一 选择题：1-6. ADC AAA

二 填空题 (1) 0.3　(2) 0.6　(3) 18

(4) $\left(\bar{X} \pm \frac{S}{\sqrt{2020}} t_{0.025}(2019)\right)$

(5) 0.1　(6) $\frac{1}{4}$

三 解答题

1 设 A: 患病 B: 阳性. $P(A)=0.05$　$P(B|A)=0.99$　$P(B|\bar{A})=0.1$

(1) $P(\bar{B}) = P(\bar{B}|A)\cdot P(A) + P(\bar{B}|\bar{A})\cdot P(\bar{A})$

$= 0.01 \times 0.05 + 0.9 \times 0.95 = 0.8555$

(2) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)\cdot P(A)}{1-P(\bar{B})} = \frac{0.99\times 0.05}{1-0.8555} = \frac{99}{289}$

或 0.34256

2 令 Y: X>3 次数. $P\{X>3\} = \int_3^{+\infty} \frac{1}{2} e^{-\frac{x}{2}} dx = e^{-\frac{3}{2}}$

则 $Y \sim B(100, e^{-\frac{3}{2}})$

$P\{Y=10\} = C_{100}^{10} (e^{-\frac{3}{2}})^{10} (1-e^{-\frac{3}{2}})^{100-10} = C_{100}^{10} e^{-15} (1-e^{-\frac{3}{2}})^{90}$

3 (1) $1 = \iint f(x,y)\,dx\,dy \Rightarrow k=6$

(2) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 3e^{-3x} & x>0 \\ 0 & x\le 0 \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} 2e^{-2y} & y>0 \\ 0 & y\le 0 \end{cases}$

(3) $f_X(x)\cdot f_Y(y) = f(x,y) \Rightarrow$ X 与 Y 独立.

(4) $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} 3e^{-3x} & x>0 \\ 0 & x\le 0 \end{cases}$

(5) $P\{X>1 | Y=2\} = \int_1^{+\infty} 3e^{-3x}\,dx = e^{-3}$

(6) $f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x)\,dx$

$z<0$ 时 $f_Z(z) = 0$

$z>0$ 时 $f_Z(z) = \int_0^z 3\cdot e^{-3x}\, 2\cdot e^{-2(z-x)}\,dx$

$= 6e^{-2z}(1-e^{-z})$.

故 $f_Z(z) = \begin{cases} 6e^{-2z}(1-e^{-z}) & z>0 \\ 0 & z\le 0 \end{cases}$

4:

X:	1	2	3.	Y:	-1	0	3	5
P_X:	0.25	0.3	0.45	P_Y:	0.35	0.25	0.2	0.2

1) $E(X) = 1\times 0.25 + 2\times 0.3 + 3\times 0.45 = 2.2$

$E(Y) = 1.25$

(2) $E(X^2) = 5.5$ $\quad E(Y^2) = 7.15$

$D(X) = 0.66$ $\quad D(Y) = 5.5875$.

(3) $Cov(X\,Y) = E(XY) - E(X)\cdot E(Y)$

$= 2.6 - 2.2\times 1.25 = -0.15$.

$\rho_{XY} = \dfrac{Cov(XY)}{\sqrt{D(X)}\sqrt{D(Y)}} = \dfrac{-0.15}{\sqrt{0.66}\cdot\sqrt{5.5875}}$

5. (1) $\mu_1 = E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_\theta^{+\infty} x\cdot e^{-(x-\theta)}\,dx = \theta+1$.

$\theta+1 = \bar{X} \Rightarrow \hat{\theta}_1 = \bar{X} - 1$.

$L(\theta) = \prod_{i=1}^n f(x_i) = \begin{cases} \prod_{i=1}^n e^{-(x_i-\theta)} & \theta \le \min\{x_i\} \\ 0 & 其它. \end{cases}$

$\Rightarrow \hat{\theta}_2 = \min\{x_1 \cdots x_n\}$

(2) $E(\hat{\theta}_1) = E(\bar{X}-1) = \theta+1-1 = \theta$ $\hat{\theta}_1$ 无偏.

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \begin{cases} 1-e^{-(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$F_{\hat{\theta}_2}(x) = 1-(1-F(x))^n = \begin{cases} 1-e^{-n(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$f_{\hat{\theta}_2}(x) = \begin{cases} ne^{-n(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$\Rightarrow E(\hat{\theta}_2) = \int_{-\infty}^{+\infty} x f_{\hat{\theta}_2}(x)\,dx = \frac{1}{n}+\theta \ne \theta$$

故有偏.

6 枢轴量: $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(99)$

置信区间: $\left(\frac{99\times 25}{\chi^2_{0.01}(99)}, \frac{99\times 25}{\chi^2_{0.99}(99)}\right)$.

(四) 证明:

$\bar{X} \sim N(0, \frac{3^2}{9}) \sim N(0,1)$ $\frac{(10-1)S^2}{4^2} \sim \chi^2(9)$

$\bar{X}$ 与 S^2 独立.

$$\frac{\bar{X}}{\sqrt{\frac{(10-1)S^2}{4^2}/9}} \sim t(9) \text{ 即 } \frac{4\bar{X}}{S} \sim t(9)$$