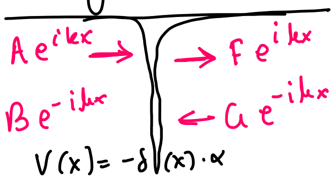
Tuesday, 4 April 2023 04:29

* Bound states ($E<0$)
* Scattering states ($E>0$)

A: amplitude of the incoming waves.

B: amplitude of the reflected wave

F: amplitude of the transmitted wave.



$G = 0$ (if we consider scattering from the left).

Two conditions & 4 unknowns:

1. $F + G = A + B$
2. $F - G = A(1+2i\beta) - B(1-2i\beta)$

(where $\beta = \frac{m\alpha}{\hbar^2 k}$)

When $G = 0$

1. $F = A + B$
2. $F = A(1+2i\beta) - B(1-2i\beta)$

$\Rightarrow$ $\boxed{B = \frac{i\beta}{1-i\beta} A}$ $\boxed{F = \frac{1}{1-i\beta} A.}$

Reflected — incoming wave. — transmitted

What is the probability of finding the particle at a specified location?

The probability is specified by $|\psi|^2$

So the relative probability that an incident particle will be reflected back is

$$\boxed{R = \frac{|B|^2}{|A|^2} = \frac{\beta^2}{1+\beta^2}}$$

R is called the Reflected Coefficient.

beam of particle

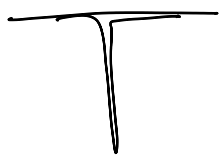
R will tell you the fraction of the number that will bounce back.

Transmission Coefficient

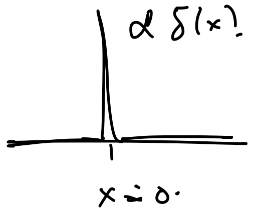
$$\boxed{T = \frac{|F|^2}{|A|^2} = \frac{1}{1+\beta^2}}$$

$$T + R = 1.$$

$$\boxed{R = \frac{1}{1 + (2\hbar^2 E/2m\alpha^2)} \; ; \; T = \frac{1}{1 + (m\alpha^2/2\hbar^2 E)}}$$



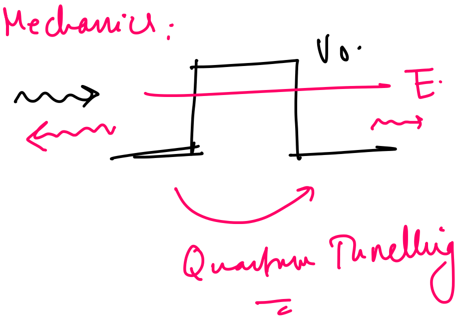
how about $V(x) = +\alpha\delta(x)$?

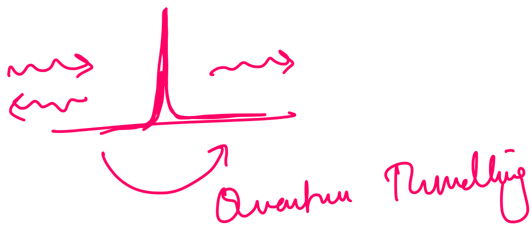


In classical Mechanics:

if $E < V_0$

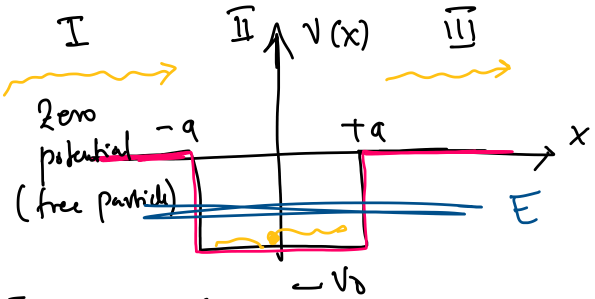
$R = 1, T = 0$





Example: Finite Square Well.

$$V(x) = \begin{cases} -V_0 & \text{for } -a < x < a \\ 0 & \text{for } |x| > a \end{cases}$$



In the region: $(x < -a)$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \quad \text{or} \quad \frac{d^2\psi}{dx^2} = k^2\psi.$$

$$k = \frac{\sqrt{-2mE}}{\hbar}$$

* Bound states
* Scattering States.

Bound States $(E<0)$

Generalised Solution:

$$\psi(x) = A\exp(-kx) + B\exp(kx)$$

as $x \to -\infty$ the first part blows up

$$\Rightarrow A = 0 \Rightarrow \psi(x) = B\exp(k \cdot x)$$

Region I] → using $x = -a$
Region II
Region III] → combine these using $x = +a$

For Region III. $(x > a)$.

Generalized solution:

$$\psi(x) = F\exp(-kx) + G\exp(kx)$$

as $x \to +\infty$ the second part blows up.

$$\Rightarrow \boxed{G = 0} \Rightarrow \boxed{\psi(x) = F\exp(-kx)}$$

Region II: $-a < x < a \qquad V(x) = -V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi(x) = E\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} = -l^2\psi$$

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar} \Big\} \text{ real \& positive}$$

Generalised Solution.

$$\psi(x) = C\sin(lx) + D\cos(lx).$$

Boundary Conditions

$$\psi(x) = \begin{cases} Fe^{-\kappa x} & (x > a) \\ D\cos(lx) & (0 < x < a) \\ \psi(-x) & (x < a). \end{cases}$$

Even parity Solution.