

# 第6讲：着色

# 上次课程内容

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- 什么是着色 (Shading) ?
- 一个简单的着色模型: Blinn-Phong反射模型
- 着色频率
  - Flat shading
  - Gouraud shading
  - Phong shading
- 图形管线 (Graphics Pipeline)
- 纹理映射 (Texture Mapping)

# 本次课程内容

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- 纹理映射
- 重心坐标 (Barycentric coordinates)
- 纹理的应用
- 期中综述报告要求已发布



# 纹理映射 (Texture Mapping)

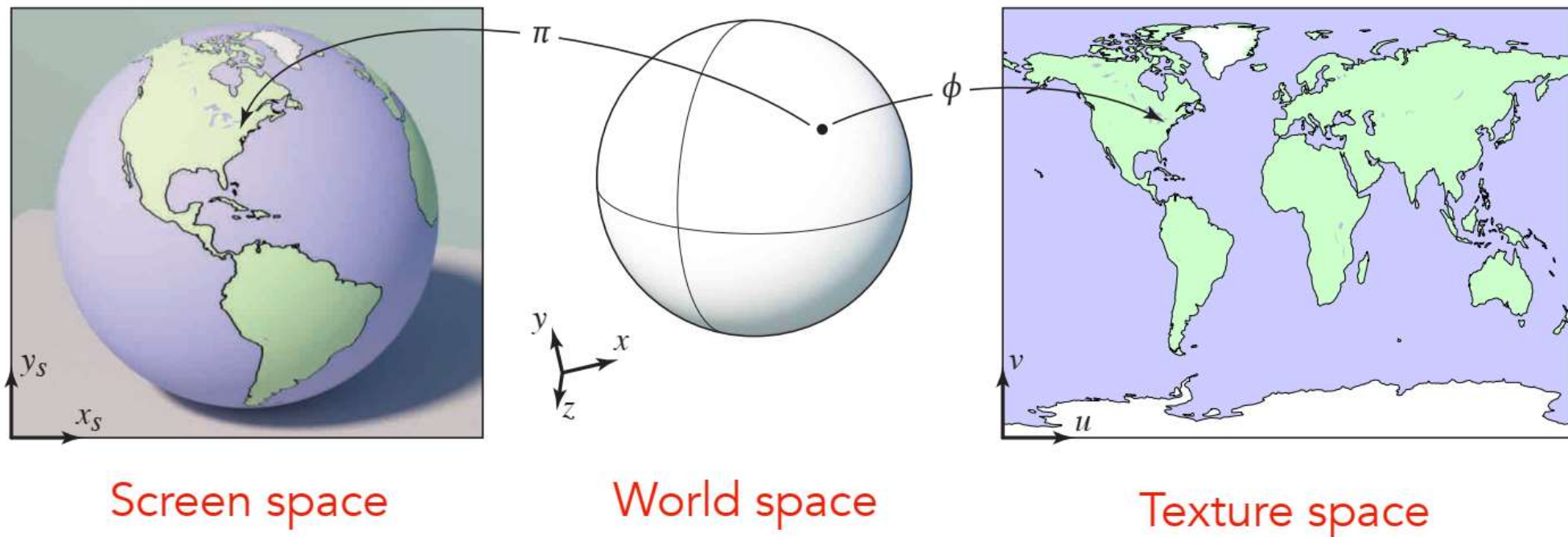


Pattern on ball

Wood grain on floor

# 表面 (surface) 是二维的

- 三维空间中的表面上的一点总可以对应于二维图像 (纹理) 上的一点





# 将纹理应用于表面

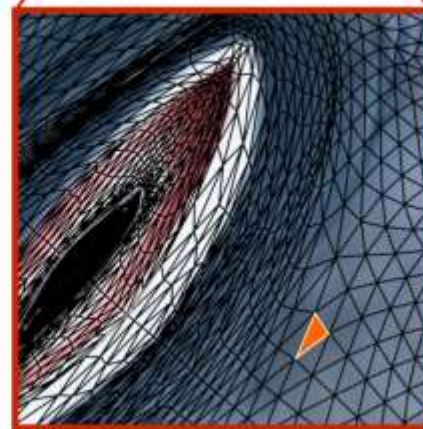
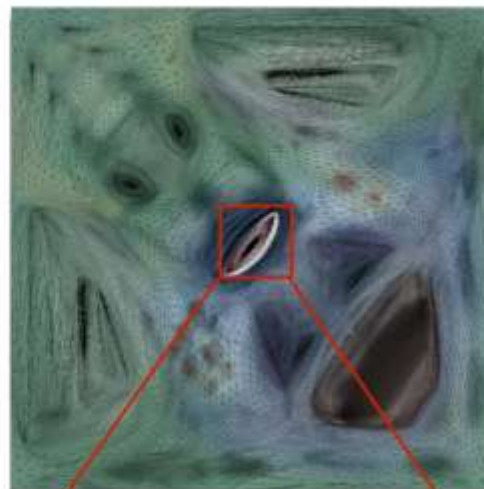
Rendering without texture



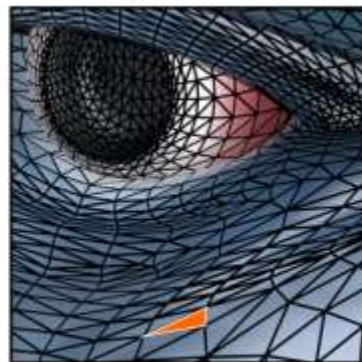
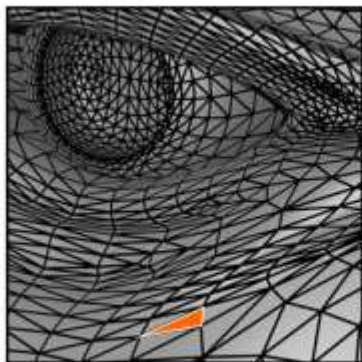
Rendering with texture



Texture



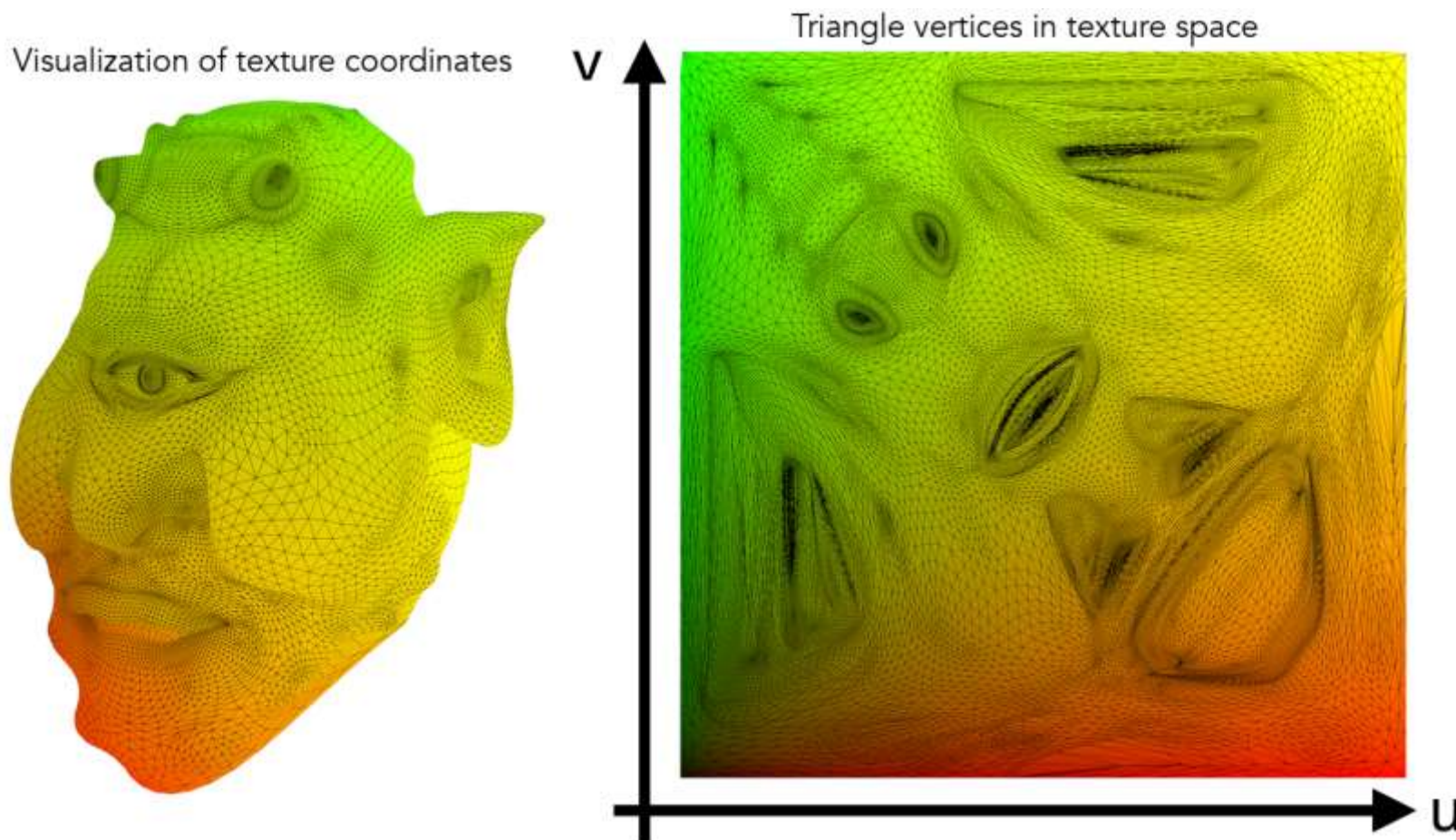
Zoom



Each triangle "copies" a piece of the texture image to the surface.

# 纹理坐标

- 每一个三角形顶点都分配有一个纹理坐标  $(u, v)$



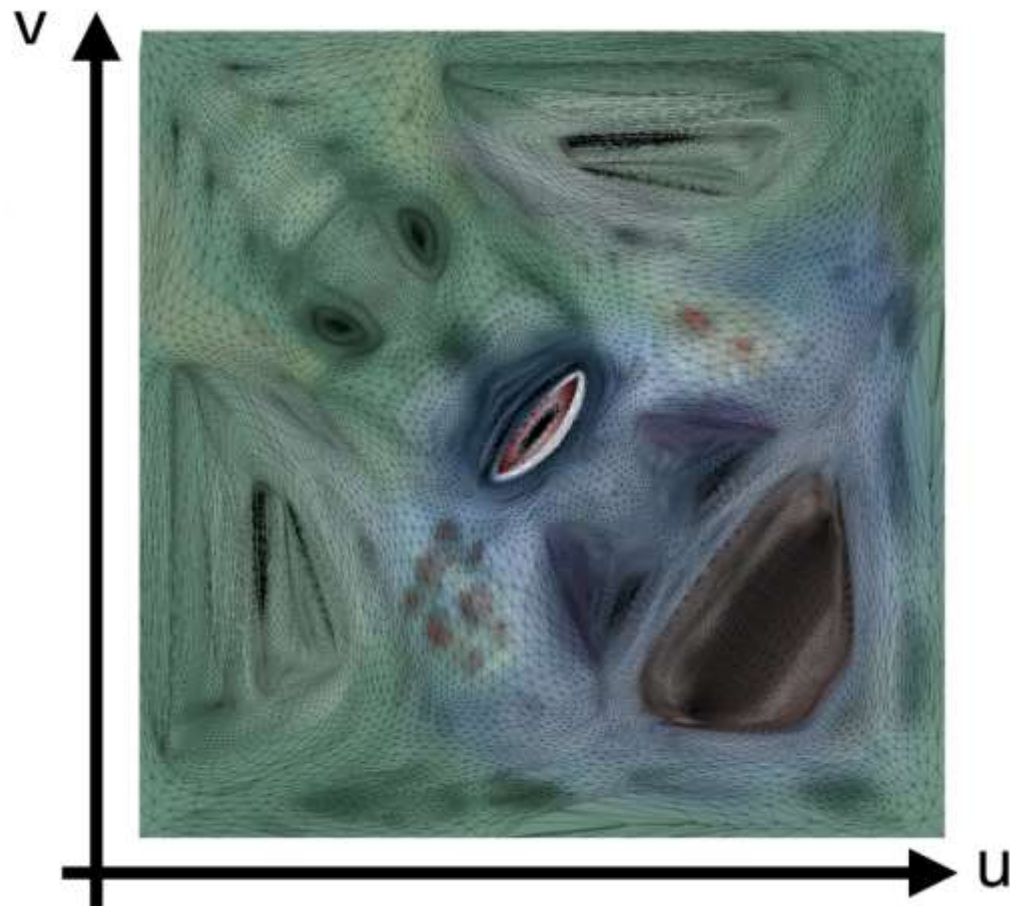


# 将纹理应用于表面

Rendered result



Triangle vertices in texture space

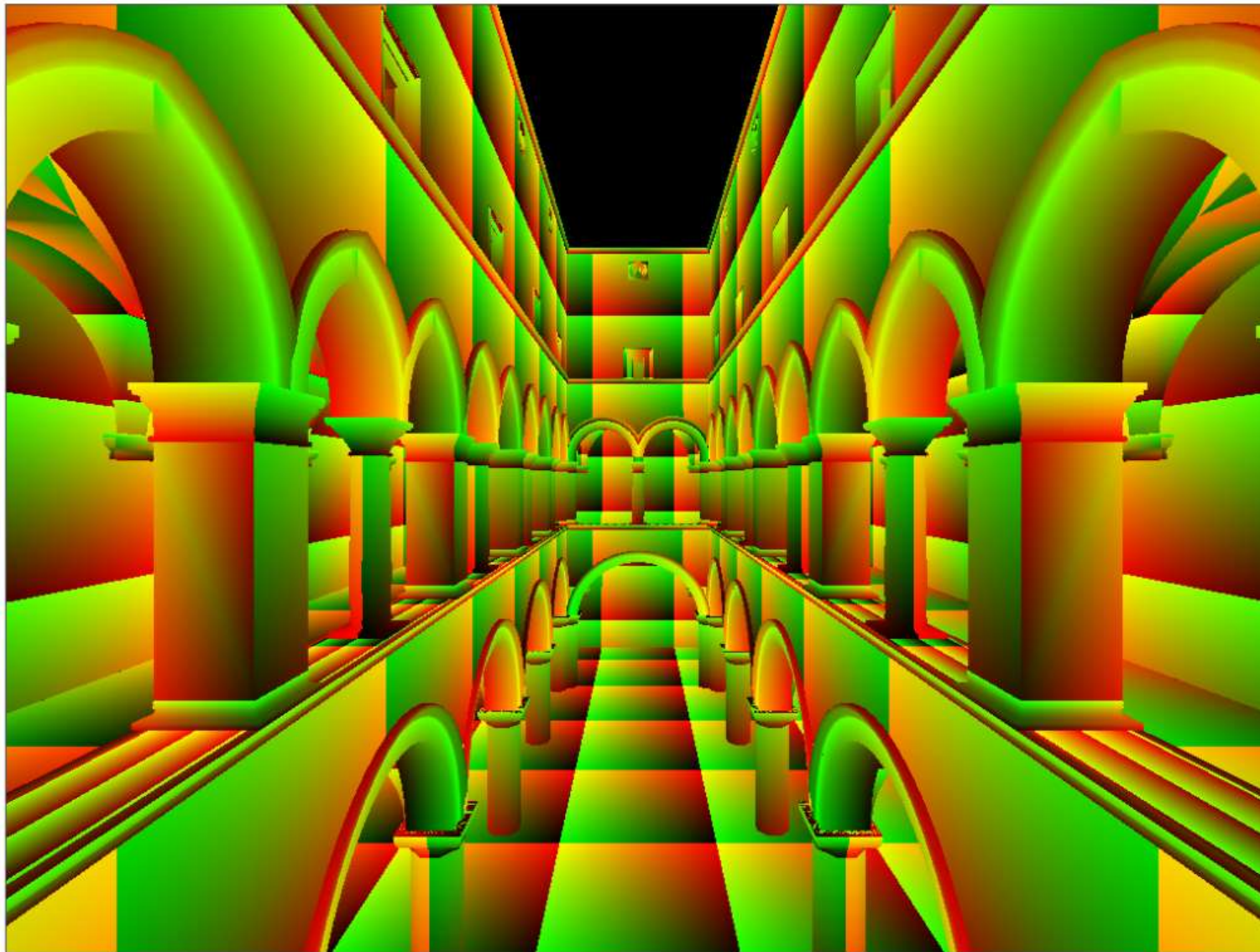




# 将纹理应用于表面



# 纹理坐标可视化





# 纹理可被重复使用



example textures  
used / **tilled**

# Q&A





# 在三角形内部进行插值

Q: 为什么需要插值?

- 通常在顶点处给定属性的值 (离散)
- 需要在三角形内部获得平滑变化的属性值 (连续)

Q: 要插值些什么属性?

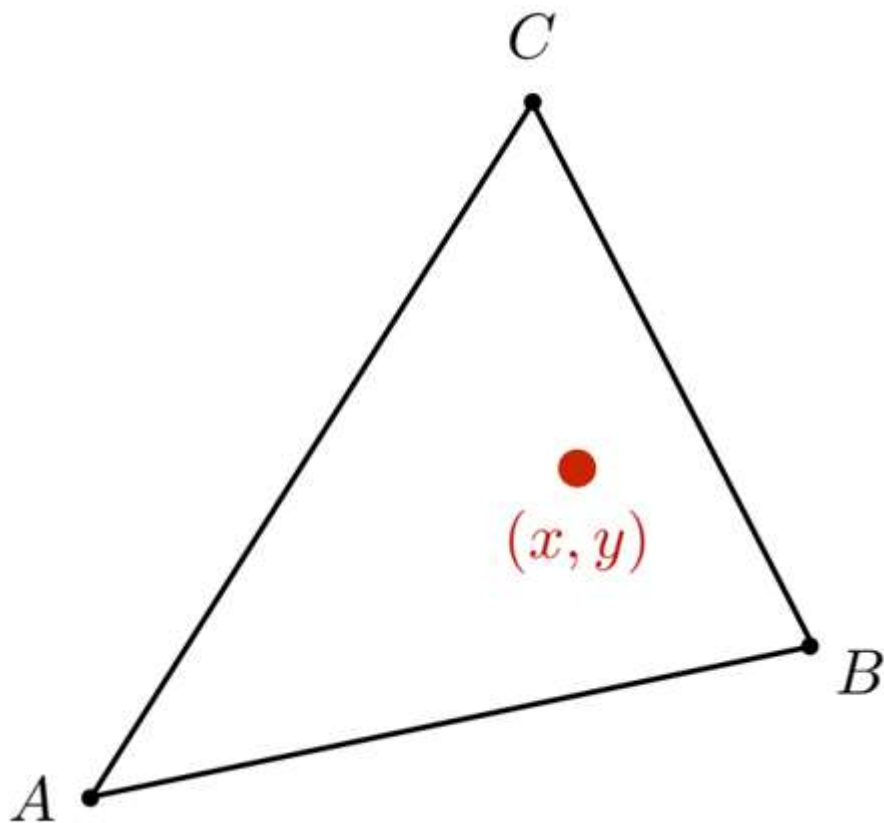
- 纹理坐标、颜色、法向量……

Q: 怎么来做插值?

- 重心坐标

# 重心坐标

- 利用三角形三个顶点建立的一套坐标系



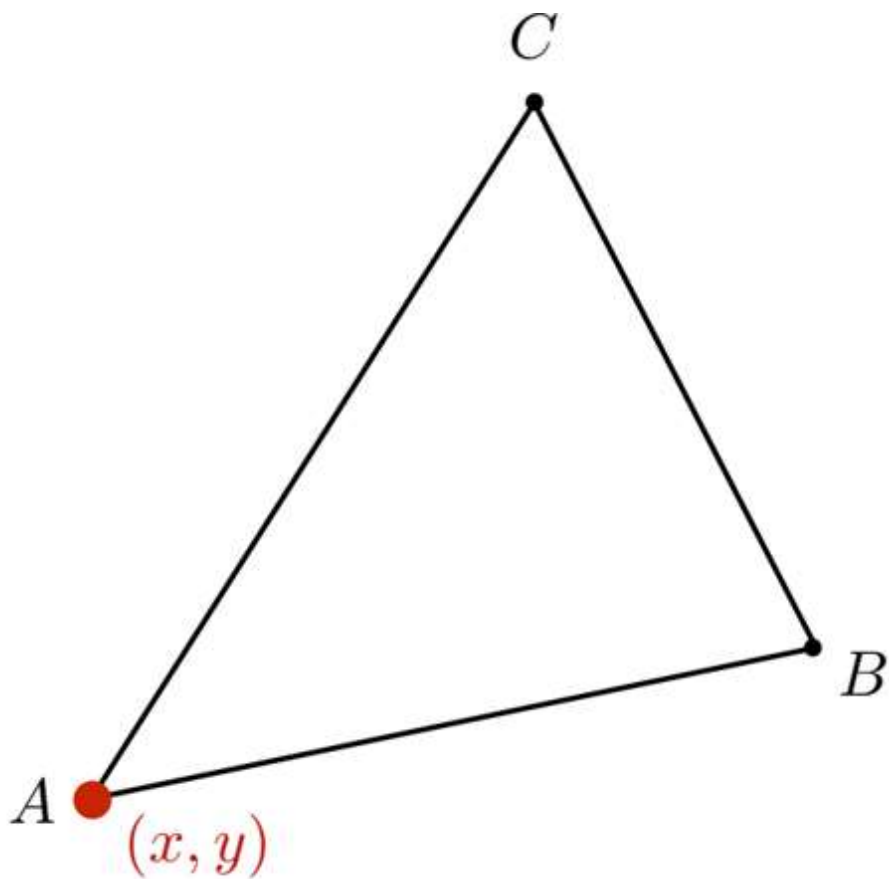
$$(x, y) = \alpha A + \beta B + \gamma C$$

$$\alpha + \beta + \gamma = 1$$

**Inside the triangle if  
all three coordinates  
are non-negative**

# 重心坐标

Q: 点A的重心坐标是什么?

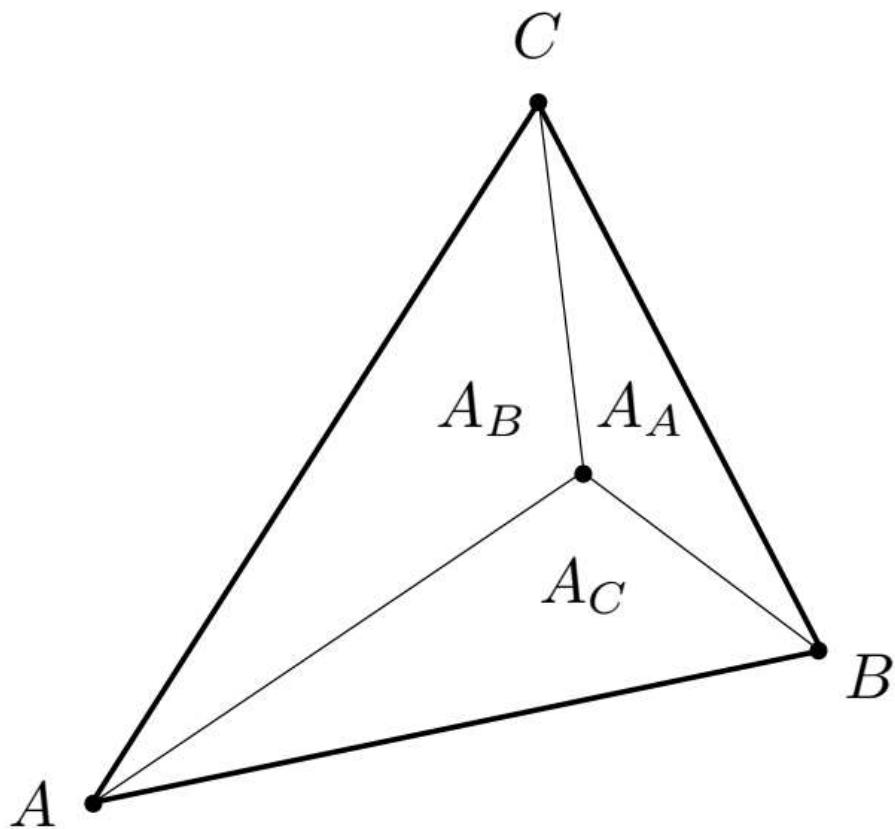


$$(\alpha, \beta, \gamma) = (1, 0, 0)$$

$$(x, y) = \alpha A + \beta B + \gamma C \\ = A$$

# 重心坐标

Q: 如何求解 $(\alpha, \beta, \gamma)$ ?



$$\alpha = \frac{A_A}{A_A + A_B + A_C}$$

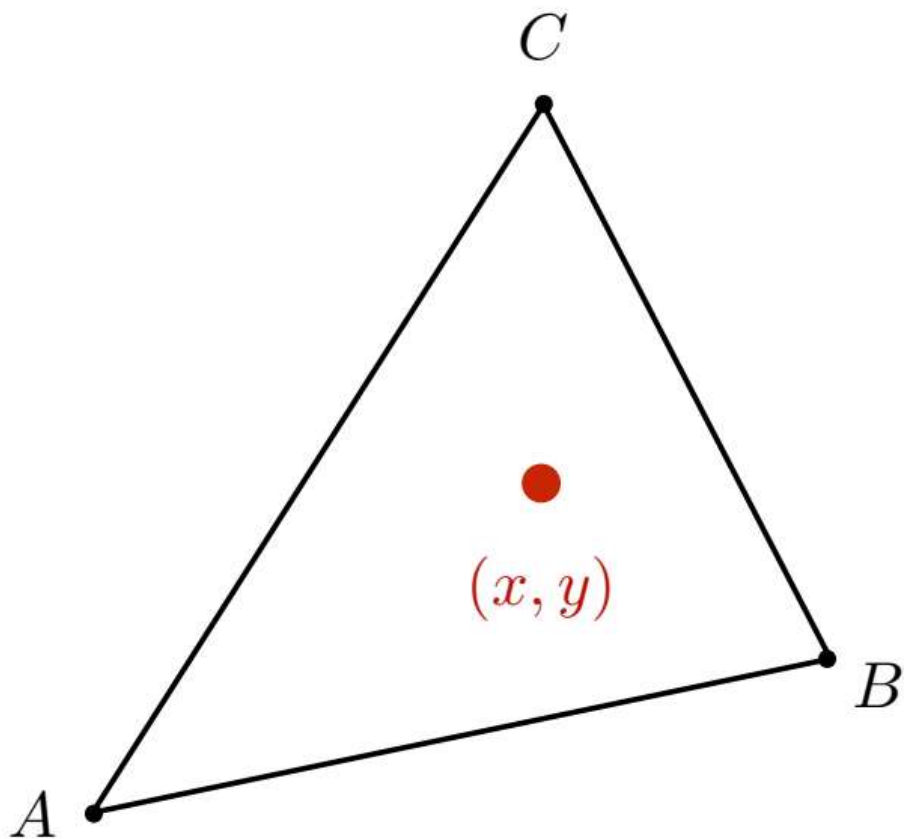
$$\beta = \frac{A_B}{A_A + A_B + A_C}$$

$$\gamma = \frac{A_C}{A_A + A_B + A_C}$$



# 重心坐标

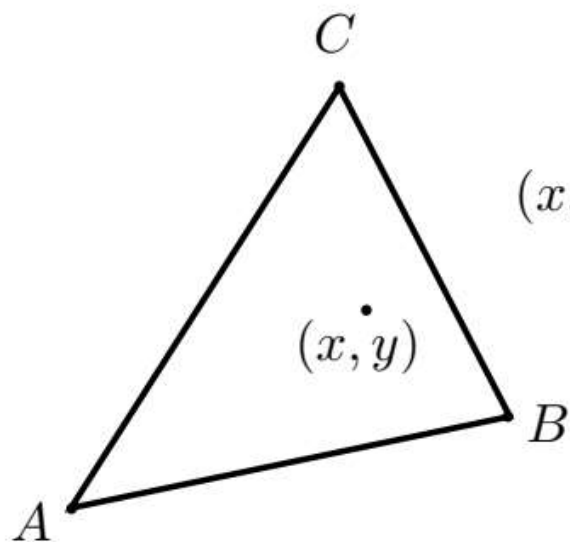
Q: 三角形重心的重心坐标是什么?



$$(\alpha, \beta, \gamma) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$(x, y) = \frac{1}{3} A + \frac{1}{3} B + \frac{1}{3} C$$

# 重心坐标



$$(x, y) = \alpha A + \beta B + \gamma C$$

$$\alpha + \beta + \gamma = 1$$

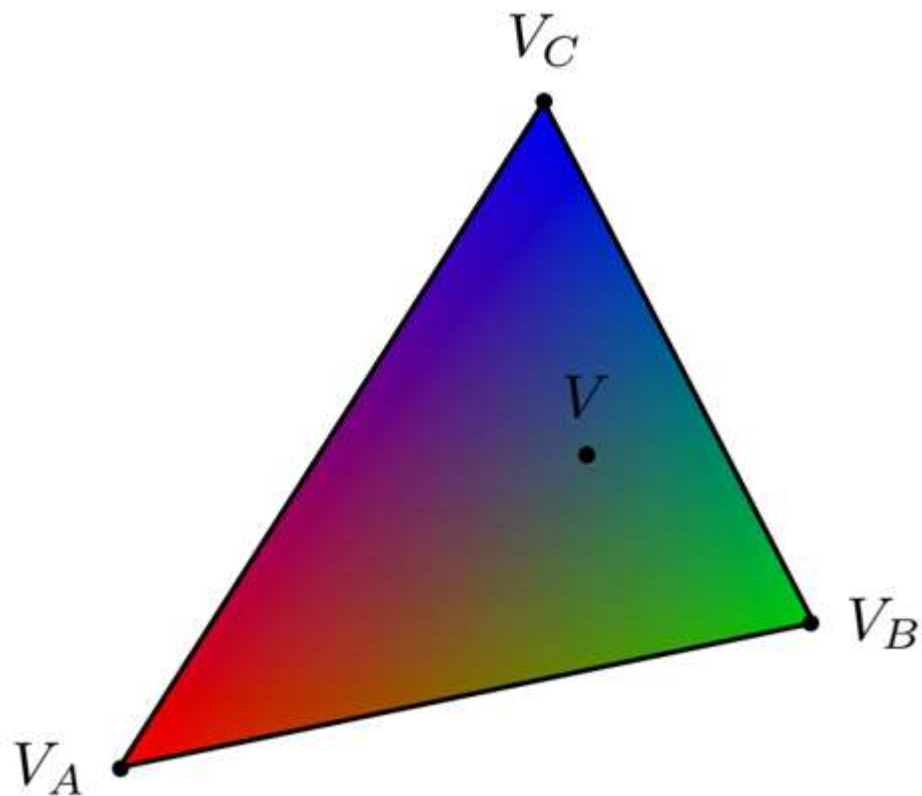
$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

# 重心坐标

- 属性值可以是纹理坐标、颜色、法向量、深度、材质属性等等



$$V = \alpha V_A + \beta V_B + \gamma V_C$$

注意：在投影变换下，重心坐标无法保持不变！

# Q&A





# 纹理的应用

```
uniform sampler2D myTexture;    // program parameter
uniform vec3 lightDir;          // program parameter
varying vec2 uv;                // per fragment value (interp. by rasterizer)
varying vec3 norm;              // per fragment value (interp. by rasterizer)

void diffuseShader()
{
    vec3 kd;
    kd = texture2d(myTexture, uv);    // material color from texture
    kd *= clamp(dot(-lightDir, norm), 0.0, 1.0); // Lambertian shading model
    gl_FragColor = vec4(kd, 1.0);     // output fragment color
}
```

# 简单的纹理映射：漫反射颜色

Usually a pixel's center

for each rasterized screen sample  $(x,y)$ :

$(u,v)$  = evaluate texture coordinate at  $(x,y)$

texcolor = texture.sample( $u,v$ );

set sample's color to texcolor;

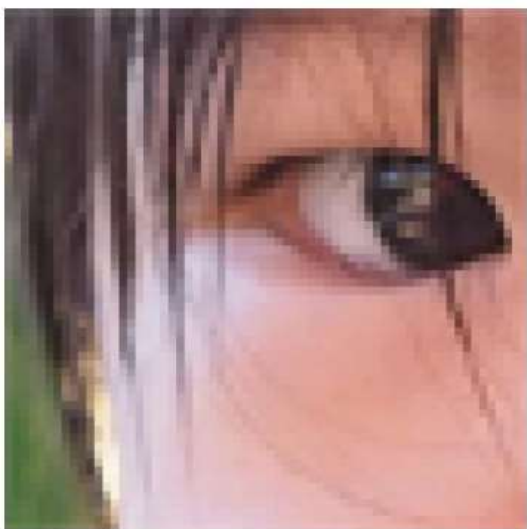
Using barycentric coordinates!

Usually the diffuse albedo  $K_d$   
(recall the Blinn-Phong reflectance model)

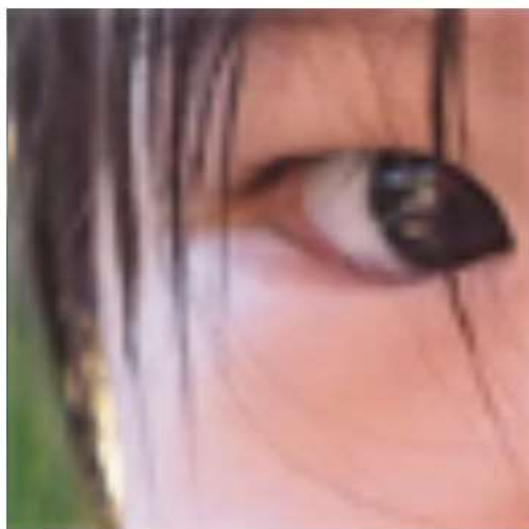
# 纹理放大

Q: 如果纹理很小（分辨率不足）会带来什么问题？

- 纹理上的一个像素（pixel）称为一个纹理元素/纹素(texel)



Nearest

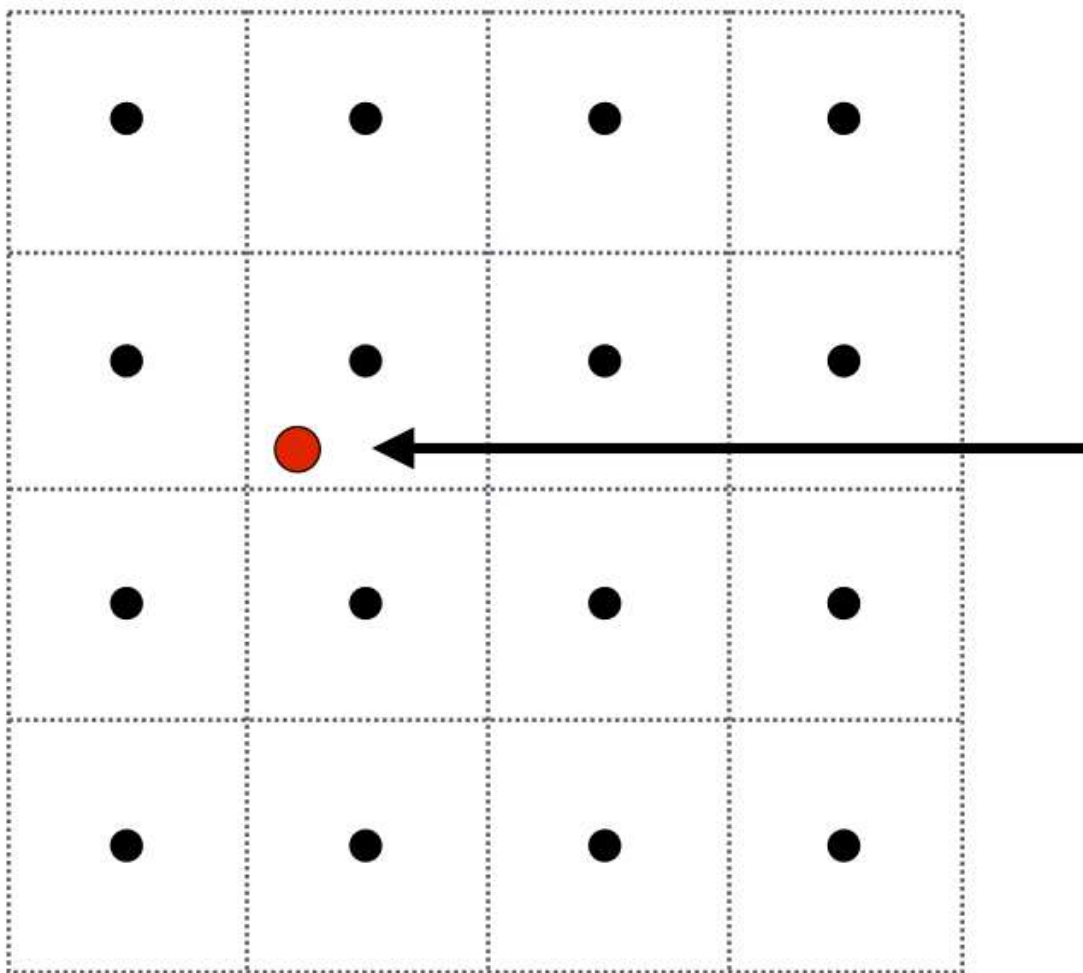


Bilinear



Bicubic

# 最近邻方法

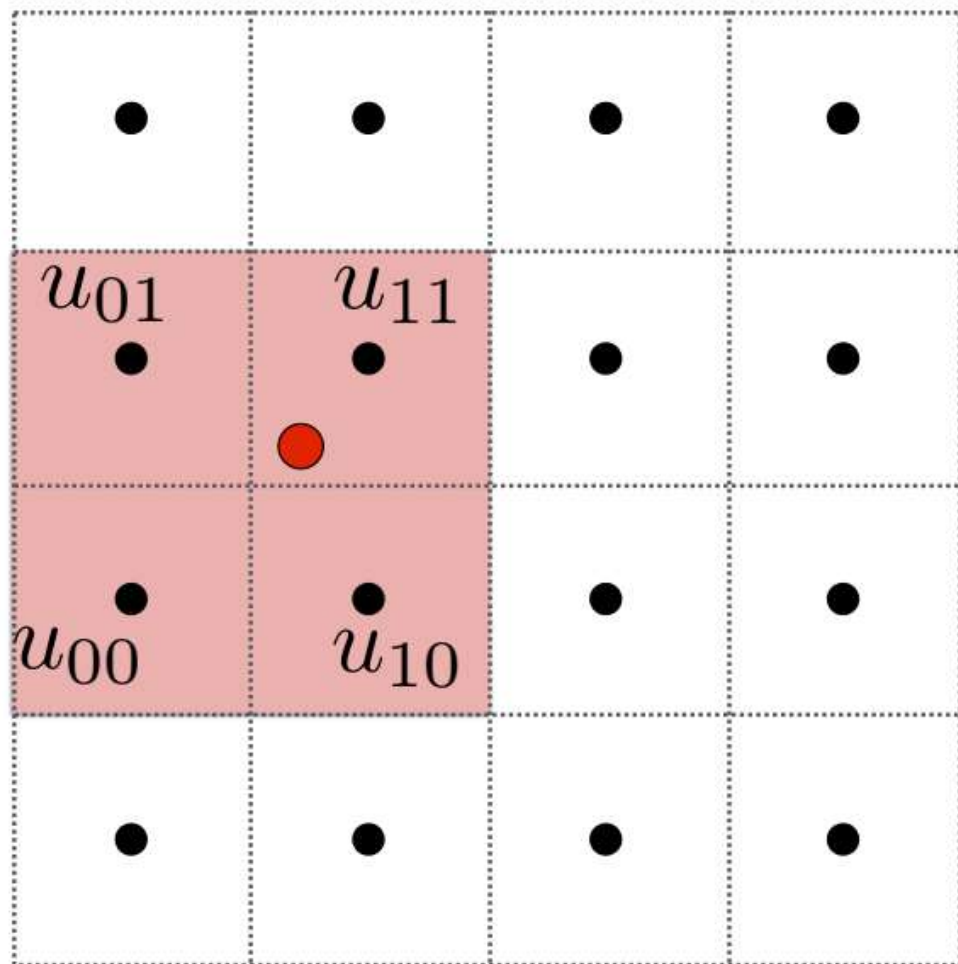


Want to sample  
texture value  $f(x,y)$  at  
red point

Black points indicate  
texture sample  
locations

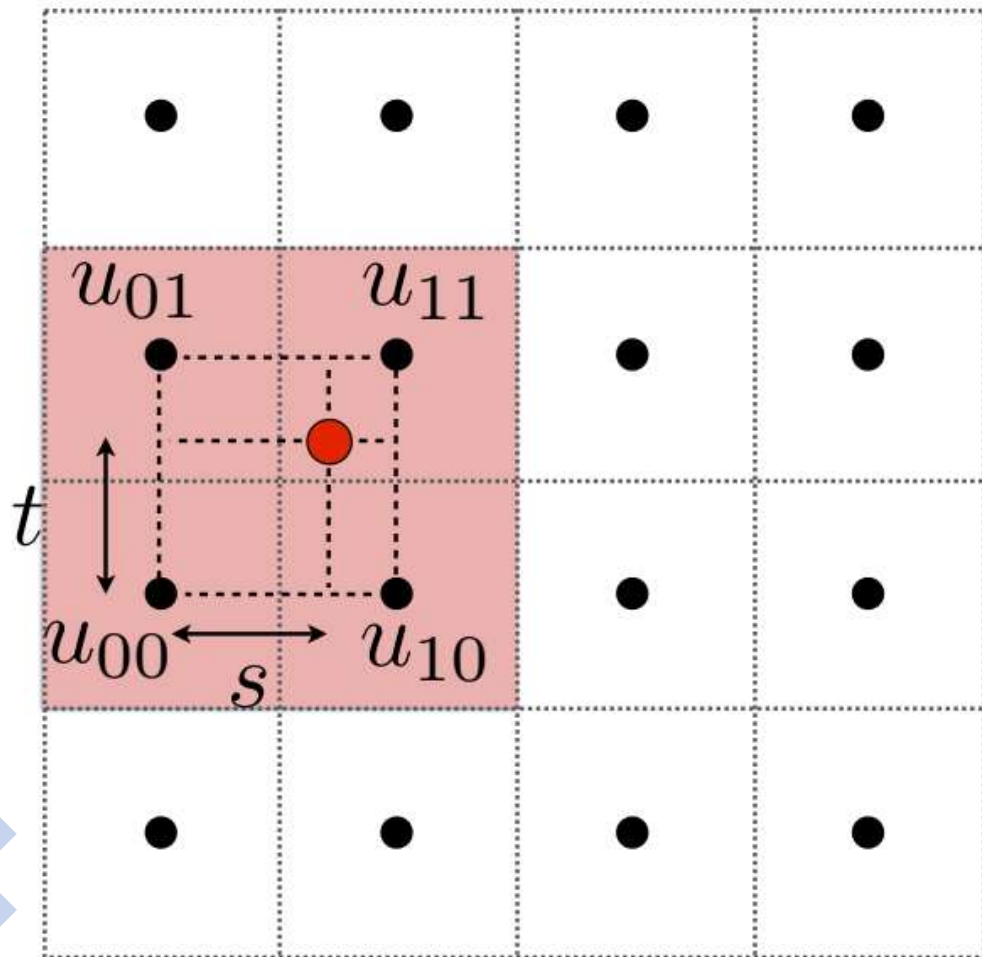


# 双线性插值



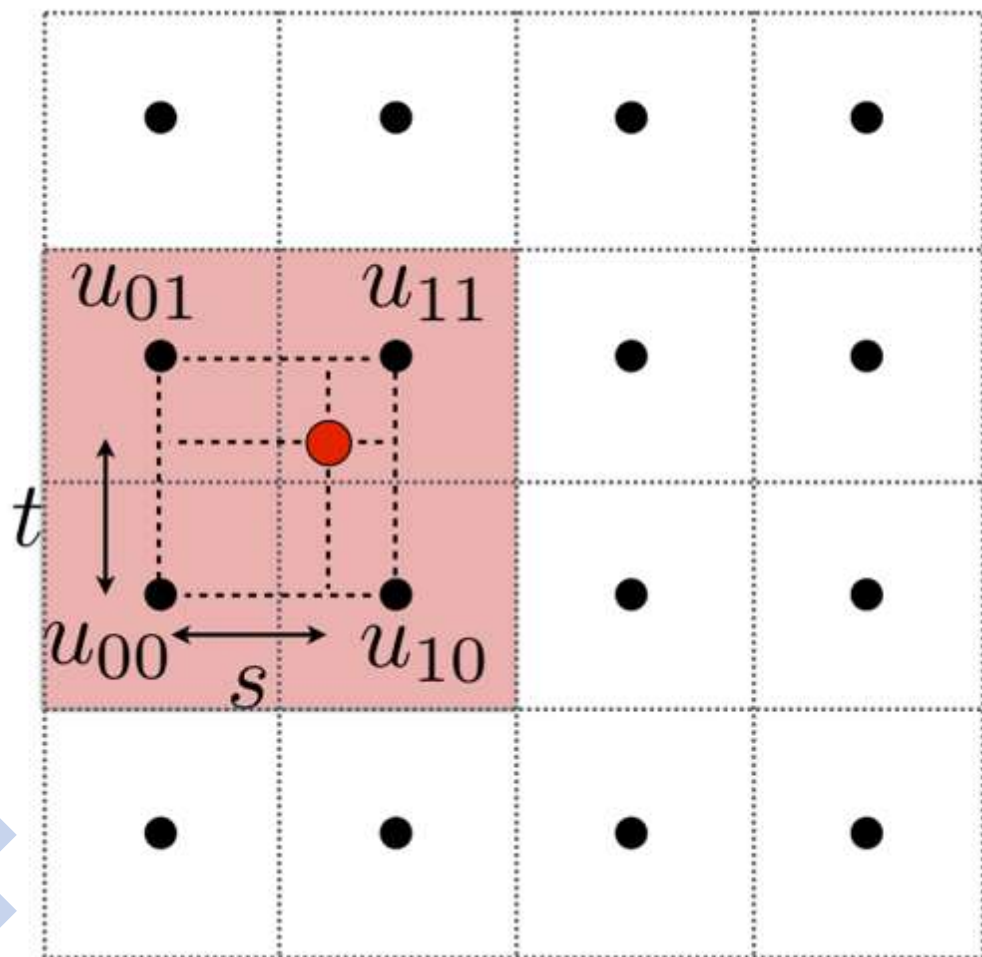
Take 4 nearest sample locations, with texture values as labeled.

# 双线性插值



And fractional offsets,  
( $s, t$ ) as shown

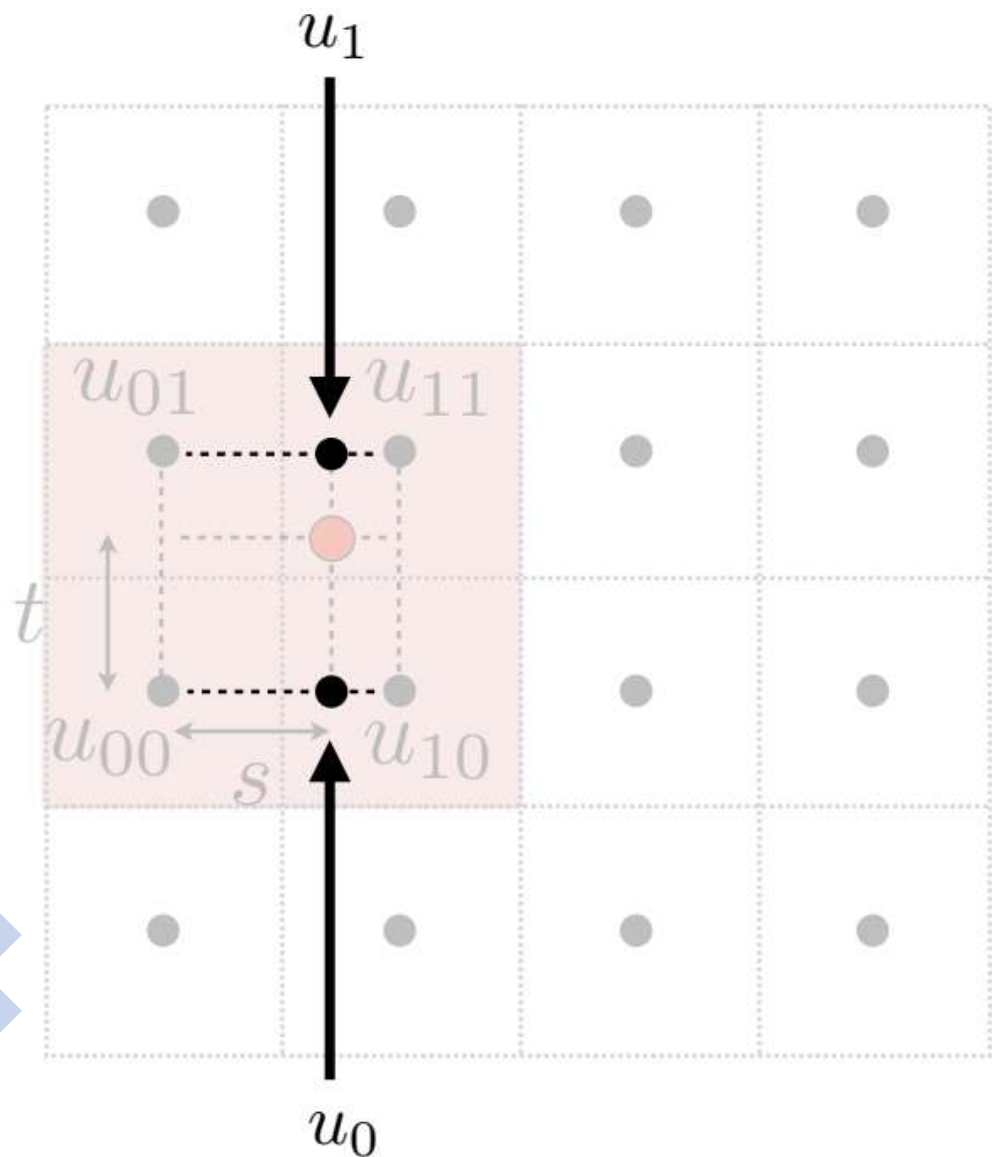
# 双线性插值



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

# 双线性插值



**Linear interpolation (1D)**

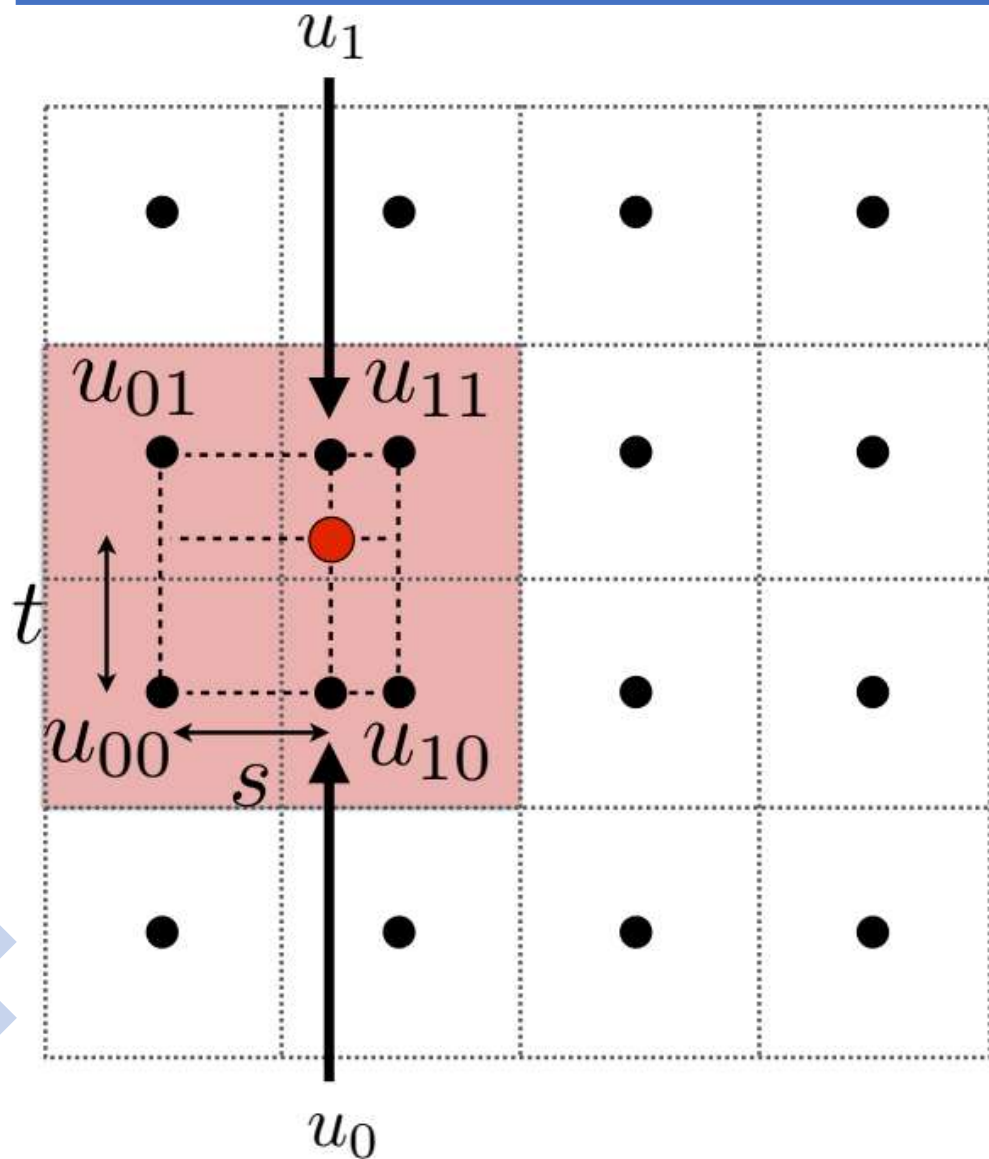
$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

**Two helper lerps (horizontal)**

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

# 双线性插值



**Linear interpolation (1D)**

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

**Two helper lerps**

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

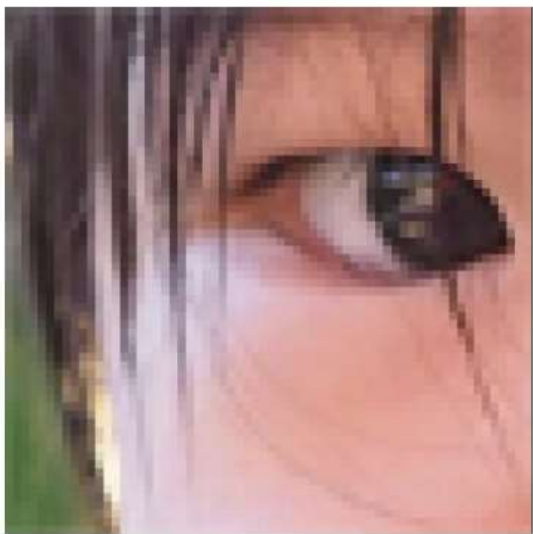
**Final vertical lerp, to get result:**

$$f(x, y) = \text{lerp}(t, u_0, u_1)$$

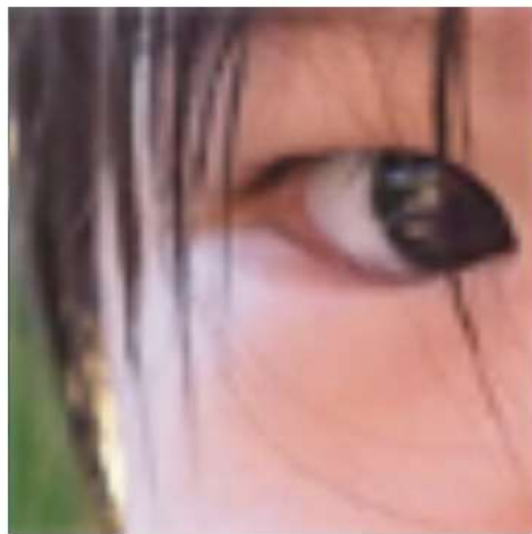


# 纹理放大

- 纹理放大是一种较简单的情况
  - 双线性插值可以在可接受的代价下生成不错的结果



Nearest



Bilinear



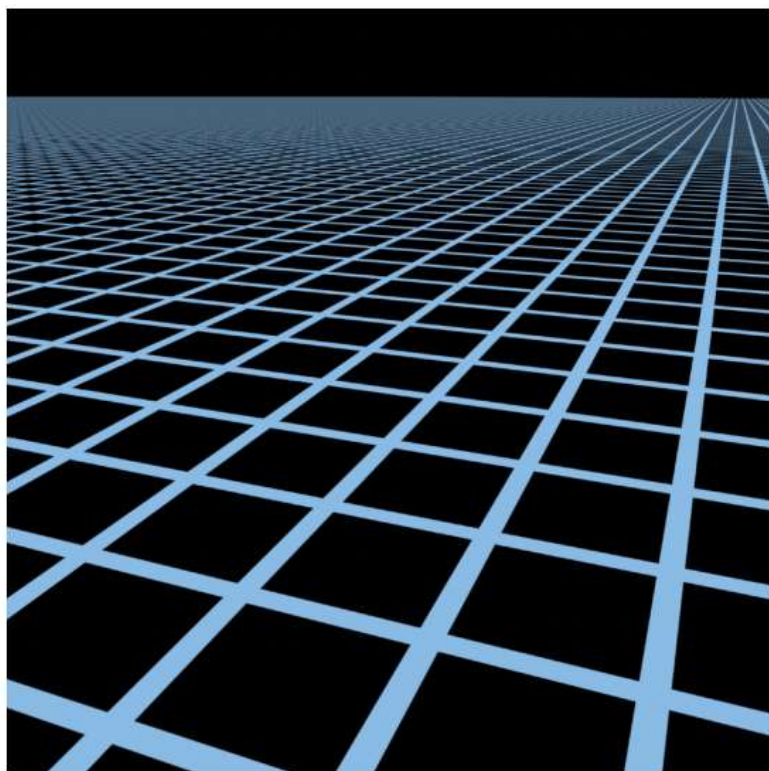
Bicubic

# Q&A

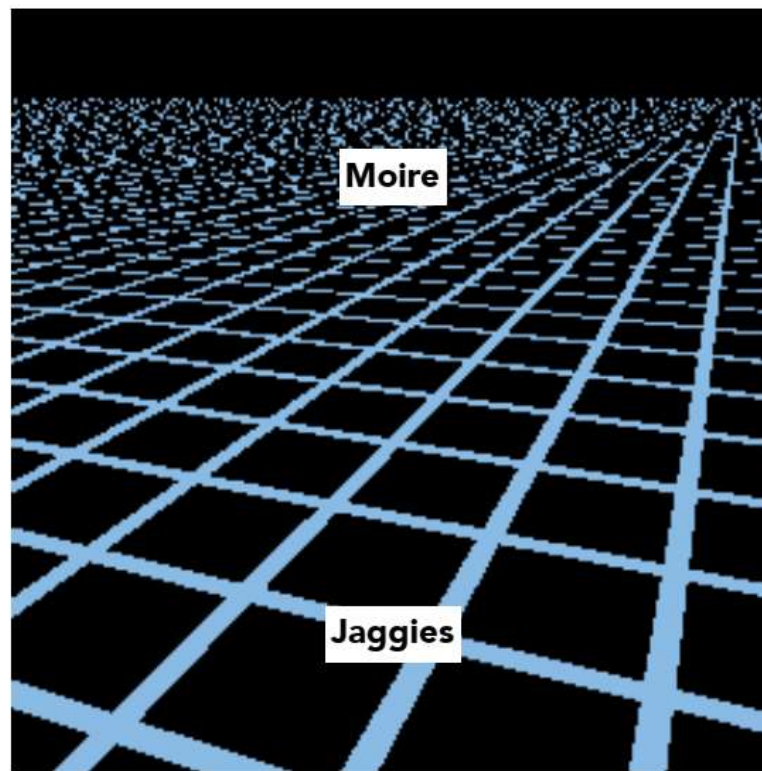


# 纹理缩小

Q: 如果纹理很大会有问题吗?

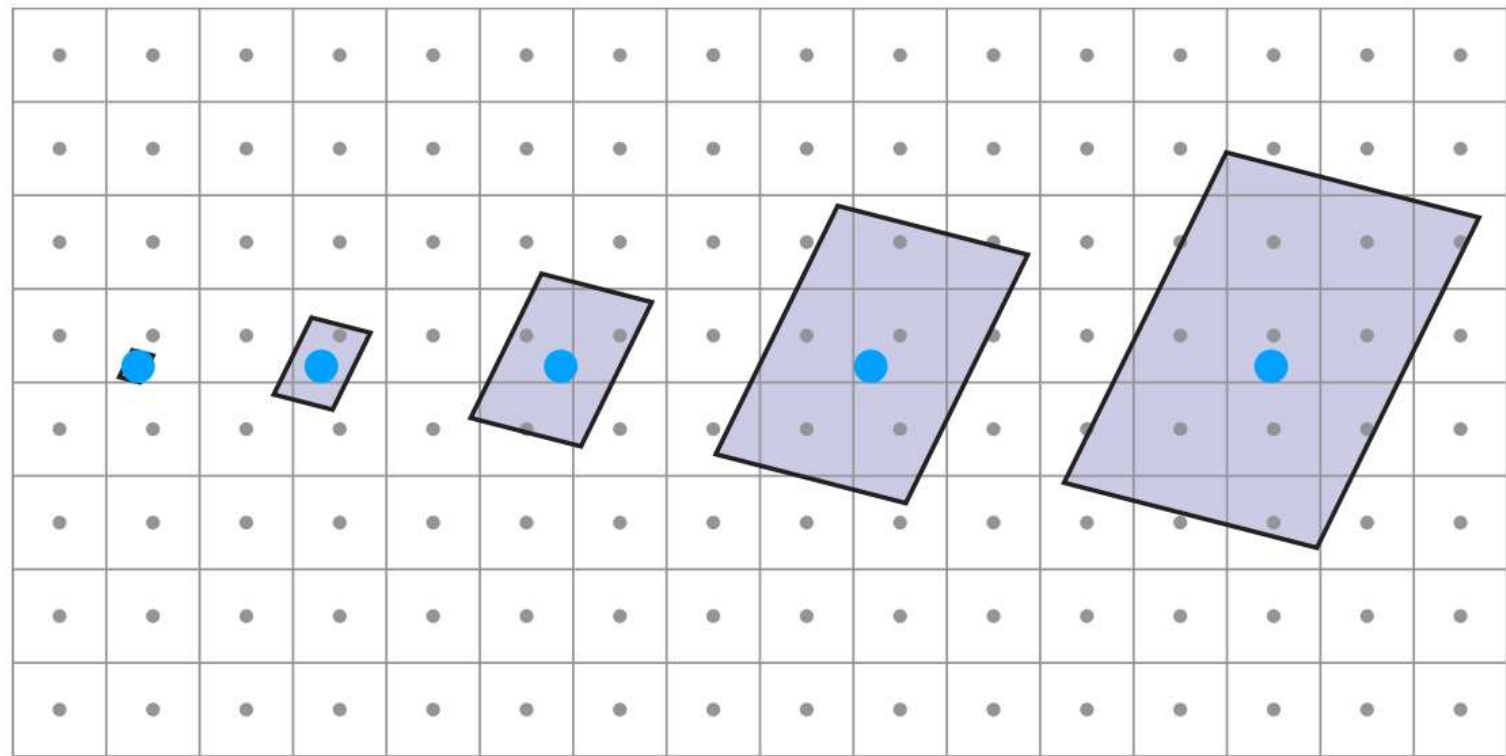


Reference



Point sampled

# 像素“覆盖”的纹理



Upsampling  
(Magnification)

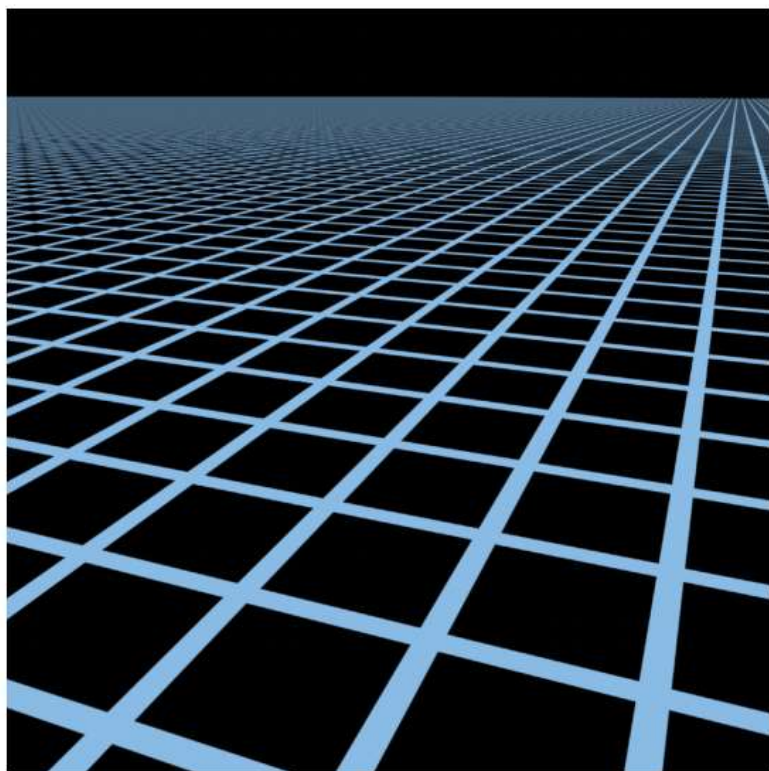


Downsampling  
(Minification)

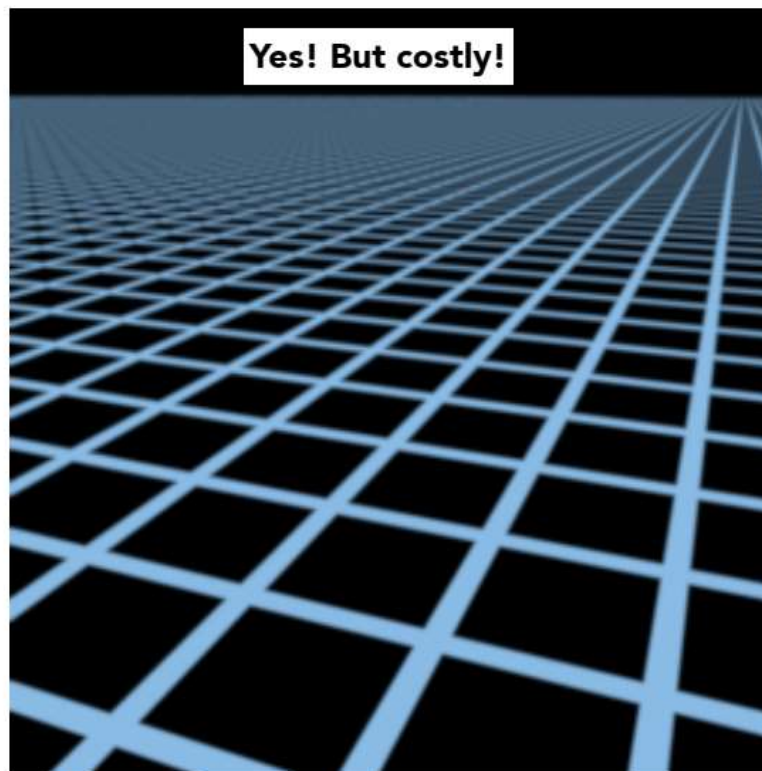


# 纹理缩小

Q: 超级采样可以用来解决这个问题吗?



Reference



512x supersampling



# 纹理缩小

Q: 超级采样可以用来解决这个问题吗?

- 可以, 质量高但代价也高
- 当纹理缩小时, 一个像素“覆盖”多个纹素
- 像素中的信号频率过高, 需要更高的采样频率

Q: 有没有其他思路来解决这个问题?

- 不做采样, 而是在一定范围内求平均值
- 点查询vs. 范围查询

# 不同像素“覆盖区域”可能不同



# Mipmap (多级贴图)

- 快速、近似、正方形的范围查询



Level 0 = 128x128



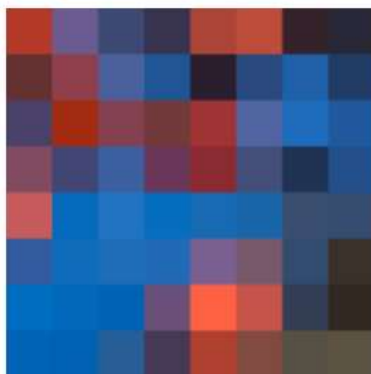
Level 1 = 64x64



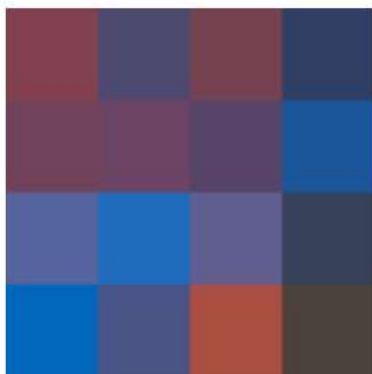
Level 2 = 32x32



Level 3 = 16x16



Level 4 = 8x8



Level 5 = 4x4

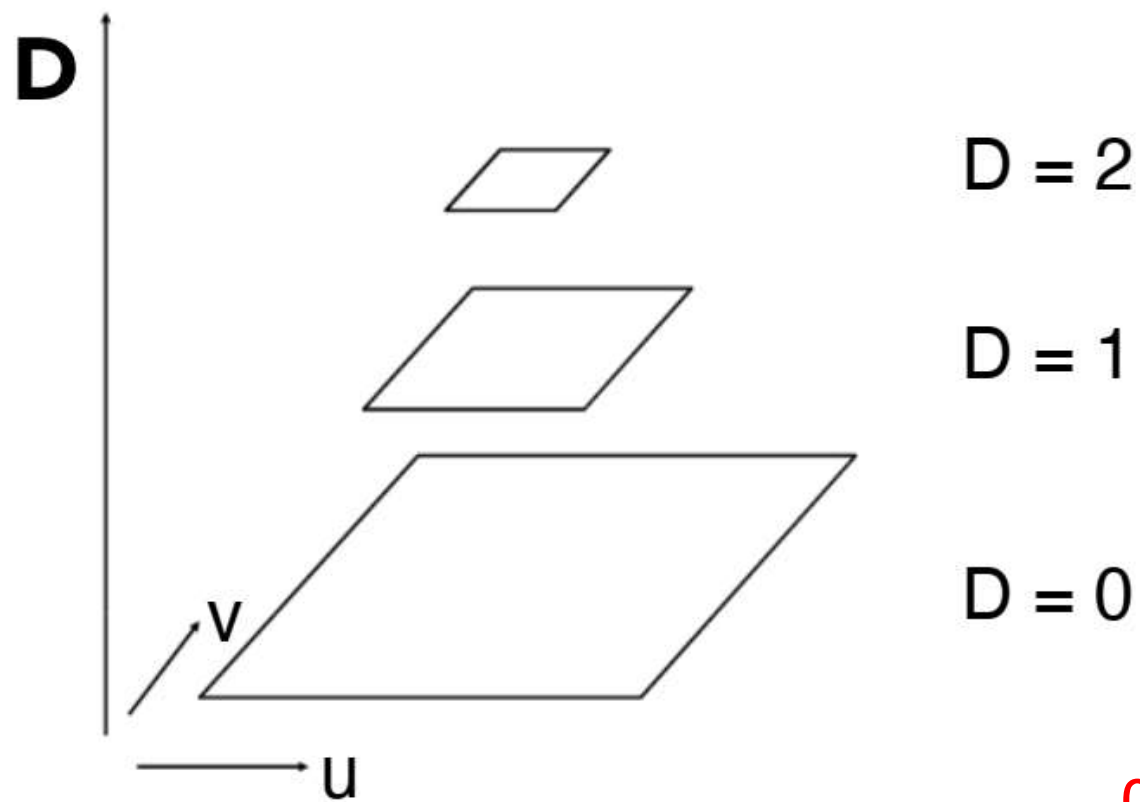


Level 6 = 2x2



Level 7 = 1x1

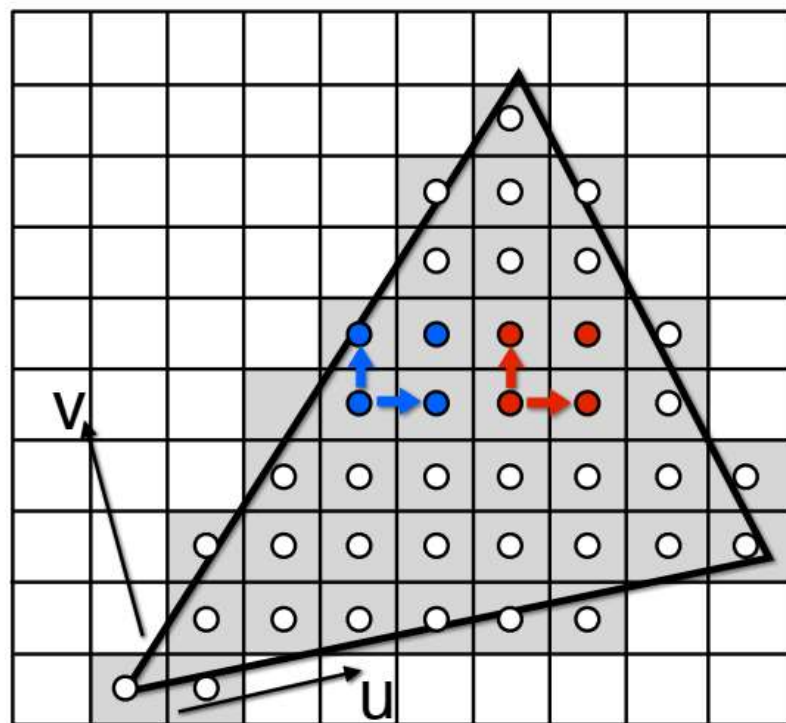
# Mipmap



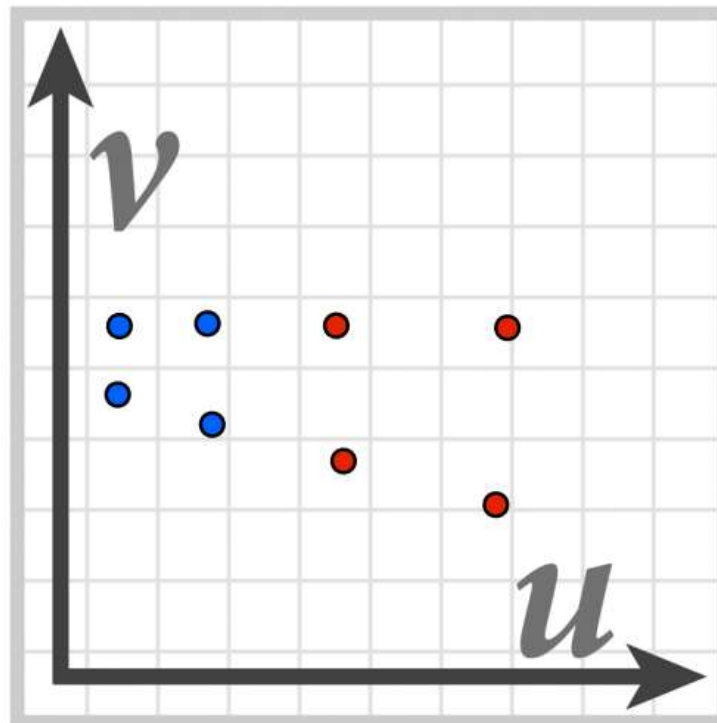
"Mip hierarchy"  
level = D

Q:mipmap的额外存储空间是多少？

# 计算像素的“覆盖”范围



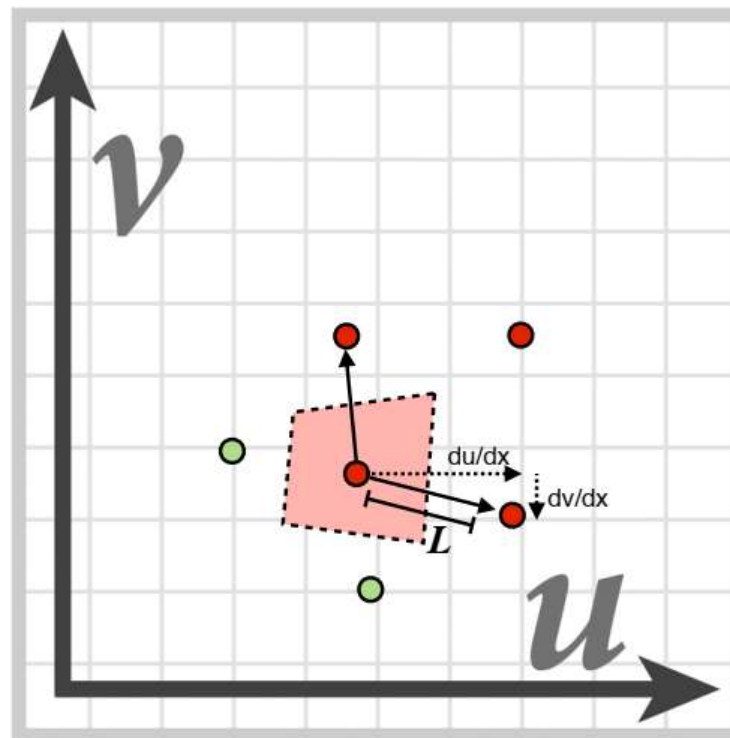
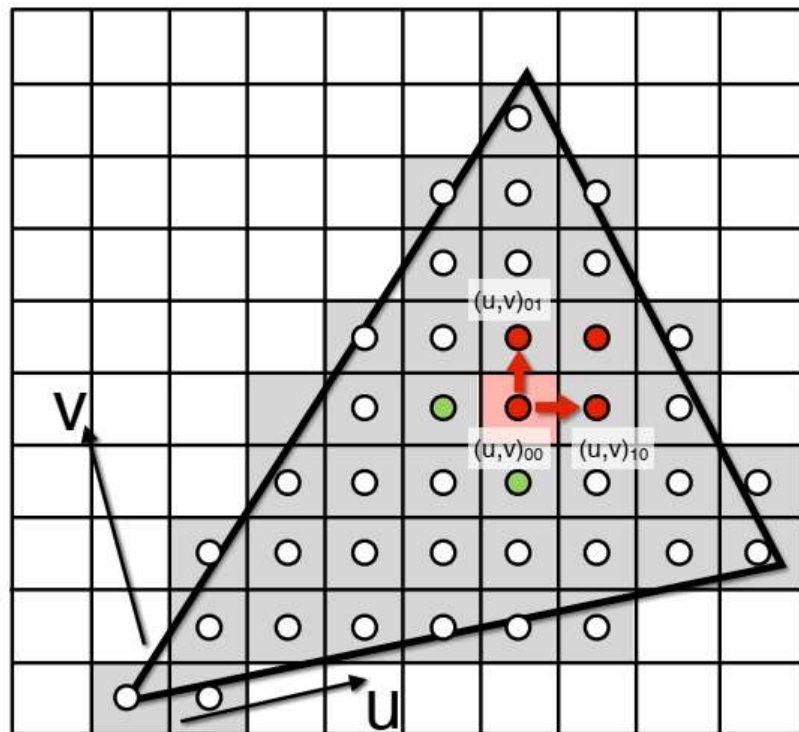
Screen space  $(x,y)$



Texture space  $(u,v)$

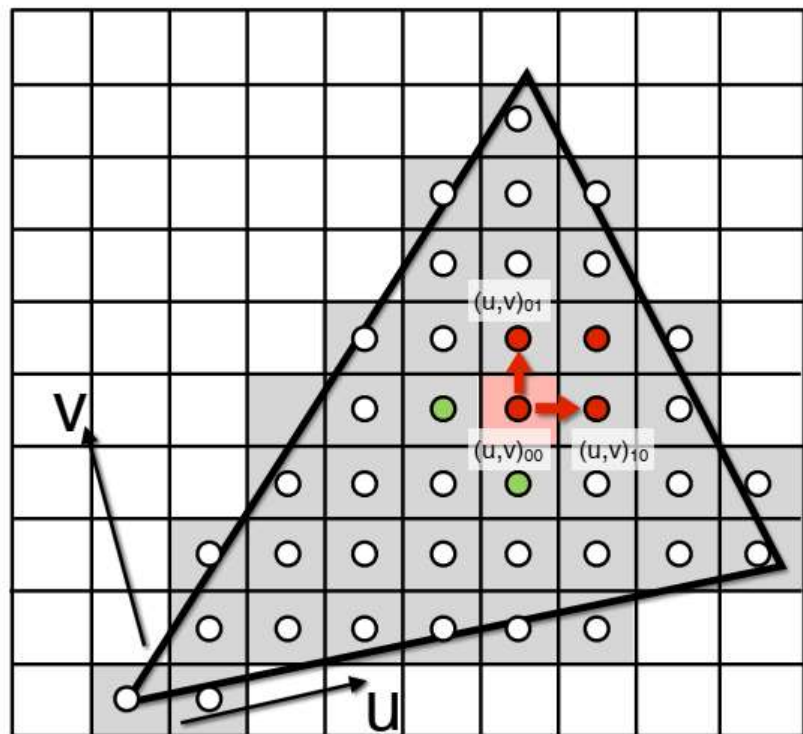


# 计算像素的“覆盖”范围

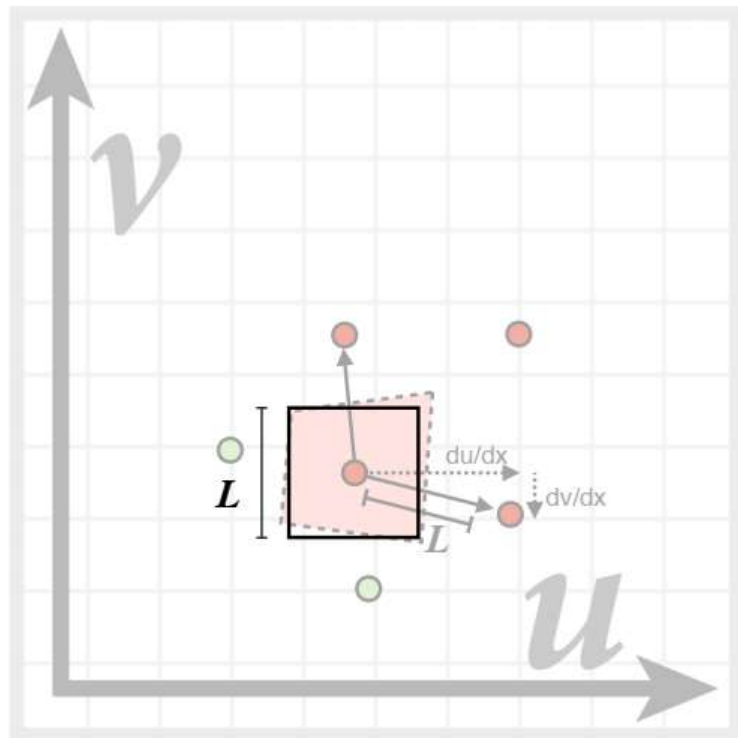


$$L = \max \left( \sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} \right)$$

# 计算像素的“覆盖”范围

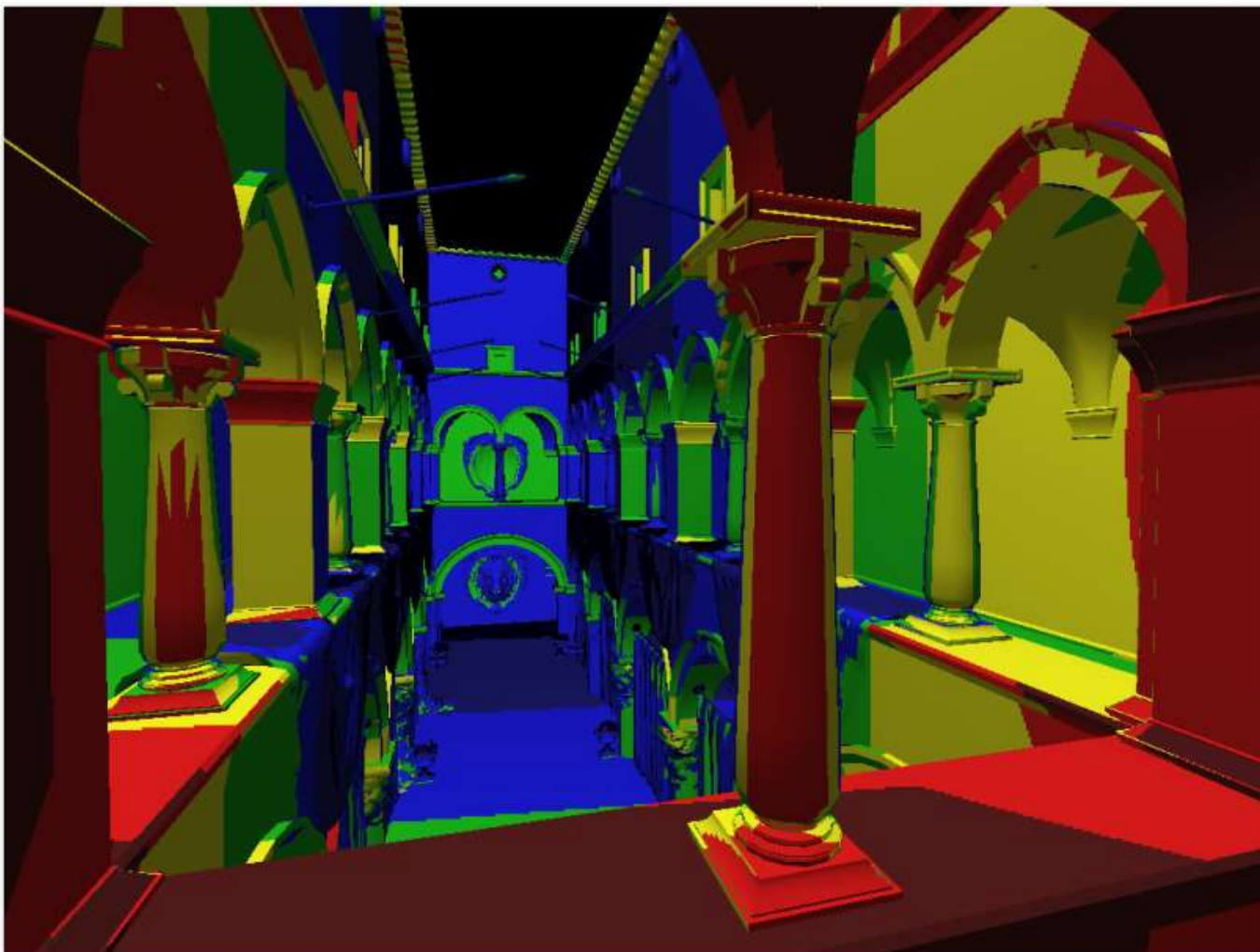


$$D = \log_2 L$$



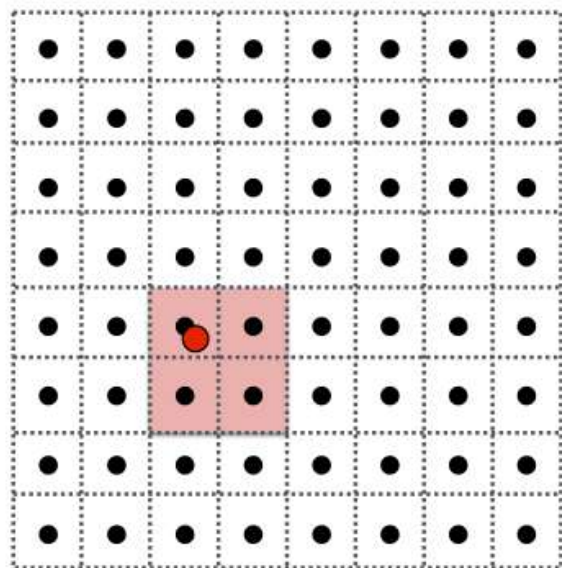
$$L = \max \left( \sqrt{\left( \frac{du}{dx} \right)^2 + \left( \frac{dv}{dx} \right)^2}, \sqrt{\left( \frac{du}{dy} \right)^2 + \left( \frac{dv}{dy} \right)^2} \right)$$

# Mipmap level 可视化



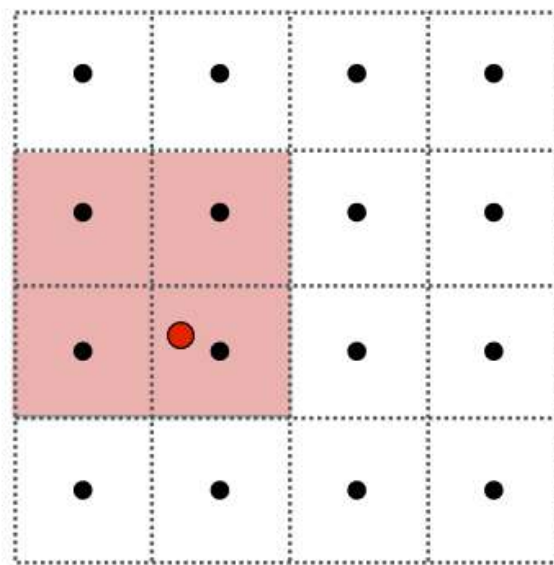
D rounded to nearest integer level

# 三线性插值



Mipmap Level D

Bilinear result



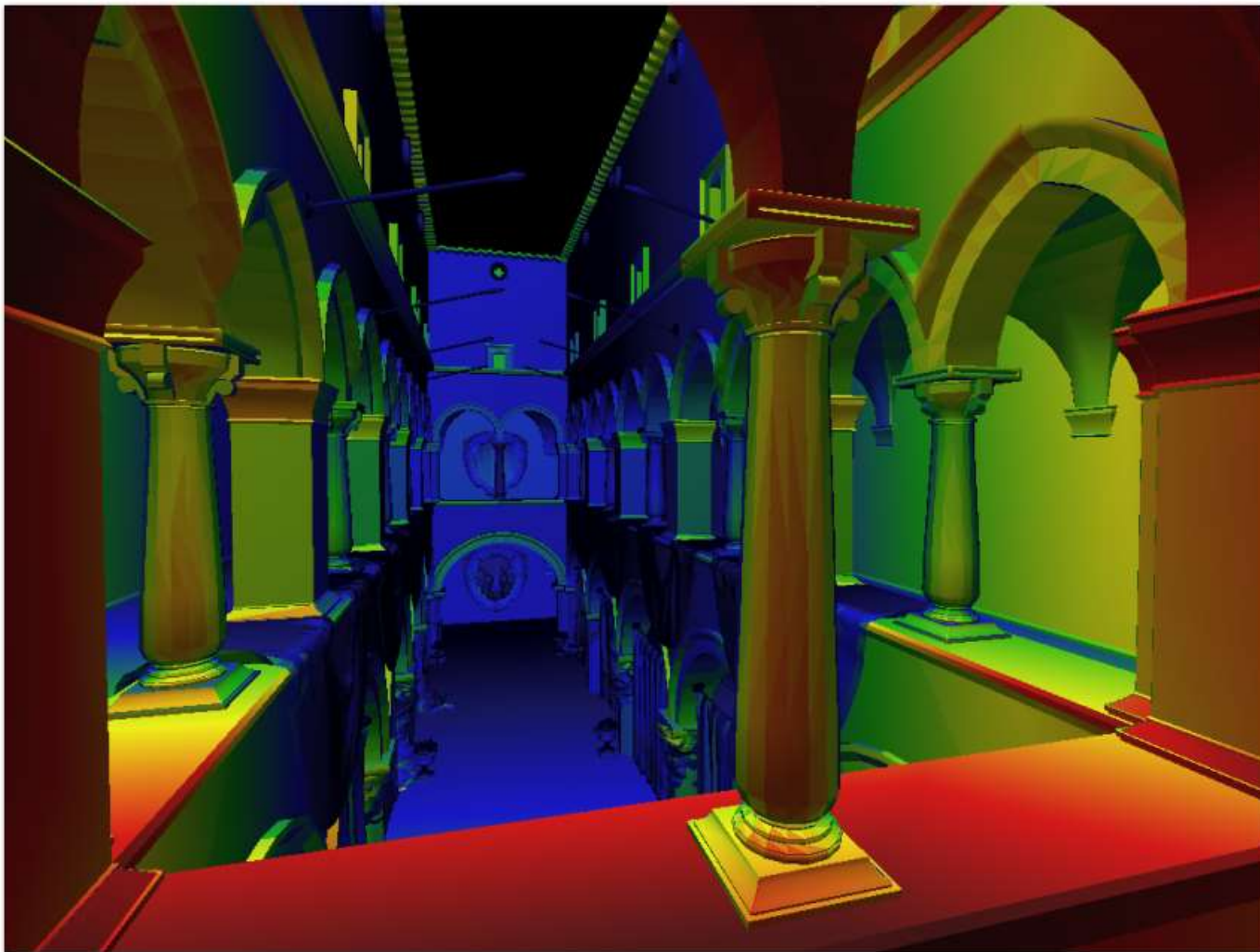
Mipmap Level D+1

Bilinear result

Linear interpolation based on continuous D value



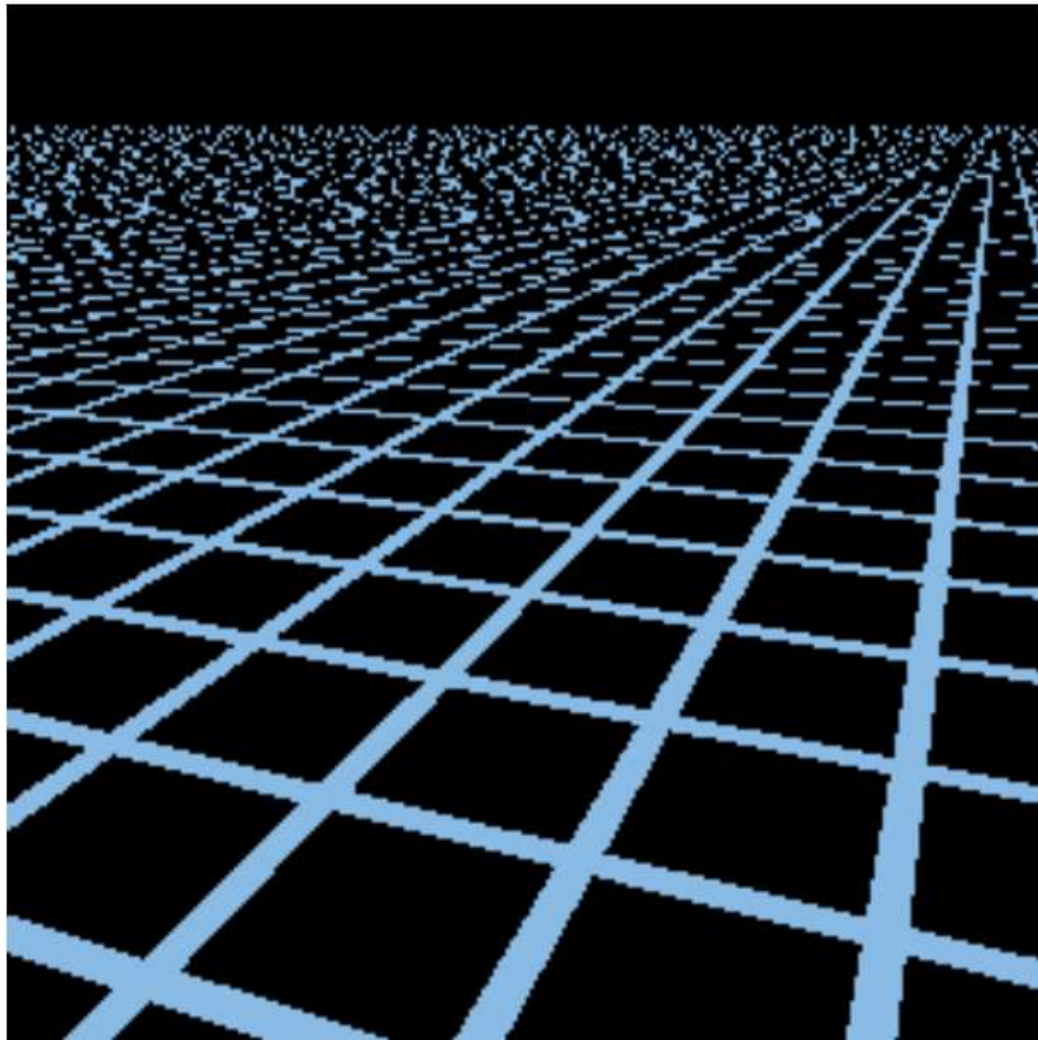
# Mipmap level 可视化



Trilinear filtering: visualization of continuous D

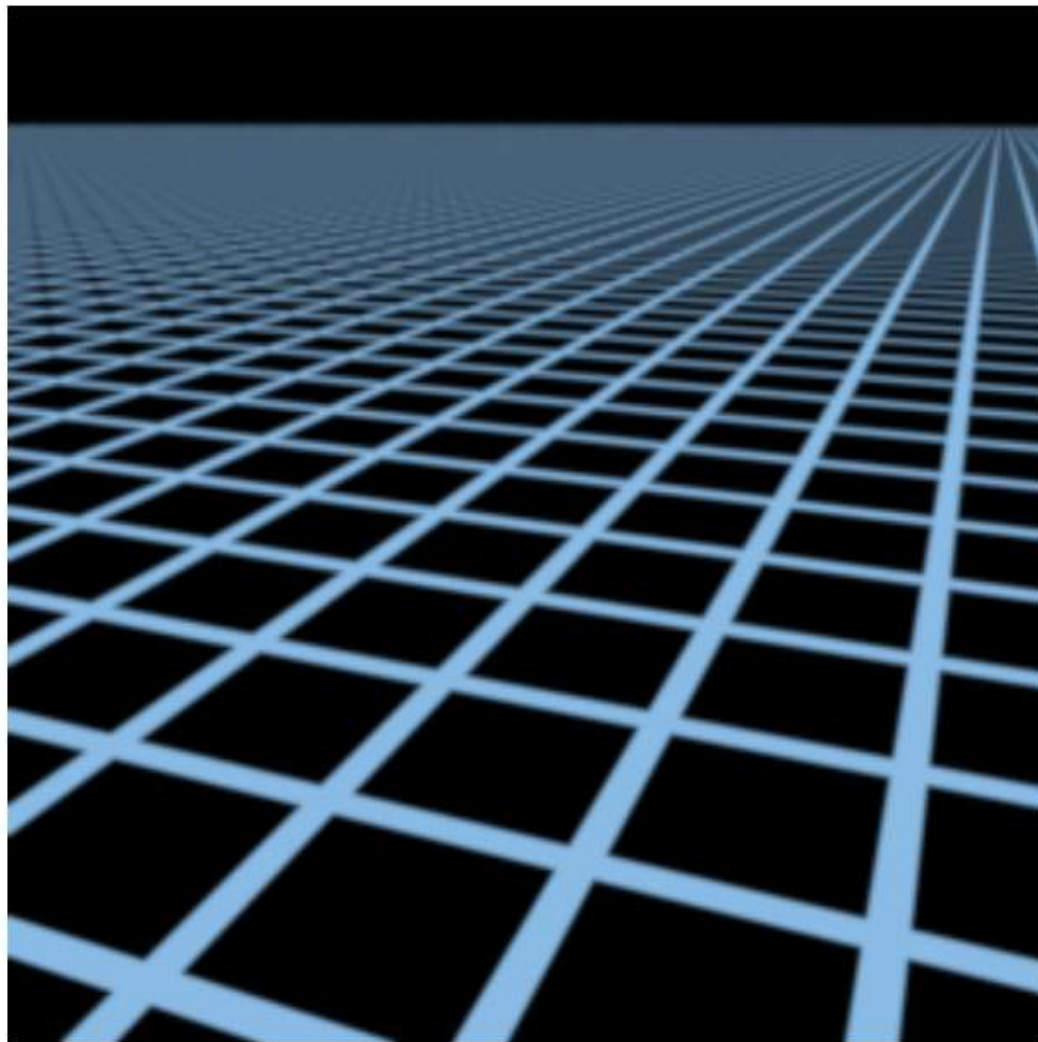


# Mipmap的问题



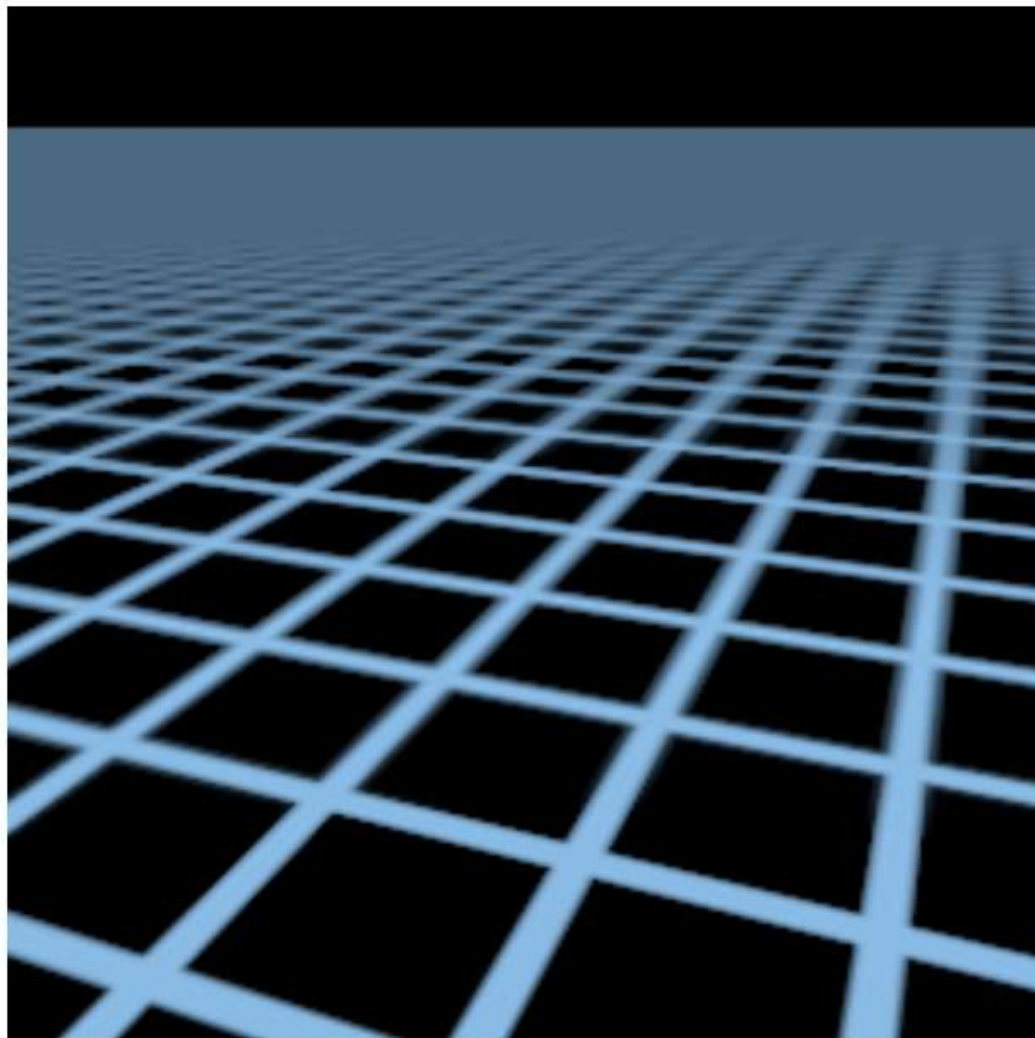
Point sampling

# Mipmap的问题



Supersampling 512x (assume this is correct)

# Mipmap的问题

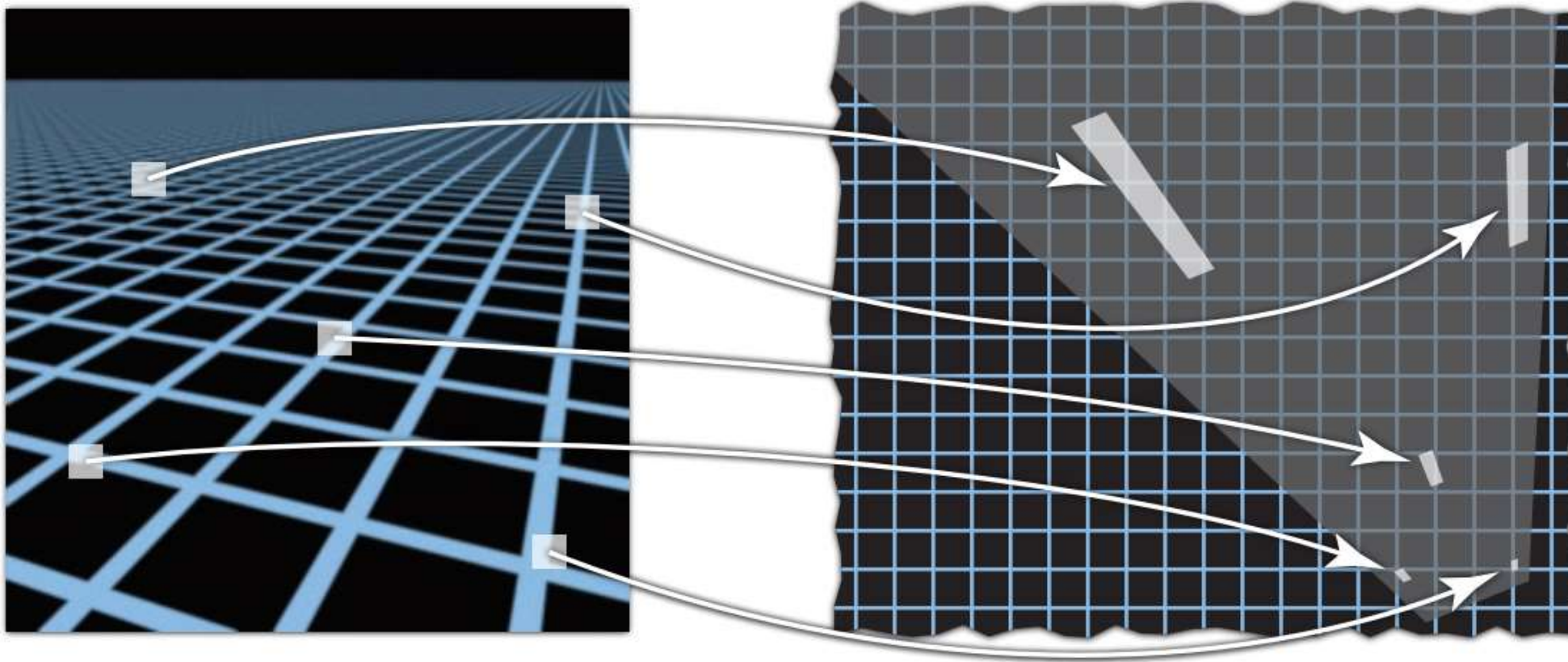


过度模糊!

Mipmap trilinear sampling



# Mipmap的问题



Screen space

Texture space

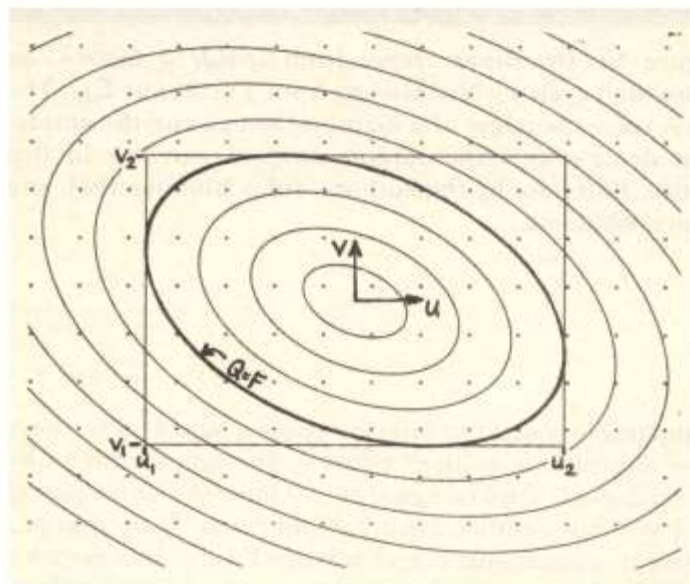
# Mipmap的问题

- 各向异性过滤 (Anisotropic Filtering)

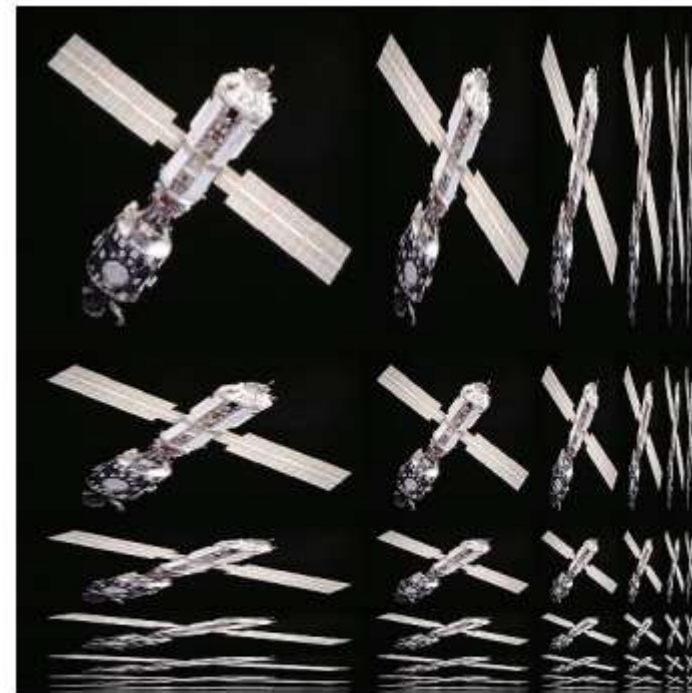
- 可以查找轴对齐的矩形区域
- 存储空间增大

- EWA过滤

- 可以处理不规则区域
- 多次查询



Greene & Heckbert '86



Wikipedia



# Q&A

