$\lambda = 30/hr = 0.0083/s = 0.5/min$! X = dist. in 5 mins

1) i) $P(X \geq 1) = 1 - P(X=0)$

$P(X=0) = 1 - \frac{\lambda^x}{x!} e^{-\lambda} = 1 - \frac{0.5^0}{0!} e^{-0.5} = 1 - 0.6065 = \underline{0.3935}$

ii) $\lambda = \frac{30}{hr} \cdot \frac{1}{60} \cdot 10 = 5/10min$! Y = dist. in 10 minutes

$$P(Y<5) = \sum_{y=0}^{4} \frac{5^y e^{-5}}{y!} = e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right)$$

$$= \underline{0.440}$$

iii) $\lambda = \frac{30}{hr} \cdot \frac{20}{60} = 10/20\ min$! Z = dist in 20 mins

$$P(Z>10) = 1 - P(Z \leq 10) = 1 - \sum_{z=0}^{10} \frac{10^z e^{-10}}{z!}$$

$$= 1 - 0.58304 = \underline{0.41696}$$

2) i) $\underline{P(A)} = P(X \geq 87) = P\left(Z \geq \frac{87-81}{\sqrt{30}}\right)$ $\qquad Z = \frac{X-\mu}{\sigma}$

$= P(Z > 1.095) = \underline{0.1368}$ (using z-table)

$\underline{P(C)} = P(X < 75)$

$= P\left(Z < \frac{75-81}{\sqrt{30}}\right) = P(Z < -1.095) = \underline{0.1368}$

$\underline{P(B)} = 1 - 0.1368 - 0.1368 = \underline{0.7265}$

ii) $P(X \geq 87 \mid X \geq 83) = P\left(Z \geq \frac{87-81}{\sqrt{30}} \mid Z \geq \frac{83-81}{\sqrt{30}}\right)$

$= P\left(Z \geq \frac{87-81}{\sqrt{30}}\right) \Big/ P\left(Z \geq \frac{2}{\sqrt{30}}\right) = \frac{0.13667}{0.3575} = \underline{0.382}$

3) i) $\mu = \frac{b+a}{2} \therefore a = 0$

$\therefore \mu = \frac{b}{2}$

$\sigma^2 = \frac{(b-a)^2}{12}$ (for uniform dist)

$\sigma^2 = \frac{b^2}{12}$

ii) $f_{X|T_0}(x) \therefore S_i = 0 \therefore X = N$

$\therefore f_{X|T_0}(x) = \frac{1}{b}, \quad 0 \le x \le b$

$S_i = 1 \therefore X = N+1$

$\therefore f_{X|T_1}(x) = \frac{1}{b}, \quad 1 \le x \le b+1$

iii) $P(X<1|T_1) = 0$ [1,2] (X at least 1 when T_1 true)

$P(X \ge 1|T_0) = \frac{a^2}{12} = \frac{1}{6}$

b=1,2] $P_e = 0.6 \cdot \frac{0}{12} + \frac{0.4}{6} = 0.0667$

b=1,6] $P(X<1|T_1) = \frac{0}{1.6} = 0$

$P(X \ge 1|T_0) = \frac{0.6}{1.6} = \frac{3}{8}$

$P_e = 0 + \frac{0.4 \cdot 3}{8} = 0.15$

$X \sim G(\mu=2, \sigma^2=4)$.

4)

i) $P(X>6 \mid X>4)$

$z = \frac{6-2}{2} = 2 \qquad z = \frac{4-2}{2} = 1$

$Q(x) = P(z>2 \mid z>1) \sim G(0,1)$

$= \frac{P(z>2)}{P(z>1)} = \frac{0,0228}{0,1587} = \boxed{0,1434}$

ii) $X \sim Exp(\lambda)$

$P(X>t+\Delta t \mid X>t) = \frac{P(X>t+\Delta t \cap X>t)}{P(X>t)} = \frac{P(X>t+\Delta t)}{P(X>t)}$

$P(X>t+\Delta t) = e^{-\lambda(t+\Delta t)}$

$P(X>t) = e^{-\lambda(t)}$

$= \frac{e^{-\lambda(t+\Delta t)}}{e^{-\lambda(t)}} = \frac{e^{-\lambda t} e^{-\lambda \Delta t}}{e^{-\lambda t}} = e^{-\lambda \Delta t} = P(X>\Delta t)$

$= L.M.S$