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补充题目：设 X 与 Y 相互独立，$P\{X=-1\}=P\{X=0\}=P\{X=1\}=\frac{1}{3}$，

$Y \sim U(0,1)$，求 $Z=X+Y$ 的分布.

24. 解：(1) $f_X(x) = \int_0^{+\infty} \frac{1}{2}(x+y)e^{-(x+y)}dy = \frac{1}{2}xe^{-x}\int_0^{+\infty} e^{-y}dy + e^{-x}\int ye^{-y}dy$

$= \frac{1}{2}xe^{-x} + \frac{1}{2}e^{-x} = \frac{x+1}{2}e^{-x}$　　$x>0$

$f_X(x) = \begin{cases} \frac{x+1}{2}e^{-x} & x>0 \\ 0 & \text{其他} \end{cases}$

同理：$f_Y(y) = \begin{cases} \frac{y+1}{2}e^{-y} & y>0 \\ 0 & \text{其他} \end{cases}$

由于 $f_X(x)\cdot f_Y(y) \neq f(x,y)$，所以 X、Y 不相互独立

(2) $Z=X+Y$　（则 $0<y<z$）

$f_Z(z) = \int_{-\infty}^{+\infty} f(z-y,y)\,dy = \begin{cases} \int_0^z f(z-y,y)\,dy & z>0 \\ 0 & \text{其他} \end{cases}$

$= \begin{cases} \int_0^z \frac{1}{2}ze^{-z}\,dy & z>0 \\ 0 & \text{其他} \end{cases} = \begin{cases} \frac{1}{2}z^2e^{-z} & z>0 \\ 0 & \text{其他} \end{cases}$

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解：$Z=\frac{Y}{X}$

$$f_Z(z)=\int_{-\infty}^{+\infty}|x|\ f(x,xz)\,dx$$

被积函数非零，当且仅当 $\begin{cases}x>0\\zx>0\end{cases} \Rightarrow \begin{cases}x>0\\z>0\end{cases}$

故 $z\le 0$ 时，$f_Z(z)=0$

$z>0$ 时 $f_Z(z)=\int_0^{+\infty} x f_X(x)\cdot f_Y(xz)\,dx$

$$=\int_0^{+\infty} x e^{-x}\cdot e^{-xz}\,dx$$

$$=\frac{1}{(1+z)^2}$$

故 $f_Z(z)=\begin{cases}\frac{1}{(1+z)^2} & z>0\\ 0 & z\le 0\end{cases}$

29. 解：

(1). $1=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}f(x,y)\,dy=\int_0^1 dx\int_0^{+\infty} be^{-(x+y)}\,dy=b-be^{-1}$

则 $b=\frac{e}{e-1}$

(2). $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,dy=\begin{cases}\int_0^{+\infty} be^{-(x+y)}dy\\ 0 & 其他\end{cases}=\begin{cases}be^{-x}\\ 0\end{cases}=\begin{cases}\frac{e}{e-1}e^{-x} & 0<x<1\\ 0 & 其他\end{cases}$

$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx=\begin{cases}\int_0^1 be^{-(x+y)}dx\\ 0 & 其他\end{cases}=\begin{cases}be^{-y}\cdot(1-e^{-1})\\ 0\end{cases}=\begin{cases}e^{-y} & y>0\\ 0 & 其他\end{cases}$

(3) $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = \begin{cases} 1 & x \geq 1 \\ \int_0^x \frac{1}{1-e^{-1}} e^{-t}\,dt & \\ 0 & x \leq 0 \end{cases} = \begin{cases} 1 & x \geq 1 \\ \frac{1}{1-e^{-1}}(1-e^{-x}) & 0<x<1 \\ 0 & x \leq 0 \end{cases}$

$F_Y(y) = \int_{-\infty}^{y} f_Y(t)\,dt = \begin{cases} \int_0^y e^{-t}\,dt & \\ 0 & y \leq 0 \end{cases} = \begin{cases} 1-e^{-y} & y>0 \\ 0 & y \leq 0 \end{cases}$

$F_U(z) = P\{M \leq z\} = P\{X \leq z, Y \leq y\} = P\{X \leq z\} \cdot P\{Y \leq z\}$

即 $F_U(z) = F_X(z) \cdot F_Y(z) = \begin{cases} 1-e^{-z} & x \geq 1, y>0 \\ \frac{1}{1-e^{-1}}(1-e^{-z})^2 & 0<x<1, y>0 \\ 0 & 其他 \end{cases}$

36. 解.

(1). $P\{X=2 \mid Y=2\} = \frac{P\{X=2, Y=2\}}{P\{Y=2\}} = \frac{0.05}{0.01+0.03+0.05\times 3+0.06} = \frac{1}{5}$

$P\{Y=3 \mid X=0\} = \frac{P\{X=0, Y=3\}}{P\{X=0\}} = \frac{0.01}{0.00+0.01+0.03+0.05+0.07+0.09} = \frac{1}{25}$

(2) $V=\max\{X,Y\}$ 的可能取值有 0, 1, 2, 3, 4, 5.

$P\{V=0\} = P\{X=0, Y=0\} = 0.00$

$P\{V=1\} = P\{X=1, Y=0\} + P\{X=0, Y=1\} + P\{X=1, Y=1\} = 0.04$

同理. $P\{V=2\} = 0.01+0.03+0.05+0.04+0.03 = 0.16$

$P\{V=3\} = 0.01+0.02+0.04+0.06+0.05\times 3 = 0.28$

$P\{V=4\} = 0.07+0.06+0.05+0.06 = 0.24$

$P\{V=5\} = 0.09+0.08+0.06+0.05 = 0.28$

则 V 的分布律为:

V	0	1	2	3	4	5
P	0	0.04	0.16	0.28	0.24	0.28

(3). 同(2). 解: $U=\min\{X,Y\}$ 的可能取值有 0, 1, 2, 3

~~当X=0~~

$P\{U=0\} = 0.00+0.01+0.03+0.05+0.07+0.09+0.01\times 3 = 0.28$

$P\{U=1\} = 0.02+0.04+0.05+0.06+0.08+0.03+0.02 = 0.30$

$P\{U=2\} = 0.05\times 3+0.06+0.04 = 0.25$

$P\{U=3\} = 0.06+0.06+0.05 = 0.17$

则 U 的分布律为:

U	0	1	2	3
P	0.28	0.30	0.25	0.17

(4). $W=X+Y$

W的可能取值有 0, 1, 2, 3, 4, 5, 6, 7, 8

$P\{W=0\} = P\{X=0, Y=0\} = 0$

$P\{W=1\} = P\{X=1, Y=0\} + P\{X=0, Y=1\} = 0.02$

$P\{W=2\} = P\{X=0, Y=2\} + P\{X=2, Y=0\} + P\{X=1, Y=1\} = 0.06$

同理: $P\{W=3\} = 0.01 + 0.03 + 0.04 + 0.05 = 0.13$

$P\{W=4\} = 0.02 + 0.05 + 0.05 + 0.07 = 0.19$

$P\{W=5\} = 0.04 + 0.05 + 0.06 + 0.09 = 0.24$

$P\{W=6\} = 0.06 + 0.05 + 0.08 = 0.19$

$P\{W=7\} = 0.06 + 0.06 = 0.12$

$P\{W=8\} = 0.05$

W的分布律为:

W	0	1	2	3	4	5	6	7	8
P	0	0.02	0.06	0.13	0.19	0.24	0.19	0.12	0.05

补充题目:

$$F_Z(z) = P\{X+Y\le z\} = P\{X=-1\}\cdot P\{X+Y\le z \mid X=-1\}$$

$$+ P\{X=0\}\cdot P\{X+Y\le z\mid X=0\}$$

$$+ P\{X=1\}\cdot P\{X+Y\le z\mid X=1\}$$

即 $F_Z(z) = P\{X=-1\}\cdot P\{Y\le z+1\} + P\{X=0\}\cdot P\{Y\le z\} + P\{X=0\}\cdot P\{Y\le z-1\}$

则 $F_Z(z) = \frac{1}{3} \times \left[F_Y(z+1) + F_Y(z) + F_Y(z-1)\right]$

由于 $Y\sim U(0,1)$ 故 $f_Y(y) = \begin{cases}1 & 0<y<1\\ 0 & \text{其他}\end{cases}$

$$F_Y(y) = \int_{-\infty}^{y} f_Y(y)\,dy = \begin{cases}0 & y<0\\ y & 0\le y<1 \\ 1 & y\ge 1\end{cases}$$

则 $F_Z(z) = \begin{cases}0 & z<-1\\ \frac{1}{3}(z+1) & -1\le z<2\\ 1 & z\ge 2\end{cases}$