21(2) 25. 26(1)(2)(3) 31 35(1)(2)(3) 36(2). 37

21(2) 解:

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{其他} \end{cases}$$

$x<0$ 时. $F(x) = 0$

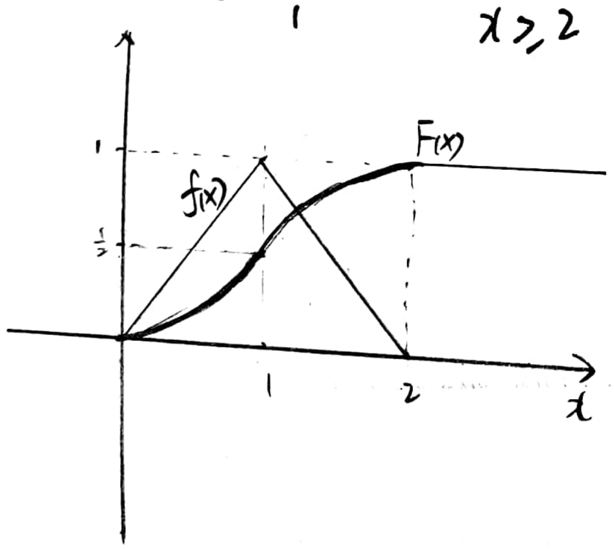
$0 \le x < 1$ 时. $F(x) = \int_0^x f(t)\,dt = \frac{1}{2}x^2$

$1 \le x < 2$ 时. $F(x) = \frac{1}{2} + \int_1^x f(t)\,dt = 2x - \frac{1}{2}x^2 + \frac{1}{2}$

$x \geq 2$ 时. $F(x) = 1$

则分布函数 $F(x)$ 为

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{1}{2}x^2 & 0 \le x < 1 \\ 2x - \frac{1}{2}x^2 + \frac{1}{2} & 1 \le x < 2 \\ 1 & x \geq 2 \end{cases}$$



25. 解：

方程有实根，$\Delta = (4k)^2 - 4\times4\times(k+2) = 16(k^2-k-2) > 0$

即 $k<-1$ 或 $k>2$

又∵ $K \sim U(0,5)$ $\quad f_K(x) = \begin{cases} 0 & 其他 \\ \frac{1}{5} & 0\le x<5 \end{cases}$

$P(k\le -1) = 0$

$P(k\ge 2) = \int_2^{+\infty} f_K(x)\,dx = \frac{3}{5}$

26. 解：$X \sim N(3,2^2)$ 故设 $Y = \frac{X-3}{2}$ 则 $Y\sim N(0,1)$

(1). $P\{2<x\le5\} = P\{-\frac{1}{2}<Y\le1\} = \Phi(1) - \Phi(-\frac{1}{2})$

$= \Phi(1) - (1-\Phi(0.5)) = 0.8413 - 1 + 0.6915$

$= 0.5328$

$P\{-4<X\le10\} = P\{-\frac{7}{2}<Y\le\frac{7}{2}\} = \Phi(\frac{7}{2}) - \Phi(-\frac{7}{2})$

$= 2\Phi(\frac{7}{2}) - 1 = 2\times0.9998 - 1 = 0.9996$

$P\{|X|>2\} = P\{Y>-0.5\} \cup P\{Y<-2.5\}$

$= 1 - \Phi(0.5) + \Phi(-2.5)$

$= 1 - (1-\Phi(0.5)) + 1 - \Phi(2.5)$

$= 0.6915 - 0.9938 + 1$

$= 0.6977$

$P\{X>3\} = P\{Y>0\} = 1 - \Phi(0) = 0.5$

(2). $P\{X>c\} = 1 - P\{X \le c\} = P\{X \le c\}$

故 $P\{X \le c\} = 0.5$

又∵ $\Phi\left(\frac{c-3}{2}\right) = 0.5$ $\quad \frac{c-3}{2} = 0$ $\quad c = 3$

(3). $P\{X>d\} = 1 - P\{X \le d\} \ge 0.9$

$\cancel{P\{X \le d\} \le 0.1}$

则 $1 - P\{Y \le \frac{d-3}{2}\} = 1 - \Phi\left(\frac{d-3}{2}\right) = \Phi\left(-\frac{d-3}{2}\right) \ge 0.9$
$= \Phi(1.28)$

则 $-\frac{d-3}{2} \ge 1.28$

$d \le 0.44$

31. 解：当 $x<0$ 时，$F(x) = 0$

当 $0 \le x < 30$ 时，$F(x) = 0.2 + 0.8 \times \frac{x}{30}$

当 $x \ge 30$ 时，$F(x) = 1$

$$F(x) = \begin{cases} 0 & x<0 \\ 0.2 + \frac{0.8x}{30} & 0 \le x < 30 \\ 1 & x \ge 30 \end{cases}$$

∵ $F(0) \ne F(0^-)$，则 $F(x)$ 不是连续型.

又因不存在一个可列点集，使得 $F(x)$ 在这个点集上 X取值的概率是1，则X也不是离散型.

X是混合型随机变量.

35. 解.

(1). $\cancel{0<Y<+\infty}$ $Y>0$.

则 $y<0$ 时 $f_Y(y)=0$

$y>0$ 时. $F_Y(y)=P\{Y\le y\}=P\{e^x\le y\}=P\{X\le \ln y\}$

$$=\cancel{\int_{-\infty}^{\ln y}}\ \Phi(\ln y)$$

则 $f_Y(y)=\Phi'(\ln y)=\varphi(\ln y)\cdot\frac{1}{y}=\frac{1}{\sqrt{2\pi}y}e^{-\frac{(\ln y)^2}{2}}$

则 $f_Y(y)=\begin{cases}0 & \text{其他}\\ \frac{1}{\sqrt{2\pi}y}e^{-\frac{(\ln y)^2}{2}} & y>0\end{cases}$

(2). $Y=2X^2+1$

$Y\ge 1$

当 $y<1$ 时, $f_Y(y)=0$

$y>1$ 时. $F_Y(y)=P\{2X^2+1\le y\}=P\{-\sqrt{\frac{y-1}{2}}\le X\le\sqrt{\frac{y-1}{2}}\}$

$$=2\Phi\left(\sqrt{\frac{y-1}{2}}\right)-1$$

$f_Y(y)=2\times\varphi\left(\sqrt{\frac{y-1}{2}}\right)\times\frac{1}{4\sqrt{\frac{y-1}{2}}}=\frac{1}{2\sqrt{\pi(y-1)}}e^{-\frac{(y-1)^2}{4}}$

(3). $Y = |X|$

$Y \geq 0$

当 $y < 0$ 时，$f_Y(y) = 0$

当 $y \geq 0$ 时 $F_Y(y) = P\{|X| \leq y\} = P\{-y \leq X \leq y\}$

$= \Phi(y) - \Phi(-y) = 2\Phi(y) - 1$

则 $y > 0$ 时 $f_Y(y) = 2 \cdot \varphi(y) = \sqrt{\frac{2}{\pi}} e^{-\frac{y^2}{2}}$

则 $f_Y(y) = \begin{cases} \sqrt{\frac{2}{\pi}} e^{-\frac{y^2}{2}} & y > 0 \\ 0 & 其他. \end{cases}$

36 (2) 解: $Y = X^2 \quad 0 < X < 1$

(4). 当 $y \leq 0$ 时，$f_Y(y) = 0$

当 $y \geq 1$ 时，$f_Y(y) = 0$

当 $0 < y < 1$ 时，$F_Y(y) = P\{X^2 \leq y\} = P\{-\sqrt{y} \leq X \leq \sqrt{y}\}$

$= P\{X \leq \sqrt{y}\} = e^{-\sqrt{y}}$

$f_Y(y) = -\frac{1}{2\sqrt{y}} e^{-\sqrt{y}}$

则 $f_Y(y) = \begin{cases} -\frac{1}{2\sqrt{y}} e^{-\sqrt{y}} & 0 < y < 1 \\ 0 & 其他. \end{cases}$

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27. $Y = \sin X \quad 0 \le Y \le 1$

当 $y > 1$ 或 $y < 0$ 时 $\quad f_Y(y) = 0$

当 $0 \le y \le 1$ 时. $\quad F_Y(y) = P\{0 \le \sin X \le y\} = P\{0 \le X \le \arcsin y\}$

$$= P\{0 \le X \le \arcsin y\} \cup \{\pi - \arcsin y \le X \le \pi\}$$

$$= \int_0^{\arcsin y} \frac{2x}{\pi^2}\,dx + \int_{\pi-\arcsin y}^{\pi} \frac{2x}{\pi^2}\,dx$$

$$= \frac{(\arcsin y)^2}{\pi^2} + \frac{\pi^2}{\pi^2} - \frac{(\pi - \arcsin y)^2}{\pi^2}$$

$$= \frac{2\arcsin y}{\pi}$$

则 $0 < y < 1$ 时, $\quad f_Y(y) = \dfrac{2}{\pi\sqrt{1-y^2}}$

则
$$f_Y(y) = \begin{cases} \dfrac{2}{\pi\sqrt{1-y^2}} & 0 < y < 1 \\ 0 & 其他. \end{cases}$$