) = n$

$Y \to \dim(Y) = m$

Ex: Roll two Dice:

X: value on the first dice.

Y: value on the Second dice.

X, Y can take 1 ... 6

$P(i, j) = 1/36$

$x = 5$, $y = 6$ or $x = 4$, $y = 3$

X/Y	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

$1/36 + 1/36 = 1/18$

$=$

What is $P(X=3)$

Two properties for joint distribution

1. $0 \leq P(x_i, y_j) \leq 1$
2. Total probability is 1.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) = 1$$

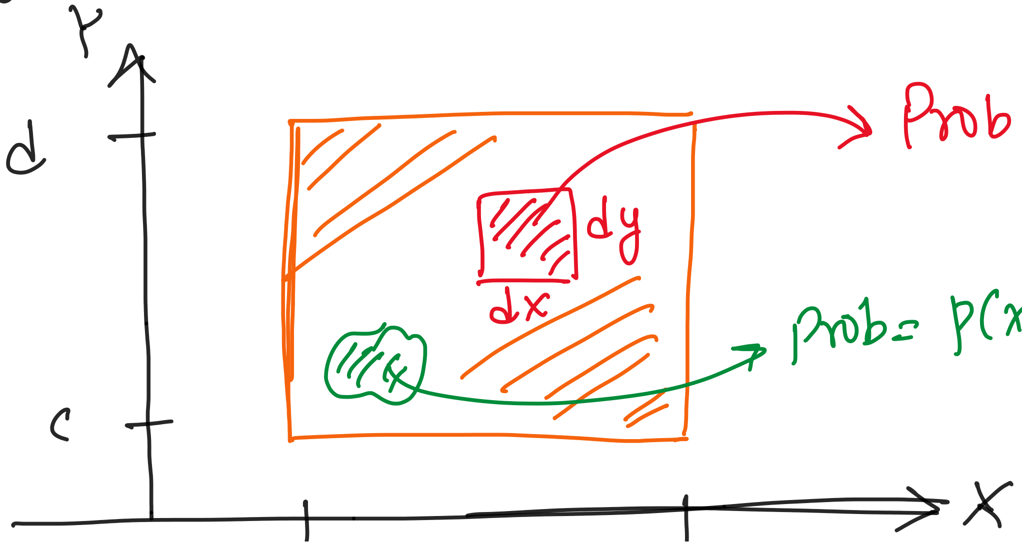
Continuous Case:

$X \in [a,b]$

$Y \in [c,d]$

Ordered pair: $X \times Y \in [a,b] \times [c,d]$

joint probability dist. $p(x,y)$ (or $f(x,y)$)



Prob $= p(x,y)\,dx\,d$

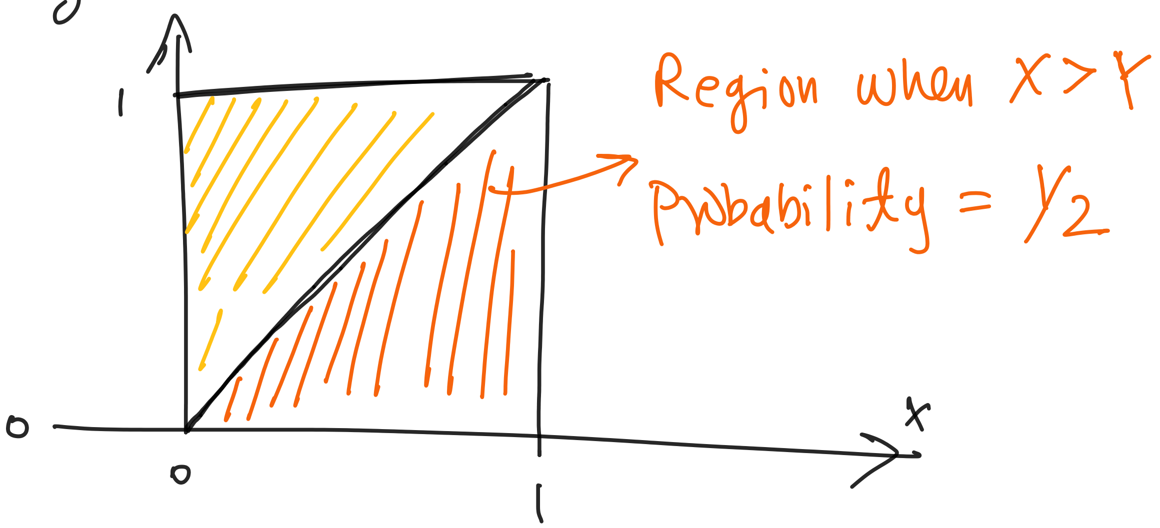
Prob $= p(x,y) \times$ area of the green region.

Two properties

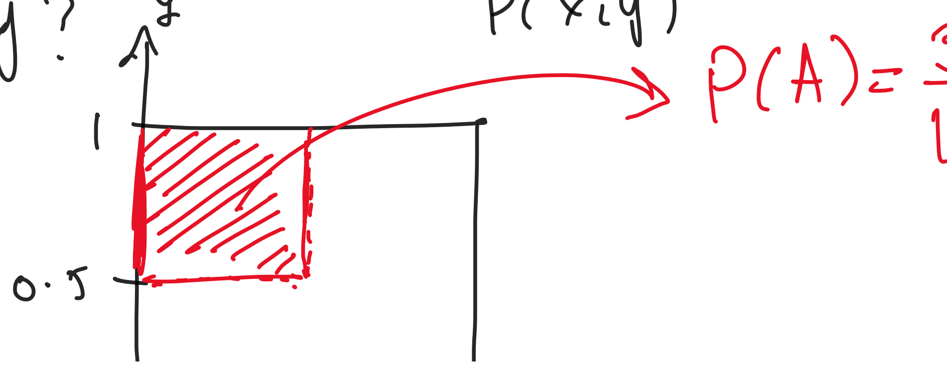
(1) $0 \leq p(x,y)$ → joint dist can take val greater than 1.

(2) $\int_a^b \int_c^d p(x,y) = 1$ (double integral)

Ex: Suppose X and Y can take values in [0,
with uniform density $p(x,y)=1$
Visualise $x > y$ and find its Prob?



Ex 2 $p(x,y) = 4xy$
Visualize: A = "X < 0.5 and Y > 0.5"
Probability?



Total Prob: $\int_0^1 \int_0^1 4xy \, dx\, dy = \int_0^1 2x^2 y \Big|_0^1 dy = \int_0^1 2y$

$= 2 y^2/2 \Big|_0^1 = 1$. QED.

$P(A) = \int_0^{0.5} \int_{0.5}^1 4xy \, dy\, dx = \int_0^{0.5} 2xy^2 \Big]_{0.5}^1 dx = \int_0^{0.5}$

$= \left[\frac{3x^2}{4} \right]_0^{0.5} = \frac{3}{16}$

Joint Cumulative Distribution

Suppose X & Y are jointly-dist. ra
variables. we use the notation

$\left. \begin{matrix} X \le x \\ Y \le y \end{matrix} \right\}$ "$X \le x$ and Y

$$F(x,y) = P(X \le x, Y \le y)$$

For the continuous case. with joint densi
$f(u,v)$ over the region $[a,b] \times [c,d]$
↑

$F(x,y) = \int^y \int^x f(u,v)\, du\, dv$.

$F(x,y) = \int_c \int_a f(u,v)\,du\,dv$

Recove the joint pdf:

$$f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \, \partial y}$$

For the discrete case:

$$F(x,y) = \sum_{x_i \le x} \sum_{y_i \le y} P(x_i, y_i)$$

Properties of the Cumulative P.dF: F(

1. $F(x,y)$ is non-decreasing
   if x and y increase then $F(x,y)$
   stay constant or increase.

2. $F(x,y) = 0$ at the lower-left
   joint range.
   If the lower-left is $(-\infty, -\infty)$ then

$$\lim_{(x,y) \to (-\infty,-\infty)} F(x,y) = 0$$

3. $\lim \; F(x,y) = 1$

$(x,y) \to (+\infty, +\infty)$

Marginal Distribution

X & Y are jointly distributed random variables and we want to consider only one of them.

We need to find Prob. density funct of X without Y. This is called the Marginal Distribution.

Roll 2 dice case:

$$P(X=5) = \sum_{j=1}^{6} P(X=5, y_j)$$

$$= 1/36 + 1/36 + \dots + 1/36 = \frac{6}{36} =$$

Next lecture:

- Marginal Dist.
- Covariances & correlation.