28 march

Tuesday, 28 March 2023 04:07

Post Midterm Contents.

1) Schrodinger Equation (Solving diff potentials
2) formalism of Quantum Mechanics

Schrodinger Equation.

$$\underbrace{-i\hbar\frac{\partial}{\partial t}}_{\text{time derivative}}\psi(x,t) = \underbrace{\hat{H}}_{\text{Hamiltonian operator}}\psi(x,t).$$

Solving the schrodinger equation

we assumed a seperable function

$$\psi(x,t) = \psi(x)f(t).$$

$$= \psi(x)e^{-i\omega t}$$

angular frequency

$\omega = E/\hbar$

time independent schrodinger eq.

$$\hat{H}\psi(x) = E(x).$$

$$\hat{H} = \underbrace{-\frac{\hbar^2}{2m}\hat{\nabla}^2}_{\text{Momentum part}} + \underbrace{\hat{V}(x)}_{\text{Potential part.}}$$

In One dimension $\hat{\nabla} \rightarrow \frac{d}{dx}$

Previously: 1) $V(x) = 0$ → free particle

bounded.
2) $V(x) = \frac{1}{2}m\omega^2x^2$
3) $V(x) = \begin{cases} \infty & \text{otherwise.} \\ 0 & -L<x<L \end{cases}$

* For a free particle case the solution is like a wave packet and the wave function is not normalisable.

* for bounded cases the wave function is normalisable. and we get stationary Solutions. Quantum number n.

* We can create a normalisable wavefunction from the superposition of free particle Solutions.


→ wave packet.

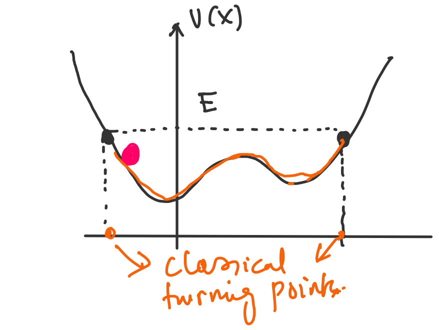
* time dependence of the wave function is different for bounded case as well

as for wave packets.

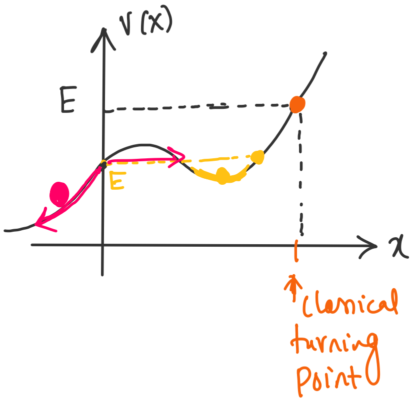
The Delta potential

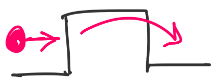
$V(x)$: potential energy.

E : energy eigenvalues.



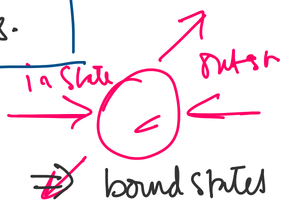
Classical Mechanics.





What happens in Quantum Mechanics.

- Bound states
- Scattering states.



$$\begin{cases} E < V(-\infty) \text{ and } V(+\infty) \Rightarrow \text{bound states} \\ E > V(-\infty) \text{ and } V(+\infty) \Rightarrow \text{Scattering states} \end{cases}$$

as $x \to \pm\infty$ $\quad V(x) \to 0$

$\Rightarrow$ if $\boxed{\begin{array}{l} E < 0 \Rightarrow \text{bound state} \\ E > 0 \Rightarrow \text{Scattering state} \end{array}}$

Delta potential:

- In this example we get the bound states as well as the scattering states.

Delta function is defined as follows.

$$\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$$

with : $\int_{-\infty}^{+\infty} \delta(x)\, dx = 1$

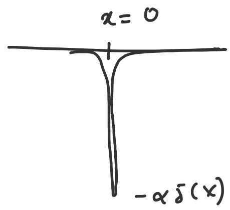
- generalised function or simply "distribution"

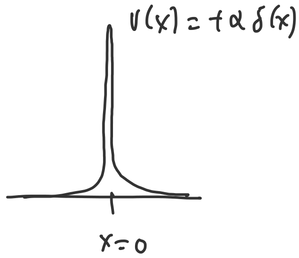
$$f(x)\delta(x-a) = f(a)\delta(x-a)$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-a)\,dx = f(a)\int_{-\infty}^{+\infty}\delta(x-a)\,dx = f(a)$$

$$\int_{-\infty}^{+\infty} \rightarrow \int_{a-\epsilon}^{a+\epsilon} \Rightarrow \lim \epsilon \rightarrow 0$$

$$\boxed{V(x) = -\alpha\,\delta(x)}$$





Schrodinger equation becomes:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \alpha\,\delta(x)\,\psi = E\psi.$$

the solution → bound states $(E<0)$
Scattering states $(E>0)$

In the region $x<0$, $V(x)=0$

$$\frac{d^2}{dx^2}\psi = -\frac{2mE}{\hbar^2}\psi = k^2\psi.$$

$$k^2 = -\frac{2mE}{\hbar^2} \Rightarrow k = \frac{\sqrt{-2mE}}{\hbar}$$

Generalised Solution:

$$\psi(x) = Ae^{-kx} + Be^{kx}$$

but the first term blows up as $x \rightarrow -\infty$

$\Rightarrow A=0$ $\quad \psi(x) = Be^{kx} \quad (x<0)$

Similarly: for $x>0$, $V(x)=0$

$$\psi(x) = Fe^{-kx} + Ge^{kx}.$$

the second term blows up as $x \rightarrow +\infty$

$\Rightarrow G=0 \Rightarrow \psi(x) = Fe^{-kx} \quad (x>0)$

Boundary Conditions -

1) ψ is always continuous.
2) $\frac{d\psi}{dx}$ is also continuous except at points where the potential is

infinite

$$\Rightarrow \boxed{F = B}$$

$$\psi(x) = \begin{cases} Be^{+kx} & (x \le 0) \\ Be^{-kx} & (x \ge 0) \end{cases} = Be^{-k|x|}$$