Homework 4 Part 2

Total: 35pt

This is an individual assignment.

Description

Create or edit this Jupyter Notebook to answer the questions below. Use simulations to answer these questions. An analytical solution can be useful to check if your simulation is correct but analytical solutions alone will not be accepted as a solution to a problem.

```
import scipy.stats as stats
import numpy as np
import numpy.random as npr
import matplotlib.pyplot as plt
%matplotlib inline
plt.style.use('ggplot')
```

Problem 5

(20 pt) Build a function that simulates the communication system in Problem 3 (of part 1). Take as input the value of b and the number of simulations. Your simulation should implement the decision rule given in (7) and return or print the probability of error.

1. Run your simulation and output the probability of error when $b = 1.2$.
2. Run your simulation and output the probability of error when $b = 1.6$.

Recall the probability of error is defined by

```
def problem3(b, numsims):
    errors = 0
    for i in range(numsims):

        #true signal simulation
        if np.random.uniform(0,1) < 0.6:
            true_signal = 1
        else:
            true_signal = 0

        noise = np.random.uniform(0,b)
        #decision rule
        if noise + true_signal < 1:
            est_signal = 0
        else:
```

```
            est_signal = 1
        if est_signal != true_signal:
            errors += 1

    probErrors = errors / numsims
    return(f"The probability of errors when b is equal to {b} is 
{probErrors}.")


problem3(1.2, 50000)
```

```
'The probability of errors when b is equal to 1.2 is 0.06838.'
```

```
problem3(1.6, 50000)
```

```
'The probability of errors when b is equal to 1.6 is 0.14884.'
```

Problem 6

(15 pt)

Read this webpage to learn about some additional properties of variance: {https://eng.libretexts.org/Bookshelves/Computer_Science/Programming_and_Computation_Fundamentals/Mathematics_for_Computer_Science_(Lehman_Leighton_and_Meyer)/04%3A_Probability/19%3A_Deviation_from_the_Mean/19.03%3A_Properties_of_Variance}

In particular, for S and T two independent RV, Variance of $S+T$ equals the variance of S plus the variance of T.

Let X_1 be a uniformly distributed over interval $[0,10]$, and X_2 be a Gaussian RV with mean $\mu=5$ and std $\sigma=3$.

1. Use the simulation to plot the empirical distribution of $Y=X_1+2X_2$
2. Use simulation to estimate the $E(Y)$ and $E(Y^2)$ and $Var(Y)$.
3. Analytically calculate the variance of Y, validate if the estimated variance equals the variance of Y.

```
numsims = 10000
X1 = np.random.uniform(0,10, size = numsims)
X2 = np.random.normal(loc = 5, scale = 3, size = numsims)
Y = X1 + 2*X2

E_Y = np.mean(Y)
E_Y_Squared = np.mean(Y**2)
Var_Y = np.var(Y)

Analytical_Var_Y = np.var(X1) + np.var(X2)


plt.hist(Y, bins = 50, density=True)
```

```
plt.title("Histogram of Y")
plt.xlabel("Value")
plt.ylabel("Density")


print(f"E(Y) is {E_Y}")
print(f"E(Y^2) is {E_Y_Squared}")
print(f"Var(Y) is {Var_Y}")
print(f"Var(Y) calculated analytically using theorem of sums of
independent variances is {Var_Y}")
```

```
E(Y) is 15.042934534556082
E(Y^2) is 271.43443653087706
Var(Y) is 45.144557119937126
Var(Y) calculated analytically using theorem of sums of independent
variances is 45.144557119937126
```