Habib University
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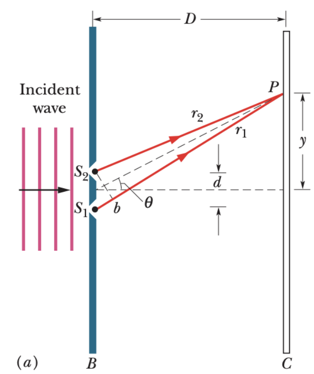
Integrated Sciences and Mathematics (iSciM)

Quantum Mechanics Spring 2023
Exercise Sheet 1
Issued : 17 January 2022 Due : 24 January 2023

Note : *Please submit your scanned solutions directly on canvas before the deadline. Answers will be uploaded soon after the deadline.*

Problem 1 (10 marks)

What is the distance on screen C in Fig between adjacent maxima near the center of the interference pattern ? The wavelength l of the light is 546 nm, the slit separation d is 0.12 mm, and the slit – screen separation D is 55 cm. Assume that θ in Fig is small enough to permit use of the approximations $\sin\theta \approx \tan\theta \approx \theta$, in which θ is expressed in radian measure.



Solution

For small θ, we can approximate path difference $\triangle r$ between r_1 and r_2 using :

$$\frac{\triangle r}{d} = \sin\theta \approx \tan\theta = \frac{y}{D}$$

Maxima occur where path difference is a multiple of wavelength and so, near the screen center they are separated by :

$$\triangle y = \frac{lD}{d} = 2.5mm$$

Problem 2 (5 marks)

A double-slit arrangement produces interference fringes for sodium light ($\lambda = 589nm$) that have an angular separation of 3.50×10^{-3} rad. For what wavelength would the angular separation be 10.0% greater ?

Solution

Again, for small angles $\theta \approx \tan\theta$ so one finds :

$$\triangle\theta \approx \frac{\triangle y}{D} = \frac{\lambda}{d}$$

Therefore, angular separation $\triangle\theta$ is proportional to wavelength λ and for a 10.0% increase in $\triangle\theta$, we need a higher wavelength by the same ratio. So the new wavelength λ' is :

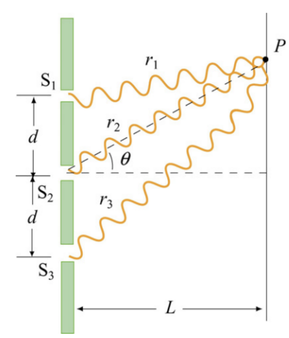
$$\lambda' = 648nm$$

Problem 3 (10 marks)

Suppose a monochromatic coherent source of light passes through three parallel slits, each separated by a distance d from its neighbor, as shown in the Figure. The waves have the same amplitude E_0 and angular frequency ω , but a constant phase difference $\phi = 2\pi d \sin\theta/\lambda$. (a) Show that the intensity is

$$I = \frac{I_0}{9}\left[1 + 2\cos\left(\frac{2\pi d \sin(\theta)}{\lambda}\right)\right]^2 \tag{1}$$

where I_0 is the maximum intensity associated with the primary maxima. (b) What is the ratio of the intensities of the primary and secondary maxima?



Solution

The three paths have different lengths which we denote by $r - d\sin\theta$, r and $r + d\sin\theta$, respectively. To calculate intensity, we need to add up all the complex amplitudes corresponding to each path and take mod square of the final amplitude. The resulting number will be proportional to the intensity.

$$I \propto \left| \exp\left(2\pi i \frac{r - d\sin\theta}{\lambda}\right) + \exp\left(2\pi i \frac{r}{\lambda}\right) + \exp\left(2\pi i \frac{r + d\sin\theta}{\lambda}\right) \right|^2$$

$$I \propto \left| \exp\left(2\pi i \frac{r}{\lambda}\right) \right|^2 \left| \exp\left(-2\pi i \frac{d\sin\theta}{\lambda}\right) + 1 + \exp\left(2\pi i \frac{d\sin\theta}{\lambda}\right) \right|^2$$

$$I \propto \left| \exp\left(-2\pi i \frac{d\sin\theta}{\lambda}\right) + 1 + \exp\left(2\pi i \frac{d\sin\theta}{\lambda}\right) \right|^2$$

Using Euler's identity $e^{i\theta} = \cos\theta + i\sin\theta$,

$$I \propto \left(1 + 2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right)^2$$

At the center of screen $\theta = 0$ so we have $I_0 \propto 9$ and so the constant of proportionality can be substituted to give,

$$I = \frac{I_0}{9}\left[1 + 2\cos\left(\frac{2\pi d\sin(\theta)}{\lambda}\right)\right]^2$$