Problems

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$\rightarrow$ Q6: $Cov(X,Y) = \sum_i \sum_j p(x_i, y_j) x_i \cdot y_j - \mu_X \mu_Y$. $\leftarrow$

μ_X

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$= E[XY - X\mu_Y - \mu_X Y + \mu_X \mu_Y]$$

$$= E[XY] - E[X\mu_Y] - E[\mu_X Y] + E[\mu_X \mu_Y]$$

$\downarrow$ $p(x_i, x_j)$ $\qquad \downarrow$ $\mu_Y E[X]$

$$\begin{cases} X = 1 \\ Y = 1 \end{cases}$$

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$= \sum_i \sum_j p(x_i y_j)(x_i - \mu_X)(y_j - \mu_Y)$$

$$= \sum_i \sum_j p(x_i y_j)(x_i y_i) - \mu_X \mu_Y. \leftarrow$$

Ex 7:

X	x:	1	2	3	4
pdf	$p_X(x)$:	1/10	2/10	3/10	4/10

Y	y:	1	2	3	4	5
pdf	$P_Y(y)$:	$1/15$	$2/15$	$3/15$	$4/15$	$5/15$.

Make a table for $X+Y$.

$P(x,y)$

$Y \backslash X$	1	2	3	4
1	$1/150$	$2/150$	$3/150$	$4/150$
2	$2/150$	$4/150$	$6/150$	$8/150$
3	$3/150$	$6/150$	$9/150$	$12/150$
4	$4/150$	$8/150$	$12/150$	$16/150$.
5	$5/150$	$10/150$	$15/150$	$20/150$.

$X+Y=$

$X+Y$ can take values from $\{2, 3, 4 \ldots\ldots 9\}$

$X+Y$	$x+y$:	2	3	4	5	6	7	8	9
	P_{X+Y}:	$1/150$	$4/150$	$10/150$	$\frac{20}{150}$	$\frac{30}{150}$	$\frac{34}{150}$	$\frac{31}{150}$	$20/150$.

$X+Y=2 \rightarrow \{(1,1)\}$

$x+y=3 \to \{(1,2),(2,1)\}.$

$x+y=4 \to \{(1,3),(2,2),(3,1)\}.$ ←

$x+y=5 \to \{(1,4),(2,3),(3,2),(4,1)\}$

$x+y=6 \to \{(1,5)(2,4),(3,3),(4,2)\}$ ←

$x+y=7 \to \{(2,5),(3,4),(4,3),\}$

$x+y=8 \to \{(4,4);(3,5)\}.$

$x+y=9 \to \{(4,5)\}.$

Ex

Binomial:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

$E(X)$. $Var(X)$.

$X \sim B(n,p)$

$$E(x) = \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \cdot \binom{n}{x} p^x (1-p)$$

we can use : $\binom{n}{x} = \binom{n-1}{x-1}$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \qquad \binom{n-1}{x-1} = \frac{(n-1)!}{(x-1)!(}$$

$$= \frac{(n-1)}{(x-1)!(}$$

$$= \sum_{x=1}^{n} x \cdot \binom{n-1}{x-1} p^x (1-p)^{n-x}$$

$$(n-1)-($$

$$= np \sum x \binom{n-1}{x-1} p^{x-1} (1-p)$$

change of variables : $r = x-1$

$$= np \underbrace{\sum_{r=0}^{n-1} \binom{n-1}{r} p^r (1-p)^{(n-1)-}}$$

formula for binomial expansion:

$$(x+y)^k = \sum^{k} \binom{k}{r} x^r y^{k-r}$$

$$E(X) = np \cdot (p + (1-p))^{n-1} = n$$

$X \sim B(n, p)$

One trial : $B(1, p)$

$X_1, X_2, \ldots X_n$.

$$X = X_1 + X_2 + \cdots + X_n.$$

$$E(X) = E[X_1 + X_2 + \cdots + X_n]$$
$$= E[X_1] + E[X_2] + \cdots + E[$$
$$= p + p + \cdots + p = n$$

$$Var(X) = Var(X_1 + \cdots + X_n).$$
$$= Var(X_1) + \cdots + Var(X_n)$$
$$= \overbrace{p(1-p)} + \cdots + p(1-$$
$$= np(1-p).$$

Normal dist.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\Big(-$$

$Z \sim N(0,1)$ X

find μ and variar

$E(Z) = 0$ —

$Var(Z) = 1$

$P(X \leq x)$ in terms o

$P(aZ + b \leq x) = P($

$$P\left(z \le \frac{x-b}{a}\right.$$

$\Phi(z)$

Normal standard di

$$= \Phi(0) - \Phi($$

$$\Phi(0.667) = 0.2$$

Part 4

$X \sim N($

$Z \sim N(0$

CLT:

Let X_j be the IQ

Selected person.

$$E(X_j) = 100$$

$$\sigma_{X_j} = 15$$

Let $\bar{X}$ be the average of (

randomly selected peopl(

$$\begin{cases} E(\bar{X}) = 100 \\ \sigma_{\bar{X}} = 15/\sqrt{100} \end{cases}$$

$$P(\bar{X} > 115)$$

$P(\bar{X} > 115)$

Standardized this in

$P(\bar{X} > 115) \approx$