2022春一A答案

一、 B D D C B D

二、 1. $\frac{5}{16}$　2. e^{-1}　3. $\frac{26}{27}$　4. $\frac{1}{5}$　5. $\frac{2}{3}$　6. $\left(\bar{X} \pm \frac{S}{\sqrt{n}} t_{0.025}(n-1)\right)$

三、1. 解：设 A：出事故. B_1：易出事故者　B_2：不易出事故者

(1) $P(A) = P(A|B_1)\cdot P(B_1) + P(A|B_2)\cdot P(B_2) = 0.4\times 0.3 + 0.2\times 0.7 = \boxed{0.26}$

(2) $P(B_1|\bar{A}) = \dfrac{P(\bar{A}|B_1)\cdot P(B_1)}{P(\bar{A})} = \dfrac{0.6\times 0.3}{1-0.26} = \boxed{\dfrac{9}{37}}$

2. (1) $f_X(x) = \int_0^{+\infty} x e^{-x} e^{-y}\, dy = \boxed{x e^{-x},\ x>0}$

$f_Y(y) = \int_0^{+\infty} x e^{-x} e^{-y}\, dx = \boxed{e^{-y},\ y>0}$

(2) 因 $f(x,y) = f_X(x)\cdot f_Y(y)$，故$\boxed{\text{独立}}$.

(3) $y>0$时，$f_{X|Y}(x|y) \overset{\text{独立}}{=} f_X(x) = \boxed{x e^{-x},\ x>0}$

(4) $P\{X\le Y\} = \int_0^{+\infty}\int_x^{+\infty} x e^{-(x+y)}\, dy\, dx = \int_0^{+\infty} x e^{-2x}\, dx = \boxed{\dfrac{1}{4}}$

3. (1) (4分)

Z_1	0	1
P_k	$\frac{1}{2}$	$\frac{1}{2}$

Z_2	0	1
P_k	$\frac{3}{4}$	$\frac{1}{4}$

(2) (4分)

$Z_2 \backslash Z_1$	0	1
0	$\frac{1}{2}$	$\frac{1}{4}$
1	0	$\frac{1}{4}$

(3) (6分) $EZ_1Z_2 = \frac{1}{4}$，$EZ_1 = \frac{1}{2}$，$EZ_2 = \frac{1}{4} \Rightarrow Cov(Z_1,Z_2) = \frac{1}{8}$

$DZ_1 = \frac{1}{4}$，$DZ_2 = \frac{3}{16} \Rightarrow \rho_{Z_1Z_2} = \boxed{\dfrac{\sqrt{3}}{3}}$

4. 设 Y 为一月内未等到服务的次数.

则 $Y \sim b(10, p) \overset{\text{近似}}{\approx} N(10p, 10p(1-p))$

其中 $p = P\{X > 20\} = e^{-\frac{20}{20}} = e^{-1}$

$$\Rightarrow P\{Y \le 3\} \approx P\left\{\frac{Y-10e^{-1}}{\sqrt{10e^{-1}(1-e^{-1})}} \le \frac{3-10e^{-1}}{\sqrt{10e^{-1}(1-e^{-1})}}\right\} = \Phi\left(\frac{3-10e^{-1}}{\sqrt{10e^{-1}(1-e^{-1})}}\right)$$

5. (1) $EX = \int_1^{+\infty} \theta x^{\theta-1}\,dx = \frac{\theta}{\theta-1} \Rightarrow \theta = \frac{EX}{EX-1} \Rightarrow \hat{\theta}_1 = \frac{\overline{X}}{\overline{X}-1}$

(2) $L(\theta) = \theta^n \left(\prod_{i=1}^{n} x_i\right)^{-\theta-1} \Rightarrow \ln L(\theta) = n\ln\theta - (\theta+1)\sum_{i=1}^{n}\ln x_i$

$\Rightarrow \frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n}\ln x_i \overset{令}{=} 0$

$\Rightarrow \theta = \frac{n}{\sum_{i=1}^{n}\ln x_i}$，验证为最大值点 $\Rightarrow \hat{\theta}_2 = \frac{n}{\sum_{i=1}^{n}\ln x_i}$

(3) $T = [\hat{\theta}_2]^{-1} = \frac{1}{n}\sum_{i=1}^{n}\ln x_i$

$\Rightarrow ET = E\ln X = \int_1^{+\infty}\ln x \cdot \theta x^{-\theta-1}\,dx = -\ln x \cdot x^{-\theta}\Big|_1^{+\infty} + \int_1^{+\infty}\frac{1}{x}\cdot x^{-\theta}\,dx = \frac{1}{\theta}$

$\Rightarrow$ 无偏

6. $H_0: \sigma^2 = 0.1$，$H_1: \sigma^2 \ne 0.1$

拒绝域 $C = \{T < \chi^2_{1-\frac{\alpha}{2}}(n-1)\} \cup \{T > \chi^2_{\frac{\alpha}{2}}(n-1)\}$

$= \{T < 12.401\} \cup \{T > 39.364\}$，其中 $T = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(24)$

检验 $T = \frac{24 \times 0.125}{0.1} = 30 \notin C$，故符合规定.