1 (1) (2) (3) 4(2) 7. 16. 22. 23 36

补充题目：设 $X_1, \cdots, X_n$ 独立同分布，且 $EX_i = \mu$，$D(X_i) = \sigma^2$

令 $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$，求 $E\bar{X}$，$D(\bar{X})$，$E[\sum_{i=1}^{n}(X_i - \bar{X})^2]$

1、解.

(1). $P(X=2) = \frac{1}{8}$

$P(X=3) = \frac{5}{8}$

$P(X=4) = \frac{1}{8}$

$P(X=9) = \frac{1}{8}$

则X的分布律为：

X	2	3	4	9
P	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$E(X) = 2\times\frac{1}{8} + 3\times\frac{5}{8} + 4\times\frac{1}{8} + \cancel{8}9\times\frac{1}{8} = \frac{\cancel{28}30}{8} = \frac{15}{4}$

(2). $P(Y=2) = \frac{2}{30}$ $P(Y=3) = \frac{15}{30}$ $P(Y=4) = \frac{4}{30}$ $P(Y=9) = \frac{9}{30}$

则Y的分布律为：

Y	2	3	4	9
P	$\frac{2}{30}$	$\frac{15}{30}$	$\frac{6}{30}$	$\frac{9}{30}$

$E(Y) = \frac{2}{30}\times 2 + \frac{15}{30}\times 3 + \frac{6}{30}\times 4 + \frac{9}{30}\times 9 = \frac{146}{30} = \frac{73}{15}$

(3). $P(X=1) = \frac{1}{6}$ $P(X=2) = \frac{1}{6}$ $P(X=\cancel{8}3) = \frac{1}{6}$ $P(X=4) = \frac{1}{6}$ $P(X=5) = \frac{1}{6}$

$P(X=7) = \frac{1}{36}$ $P(X=8) = \frac{1}{36}$ $P(X=9) = \frac{1}{36}$ $P(X=10) = \frac{1}{36}$ $P(X=11) = \frac{1}{36}$ $P(X=12) = \frac{1}{36}$

X的分布律为：

X	1	2	3	4	5	~~6~~ 7	8	9	10	11	12
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$E(X) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{7}{36} + \frac{8}{36} + \frac{9}{36} + \frac{10}{36} + \frac{11}{36} + \frac{12}{36}$

$E(X) = \frac{147}{36} = \frac{49}{12}$

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4(2) $P(X=1) = \frac{1}{2}$

$P(X=2) = \frac{1}{2} \times \frac{1}{3}$

$P(X=3) = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$

$\vdots$

则 $P(X=k) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \frac{k-1}{k} \times \frac{1}{k+1} = \frac{1}{k(k+1)}$

由于 $\sum_{k=1}^{\infty} k P\{X=k\} = \sum_{k=1}^{\infty} k \cdot \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \frac{1}{k+1} = \infty$

$\sum_{k=1}^{\infty} kP(X=k)$ 不绝对收敛.

故 X 的数学期望不存在

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7. 解.

(1) $f(x) = \begin{cases} e^{-x} & x>0 \\ 0 & x\le 0 \end{cases}$

(i) $Y = 2X$

$$E(Y) = E(2X) = \int_{-\infty}^{+\infty} 2x\cdot f(x)\,dx = \int_0^{+\infty} 2x\cdot e^{-x}\,dx$$

$$= -2\int_0^{+\infty} x\,de^{-x} = -2\left(xe^{-x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-x}dx\right)$$

$$= -2e^{-x}\Big|_0^{+\infty} = 2$$

(ii) $Y = e^{-2X}$

$$EY = E(e^{-2X}) = \int_{-\infty}^{+\infty} e^{-2X}\cdot f(x) = \int_0^{+\infty} e^{-2x}\cdot e^{-x}\,dx = \int_0^{+\infty} e^{-3x}dx$$

$$= -\frac{1}{3}e^{-3x}\Big|_0^{+\infty} = \frac{1}{3}$$

(2). $X_i \sim U(0,1)$ $i=1,2,\cdots,n$

$F(x) = \begin{cases} 0 & x<0 \\ x & 0\le x<1 \\ 1 & x\ge 1 \end{cases}$

(i) 由于 $X_1, X_2, \cdots, X_n$ 相互独立, $U = \max\{X_1, X_2, \cdots, X_n\}$

$F_U(u) = \begin{cases} 0 & u<0 \\ u^n & 0\le u<1 \\ 1 & u\ge 1 \end{cases}$

$f_U(u) = \begin{cases} nu^{n-1} & 0<u<1 \\ 0 & 其他 \end{cases}$

$$E(U) = \int_{-\infty}^{+\infty} u f_U(u)\,du = \int_0^1 nu^n\,du = \frac{n}{n+1}$$

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(ii). $V=\min\{X_1, X_2, \cdots, X_n\}$

$$F_V(v)=\begin{cases}0 & v<0\\ 1-(1-v)^n & 0\le v<1\\ 1 & v\ge 1\end{cases}$$

$$f_V(v)=\begin{cases}n(1-v)^{n-1} & 0<v<1\\ 0 & \text{其他}\end{cases}$$

$$E(V)=\int_{-\infty}^{+\infty} v\cdot f_V(v)=\int_0^1 n(1-v)^{n-1}\cdot v\,dv=-\int_0^1 v\,d(1-v)^n$$

$$=-v(1-v)^n\Big|_0^1+\int_0^1(1-v)^n dv$$

$$=-\frac{1}{n+1}(1-v)^{n+1}\Big|_0^1=\frac{1}{n+1}$$

16. 解:

(1) $P(X=1)=\frac{1}{n}$

$P(X=2)=\frac{n-1}{n}\cdot\frac{1}{n-1}=\frac{1}{n}$

$P(X=3)=\frac{n-1}{n}\cdot\frac{n-2}{n-1}\cdot\frac{1}{n-2}=\frac{1}{n}$

$\vdots$

$P(X=k)=\frac{1}{n}\quad 1\le k\le n$

$E(X)=1\times\frac{1}{n}+2\times\frac{1}{n}+\cdots+n\times\frac{1}{n}=\frac{1+n}{2}$

(2). 设 $X_i=\begin{cases}1 & \text{前 } i-1 \text{ 次都未打开}\\ 0 & \text{前 } i-1 \text{ 次有一次成功}\end{cases}$

$E(X_1)=1$

$E(X_i)=\frac{n-1}{n}\cdot\frac{n-2}{n-1}\cdot\cdots\frac{n-i+1}{n-i-2}=\frac{n-i+1}{n}\quad i=2,3,\cdots,n$

$E(X)=1+\sum_{i=2}^{n}E(X_i)=1+\sum_{i=2}^{n}\frac{n-i+1}{n}=\frac{n+1}{2}$

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22. 解.

(1). $E(X_i) = i \qquad i = 1, 2, 3, 4$

$E(X) = \sum_{i=1}^{4} E(X_i) = 1 + 2 + 3 + 4 =$

$D(X_i) = 5 - i$

$Y = 2X_1 - X_2 + 3X_3 - \frac{1}{2}X_4$

$E(Y) = 2E(X_1) - E(X_2) + 3E(X_3) - \frac{1}{2}E(X_4)$

$= 2 \times 1 - 2 + 3 \times 3 - \frac{1}{2} \times 4$

$= 7$

$D(Y) = 4D(X_1) + D(X_2) + 9D(X_3) + \frac{1}{4}D(X_4)$

$= 4 \times 4 + 3 + 9 \times 2 + \frac{1}{4}$

$= 37.25$

(2). $X \sim N(720, 30^2) \qquad Y \sim N(640, 25^2)$

X 与 Y 相互独立. 故 Z_1, Z_2 均服从正态分布

$E(Z_1) = E(2X + Y) = 2E(X) + E(Y) = 720 \times 2 + 640 = 2080$

$D(Z_1) = D(2X + Y) = 4D(X) + D(Y) = 4 \times 30^2 + 25^2 = 4225$

$E(Z_2) = E(X - Y) = E(X) - E(Y) = 720 - 640 = 80$

$D(Z_2) = D(X - Y) = D(X) + D(Y) = 30^2 + 25^2 = 1525$

故 $Z_1 \sim N(2080, 4225) \qquad Z_2 \sim N(80, 1525)$ 同理 $X + Y \sim N(1360, 1525)$

$P\{X > Y\} = P\{X - Y > 0\} = P\{Z_2 > 0\} = 1 - P\{Z_2 \le 0\} = 1 - \Phi\left(\frac{0 - 80}{\sqrt{1525}}\right)$

$= \Phi(2.0486) = 0.97\cancel{8}98$

$P\{X + Y > 1600\} = 1 - P\{X + Y \le 1600\} = 1 - \Phi\left(\frac{1600 - 1360}{\sqrt{1525}}\right) \approx 1 - \Phi(1.02)$

$= 1 - 0.8461 = 0.1539$

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23. 解.

(1) 设总销量为 X.

$X = X_1 + X_2 + X_3 + X_4 + X_5$. 且 X_1, X_2, X_3, X_4, X_5 相互独立

$E(X) = EX_1 + EX_2 + EX_3 + EX_4 + EX_5 = 200 + 240 + 180 + 260 + 320$

$= 1200$

$D(X) = DX_1 + DX_2 + DX_3 + DX_4 + DX_5 = 225 + 240 + 225 + 265 + 270$

$= 1225$

(2) 设仓库至少储存 n kg 产品.

则 $P\{X \le n\} > 0.99$.

因为 $X \sim N(1200, 1225)$.

$\Phi\left(\frac{n-1200}{\sqrt{1225}}\right) > 0.99 = \Phi(2.33)$

故 $\frac{n-1200}{\sqrt{1225}} > 2.33$

$n > 1281.55$.

故应至少储存 1281.55 kg 产品

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36. 解：设 X 为每一百毫升血液中白细胞数目.

则 $u = E(X) = 7300$

$\sigma = \sqrt{D(X)} = 700$

$p = P\{5200 < X < 9400\}$

$= P\{7300 - 2100 < X < 7300 + 2100\}$

$= P\{|X - 7300| < 2100\} = P\{|X - u| < 3\sigma\}$

又因为 $P\{|X - u| < \varepsilon\} \geq 1 - \frac{\sigma^2}{\varepsilon^2}$

$p \geq 1 - \frac{\sigma^2}{(3\sigma)^2} = \frac{8}{9}$

补充题目：

解：$E(\bar{X}) = E(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n} E(\sum_{i=1}^{n} X_i) = \frac{1}{n} \times (u + u + \cdots + u) = u$

$D(\bar{X}) = D(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n^2} D(\sum_{i=1}^{n} X_i) = \frac{1}{n^2} \times (\sigma^2 + \sigma^2 + \cdots + \sigma^2)$

$= \frac{\sigma^2}{n}$

$E[\sum_{i=1}^{n}(X_i - \bar{X})^2]$ ~~$= \sum_{i=1}^{n} E[(X_i - \bar{X})^2] = \sum_{i=1}^{n} D(X_i) = n\sigma$~~

$= E(\sum_{i=1}^{n} X_i^2 - n\bar{X}^2) = \sum_{i=1}^{n} E(X_i^2) - nE(\bar{X}^2)$

又因 $E(X_i^2) = D(X_i) + [E(X_i)]^2 = \sigma^2 + \mu^2$

$E(\bar{X}^2) = D(\bar{X}) + [E(\bar{X})]^2 = \frac{\sigma^2}{n} + \mu^2$

故 $E[\sum_{i=1}^{n}(X_i - \bar{X})^2] = n(\sigma^2 + \mu^2) - n \cdot (\frac{\sigma^2}{n} + \mu^2) = (n-1)\sigma^2$

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