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19.(2) 解: 设 A="从第二盒中取到白球"

设 B_i="从第一盒中取 i 只白球, $2-i$ 只红球". $(i=0,1,2)$

~~故 $P(A)=$~~ 故 $A=A\Omega=A(B_0\cup B_1\cup B_2)$

$$P(A)=\sum_{i=0}^{2}P(AB_i)=\sum_{i=0}^{2}P(B_i)P(A|B_i)$$

$$=\frac{C_5^2}{C_9^2}\times\frac{5}{11}+\frac{C_5^1C_4^1}{C_9^2}\times\frac{6}{11}+\frac{C_4^2}{C_9^2}\times\frac{7}{11}$$

$$=\frac{53}{99}$$

22. 解:

(1). 设 A="他取得该资格"

$$P(A)=1-P(\bar{A})=\boxed{1-(1-P)\times\frac{P}{2}=\frac{P^2}{2}-\frac{P}{2}+1}$$

$$=1-(1-P)\times\left(1-\frac{P}{2}\right)=\frac{3}{2}P-\frac{P^2}{2}$$

(2).

(2). 设 B="他第二次及格"

$$P(B)=P\times P+(1-P)\times\frac{P}{2}=\frac{P^2}{2}+\frac{P}{2}$$

设 C="他第一次及格"

$$P(C|B)=\frac{P(BC)}{P(B)}=\frac{P^2}{\frac{P^2}{2}+\frac{P}{2}}=\frac{2P}{P+1}$$

23. 解:

设C = "接收站收到信息A"

$$P(C) = \frac{2}{3}\times 0.98 + \frac{1}{3}\times 0.01 = \frac{1.97}{3}$$

设D = "原发信息是A"

$$P(D) = \frac{2}{3}$$

故 $$P(D|C) = \frac{P(D)}{P(C)} = \frac{\frac{2}{3}\times 0.98}{\frac{1.97}{3}} = \frac{196}{197}$$

36. 解: 设 A = "三人中至少有一人能将此密码译出"

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{4}{5}\times\frac{2}{3}\times\frac{3}{4} = \frac{3}{5}$$

38. 解: 设 A = "投掷r次都得到国徽"

设 B = "该硬币是正品"

由贝叶斯公式得:

$$P(B|A) = \frac{P(B)\,P(A|B)}{P(A|B)\cdot P(B) + P(A|\bar{B})\,P(\bar{B})} = \frac{(\frac{1}{2})^r\times\frac{m}{m+n}}{\frac{m}{m+n}\times(\frac{1}{2})^r + \frac{n}{m+n}\times 1} = \frac{m}{m+2^r n}$$

39. 解:

$$P(A_1|B) = \frac{\cancel{P(A_1 B)}\ P(B|A_1)\,P(A_1)}{P(B|A_1)\cdot P(A_1) + P(B|A_2)\,P(A_2) + P(B|A_3)\,P(A_3)} = \frac{0.8\times0.98^3}{0.8\times0.98^3+0.15\times0.9^3+0.05\times0.1^3}$$

$$\approx 0.8731$$

$$P(A_2|B) = \frac{P(B|A_2)\ P(A_2)}{P(B|A_1)\cdot P(A_1) + P(B|A_2)\,P(A_2) + P(B|A_3)\,P(A_3)} = \frac{0.15\times0.9^3}{0.8\times0.98^3+0.15\times0.9^3+0.05\times0.1^3}$$

$$\simeq 0.1268$$

$$P(A_3|B) = \frac{P(B|A_3)\,P(A_3)}{P(B|A_1)P(A_1) + P(B|A_2)\,P(A_2) + P(B|A_3)\,P(A_3)} = \frac{0.05\times0.1^3}{0.8\times0.98^3+0.15\times0.9^3+0.05\times0.1^3}$$

$$= 5.798\times10^{-5}$$

40. 解:

设 A = "输入的是 AAAA", B = "输入的是 BBBB", C = "输入的是 CCCC"

设 D = "输出为 ABCD" 则 ~~P(D|X) 关~~

$$\cancel{P(A)}\ P(A|D) = \frac{P(A)\cdot P(D|A)}{P(A)\,P(D|A) + P(B)\,P(D|B) + P(C)\,P(D|C)}$$

$$= \frac{P_1\times a^2\times(\frac{1-a}{2})^2}{P_1\times a^2(\frac{1-a}{2})^2 + P_2\times a\times(\frac{1-a}{2})^3 + P_3\cdot a\cdot(\frac{1-a}{2})^3}$$

$$= \frac{P_1 a}{P_1 a + (P_2+P_3)\frac{1-a}{2}} = \frac{P_1 a}{P_1 a + (1-P_1)\frac{1-a}{2}} = \frac{2P_1 a}{3P_1 a - P_1 - a + 1}$$

性质 证明.

性质1. $P(\phi|A)=0$

证明: $P(\phi|A) = P(A)\cdot P(\phi A) = P(A)\cdot P(\phi) = 0$

性质2. 若 $A_1, A_2, A_3 \cdots, A_n$ 是两两互不相容的事件，

则有 $P(A_1 \cup A_2 \cup \cdots \cup A_n | \cancel{A}B) = P(A_1|B) + P(A_2|B) + \cdots + P(A_n|B)$

证明: $P(A_1 \cup A_2 \cup \cdots \cup A_n|B) = P(\bigcup_{i=1}^{n} A_i|B) = \sum_{i=1}^{n} P(A_i|B)$

性质3. 设 A, B, C 是三个事件，若 $A \subset B$，则有

$P(B-A|C) = P(B|C) - P(A|C)$ $P(B) \geq P(A)$

证明: $\cancel{P(B-A)}$ ~~B逆~~

$A \subset B$.

则 ~~A与B-A~~ $B = A \cup (B-A)$ A与$B-A$互不相容

$P(B|C) = P(A|C) + P(B-A|C)$

故 $P(B-A|C) = P(B|C) - P(A|C)$

由非负性得: $P(B-A|C) \geq 0$

则 $P(B) \geq P(C)$

性质4. 对于任一事件A，事件B. $P(A|B) \le 1$

证：因 $A \subset S$，由性质3得，

$$P(A|B) \le P(S|B) = 1$$

性质5 对于任一事件A,B有 $P(\bar{A}|B) = 1 - P(A|B)$

证明：因 $A \cup \bar{A} = S$，且 $A\bar{A} = \phi$.

$$1 = P(S|B) = P(A \cup \bar{A}|B) = P(A|B) + P(\bar{A}|B)$$

故 $P(\bar{A}|B) = 1 - P(A|B)$

性质6. 对于 $\forall$ A,B事件,C事件,有.

$$P(A \cup B|C) = P(A|C) + P(B|C) - P(AB|C)$$

证明：$A \cup B = A \cup (B - AB)$

且 $A \cap (B - AB) = \phi$

$$P(A \cup B|C) = P(A \cup (B - AB)|C)$$

$$= P(A|C) + P(B - AB|C)$$

$$= P(A|C) + P(B|C) - P(AB|C)$$