26(2) 30 32 33 34

补充：(1) 将随机变量 X 的 k 阶中心矩用它的原点矩表示出来

(2) 求 $Exp(\lambda)$ 的中位数

26. 解：

(2) (i) $Z=5X-Y+15$ X, Y 相互独立.

$E(Z)=E(5X-Y+15)=5E(X)-E(Y)+15=29$

$D(Z)=D(5X-Y+15)=25D(X)+D(Y)=109$

(ii) X 与 Y 不相关

$Cov(X,Y)=0$

$E(Z)=E(5X-Y+15)=5E(X)-E(Y)+15=29$

$D(Z)=109$

(iii) X 与 Y 的相关系数为 0.25

$\rho_{XY}=0.25$

$Cov(X,Y)=\sqrt{D(X)}\sqrt{D(Y)}\cdot\rho_{XY}=1.5$

$D(Z)=25D(X)+D(Y)-10Cov(X,Y)=94$

30. 解：X、Y 的分布律分别为：

X	0	1
p_k	$P(\bar{A})$	$P(A)$

Y	0	1
p_k	$P(\bar{B})$	$P(B)$

则 $P\{XY=1\} = P\{X=1, Y=1\} = P\{AB\}$

则 XY 的分布律为

XY	0	1
p_k	$1-P(AB)$	$P(AB)$

则 $E(X)=P(A)$ $\quad E(Y)=P(B)$ $\quad E(XY)=P(AB)$

由假设 $\rho_{XY}=0$ 得，$E(XY)=E(X)E(Y)$

即 $P(AB) = P(A)\cdot P(B)$

则 A 与 B 相互独立.

$P\{X=1, Y=1\} = P(AB) = P(A)\cdot P(B) = P\{X=1\}\cdot P\{Y=1\}$

$P\{X=1, Y=0\} = P\{A\bar{B}\} = P(A)\, P(\bar{B}) = P\{X=1\}\, P\{Y=0\}$

$P\{X=0, Y=0\} = P\{\bar{A}\bar{B}\} = P(\bar{A})\cdot P(\bar{B}) = P\{X=0\}\, P\{Y=0\}$

$P\{X=0, Y=1\} = P\{\bar{A}B\} = P(\bar{A})\cdot P(B) = P\{X=0\}\cdot P\{Y=1\}$

32. 解:

$$E(X) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x f(x,y)\, dx\, dy$$

$$= \int_0^2 dx \int_0^2 x \cdot \frac{1}{8}(x+y)\, dy$$

$$= \frac{7}{6}$$

$$E(Y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y f(x,y)\, dx\, dy$$

$$= \int_0^2 dy \int_0^2 y\, \frac{1}{8}(x+y)\, dy$$

$$= \frac{7}{6}$$

$$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy f(x,y)\, dx\, dy$$

$$= \int_0^2\int_0^2 xy \cdot \frac{1}{8}(x+y)\, dx\, dy$$

$$= \frac{4}{3}$$

$$E(X^2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2 f(x,y)\, dx\, dy$$

$$= \int_0^2\int_0^2 x^2 \frac{1}{8}(x+y)\, dx\, dy$$

$= \frac{5}{3}$ 　则 $D(X) = E(X^2) - [E(X)]^2 = \frac{11}{36}$

同理 $E(Y^2) = \frac{5}{3}$ 　 $D(Y) = \frac{11}{36}$

$$Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{4}{3} - \frac{7}{6}\times\frac{7}{6} = -\frac{1}{36}$$

$$D(X+Y) = D(X) + D(Y) + 2Cov(X,Y) = \frac{11}{36}\times 2 + 2\times\left(-\frac{1}{36}\right) = \frac{5}{9}$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{D(X)D(Y)}} = -\frac{1}{11}$$

3、解.

$$Cov(Z_1,Z_2) = Cov(\alpha X+\beta Y,\ \alpha X-\beta Y)$$

$$= \alpha^2 Cov(X,X) \cancel{-2\alpha\beta Cov(X,Y)} - \beta^2 Cov(Y,Y)$$

$$= \alpha^2 D(X) - \beta^2 D(Y)$$

$$= \alpha^2\sigma^2 - \beta^2\sigma^2$$

$$D(Z_1) = D(\alpha X+\beta Y) = \alpha^2 D(X) + \beta^2 D(Y) + 2Cov(\alpha X,\beta Y)。$$

$$= \alpha^2\sigma^2 + \beta^2\sigma^2$$

$$D(Z_2) = D(\alpha X-\beta Y) = \alpha^2 D(X) + \beta^2 D(Y) - 2Cov(\alpha X,\beta Y)$$

$$= \alpha^2\sigma^2 + \beta^2\sigma^2$$

$$\rho_{XY} = \frac{Cov(Z_1,Z_2)}{\sqrt{D(Z_1)D(Z_2)}} = \frac{\alpha^2\sigma^2-\beta^2\sigma^2}{\sqrt{(\alpha^2\sigma^2+\beta^2\sigma^2)^2}} = \frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}$$

4. 解

(1). $E(W) = E[(aX+3Y)^2] = E(a^2X^2 + 6aXY + 9Y^2)$

$= a^2E(X^2) + 9E(Y^2) + 6aE(XY)$

$E(X^2) = D(X) + [E(X)]^2 = 4$

$E(Y^2) = D(Y) + [E(Y)]^2 = 16$

$E(XY) = Cov(X,Y) + E(X)E(Y) = \rho_{XY} \cdot \sqrt{D(X)D(Y)} = -4$

$E(W) = 4a^2 + 9\times 16 - 24a$

$= 4(a-3)^2 + 108$

(2). 故当 $a=3$ 时, $\min E(X) = 108$

(2) $E(WY) = \cancel{E[(X-aY)(X+aY)]} = \cancel{E(X^2 - a^2Y^2)}$

$= \cancel{E(X^2) - a^2 E(Y^2)}$

$\cancel{= D(X)}$

(2) $Cov(W,V) = Cov(X-aY,\ X+aY)$

$= Cov(X,X) - a^2 Cov(Y,Y)$

$= \sigma_X^2 - a^2\sigma_Y^2$

$= 0$　　W, V 不相关.

因为 (X,Y) 是二维正态变量, 而 W 与 V 分别是 X, Y 的线性组合.

则 (W,V) 也是二维正态变量.

综上. W, V 相互独立.

补充题目

解 (1) ~~$E(X=E$~~

$$E(X-EX)^k = E\left[\sum_{i=0}^{k}(-1)^{k-i}[E(X)]^{k-i} C_k^i X^i\right]$$

$$= \sum_{i=0}^{k}(-1)^{k-i}[E(X)]^{k-i} C_k^i E(X^i)$$

(2) $X \sim Exp(\lambda)$.

$$f(x) = \begin{cases} 0 & x \leq 0 \\ \lambda e^{-\lambda x} & x > 0 \end{cases}$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases}$$

中位数 m 满足

$$P\{X > m\} = 1 - F(m) = \frac{1}{2}$$

$$1 - F(m) = 1 - (1 - e^{-\lambda m}) = e^{-\lambda m} = \frac{1}{2}$$

则 $m = \frac{\ln 2}{\lambda}$