2.3.9

$$a \text{ is a unit in } \mathbb{Z}_n$$
$$\exists b \in \mathbb{Z}_n \quad ab \equiv 1 \pmod{n}$$
$$n \mid ab - 1$$
$$\Downarrow$$
$$\exists k \in \mathbb{Z} \quad ab - 1 = nk$$
$$ab - nk = 1$$
$$ab + n(-k) = 1$$
$$\gcd(a, n) = 1$$

$$\forall c \in \mathbb{Z}_n \setminus \{0\} \quad ac \neq 0$$
$$\Updownarrow$$
$$\nexists c \in \mathbb{Z}_n \quad ac \equiv 0$$
$$\Uparrow$$
$$(c \in \mathbb{Z}_n \text{ and } ac \equiv 0) \Rightarrow (c \equiv 0)$$

$$\gcd(a, n) = 1$$
$$\exists k \in \mathbb{Z}, r \in \mathbb{Z}_n \quad nk + r = c$$
$$\exists m \in \mathbb{Z} ac = mn$$
$$a(nk + r) = mn$$
$$ar = n(m - ak)$$
$$n \mid ar$$
$$\gcd(a, n) = 1$$
$$n \mid r \text{ by Euclid's lemma}$$
$$r = 0$$
$$c = nk$$
$$c \equiv 0$$