Tuesday, 11 April 2023 04:30

- Schrodinger equation.

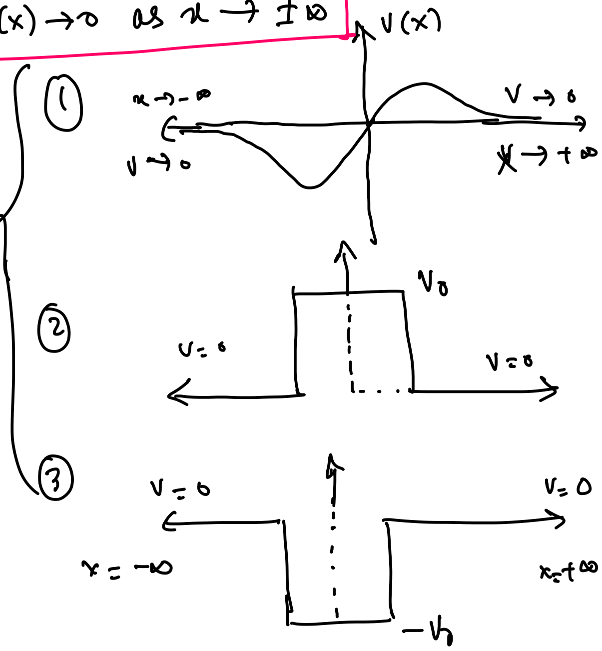
$$\underbrace{-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2} + V(x)\psi}_{} = E\psi(x)$$

energy states

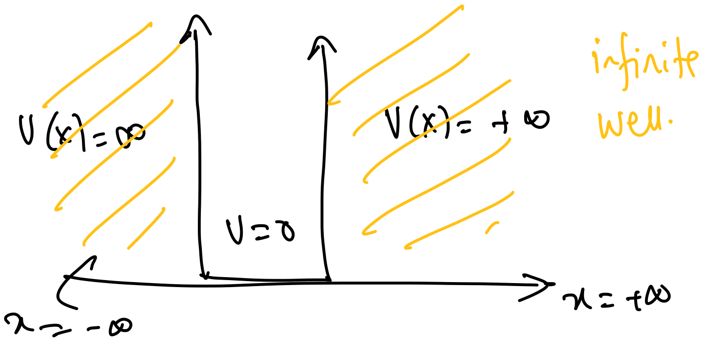
- Solve the eq.
- Construct wavefunction
- Energy eigenstates
- V (x) potential.

V (x) is a localised potential in some region of space.. $\boxed{V(x) \to 0 \text{ as } x \to \pm\infty}$

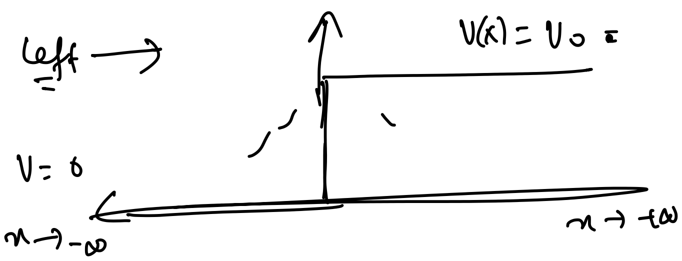
Scattering Solutions



How about other types of potential.



In this example you will not get scattering solution, you will only get "Bounded Solutions"



Bounded States

- Bounded states are states that are localised in some region of space.
- the wavefunction are normalisable and have profiles that drop off exponentially far from the potential

$$\psi(x) \sim e^{-\lambda|x|} \quad \text{as } |x| \to \infty$$

- The energy of the state

$$E = \frac{-\hbar^2 \lambda^2}{2m}$$

- In particular bound states have $E < 0$
- It is this property that ensures that the particle is trapped within the potential and cannot escape to infinity.
- In the absence of a potential, a solution which decays exponentially to the left will grow exponentially to the far right. But for the state to be normalisable the potential has to turn this behaviour around ~~as~~ so the wave function decreases as both $x \to -\infty$ and $x \to +\infty$. This will only happen for specific values of λ. $\Rightarrow$ Therefore the spectrum of bound states are always discrete

(quantum)

Scattering States:

- The scattering states are not localised is space.
- wavefunction are not normalisable
- asymptotically, far from the potential scattering states take the form of plane waves. In 1 dimension

Right Moving: $\psi \sim e^{ikx}$) free left moving: $\psi \sim e^{-ikx}$ } particle state

for $k>0$

- the time dependence $\psi \sim e^{\pm ikx - i\underline{Et}/\hbar}$
- Solving the Schrodinger eq in the asymptotic region with $V=0$ gives the energy:

$$E = \frac{\hbar^2 k^2}{2m}$$

- $E>0$

Scattering Matrix (S-Matrix)