Habib University
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Integrated Sciences and Mathematics (iSciM)

Quantum Mechanics Spring 2023
Exercise Sheet 3
Issued : 24 January 2022 Due : 31 January 2023

Note : *Please submit your scanned solutions directly on canvas before the deadline.*

Problem 1 (15 marks)

Consider the wave function :

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t} \tag{1}$$

where A, λ and ω are positive real numbers.

1. Normalise Ψ
2. Determine the expectation values of x and x^2
3. Find the standard deviation of x.Sketch the graph of Ψ as a function of x, and mark the points $(\langle x\rangle + \sigma)$ and $(\langle x\rangle - \sigma)$ to illustrate the sense in which σ represent the "spread" of x. What is the probability that the particle would be found outside this range ?

Problem 2 (15 marks)

At the time $t = 0$, the particle waver function is represented by :

$$\Psi(x,0) = \begin{cases} Ax/a & \text{if } 0 \le x \le a \\ A(b-x)/(b-a) & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where A, a, and b are constants.

1. Normalise Ψ, that is A in terms of a and b.
2. Sketch $\Psi(x,0)$ as a function of x.
3. Where is the particle most likely to be found at $t = 0$?
4. what is the probability of finding the particle to the left of a ? Check your results in the limiting cases when $b = a$ and $b = 2a$.
5. What is the expectation value of x ?