第12讲：光线追踪

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上次课程内容

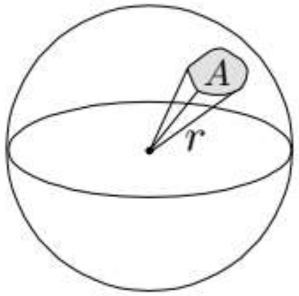
- 光线追踪的加速技术（利用AABB加速光线追踪）
  - 空间划分（均匀格子、KD-Tree）
  - 物体划分（层次包围盒BVH）
- 辐射度量学（Radiometry）
  - Radiant energy
  - Radiant flux
  - Radiant intensity
  - Irradiance
  - Radiance

本次课程内容

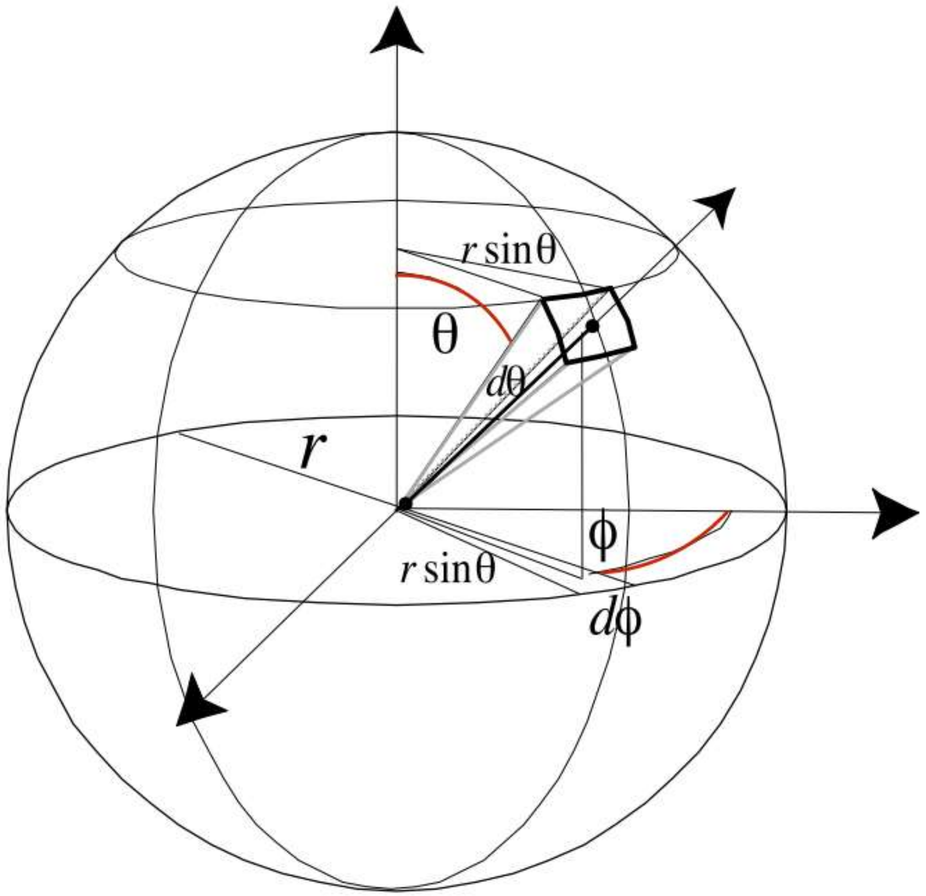
- 辐射度量学（Radiometry）
- 光线传播理论
  - 反射方程
  - 渲染方程
- 全局光照
- 概率知识

辐射度量学回顾

- Radiant energy$(Q)[J]$
  - 电磁辐射的能量
- Radiant flux/power$\left(\Phi = \frac{dQ}{dt}\right)[W][lm]$
  - 每单位时间的Radiant energy
- Radiant intensity$(I(\omega) = \frac{d\Phi}{d\omega})[\frac{W}{sr}][\frac{lm}{sr}]$
  - 每单位立体角的Radiant flux/power
  - 立体角$(\Omega = \frac{A}{r^2})$



微分立体角



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

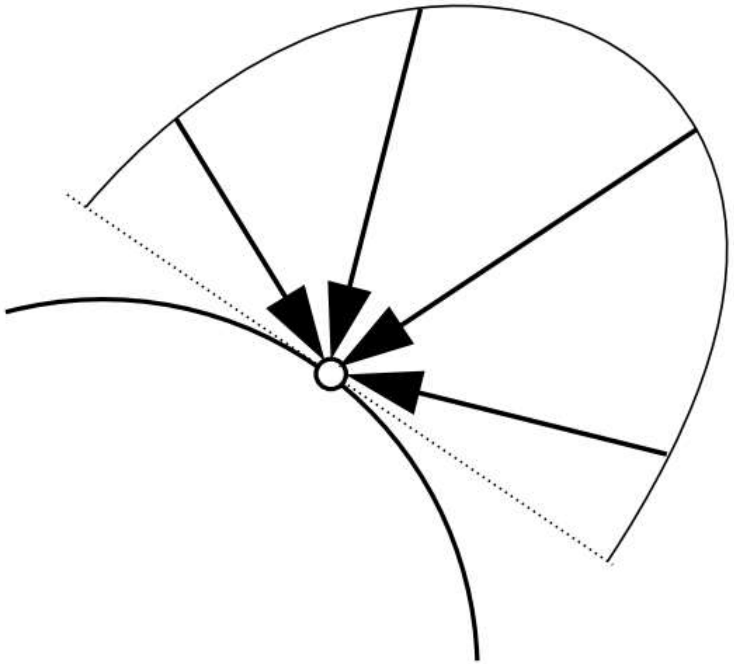
$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Irradiance

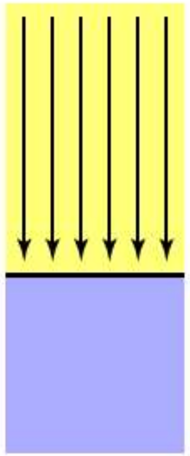
- 定义：入射到表面一点上每单位面积的Radiant flux/power，单位瓦特每平方米

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$

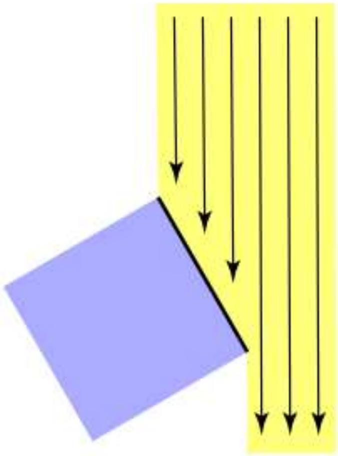


兰伯特余弦定理（Lambert’s Cosine Law）



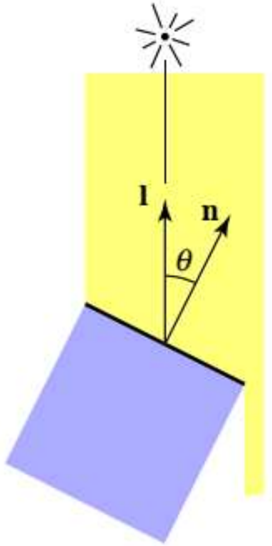
Top face of cube receives a certain amount of light

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube intercepts half the light

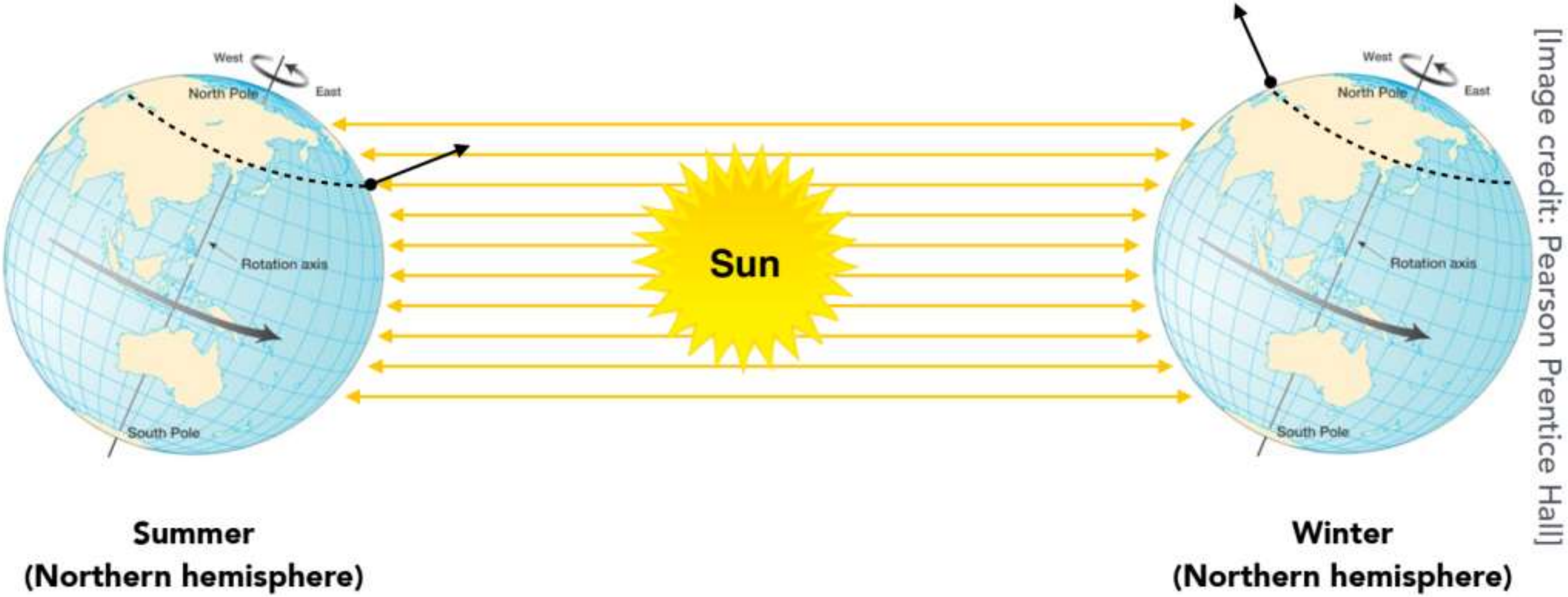
$$E = \frac{1}{2}\frac{\Phi}{A}$$



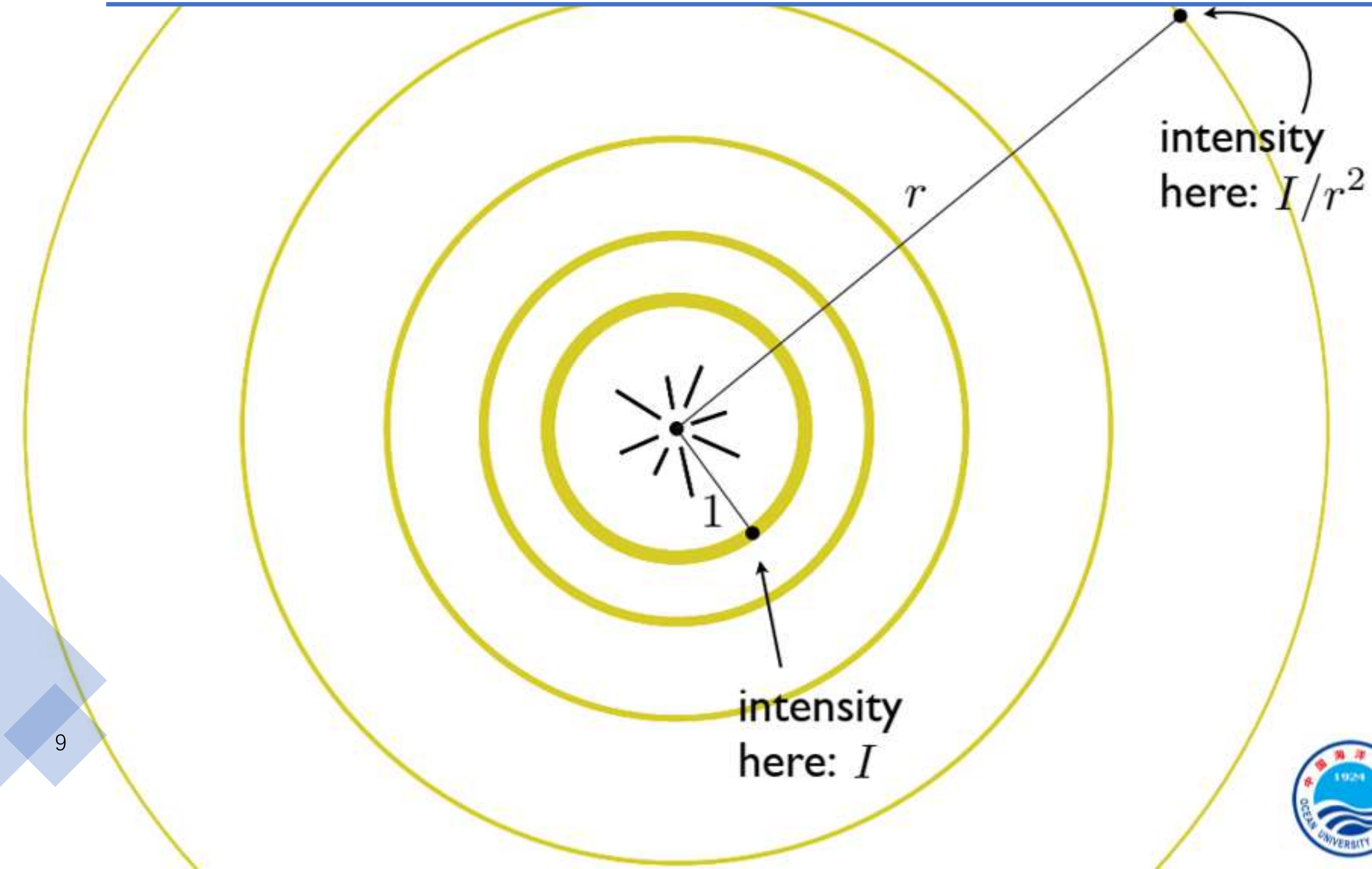
In general, light per unit area is proportional to cos θ = l • n

$$E = \frac{\Phi}{A}\cos\theta$$

为什么有四季?


West
North Pole
East
Rotation axis
South Pole
Sun
West
North Pole
East
Rotation axis
South Pole
Summer
(Northern hemisphere)
Winter
(Northern hemisphere)
[Image credit: Pearson Prentice Hall]

光的衰减



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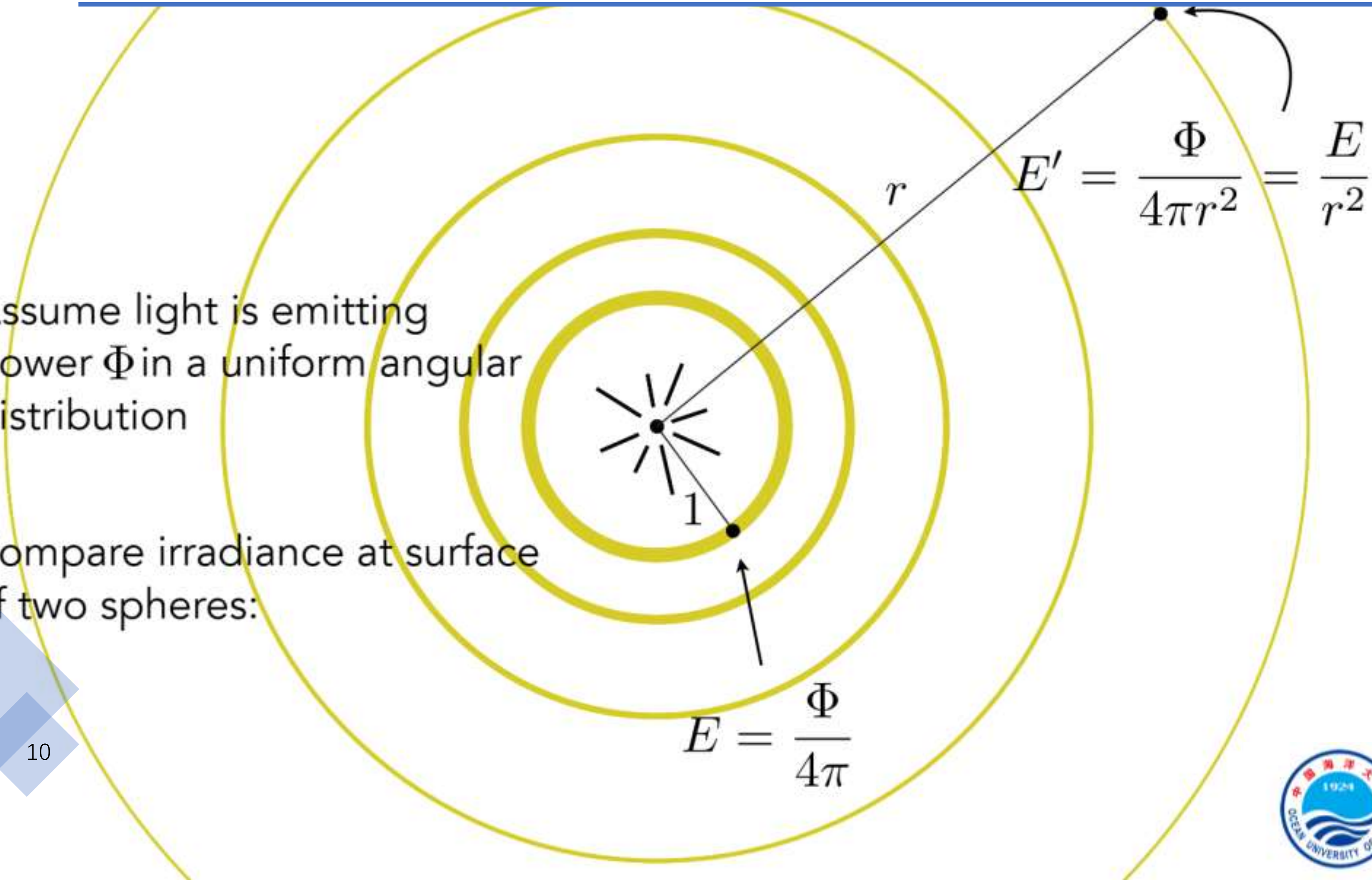
Irradiance的衰减

Assume light is emitting power Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:



Radiance

- 描述环境中光的分布

➢ radiance是与光线相关的量

➢ 渲染就是计算radiance

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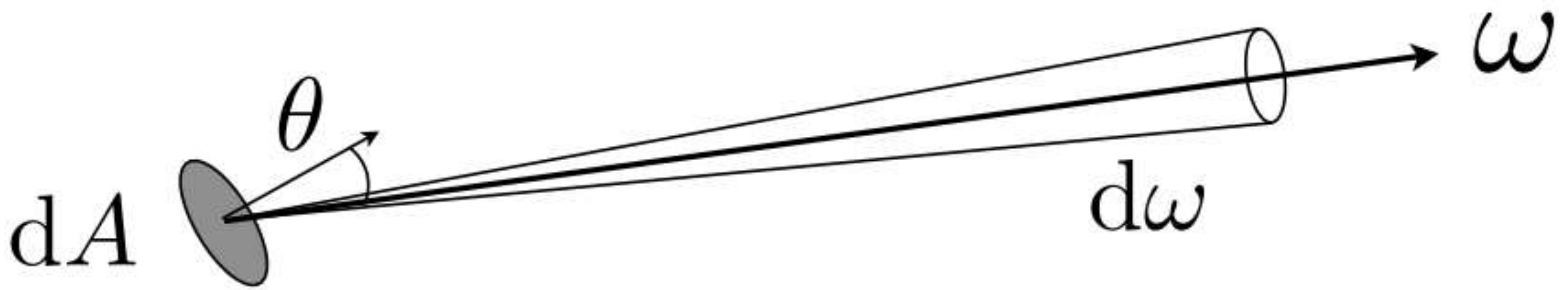
Radiance

- 定义：每单位立体角每单位投影面积的Radiant flux/power ，单位瓦特每球面度每平方米



$$L(\mathrm{p},\omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p},\omega)}{\mathrm{d}\omega\,\mathrm{d}A\cos\theta}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr\,m^2}}\right]\left[\frac{\mathrm{cd}}{\mathrm{m^2}}=\frac{\mathrm{lm}}{\mathrm{sr\,m^2}}=\mathrm{nit}\right]$$

Radiance

- 定义：每单位立体角每单位投影面积的Radiant flux/power

$$L(\mathrm{p},\omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p},\omega)}{\mathrm{d}\omega\,\mathrm{d}A\cos\theta}$$

- Irradiance:每单位投影面积的Radiant flux/power
- Radiant intensity:每单位立体角的Radiant flux/power
- Radiance:
  - 每单位立体角的Irradiance
  - 每单位投影面积的Radiant intensity

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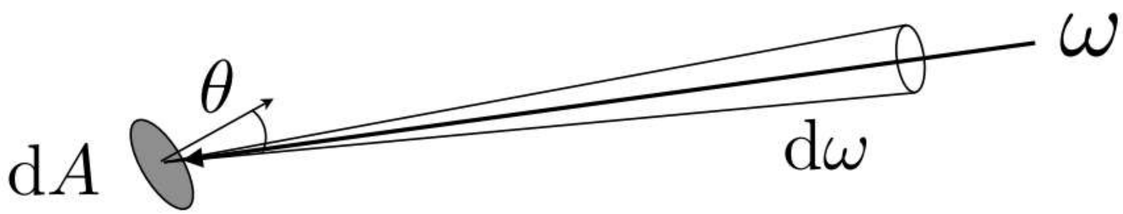
Incident Radiance

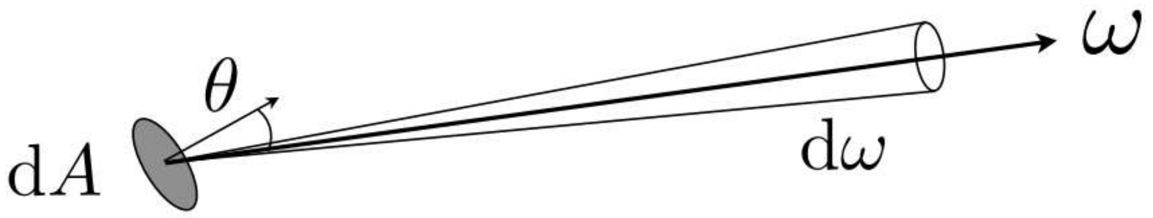
- 沿着给定光线（入射方向）到达表面的每单位立体角的Irradiance



$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

Exiting Radiance

- 沿着给定光线（出射方向）离开表面的每单位投影面积的Radiant intensity



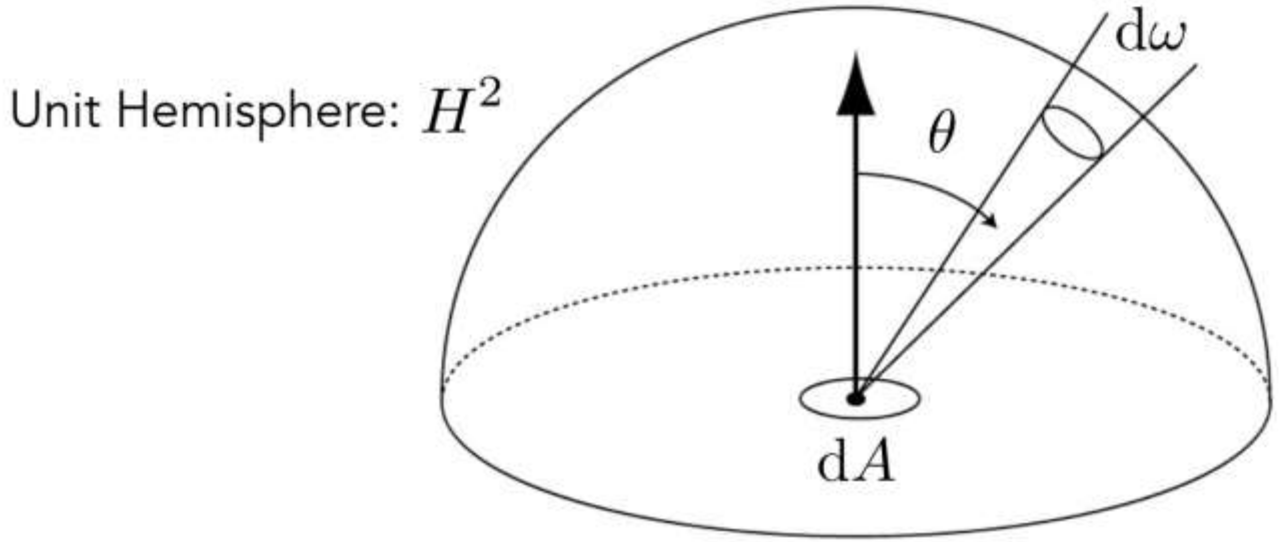
$$L(\mathrm{p},\omega)=\frac{\mathrm{d}I(\mathrm{p},\omega)}{\mathrm{d}A\cos\theta}$$

Irradiance vs. Radiance

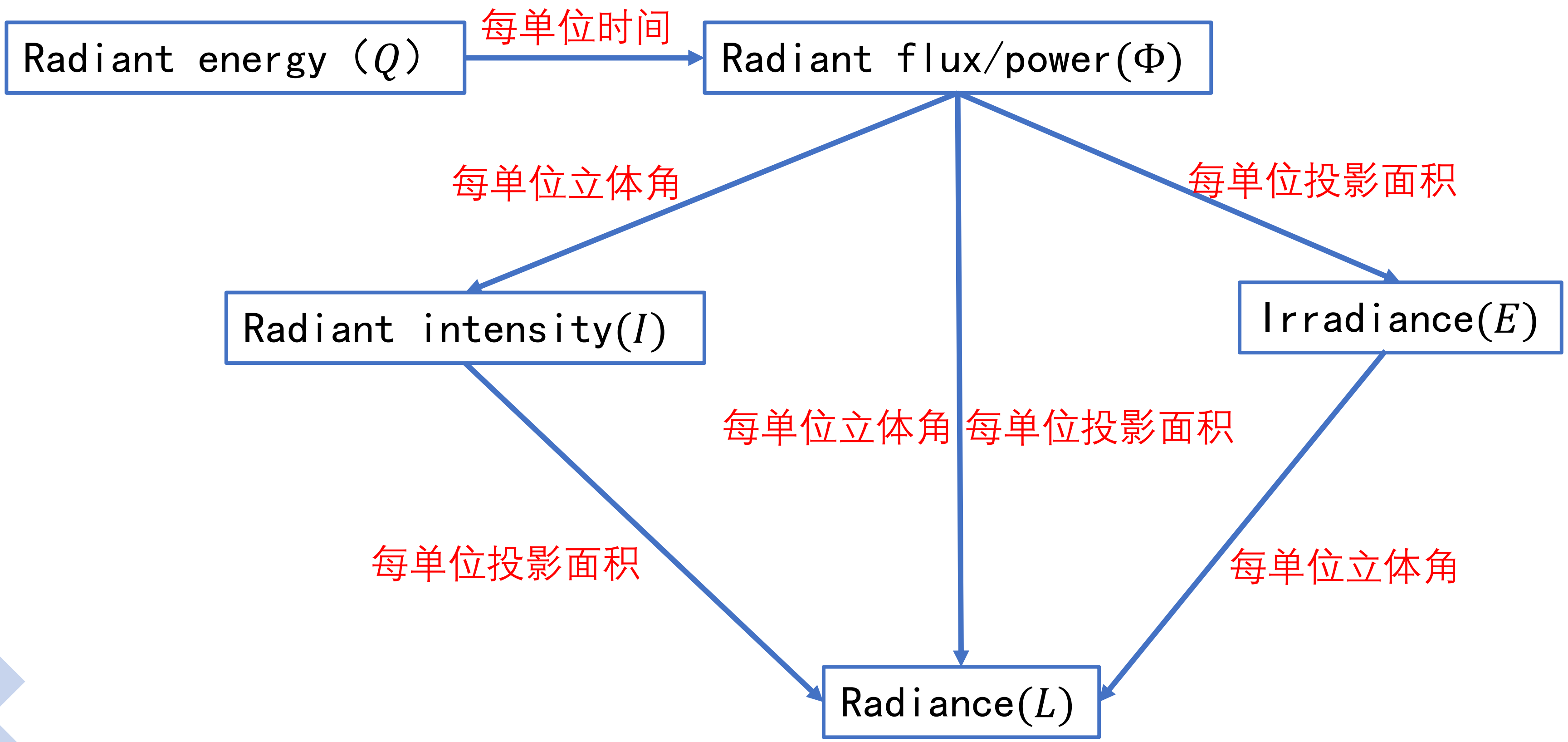
- Irradiance:通过面积dA接收的所有Radiant flux/power
- Radiance:通过面积dA接收的从$d\omega$方向来的Radiant flux/power

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$



总结

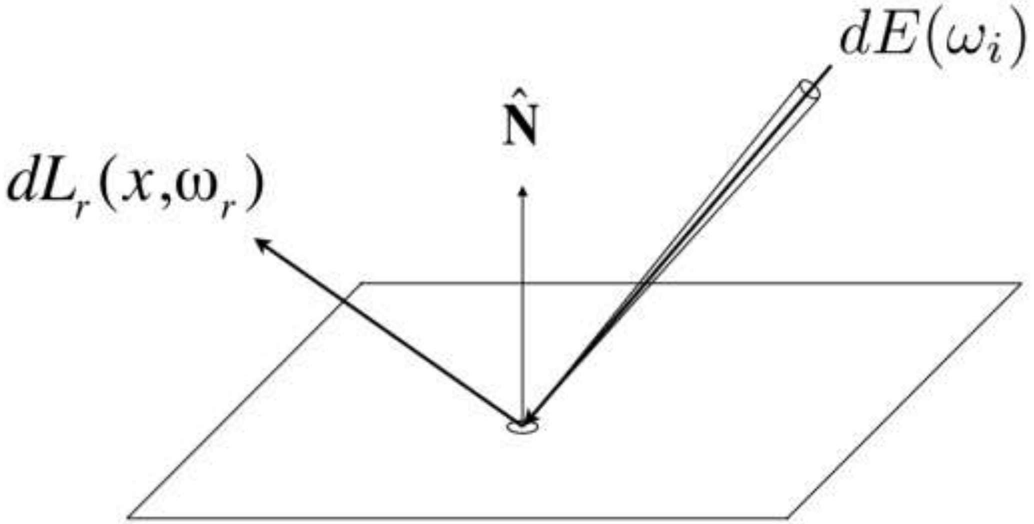


Radiant energy（Q）
每单位时间
Radiant flux/power(Φ)
每单位立体角
每单位投影面积
Radiant intensity(I)
Irradiance(E)
每单位立体角
每单位投影面积
每单位投影面积
每单位立体角
Radiance(L)

Q&A

一点处的反射

- 从ω_i方向来的Radiance，变成了dA接收到的Radiant flux/power E
- 之后E将变成射向任意方向ω_r的Radiance



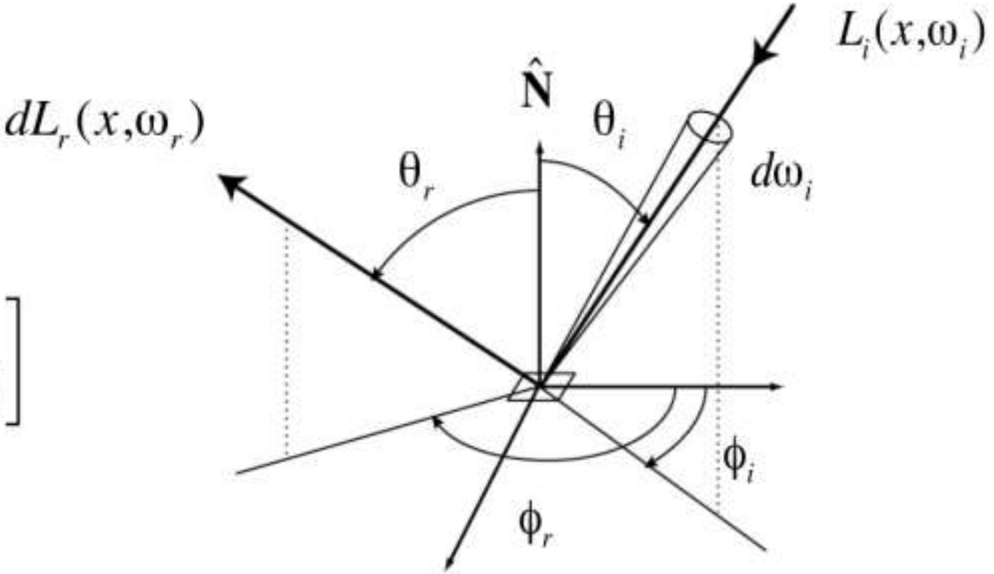
Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\, cos\, \theta_i\, d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$): $dL_r(\omega_r)$

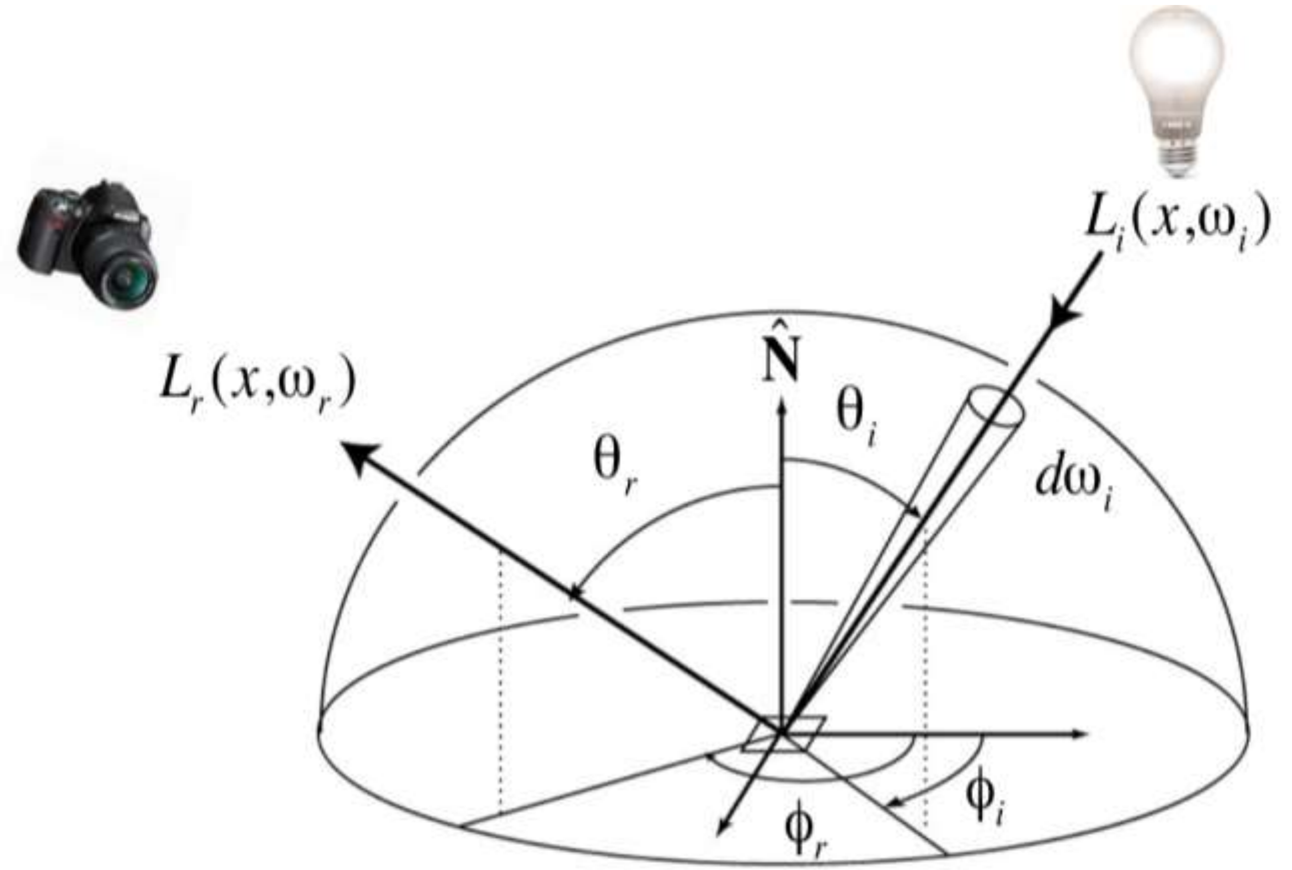
BRDF(双向反射分布函数)

- Bidirectional Reflectance Distribution Function
- 代表了有多少光可以由任一入射方向反射到任一出射方向

$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$



反射方程

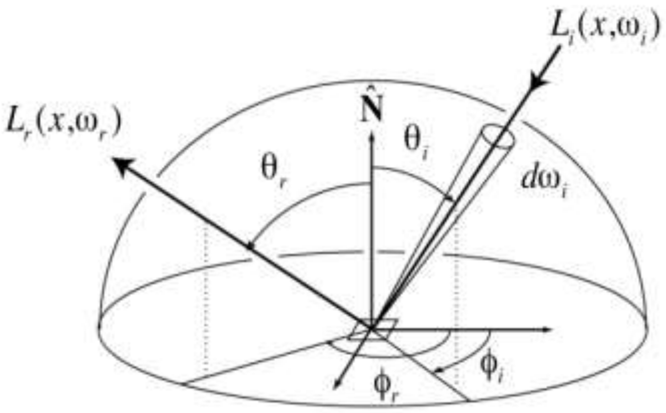


$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \rightarrow \omega_r)\, L_i(\mathrm{p},\omega_i)\, \cos\theta_i\, \mathrm{d}\omega_i$$

反射方程：递归方程

- 反射的Radiance取决于入射的Radiance

$$\boxed{L_r(\mathrm{p},\omega_r)} = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r) \boxed{L_i(\mathrm{p},\omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



- 在场景中的另外一点，入射的Radiance取决于反射的Radiance

渲染方程

- 反射方程

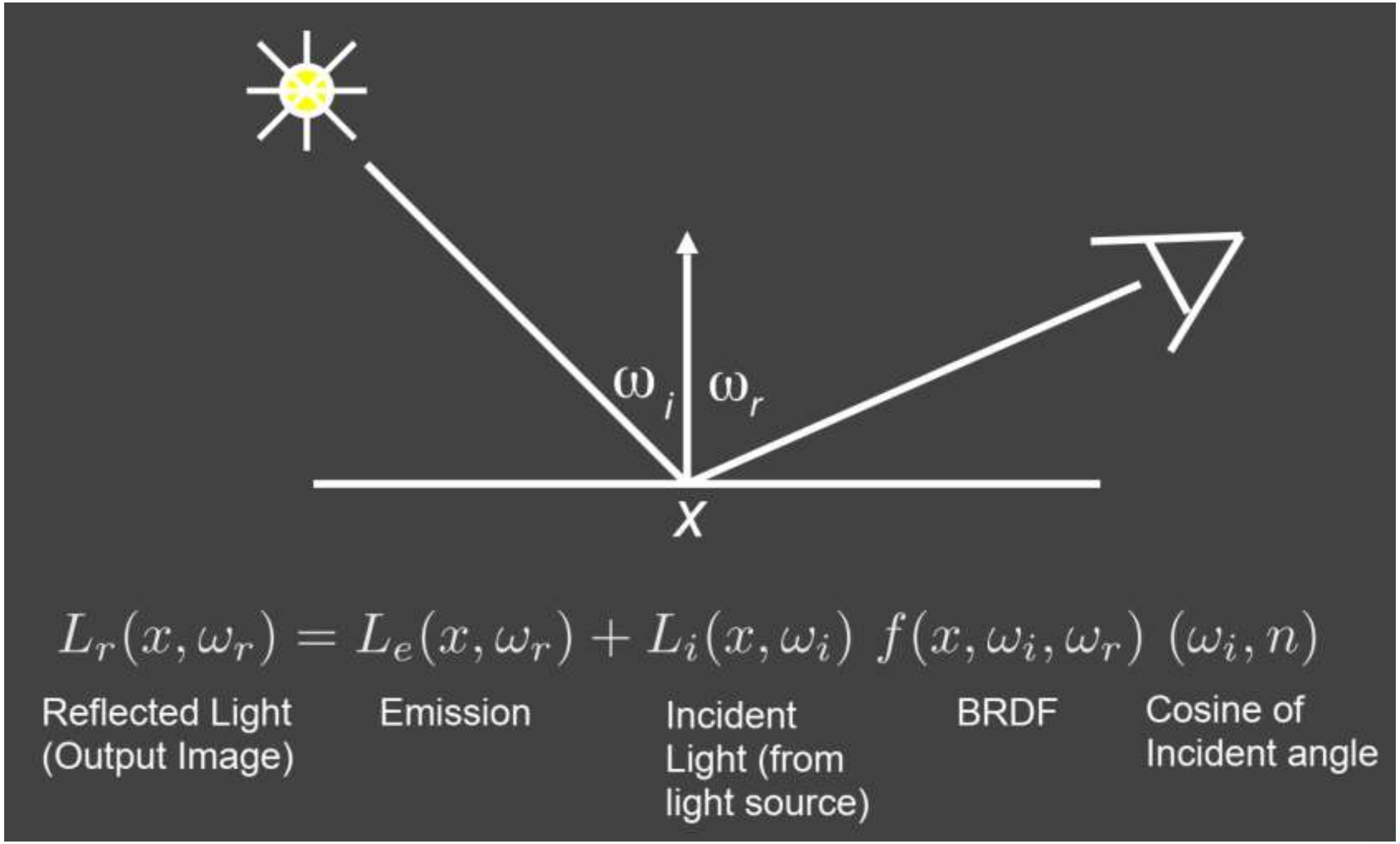
$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

- 渲染方程（假设所有方向都指向外侧）

$$\boxed{L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)\,\mathrm{d}\omega_i}$$

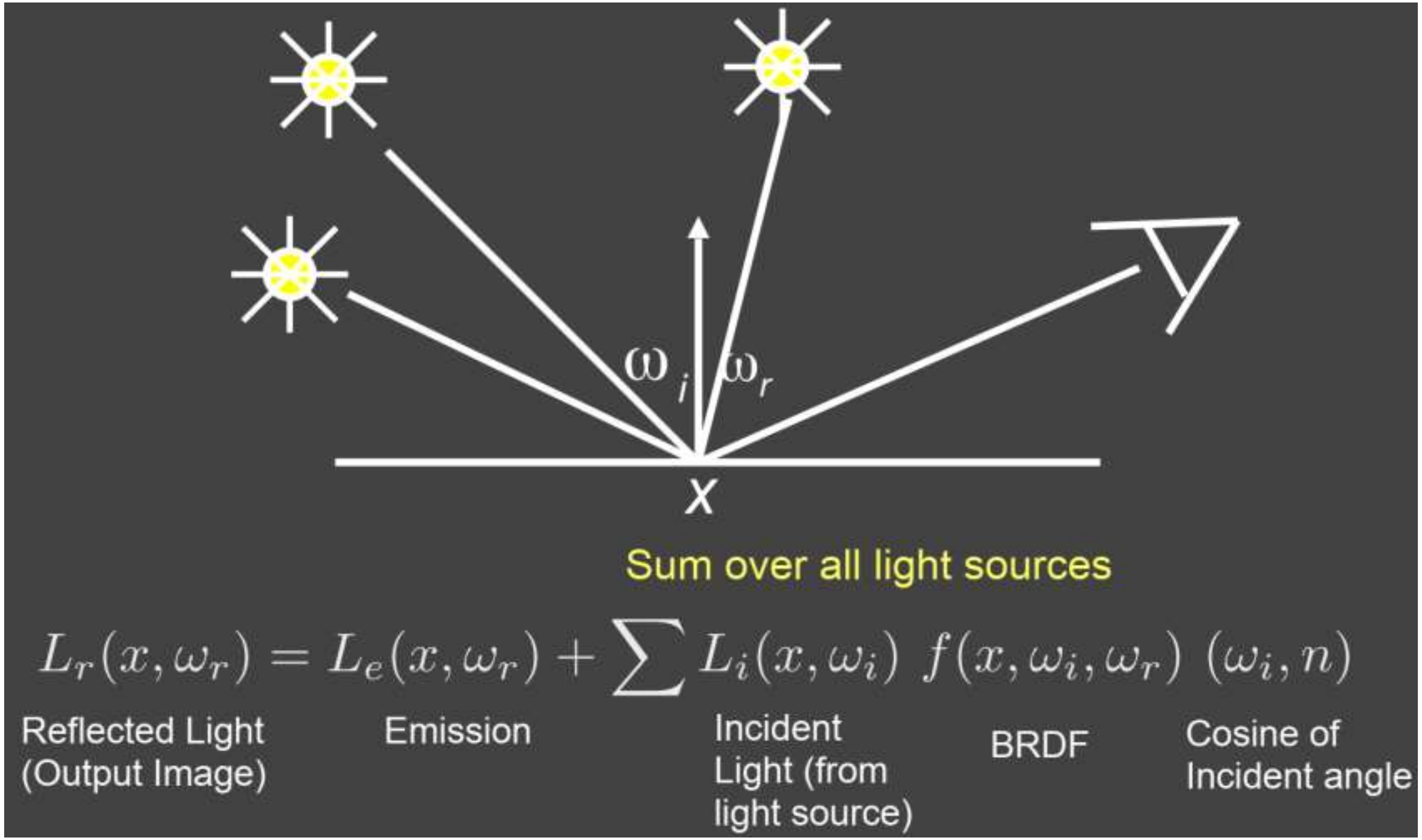
Q:如何求解？

渲染方程的理解

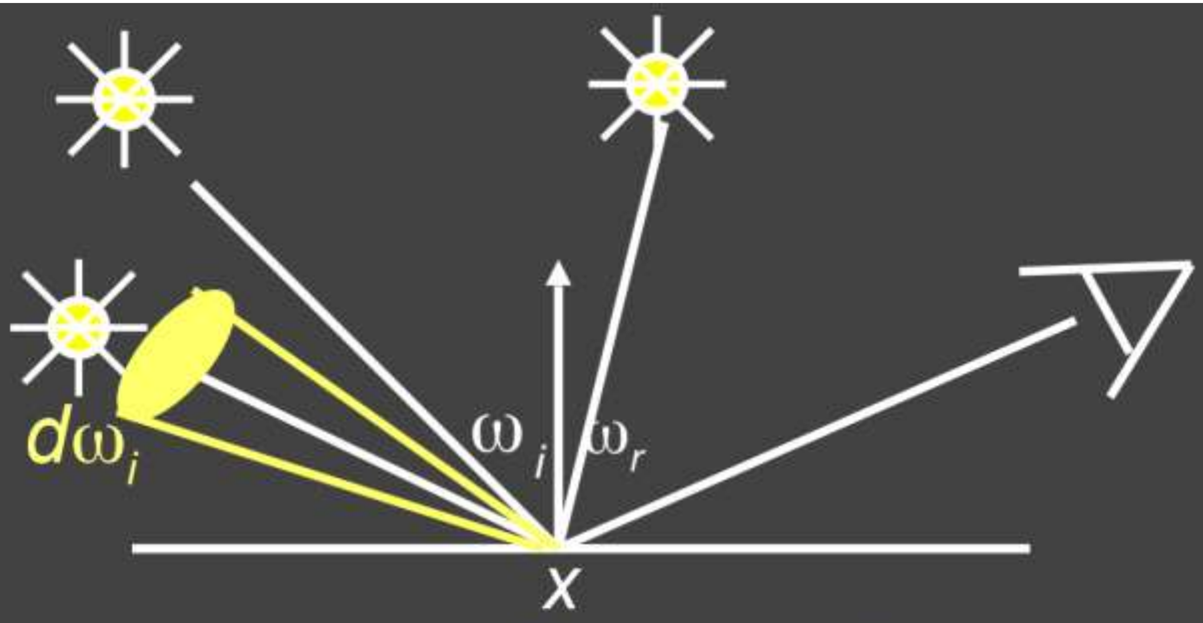


ω i
ω r
x
$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$
Reflected Light
(Output Image)
Emission
Incident
Light (from
light source)
BRDF
Cosine of
Incident angle

渲染方程的理解


ω i
ω r
x
Sum over all light sources
$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$
Reflected Light (Output Image)
Emission
Incident Light (from light source)
BRDF
Cosine of Incident angle

渲染方程的理解

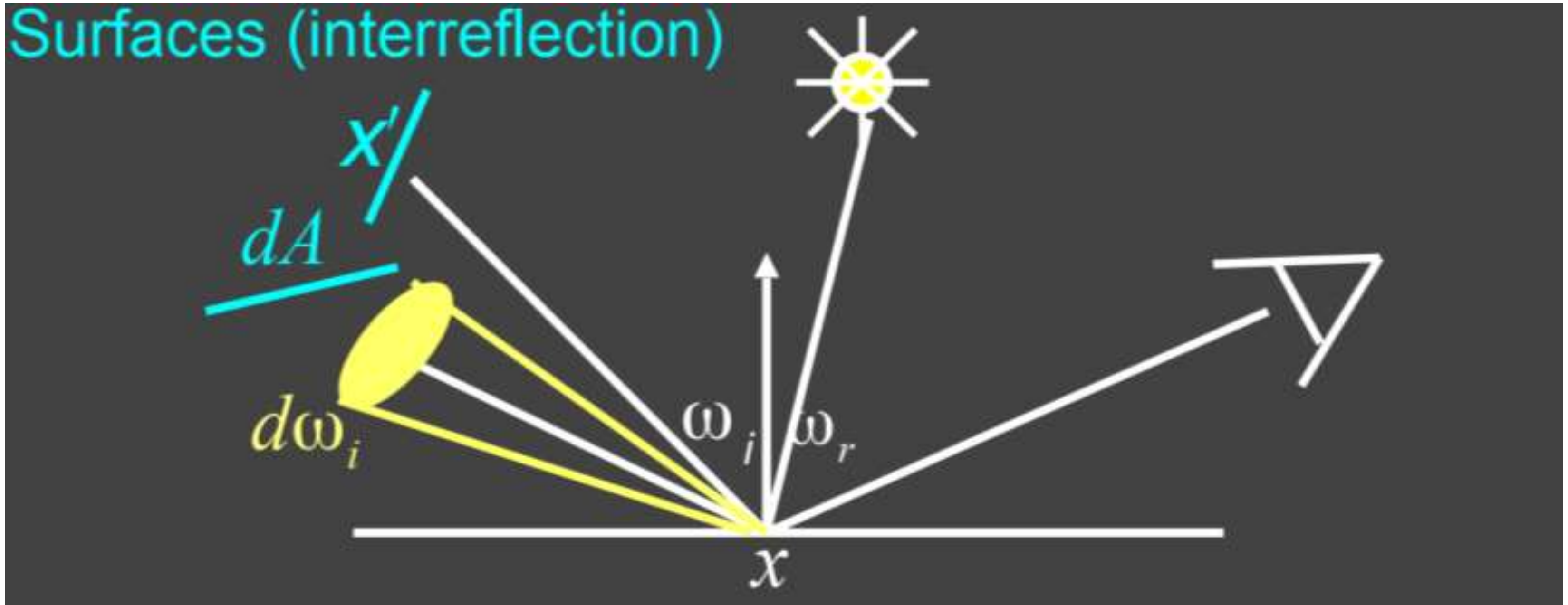


Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ \cos\theta_i\, \mathrm{d}\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

渲染方程的理解



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

渲染方程的理解

Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

渲染方程的理解

- 积分形式

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i)\ \boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

$$l(u) = e(u) + \int l(v)\ \underbrace{\boxed{K(u,v)dv}}_{\text{Kernel of equation}}$$

渲染方程的理解

- 离散化为一个简单的矩阵方程（L, E为向量，K是光传输矩阵，可理解为一个反射操作符）

$$I(u) = e(u) + \int I(v) \underbrace{\boxed{K(u,v)dv}}_{\text{Kernel of equation}}$$

$$\Downarrow$$

$$L = E + KL$$

渲染方程的理解

- 求解L

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2E + K^3E + \ldots$$

光线追踪

$$L = E + KE + K^2E + K^3E + \ldots$$

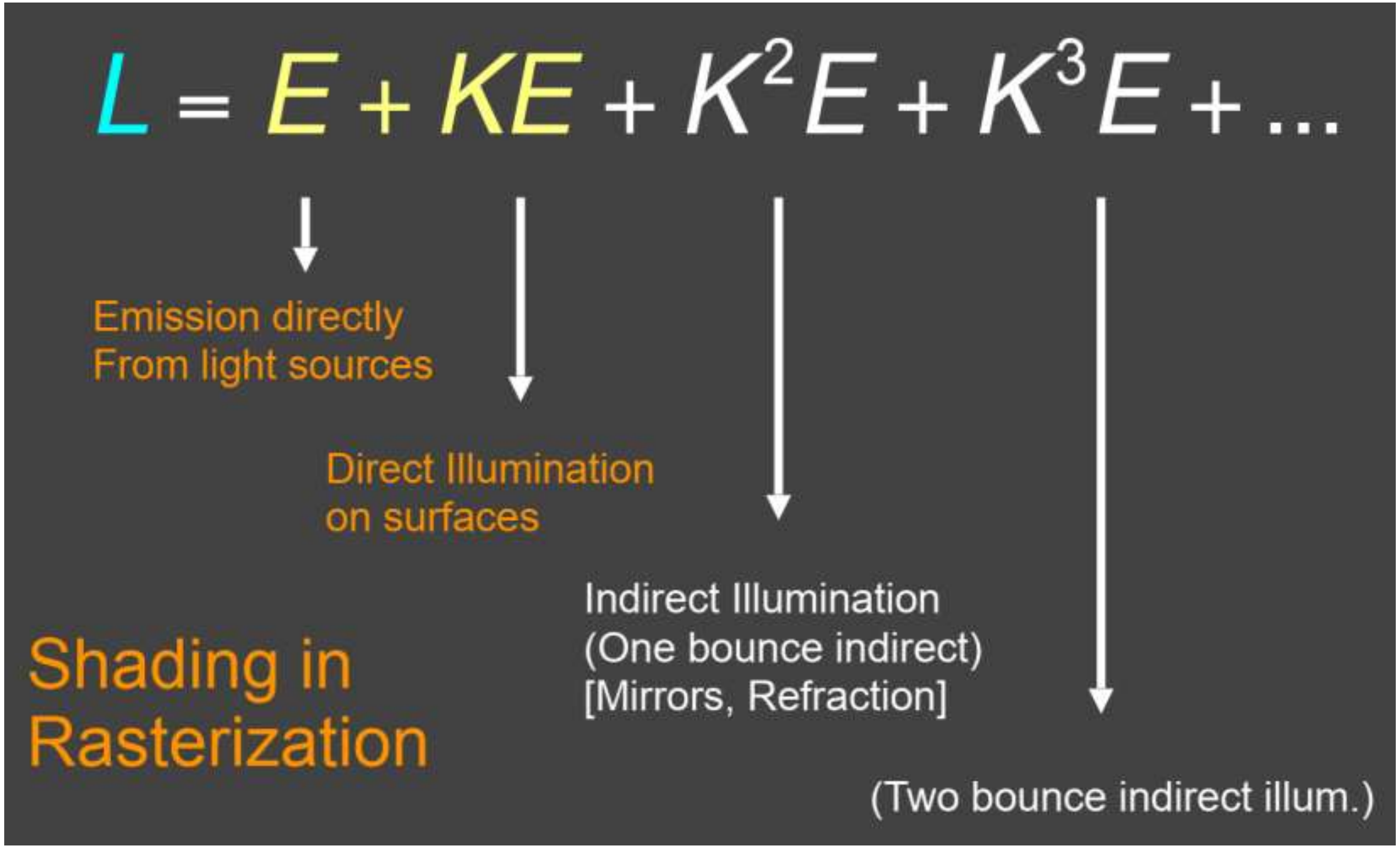
Emission directly
From light sources

Direct Illumination
on surfaces

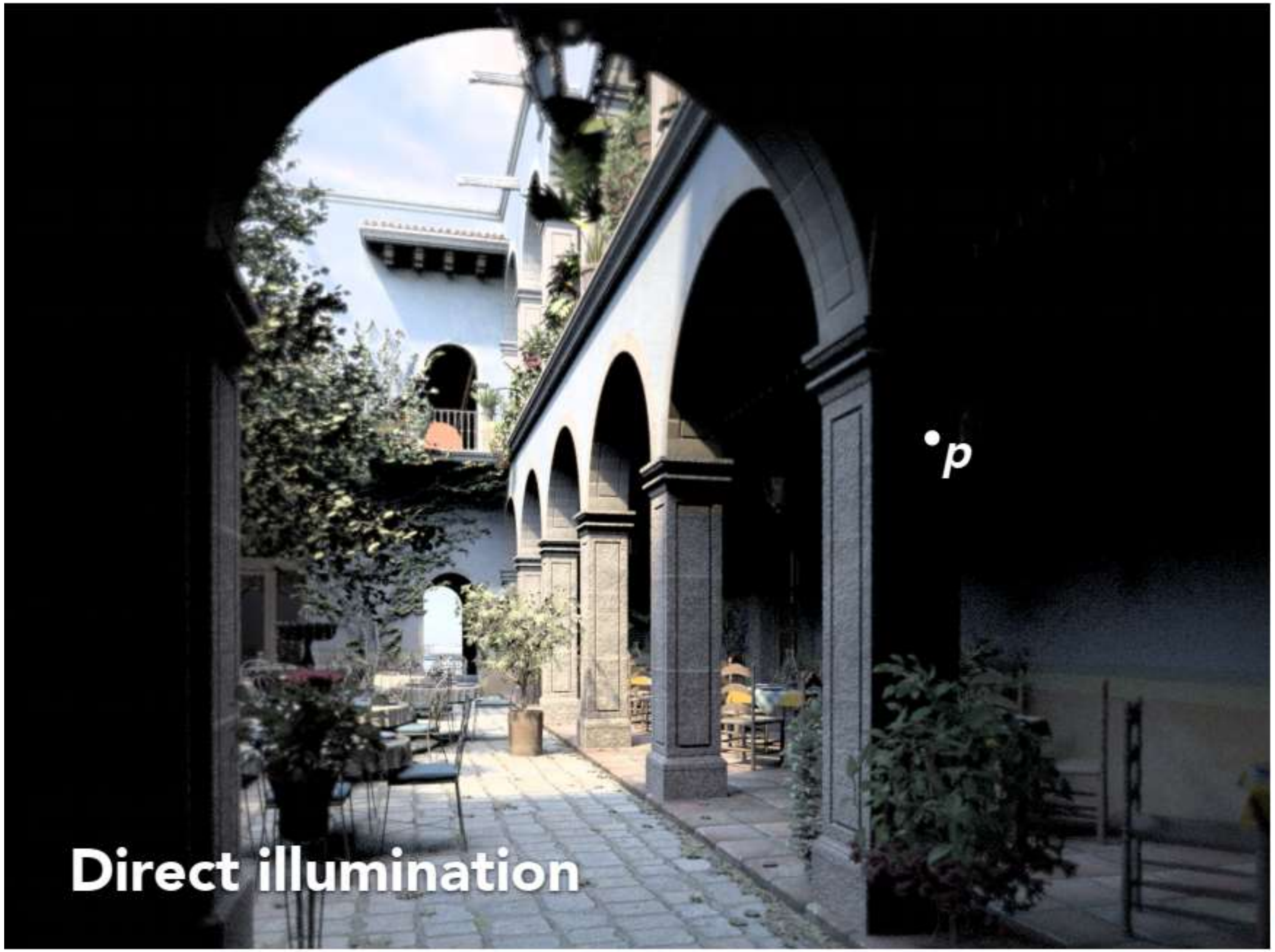
Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

光线追踪


$L = E + KE + K^2E + K^3E + \ldots$
Emission directly
From light sources
Direct Illumination
on surfaces
Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]
Shading in
Rasterization
(Two bounce indirect illum.)

光线追踪



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光线追踪

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光线追踪

Q&A

概率知识

- 随机变量X
- 概率密度函数PDF（probability density function）

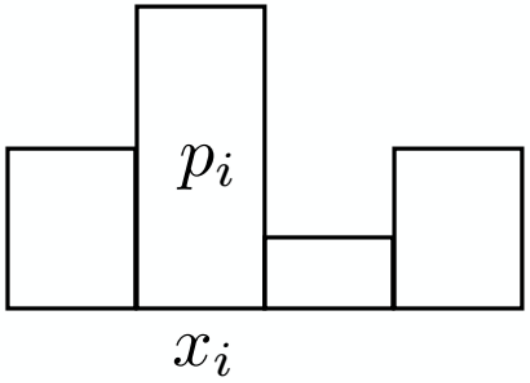
$$X\sim p(x)$$

- 例子：均匀的PDF

- ➢ X取值1、2、3、4、5、6
- ➢ $p(1)=p(2)=p(3)=p(4)=p(5)=p(6)$

概率知识

- 概率分布



- 满足

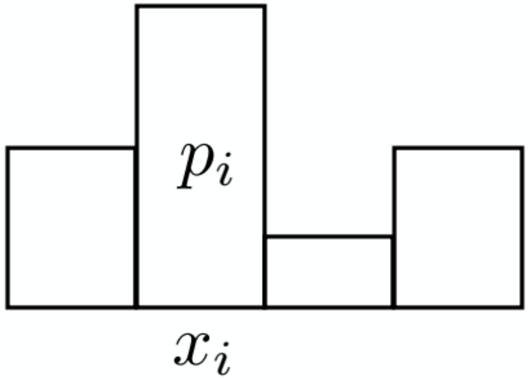
$$p_i \geq 0$$

$$\sum_{i=1}^{n} p_i = 1$$

$$p_i = \frac{1}{6}$$

概率知识

- 期望



$$E[X] = \sum_{i=1}^{n} x_i p_i$$

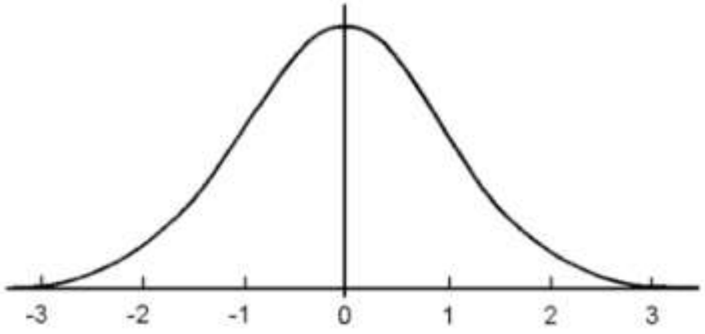
- 例子

$$E[X] = \sum_{i=1}^{n} \frac{i}{6}$$
$$= (1+2+3+4+5+6)/6 = 3.5$$

概率知识

- 连续型随机变量的概率密度函数PDF

$$X \sim p(x)$$



- 满足 $p(x) \geq 0 \text{ and } \int p(x)\, dx = 1$
- 期望 $E[X] = \int x\, p(x)\, dx$

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Q&A

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