

2. 解:

分布律:

X \ Y	0	1	2
0	0	0	$\frac{1}{35}$
1	0	$\frac{6}{35}$	$\frac{6}{35}$
2	$\frac{3}{35}$	$\frac{12}{35}$	$\frac{3}{35}$
3	$\frac{2}{35}$	$\frac{2}{35}$	0

3. 解:

$$(1). F(x, y) = \int_0^x \int_2^y k(6-x-y) dy dx$$

$$\text{有 } \int_0^2 \int_2^4 k(6-x-y) dx dy = 1$$

$$\text{即 } 8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$f(x, y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 < x < 2, 2 < y < 4 \\ 0 & \text{其他} \end{cases}$$

$$(2). P\{X < 1, Y < 3\}$$

$$= \int_2^3 dy \int_0^1 \frac{1}{8}(6-x-y) dx = \frac{3}{8}$$

$$(3). P\{X < 1.5\}$$

$$= \int_2^4 dy \int_0^{1.5} \frac{1}{8}(6-x-y) dx = \frac{27}{32}$$

$$(4). P\{X + Y \leq 4\}$$

$$= \int_0^2 dx \int_2^{4-x} \frac{1}{8}(6-x-y) dy = \frac{2}{3}$$



6. 解:

$$P_0(x=0, Y=0) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(x=0, Y=1) = \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$$

$$P(x=1, Y=1) = C_2^1 \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{4}$$

$$P(x=1, Y=2) = C_2^1 \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{4}$$

$$P(x=2, Y=2) = C_2^2 \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$$

$$P(x=2, Y=3) = C_2^2 \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$$

分布律为:

X \ Y	0	1	2	3	$P\{X=i\}$
0	$\frac{1}{8}$	$\frac{1}{8}$	0	0	$\frac{1}{4}$
1	0	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$
2	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
$P\{Y=j\}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

9. 解:

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$$(1) 1 = \int_{-1}^1 dx \int_{x^2}^1 cx^2y dy$$

$$= \int_{-1}^1 \frac{1}{2} cx^2 (1 - x^4) dx$$

$$= \frac{4}{21} C$$

$$\text{故 } C = \frac{21}{4}$$

$$(2) f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{x^2}^1 \frac{21}{4} x^2 y dy & -1 \leq x \leq 1 \\ 0 & \text{其他} \end{cases} = \begin{cases} \frac{21}{8} x^2 (1 - x^4) & -1 \leq x \leq 1 \\ 0 & \text{其他} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_{-\sqrt{y}}^{\sqrt{y}} \frac{21}{4} x^2 y dx & 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases} = \begin{cases} \frac{7}{2} y^{\frac{5}{2}} & 0 \leq y \leq 1 \\ 0 & \text{其他} \end{cases}$$

