ch6. 2 4(1)(2) 6 7. 9

2. 解:

(1) $\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$

因为总体 $X \sim N(12, 4)$. 故 $\bar{X} \sim N(12, \frac{4}{5})$

则 $P\{|\bar{X}-12|>1\} = 1 - P\{|\bar{X}-12| \le 1\}$

$= 1 - P\{-1 \le \bar{X}-12 \le 1\}$

$= 1 - P\{\frac{-1}{\sqrt{\frac{4}{5}}} \le \frac{\bar{X}-12}{\sqrt{\frac{4}{5}}} \le \frac{1}{\sqrt{\frac{4}{5}}}\}$

$= 1 - [\Phi(\frac{1}{\sqrt{\frac{4}{5}}}) - \Phi(\frac{-1}{\sqrt{\frac{4}{5}}})]$

$= 2 - 2\Phi(1.12)$

$= 0.2628$

(2) X_i 的分布函数为 $\Phi(\frac{x-12}{2})$, 则 $M = \max\{X_1, X_2, X_3, X_4, X_5\}$ 的分布函数为 $F_M(x) = [\Phi(\frac{x-12}{2})]^5$

$P\{\max\{X_1, X_2, X_3, X_4, X_5\} > 15\} = P\{M > 15\}$

$= 1 - P\{M \le 15\} = 1 - F_M(15)$

$= 1 - [\Phi(\frac{15-12}{2})]^5 = 1 - 0.9332^5$

$= 0.2923$

同理. 若记 $N = \min\{X_1, X_2, X_3, X_4, X_5\}$. 则 N 的分布函数为 $F_N(x) = 1 - [1-\Phi(\frac{x-12}{2})]^5$

故 $P\{\min\{X_1, X_2, X_3, X_4, X_5\} < 10\} = P\{N \le 10\}$

$= F_N(10) = 1 - [1 - \Phi(\frac{10-12}{2})]^5 = 1 - [1 - \Phi(-1)]^5$

$= 1 - [\Phi(1)]^5 = 1 - 0.8413^5 = 0.5785.$

4解、

(1). 因 样本 $X_1, X_2, \cdots$; 来自总体 $N(0,1)$

则 $X_1+X_2+X_3 \sim N(0,3)$ $\quad X_4+X_5+X_6 \sim N(0,3)$

二者相互独立.

则 $\dfrac{X_1+X_2+X_3}{\sqrt{3}} \sim N(0,1)$ $\quad \dfrac{X_4+X_5+X_6}{\sqrt{3}} \sim N(0,\sqrt{3})$

二者相互独立且服从 $N(0,1)$

故 $\left(\dfrac{X_1+X_2+X_3}{\sqrt{3}}\right)^2+\left(\dfrac{X_4+X_5+X_6}{\sqrt{3}}\right)^2 \sim \chi^2(2)$

则 $\frac{1}{3}Y \sim \chi^2(2)$. $\quad C=\frac{1}{3}$

(2) 样本 $X_1, X_2, \cdots, X_5$ 来自总体 $N(0,1)$

X_3, X_4, X_5 相互独立. 故 $X_3^2+X_4^2+X_5^2 \sim \chi^2(3)$

$X_1+X_2 \sim N(0,2)$

则 $\dfrac{X_1+X_2}{\sqrt{2}} \sim N(0,1)$

则 $$\frac{\frac{X_1+X_2}{\sqrt{2}}}{\sqrt{\frac{X_3^2+X_4^2+X_5^2}{3}}}=\sqrt{\frac{3}{2}}\,\frac{X_1+X_2}{\sqrt{X_3^2+X_4^2+X_5^2}} \sim t(3)$$

则 $$Y=\sqrt{\frac{3}{2}}\,\frac{(X_1+X_2)}{(X_3^2+X_4^2+X_5^2)^{\frac{1}{2}}}$$

$C=\sqrt{\dfrac{3}{2}}$

6. 解：$X \sim b(1, p)$. $X_1, X_2, \cdots, X_n$ 是来自 X 的样本

(1) $X_1, X_2, \cdots, X_n$ 相互独立，且 $X_i \sim b(1, p)$, $i=1, 2, \cdots n$.

即 X_i 的分布律为 $P\{X_i = x_i\} = p^{x_i}(1-p)^{1-x_i}$ $x_i = 0, 1$

则 $(X_1, X_2, \cdots, X_n)$ 的分布律为：

$P\{X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n\}$

$= \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$

$= p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$

(2) $X_1, X_2, \cdots X_n$ 相互独立，且 $X_i \sim b(1, p)$, $i=1, 2, \cdots, n$.

则 $\sum_{i=1}^{n} X_i \sim b(n, p)$

其分布律为

$P\{\sum_{i=1}^{n} X_i = k\} = C_n^k p^k (1-p)^{n-k}$ $k=0, 1, 2, \cdots, n$

(3) 由于 $X \sim b(1, p)$. $E(X) = p$. $D(X) = p(1-p)$

故 $E(\bar{X}) = p$. $D(\bar{X}) = \frac{p(1-p)}{n}$

$E(S^2) = D(X) = p(1-p)$

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7. 解:

$x_1, x_2, \cdots, x_{10}$ 是来自 X 的样本, $X \sim \chi^2(n)$

则 $E(X) = n \quad D(X) = 2n$.

故 $E\bar{X} = n \quad D(\bar{X}) = \frac{2n}{10} = \frac{n}{5}$

$E(S^2) = D(X) = 2n$.

9. 解:

(1) 由于 $\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$

$n = 16$.

故 $\frac{15S^2}{\sigma^2} \sim \chi^2(15)$

则 $P\{S^2/\sigma^2 \le 2.041\} = P\{15S^2/\sigma^2 \le 15 \times 2.041\}$

$= P\{15S^2/\sigma^2 \le 30.615\} = 1 - P\{15S^2/\sigma^2 > 30.615\}$

又因 $\chi^2_{0.01}(15) = 30.578$.

则 $P\{S^2/\sigma^2 \le 2.041\} = 1 - 0.01 = 0.99$

(2). 因为 $D(\frac{15S^2}{\sigma^2}) = 30$.

$\frac{15^2}{\sigma^2} D(S^2) = 30$

故 $D(S^2) = \frac{2\sigma^2}{15}$

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