7.解.

(1) 设第 i 只蛋糕的价格为 X_i. 故 X_i 的分布律为

X_i	1	1.2	1.5
P	0.3	0.2	0.5

$$E(X_i) = 1\times0.3 + 1.2\times0.2 + 1.5\times0.5 = 1.29$$

$$E(X_i^2) = 1\times0.3 + 1.2^2\times0.2 + 1.5^2\times0.5 = 1.713$$

$$D(X_i) = E(X_i^2) - [E(X_i)]^2 = 0.0489.$$

以 X 为总收入. 则 $X = \sum_{i=1}^{300} X_i$. 由中心极限定理得.

$$P\{X \geq 400\} = P\{400 \leq X < \infty\}$$

$$= P\left\{\frac{400-300\times1.29}{\sqrt{300}\cdot\sqrt{0.0489}} \leq \frac{\sum_{i=1}^{300} X_i - 300\times1.29}{\sqrt{300}\cdot\sqrt{0.0489}} < \frac{\infty-300\times1.29}{\sqrt{300}\sqrt{0.0489}}\right\}$$

$$\approx 1-\Phi(3.39) = 1-0.9997 = 0.0003$$

(2). 设 Y 为 300 只蛋糕中售价为 1.2 元的蛋糕的只数. 则 $Y \sim b(300, 0.2)$

$$E(Y) = 300\times0.2 = 60 \qquad D(Y) = 300\times0.2\times0.8 = 48$$

由棣莫弗-拉普拉斯定理得.

$$P\{Y>60\} = 1-P\{P\leq60\}$$

$$= 1-P\left\{\frac{Y-\cancel{300\times0.2}\,60}{\sqrt{48}} \leq \frac{60-60}{\sqrt{48}}\right\}$$

$$\approx 1-\Phi(0)$$

$$=0.5$$

11. 解.

$E(\bar{X}) = 5$. $D(\bar{X}) = D(\bar{Y}) = 0.3/80$

(1) 由中心极限定理知 $\bar{X}$ 近似服从 $N(5, \frac{0.3}{80})$.

故 $P\{4.9 < \bar{X} < 5.1\}$

$$= P\left\{\frac{4.9-5}{\sqrt{\frac{0.3}{80}}} < \frac{\bar{X}-5}{\sqrt{\frac{0.3}{80}}} < \frac{5.1-5}{\sqrt{0.3/80}}\right\}$$

$$\approx \Phi\left(\frac{5.1-5}{\sqrt{0.3/80}}\right) - \Phi\left(\frac{4.9-5}{\sqrt{0.3/80}}\right)$$

$$= 2\Phi(1.63) - 1$$

$$= 0.8968$$

(2). $E(\bar{X}-\bar{Y}) = E(\bar{X}) - E(\bar{Y}) = 0$. $D(\bar{X}-\bar{Y}) = D(\bar{X}) + D(\bar{Y}) = \frac{0.3}{40}$.

由中心极限定理得.

$P\{-0.1 < \bar{X} - \bar{Y} < 0.1\}$

$$= P\left\{\frac{-0.1-0}{\sqrt{0.3/40}} < \frac{(\bar{X}-\bar{Y})-0}{\sqrt{0.3/40}} < \frac{0.1-0}{\sqrt{0.3/40}}\right\}$$

$$\approx \Phi\left(\frac{0.1-0}{\sqrt{0.3/40}}\right) - \Phi\left(\frac{-0.1-0}{\sqrt{0.3/40}}\right)$$

$$= 2\Phi(1.15) - 1$$

$$= 0.7498$$

12. 解

设需要车位数为 n，且设第 i $(i=1,2,\cdots,200)$ 户有车辆数为 X_i，则由 X_i 的分布律知

$$E(X_i)=0\times0.1+1\times0.6+2\times0.3=1.2$$

$$E(X_i^2)=0^2\times0.1+1^2\times0.6+2^2\times0.3=1.8$$

故 $D(X_i)=E(X_i^2)-[E(X_i)]^2=1.8-1.2^2=0.36$

这200户每户占有车位数相互独立，从而近似地有

$$\sum_{i=1}^{200}X_i\sim N(200\times1.2,\ 200\times0.36)$$

今要求车位数 n 满足 $0.95\le P(\sum_{i=1}^{200}X_i\le n)$

则 $0.95\le\Phi\left(\frac{n-200\times1.2}{\sqrt{200\times0.36}}\right)=\Phi\left(\frac{n-240}{\sqrt{72}}\right)$

又因 $0.95=\Phi(1.645)$

则 $\frac{n-240}{\sqrt{72}}\ge1.645$

则 $n\ge253.96$

故至少需254个车位