

Homework 1 Part 2

This is an individual assignment.

Description

Create or edit this Jupyter Notebook to answer the questions below. Use simulations to answer these questions. An analytical solution can be useful to check if your simulation is correct but analytical solutions alone will not be accepted as a solution to a problem.

In [1]:

```
import numpy as np
import random
```

Problem 1

Consider repeatedly rolling a fair 4-sided die.

1. Create a simulation to compute the probability that the top face will be 4 at least once on four rolls of the die?
2. Create a simulation to compute the probability that the top face will be 4 at least once on 20 rolls of the die?
3. Create a simulation to compute how many rolls of the die would you have to do to be 90% confident that you would see at least one 4?
4. Using the formula you have computed in problem 2 part 4, make a Python function that takes in the target value p and outputs the required number of rolls of an integer.
 - A. Find the values for $p = 0.95$ and $p = 0.99$.
 - B. Use your simulation to verify that the number of rolls you specified is sufficient to achieve $p \geq 0.95$.

In [2]:

```
#1)
num_sims = 100000
rolls = 4
faces = [1,2,3,4,5,6]
event = 0

for i in range(num_sims):
    dice = random.choices(faces, k = rolls)
    if 4 in dice:
        event += 1

print ("Probability that the top face will be 4 at least once on four rolls is: ", event
/ num_sims)
```

Probability that the top face will be 4 at least once on four rolls is: 0.51531

In [3]:

```
# 2)
num_sims = 100000
rolls = 13
faces = [1,2,3,4,5,6]
event = 0

for i in range(num_sims):
    dice = random.choices(faces, k = rolls)
    if 4 in dice:
        event += 1

print ("Probability that the top face will be 4 at least once on 20 rolls is: ", event /
```

```
num_sims)
```

Probability that the top face will be 4 at least once on 20 rolls is: 0.9061

In [4]:

```
#3)
faces = [1,2,3,4,5,6]
num_sims = 10000
target_prob = 0.9
true_prob = 0
rolls = 0
event = 0

while true_prob < target_prob:
    event = 0
    rolls += 1
    for i in range(num_sims):
        dice = random.choices(faces, k = rolls)
        if 4 in dice:
            event += 1
    true_prob = event / num_sims

print ("Rolls needed to be 90% confident that you would see at least one 4: ", rolls)
```

Rolls needed to be 90% confident that you would see at least one 4: 13

In [5]:

```
#4) Function Definition
def needed_rolls(target_prob):
    faces = [1,2,3,4,5,6]
    num_sims = 100000
    true_prob = 0
    rolls = 0
    event = 0

    while true_prob < target_prob:
        event = 0
        rolls += 1
        for i in range(num_sims):
            dice = random.choices(faces, k = rolls)
            if 4 in dice:
                event += 1
        true_prob = event / num_sims

    print ("Rolls needed to be " + str(target_prob) + "% confident that you would see at
least one 4: ", rolls)

#4.1) Finding values for 0.95 and 0.99

needed_rolls(0.95)
needed_rolls(0.99)
```

Rolls needed to be 0.95% confident that you would see at least one 4: 17
Rolls needed to be 0.99% confident that you would see at least one 4: 26

In [6]:

```
#4.2) Verifying for p =0.95 that 17 rolls are needed
num_sims = 10000
rolls = 17
faces = [1,2,3,4,5,6]
event = 0

for i in range(num_sims):
    dice = random.choices(faces, k = rolls)
    if 4 in dice:
        event += 1

print ("Probability that the top face will be 4 at least once on 17 rolls is: ", event /
```

```
num_sims)
```

Probability that the top face will be 4 at least once on 17 rolls is: 0.9589

Problem 2

Create a simulation function where you will roll a fair 6-sided die twice. Use simulation to find out the probability of getting a 4 or 6 on the first toss and a 1,2,3, or 5 on the second toss.

In [7]:

```
num_sims = 100000
faces = [1,2,3,4,5,6]
toss1 = [4,6]
toss2 = [1,2,3,5]
event = 0

for i in range(num_sims):
    dice = random.choice(faces)
    if dice in toss1:
        dice = random.choice(faces)
        if dice in toss2:
            event+=1

print("Probability of getting a 4 or 6 on the first toss and a 1,2,3, or 5 on the second
toss: ", event / num_sims)
```

Probability of getting a 4 or 6 on the first toss and a 1,2,3, or 5 on the second toss:
0.22159

Problem 3

Suppose that you have a bag with 3 coins. One of them is a fair coin, but the others are biased trick coins. When flipped, the three coins come up heads with probability $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$, respectively.

Consider the experiment where you pick one coin at random and flip it three times. Let H_i be the event that the coin comes up heads on flip i . What is the probability of the outcome $H_1 \cap H_2 ?$

$$\cap \overline{H_3}$$

With small modification in your code, find out the probability of the outcome $H_1 \cap \overline{H_2} .$
 $\cap \overline{H_3}$

Use simulation to find out the probability.

In [8]:

```
#Probability of first 2 flips being heads but the third is not

num_sims = 100000
event = 0
#set of coins denoted by the probability of them landing heads
coins = ['half', 'fourth', 'sixth']

for sim in range(num_sims):
    coin = random.choice(coins)
    if coin == 'half':
        S = ['H', 'T']
    if coin == 'fourth':
        S = ['H', 'T', 'T', 'T']
    if coin == 'sixth':
        S = ['H', 'T', 'T', 'T', 'T', 'T']
    values = random.choices(S, k = 3)
    if values[0] == 'H' and values[1] == 'H' and values[2] == 'T':
        event += 1
```

```
print("Probability of that first two flips are heads and last is tails with given coins: ", event / num_sims)
```

Probability of that first two flips are heads and last is tails with given coins: 0.0650
1

In [9]:

```
#Probability of HTT
num_sims = 100000
event = 0
#set of coins denoted by the probability of them landing heads
coins = ['half', 'fourth', 'sixth']

for sim in range(num_sims):
    coin = random.choice(coins)
    if coin == 'half':
        S = ['H', 'T']
    if coin == 'fourth':
        S = ['H', 'T', 'T', 'T']
    if coin == 'sixth':
        S = ['H', 'T', 'T', 'T', 'T', 'T']
    values = random.choices(S, k = 3)
    if values[0] == 'H' and values[1] == 'T' and values[2] == 'T':
        event += 1

print("Probability of HTT with given coins: ", event / num_sims)
```

Probability of HTT with given coins: 0.1292

Submit Your Solutions

Confirm that you've successfully completed the assignment.

Along with the Notebook, include a PDF of the notebook with your solutions.

`add` and `commit` the final version of your work, and `push` your PDF file to your GitHub repository.

Submit the URL of your GitHub Repository as your assignment submission on Canvas.