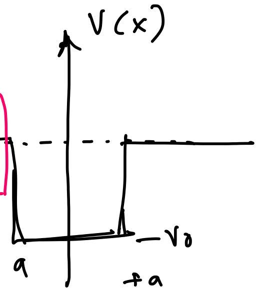
6 April

Thursday, 6 April 2023 03:59

Finite Square potential:

$$\psi(x) = \begin{cases} \boxed{\begin{array}{ll} F e^{-kx} & (x > a) \\ D\cos(lx) & (0 < x < a) \end{array}} \\ \psi(-x) & (x < 0) \end{cases}$$



We are using the even parity solution.

$\psi(x) = \psi(-x)$ if we have an even potential function.

$\Rightarrow$ Symmetry.

The continuity of $\psi(x)$ at $x = a$ says that

$$F e^{-ka} = D\cos(la) \rightarrow (i)$$

Similarly the continuity of $d\psi/dx$ says that

$$-kFe^{-ka} = -lD\sin(la) \text{ --- } (ii)$$

if I divide (ii) by (i). I get.

$$\boxed{k = l\tan(la)}$$

This is the formula for allowed energies.

k is a function of E

l is also a function of E

$$k = \frac{\sqrt{-2mE}}{\hbar} \quad ; \quad l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

How do we solve this eq:

$$\boxed{k = l\tan(la)} \rightarrow (*)$$

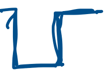
Let adopt new notations-

$$z \equiv la \quad \text{and} \quad z_0 \equiv \frac{a}{\hbar}\sqrt{2mV_0}$$

$$k^2 + l^2 = 2mV_0/\hbar^2$$

$$ka = \sqrt{z_0^2 - z^2}$$

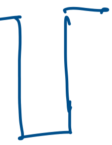
Shallow



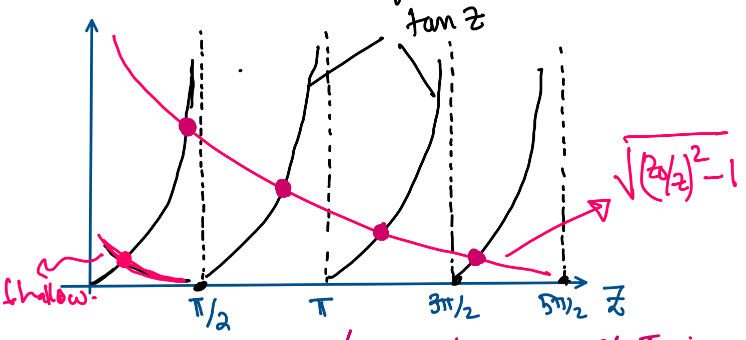
the eq (*) Now becomes

$$\tan z = \sqrt{(z_0/z)^2 - 1}$$

deep.



Transcendental equation-



● - Solutions / allowed energy states.

- Wide deep well:

if z_0 is very large the intersection occur just slighty below $z_n = n\pi/2$ with n= odd it follow that

$$\boxed{E_n + V_0 \simeq \frac{n^2\pi^2\hbar^2}{2m(2a)^2}}$$

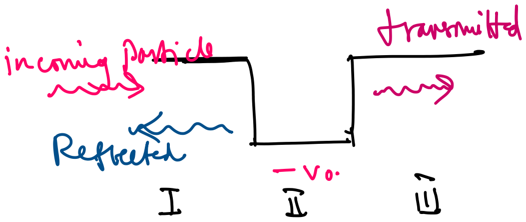
Very similar to infinit potential well.

(2) Shallow potential.

You will have at least one bound state

(Scattering Solution)

* $E > 0$



when $V(x) = 0$ in region I

The Generalised solution is

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \qquad (x < -a)$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

Region II:

$$V(x) = -V_0.$$

$$\psi(x) = C\sin(lx) + D\cos(lx) \qquad (-a < x < a)$$

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

Region III:

$$\psi(x) = Fe^{ikx} + \cancel{Ge^{-ikx}}^{\,0}$$

We apply the boundary condition.

at $x=-a$: $\psi(x)$ should be continuous.

(1) $Ae^{-ika}+Be^{ika}=-C\sin(la)+D\cos(la$

the derivative should be continuous as well.

(2) $ik\left[Ae^{-ika}-Be^{ika}\right]=l\left[C\cos(la)+D\sin(la)\right]$

(3) at $x=+a$:

$$C\sin(la)\mp D\cos(la)=Fe^{ika}$$

(4) $d\psi/dx$ at $x=+a$

$$l\left[C\cos(la)-D\sin(la)\right]=ikFe^{ika}$$

We can use two of these to eliminate C and D. and solve the remaining two for B and F.

$$B=i\frac{\sin(la)}{2kl}(l^2-k^2)F$$

$$F=\frac{e^{-2ika}A}{\cos(2la)-i\frac{\sin(2la)}{2kl}(k^2+l^2)}$$

Transmission:

$$T=|F|^2/|A|^2$$

$$T^{-1}=1+\frac{V_0^2}{4E(E+V_0)}\sin^2\left(\underbrace{\frac{2a}{\hbar}\sqrt{2m(E+V_0)}}_{=0}\right)$$

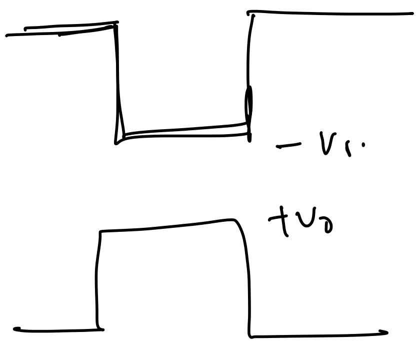
When $T=1$ (well becomes "transparent" whenever the argument of sin is zero

When this happens.

$$\frac{2a}{\hbar}\sqrt{2m(E+V_0)}=n\pi$$

$$\Rightarrow\ \boxed{E_n\mp V_0=\frac{n^2\pi^2\hbar^2}{2m(2a)^2}}$$

infinite square well:



$-v_i$
$+v_o$