Habib University
Dhanani School of Science and Engineering

Dr. Muntazir Abidi
Integrated Sciences and Mathematics (iSciM)

Quantum Mechanics Springs 2023
MidTerm Exam
14 March 2023

NOTE : *You must attempt all the Multiple Choice Questions (MCQs), which carry equal weightage. Other than MCQs, there are three problems provided, out of which you must choose and solve any two. Notes are not allowed.*

Total Marks : 50

MCQs (20 marks)

1. Which of the following best describes the time-independent Schrödinger equation ?
   (a) An equation that describes the time evolution of a particle's wavefunction.
   (b) An equation that describes the energy levels of a particle in a potential field.
   (c) An equation that describes the probability density of finding a particle in a particular state.
   (d) An equation that describes the position and momentum of a particle.

   Answer : B
2. Which of the following is not a common interpretation of the wavefunction ?
   (a) The wavefunction represents the probability density of finding a particle in a particular state.
   (b) The wavefunction represents the actual physical state of a particle.
   (c) The wavefunction represents the amplitude of a particle's wave-like behaviour.
   (d) The wavefunction represents the information available about a particle's state.

   Answer : B
3. Which of the following is an eigenfunction of the Schrödinger equation ?
   (a) Any function that satisfies the equation.
   (b) A function that produces the same function when multiplied by a constant.
   (c) A function that, when acted upon by the Schrödinger operator, produces a scalar multiple of itself.
   (d) A function that, when differentiated, produces a scalar multiple of itself.

   Answer : C
4. Which of the following statements about the uncertainty principle is false ?
   (a) It is a fundamental principle of quantum mechanics.
   (b) It is related to the position and momentum of a particle.
   (c) It is related to the energy and time of a particle.
   (d) It is related to a particle's spin and angular momentum.

   Answer : D
5. Which of the following best describes the role of observation in quantum mechanics ?
   (a) Observation causes a particle to take on a particular state.

(b) Observation does not affect the state of a particle.

(c) Observation determines the probability of finding a particle in a particular state.

(d) Observation determines the energy level of a particle.

Answer : C

6. Which of the following statements about superposition is true ?

(a) It is the principle that describes the probability of finding a particle in a particular state.

(b) It is the principle that describes particles' ability to be simultaneously in multiple states.

(c) It is the principle that describes the energy levels of a quantum system.

(d) It is the principle that describes the force between particles.

Answer : B

7. What is the Schrödinger equation ?

(a) An equation that describes the motion of particles in a magnetic field.

(b) An equation that describes the motion of particles in a gravitational field.

(c) An equation that describes the motion of particles in a quantum system.

(d) An equation that describes the motion of particles in a classical system.

Answer : C

8. Which of the following best describes the Heisenberg Uncertainty Principle ?

(a) The position and momentum of a particle cannot both be known with complete accuracy.

(b) The energy and momentum of a particle cannot both be known with complete accuracy.

(c) The position and energy of a particle cannot both be known with complete accuracy.

(d) A particle's spin and angular momentum cannot both be known with complete accuracy.

Answer : A

9. Which of the following statements about probability conservation in quantum mechanics is true ?

(a) The probability of finding a particle in a particular state is always conserved.

(b) The probability of finding a particle in a particular state can change over time.

(c) The total probability of finding a particle in any state is always conserved.

(d) The probability of finding a particle in any state can change over time.

Answer : C

10. Which of the following best describes Planck's constant ?

(a) A constant that relates the energy and frequency of a photon.

(b) A constant that relates the wavelength and momentum of a photon.

(c) A constant that relates the energy and wavelength of a photon.

(d) A constant that relates the frequency and momentum of a photon.

Answer : A

Problem : Fourier Transforms and Expectation Values (15 points)

Function $f(x)$ and its Fourier transform $\tilde{f}(k)$ are related by,

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk, e^{ikx} \tilde{f}(k)$$
$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx, e^{-ikx} f(x)$$

By definition, the expectation is defined as :

$$\langle x \rangle = \int dx, P(x)x = \int dx, |\psi(x)|^2 x$$

Using the definition of expectation values of (powers of) the momentum operator,

$$\langle \hat{p}^n \rangle = \int dx, \psi^*(x)\hat{p}^n\psi(x)$$

the form of the momentum operator,

$$\hat{p} = -i\frac{\partial}{\partial x}$$

and the definition of the Fourier transform,

1. **(5 points) :** Show that

$$\langle \hat{p} \rangle = \int \frac{dk}{2\pi} |\tilde{\psi}(k)|^2 k,$$

2. **(5 points) :** and that

$$\langle \hat{p}^2 \rangle = \int \frac{dk}{2\pi} |\tilde{\psi}(k)|^2 k^2,$$

3. **(5 points) :** and, in general, that

$$\langle f(\hat{p}) \rangle = \int \frac{dk}{2\pi} |\tilde{\psi}(k)|^2 f(nk).$$

The relation $P(k) = |\tilde{\psi}(k)|^2$, as discussed in lecture, thus follows from the Born relation,

$$P(x) = |\psi(x)|^2.$$

Problem : Superposition in the Infinite Well (20 points)

Verify (on scratch paper, no need to turn this in) the results stated in lecture for the eigenvalues of the energy operator for the infinite potential well of width L,

$$V(x) = \begin{cases} 0 & 0 \leq x \leq L \\ \infty & \text{else} \end{cases}$$

i.e. remind yourself why the energy eigenvalues are,

$$E_n = \frac{2k^2(n+1)\pi^2}{2mL^2}$$

Now suppose that at $t = 0$ we place a particle in an infinite well in the state

$$\psi_A(x,0) = \frac{1}{6}\left(\varphi_0(x) + \frac{1}{3}\varphi_1(x) + \frac{1}{2}\varphi_2(x)\right)$$

Note : Each step below requires relatively little computation. You will not need the functional form of the energy eigenfunctions $\varphi_n(x)$ to complete them, only the energy eigenvalues.

1. **(2 points) :** How does ψ_A evolve with time? Write down the expression for $\psi_A(x,t)$.
2. **(4 points) :** Calculate the expectation value of the energy, $\langle\hat{E}\rangle$, for the particle described by $\psi_A(x,t)$. Write your answer in terms of E_0. Does this quantity change with time?
3. **(2 points) :** What is the probability of measuring the energy to equal $\langle\hat{E}\rangle$ as a result of a single measurement at $t = 0$? At a later time, $t = t_1$?
4. **(2 points) :** What energy values will be observed due to a single measurement at $t = 0$ and with what probabilities? How do these probabilities change with time?
5. **(2 points) :** The energy of the particle is found to be E_2 as a result of a single measurement at $t = t_1$. Write down the wave function $\psi_A(x,t)$, which describes the state of the particle for $t > t_1$. What energy values will be observed and with what probabilities at a time $t_2 > t_1$?
6. **(4 points) :** Construct another normalized wave function $\psi_B(x,0)$ which is linearly independent of $\psi_A(x,0)$, but yields the same value of $\langle\hat{E}\rangle$ as well as the same set of measured energies with the same probabilities.
7. **(4 points) :** Construct another normalized wave function $\psi_C(x,0)$ which is linearly independent of $\psi_A(x,0)$, yields the same value of $\langle\hat{E}\rangle$, but allows a different set of measured energies (which may include some but not all of E_0, E_1 and E_2, plus others).