Trajectory Tracking Control of Omnidirectional Wheeled Mobile Manipulators: Robust Neural Network-Based Sliding Mode Approach

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***Abstract*—This paper addresses the robust trajectory tracking problem for a redundantly actuated omnidirectional mobile manipulator in the presence of uncertainties and disturbances. The development of control algorithms is based on sliding mode control (SMC) technique. First, a dynamic model is derived based on the practical omnidirectional mobile manipulator system. Then, a SMC scheme, based on the fixed large upper boundedness of the system dynamics (FLUBSMC), is designed to ensure trajectory tracking of the closed-loop system. However, the FLUBSMC scheme has inherent deficiency, which needs computing the upper boundedness of the system dynamics, and may cause high noise amplification and high control cost, particularly for the complex dynamics of the omnidirectional mobile manipulator system. Therefore, a robust neural network (NN)-based sliding mode controller (NNSMC), which uses an NN to identify the unstructured system dynamics directly, is further proposed to overcome the disadvantages of FLUBSMC and reduce the online computing burden of conventional NN adaptive controllers. Using learning ability of NN, NNSMC can coordinately control the omnidirectional mobile platform and the mounted manipulator with different dynamics effectively. The stability of the closed-loop system, the convergence of the NN weight-updating process, and the boundedness of the NN weight estimation errors are all strictly guaranteed. Then, in order to accelerate the NN learning efficiency, a partitioned NN structure is applied. Finally, simulation examples are given to demonstrate the proposed NNSMC approach can guarantee the whole system's convergence to the desired manifold with prescribed performance.**

***Index Terms*—Omnidirectional mobile manipulators, robust neural network (NN), sliding mode control (SMC), trajectory tracking control, uncertainties.**

I. Introduction

A MOBILE manipulator is ordinarily regarded as a manipulator mounted on a moving platform. Due to the mobility of the platform, this system with motion redundancy can perform manipulations in a much larger workspace than a fixed manipulator. Among many types of the mobile manipulators, the omnidirectional mobile platform, which is considered as holonomic parallel system, has 3 degrees of freedom (DOF) in the horizontal motion plane. Therefore, it has more advantages than the nonholomic differential-driven platform [1], [2] which is provided with only two independent driving inputs. Thus, it can fully use the null space motions in cabined work environment and improve overall dynamic properties of the manipulator [3]–[5].

Generally, the omnidirectional mobile platform has closed-chain mechanism with redundant actuation. The redundant actuation provides an effective means for eliminating singularities, and improving the performance such as Cartesian stiffness and homogeneous output forces. At the same time, a major difficulty, which prevents vast control strategies from applying in redundant actuated systems, is the lack of an efficient dynamic model and control method. In order to solve the aforementioned problems, there is a stringent need to derive an efficient and correct dynamic model. Without dynamic control, it is difficult to perform coordinated motion of the mobile platform and the dynamically controlled manipulator.

There are many researches about dynamic modeling and control of omnidirectional mobile manipulators with redundant or nonredundant actuation [6]–[9]. The decentralized control structure for cooperative tasks of multiple mobile manipulators was addressed in [6]. Because of the complex mechanism of the platform and intense coupling effect between the platform and manipulator, Tan and Xi [7] regarded the platform as a simplified motion point and proposed a unified model approach for planning and control. In [8], three lateral orthogonal wheel modules were used in the omnidirectional mobile manipulator for dynamic modeling and motion control. However, the researches on robust tracking control considering uncertainties and disturbances are scarce. In recent years, several results about dynamic modeling and control of holonomic systems were published [10]–[12], which provided effective means for such problems of omnidirectional mobile manipulators, but the robust control schemes for these uncertain system dynamics are not fully explored.

Along with the development of classical adaptive control algorithms, using regressor matrix to design adaptive control schemes has become popular. In such cases, a nonlinear dynamic system with unknown (or uncertain) system parameters

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is expressed by a product of a regressor matrix and an unknown parameter vector. Li *et al.* [13] applied robust adaptive technique to holonomic system with uncertainties and got satisfied performances. Learning control was presented in [14], but it was applicable only when the task was repetitive. An adaptive fuzzy approach was presented in [15], but the asymptotic convergence performance depended on a strong assumption that the approximation error plus external disturbance belonged to $L_2 \cap L_\infty$, and the tracking accuracy depended on the choice of the number of the fuzzy sets. Therefore, these works mainly consider the simple case that the dynamic models are assumed to be perfect, and update laws are then employed to estimate the unknown parameters, which are assumed to be constant or vary slowly.

It is well known that the main advantage of using sliding mode control (SMC) is strong robustness with respect to system uncertainties and external disturbances. SMC is a special discontinuous control technique applicable to various practical systems [16]. By designing switch functions of state variables or output variables to form sliding surfaces, SMC under matching condition can guarantee that when tracking trajectories reach the sliding surfaces, the switch functions keep the trajectories on the surfaces, thus yielding desired system dynamics. Therefore, it is attractive for many highly nonlinear uncertain systems, such as the holonomic and nonholonomic constrained mechanical systems [17], [18].

Neural network (NN), one of the most popular intelligent computation approaches, has an inherent learning ability and can approximate a nonlinear continuous function to arbitrary accuracy. This feature is crucial in the controller design for complex model identifying and unstructured uncertainties compensating. Thus, in the past two decades, the development of intelligent control, particularly NN control (NNC) in robotic fields has attracted considerable interest. Polycarpou [19] presented a systematic methodology to identify a nonlinear system using an NN. Jin *et al.* [20] developed a rigorous theoretical basis to design a stable NN-based controller for simple manipulator system. Lewis *et al.* [21] proposed an NNC scheme that guaranteed the closed-loop performance in terms of small tracking errors and bounded controls. An NN-based control methodology was proposed for the joint space position control of a mobile manipulator in [22], which was composed by a linear control term (classical PID) and an NN compensation term. Hu and Woo [23] proposed a control scheme which consisted of an SMC and an NNC, and their contribution proportion was determined by a fuzzy supervisory controller. Adaptive control scheme of robot manipulators using Chebyshev NN under actuator constraints was developed in [24]. An adaptive wavelet NNs control scheme of mobile robots with unmodeled dynamics and disturbances was addressed in [25].

However, there is little research work on the dynamic model and robust control of redundantly actuated holonomic constrained mechanical systems with uncertainties, such as the redundantly actuated omnidirectional wheeled mobile manipulator whose dynamic property is similar to serial-parallel manipulator in essence.

Hence, in this paper, we focus on a redundantly actuated omnidirectional mobile manipulator with holonomic kinematic constraints. The dynamic properties of the mechanical system are used to form a conservative upper boundedness of the system dynamics. Then, an SMC algorithm with this fixed large upper boundedness (FLUBSMC) is derived to guarantee that given trajectories can be tracked in the presence of structured and/or unstructured uncertainties. Although using fixed large boundedness can guarantee good performance, this control scheme is conservative in essence and cannot be applied in practical systems directly.

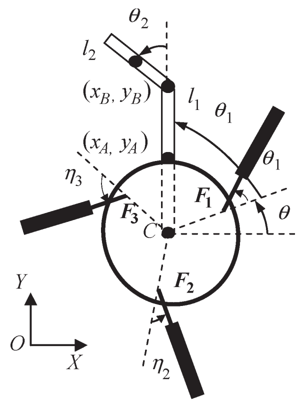

Fig. 1. Mobile manipulator assembly.

Therefore, a robust control scheme using an NN combined with an SMC (NNSMC) is further proposed to solve the aforementioned problem. SMC is designed to be robust to disturbance with a guarantee of the stability of the system. NN approximates the system dynamics directly and overcomes the structured uncertainty by learning. Therefore, the control can coordinately track the trajectory of the mobile platform and the manipulator with different dynamics effectively. Furthermore, no preliminary learning stage is required for the NN weights. The control scheme is capable of disturbance rejection in the presence of unknown bounded disturbances. The tracking stability analysis of the closed-loop system is proved by the Lyapunov theory. The convergence of the NN learning process and the boundedness of the NN weight estimation errors are all rigorously proven. In comparison to the traditional NNC, NNSMC improves the robustness of the system.

The rest of this paper is organized as follows: The dynamic model of the redundantly actuated omnidirectional mobile manipulator is derived, and some preliminaries are given in Section II. Section III presents some remarks and the design process of FLUBSMC. Then, in order to overcome the disadvantages of FLUBSMC, NNSMC is further proposed, and its stability is strictly proved, and then the partitioned NN structure is adopted to reduce the computing burden of the integrated NN in Section IV. In Section V, an illustrative example is applied to validate the dynamic model, the FLUBSMC and the NNSMC algorithms. Finally, conclusions are given in Section VI.

II. System Descriptions

The mobile manipulator considered here moves on the horizontal plane which is in the global frame OXY. It is cylinder-shaped; three identical castor wheels are axed symmetrically on the bottom of the platform, and a two-links manipulator locates on the gravity center C (which coordinate is defined

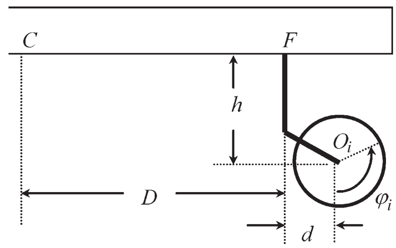

Fig. 2. Side view of wheel module i.

as $[x \quad y \quad \theta]^{\mathrm{T}}$) of the platform, as shown in Fig. 1. The angle between two neighboring wheels is $2\pi/3$. The wheels 1, 2, and 3 are assigned clockwise. l_1 and l_2 are the lengths of the two links, and θ_1 and θ_2 are the angle displacements of those two links, respectively.

The side view is shown in Fig. 2. The radius of each wheel is r, and the horizontal distance between C and the center F_i of the vertical axis of each wheel is D. The offset d is the horizontal distance between the center of each wheel O_i and F_i. The angle displacements for wheel rolling and steering are φ_i and η_i, respectively.

A. Kinematic Model

According to the aforementioned description, we first define the following state variables for easy reference:

$$
\begin{aligned}
q &= \begin{bmatrix} q_1^{\mathrm{T}} & q_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\
&= [x \quad y \quad \theta \quad \theta_1 \quad \theta_2 \quad \varphi_1 \quad \eta_1 \quad \varphi_2 \quad \eta_2 \quad \varphi_3 \quad \eta_3]^{\mathrm{T}} \\
q_1 &= [x \quad y \quad \theta \quad \theta_1 \quad \theta_2]^{\mathrm{T}} \\
q_2 &= [\varphi_1 \quad \eta_1 \quad \varphi_2 \quad \eta_2 \quad \varphi_3 \quad \eta_3]^{\mathrm{T}} \\
\zeta &= [\theta_1 \quad \theta_2 \quad \varphi_1 \quad \eta_1 \quad \varphi_2 \quad \eta_2 \quad \varphi_3 \quad \eta_3]^{\mathrm{T}}
\end{aligned}
$$

q	Generalized coordinates of the mobile manipulator.
q_1	Pose of the mobile manipulator.
q_2	Drive variables of the platform.
ζ	Drive variables of the mobile manipulator.
$0_{m\times n}$	$m \times n$ zero matrix.
$I_{n\times n}$	$n \times n$ identity matrix.
$\lambda_{\min}(*)$ and $\lambda_{\max}(*)$	Minimum and maximum eigenvalues of $(*)$.

Thus, the whole mobile manipulator's kinematic model can be described as

$$\dot{\zeta} = J_{q_1}(\beta, \eta)\dot{q}_1 \tag{1}$$

$$J_{q_1}(\beta, \eta) = \begin{bmatrix} 0_{2\times 3} & I_2 \\ J_1 & 0_{2\times 2} \\ J_2 & 0_{2\times 2} \\ J_3 & 0_{2\times 2} \end{bmatrix} \tag{2}$$

where

$$J_i = \begin{bmatrix} -\frac{1}{r}\cos\beta_i & -\frac{1}{r}\sin\beta_i & -\frac{D}{r}\sin\eta_i \\ \frac{1}{d}\sin\beta_i & -\frac{1}{d}\cos\beta_i & -\frac{D}{d}\cos\eta_i - 1 \end{bmatrix}, \qquad (i=1,2,3).$$

Property 1: For $J_{q_1}(\beta, \eta)$, if $\delta = (d/D) \leq (1/2)$, rank $J_{q_1}(\beta, \eta) = 5$ holds.

Proof: Calculate $\det(J_{q_1}^{\mathrm{T}}(\beta, \eta)J_{q_1}(\beta, \eta))$ as follows:

$$
\begin{aligned}
&\det\left(J_{q_1}^{\mathrm{T}}(\beta, \eta)J_{q_1}(\beta, \eta)\right) \\
&= \frac{D^6}{r^9 d^9}\Bigg\{27 + 18\delta(\cos\eta_1 + \cos\eta_2 + \cos\eta_3) + 18\delta^2 - 6\delta^2 \\
&\quad \times \left[\cos\left(\eta_1 - \eta_2 + \frac{2\pi}{3}\right) + \cos\left(\eta_2 - \eta_3 + \frac{2\pi}{3}\right)\right. \\
&\quad \left.+ \cos\left(\eta_3 - \eta_1 + \frac{2\pi}{3}\right)\right]\Bigg\}.
\end{aligned}
$$

Only when the expressions $(\cos\eta_1 + \cos\eta_2 + \cos\eta_3) = -3$ and $\cos(\eta_1 - \eta_2 + (2\pi/3)) + \cos(\eta_2 - \eta_3 + (2\pi/3)) + \cos(\eta_3 - \eta_1 + (2\pi/3)) = 3$, the expression $\det(J_{q_1}^{\mathrm{T}}(\beta, \eta)J_{q_1}(\beta, \eta))$ can achieve its minimum. Thus, $\det(J_{q_1}^{\mathrm{T}}(\beta, \eta)J_{q_1}(\beta, \eta)) \geq (D^6/r^9 d^9)(9 - 18\delta)$ can be guaranteed. Therefore, rank $J_{q_1}(\beta, \eta) = 5$ can be always held, if $\delta \leq 1/2$. ■

B. Dynamic Model

In most of the dynamic model researches about the omnidirectional mobile manipulator systems, the redundantly actuated property of the platform was not emphasized. Moreover, the integrated dynamic model of the omnidirectional mobile manipulator which was derived from the driving wheels was not explicitly addressed. According to Lagrange theory [26], based on the kinematic model, the dynamic equations of the mechanical system with external disturbances can be derived as

$$M(q)\ddot{q} + C(q, q)\dot{q} + G(q) + d(t) = B(q)u + A^{\mathrm{T}}(q)\lambda \tag{3}$$

where $u \in R^n$ is the vector of generalized input torques, $M(q) \in R^{n\times n}$ is a symmetric bounded positive definite inertia matrix, $C(q, \dot{q})\dot{q} \in R^n$ represents the vector of centripetal and Coriolis torques, $G(q) \in R^n$ is the gravitational torque vector, $d(t) \in R^n$ denotes the external disturbances, $B(q) \in R^{n\times n}$ is a full rank input transformation matrix and is assumed to be known because it is the function of fixed geometry of the system, $A(q) \in R^{m\times n}$ is the kinematic constraints matrix, and $\lambda \in R^m$ is a constraint force vector. The details of the dynamic model description are addressed in appendix.

The kinematic constraints are considered independent of time and can be expressed as

$$A(q)\dot{q} = 0 \tag{4}$$

where the effect of the kinematic constraints can be viewed as restricting the dynamics on a constraint manifold as

$$\Omega = \{(q, \dot{q}) \in R^n \times R^n : A(q)\dot{q} = 0\}. \tag{5}$$

The presence of m kinematic constraints causes the system to lose m DOF; hence, the system only has $n - m$ DOF left. Based on the kinematic model, we know $q_1(t) \in R^{n-m}$ is the vector of the independent generalized coordinates of the mobile manipulator system, and $q_2(t) \in R^m$ is the vector of driving variables of the platform.

Property 2: Define $H = [I_3 \quad 0_{3\times 2}]$ and $R(q) = \left[\begin{smallmatrix} H \\ J_{q_1} \end{smallmatrix}\right]$. Based on Property 1, the matrix $R^{\mathrm{T}}(q)B(q)$ is of full rank, so the Moore-Penrose inverse matrix of $R(q)$ always exists.

In order to reduce complexity of the whole system, applying the projection of the dynamic model into the Null space of the constraints, we can obtain

$$\begin{aligned} \dot{q} &= R(q)\dot{q}_1 \\ \bar{M}(q)\ddot{q}_1 + \bar{C}(q,\dot{q})\dot{q}_1 + \bar{G}(q) + \bar{d} &= \bar{E}u \end{aligned} \tag{6}$$

where $\bar{M}(q) = R^{\mathrm{T}}(q)M(q)R(q)$, $\bar{C}(q,\dot{q}) = R^{\mathrm{T}}(q)C_1(q,\dot{q})$, $C_1(q,\dot{q}) = M(q)\dot{R}(q) + C(q,\dot{q})R(q)$, $\bar{G}(q) = R^{\mathrm{T}}(q)G(q)$, $\bar{d} = R^{\mathrm{T}}(q)d$, $\bar{E}(q) = R^{\mathrm{T}}(q)B(q)$. Equations (4) and (5) lead to $A(q)R(q)\dot{q}_1 = 0$. In other words, $A(q)R(q) = 0$, since $A(q)R(q)$ is of full columns and the configuration of the whole system has $n-m$ DOF. For (6), some fundamental properties are addressed below.

Property 3: The matrix $\bar{M}(q) = R^{\mathrm{T}}(q)M(q)R(q)$ is symmetric and positive definite.

Property 4: $\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q})$ is skew symmetric, i.e., $\xi^{\mathrm{T}}(\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q}))\xi = 0, \forall \xi \in R^{n-m}$.

Property 5: There exist positive scalars $\beta_i (i = 1,\ldots,5)$ such that $\forall q \in R^n$, $\forall \dot{q} \in R^n$: $\|M(q)\| \le \beta_1 < \infty$, $\|C(q,\dot{q})\| \le \beta_2 + \beta_3\|\dot{q}\|$, $\|G(q)\| \le \beta_4$ and $\sup_{t\ge 0}\|d(t)\| \le \beta_5$.

It should be noted that the whole system (6) consists of a new dynamic model together with a pure kinematic relationship. This structure makes it possible to design the SMC law.

III. FLUBSMC Design

This section considers the trajectory tracking problem of the redundantly actuated mobile manipulator system discussed above. Based on the fixed large upper boundedness of the system dynamics, an SMC scheme, (FLUBSMC), is designed to ensure trajectory tracking of the closed-loop system. FLUBSMC consists of two components: fixed large upper boundedness and SMC. In order to develop FLUBSMC, the following assumptions are required throughout this section.

Assumption 1: The vectors $q_1, \dot{q}_1$ are bounded and uniformly continuous, and have bounded and uniformly continuous derivatives up to the second order. Moreover, the matrices $R(q)$ and $\dot{R}(q)$ are also bounded as $\|R(q)\| \le \beta_6$, $\|\dot{R}(q)\| \le \beta_7\|\dot{q}\|$, where $\beta_i (i = 6,7)$ are positive constants.

Remark 1: Property 1 and the structure of $R(q)$ can guarantee all $n-m$ DOF are independently actuated, which always holds for redundantly actuated holonomic mechanical systems such as parallel manipulator and so on.

Remark 2: According to Li *et al.*'s [13] claim, if $q_1(t)$ is bounded, then $R(q)$ is bounded. If $\dot{q}_1(t)$ is bounded, then $\dot{R}(q)$ is bounded. The purpose of Assumption 1 is to try to make full use of the available knowledge to reduce control gains.

A trajectory tracking control objective for perturbed mobile manipulator system is formulated as: on the basis of the vector q_1 and $\dot{q}_1$, given a desired q_{1d} and $\dot{q}_{1d}$, develop a controller such that for any $(q(0),\dot{q}(0)) \in \Omega$ as in (5), $q_1(t)$ and $\dot{q}_1(t)$ can asymptotically converge to a manifold Ω_d specified as:

$$\Omega_d = \{(q,\dot{q}) | q_1(t) = q_{1d}(t), \dot{q}_1(t) = \dot{q}_{1d}(t), q(t) = R(q)q_1(t)\} \tag{7}$$

where the vector $q_1(t)$ can be considered as $n-m$ "output equations" of the system.

Assumption 2: The desired reference trajectory q_{1d} is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. Therefore, there exists a constant q_{1B}, such that $\|q_{1d}^{\mathrm{T}} \quad \dot{q}_{1d}^{\mathrm{T}} \quad \ddot{q}_{1d}^{\mathrm{T}}\| \le q_{1B}$ always holds.

In the following, some variables are defined as:

$$e = q_1 - q_{1d} \tag{8}$$

$$\dot{q}_{1r} = \dot{q}_{1d} - \Lambda e \tag{9}$$

where e and q_{1r} denote the tracking error and a set of auxiliary signals, respectively. Λ is a positive definite matrix which eigenvalues are strictly in the right-hand of complex plane. Then, a sliding variable is defined as

$$s = \dot{q}_1 - \dot{q}_{1r} = \dot{e} + \Lambda e. \tag{10}$$

Remark 3: From Assumptions 1 and 2, there exist positive scalars $\beta_i (i = 8,9)$ such that $\forall \dot{q}_{1r} \in R^{n-m}$, $\forall \ddot{q}_{1r} \in R^{n-m}$: $\|\dot{q}_{1r}\| \le \beta_8 + \Lambda\|e\|$, $\|\ddot{q}_{1r}\| \le \beta_9 + \Lambda\|\dot{e}\|$.

When the sliding surface $s = 0$, according to the theory of SMC, the sliding mode is governed by the following differential equation:

$$\dot{e} = -\Lambda e. \tag{11}$$

Obviously, the behavior of the system on the sliding surface is determined by the structure of the matrix Λ. In other words, when $s = 0$, the tracking error transient response is completely governed by the aforementioned equation.

Based on (3) and the first equation of (6), the closed-loop system in terms of the sliding variable s can be obtained by

$$\begin{aligned} M(q)R(q)\dot{s} = B(q)u - (M(q)R(q)\ddot{q}_{1r} + C_1(q,\dot{q})\dot{q}_{1r} \\ + G(q) + d(t)) - C_1(q,\dot{q})s + A^{\mathrm{T}}(q)\lambda. \end{aligned} \tag{12}$$

Multiplying left $R^{\mathrm{T}}(q)$ and using Property 3 lead to

$$\bar{M}(q)\dot{s} = \bar{E}(q)u - \bar{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q,t) - \bar{C}(q,\dot{q})s \tag{13}$$

$$\bar{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q,t) = (\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q) + R^{\mathrm{T}}(q)d(t) \tag{14}$$

$$\begin{aligned} \underline{H}(\ddot{q}_{1r},\dot{q}_{1r},\dot{q},q) &= R^{\mathrm{T}}(q)\left(M(q)R(q)\ddot{q}_{1r}\right. \\ &\quad \left. + C_1(q,\dot{q})\dot{q}_{1r} + G(q)\right) \\ &= R^{\mathrm{T}}(q)\left(M(q)R(q)(\ddot{q}_{1d} - \Lambda\dot{e})\right. \\ &\quad \left. + C_1(q,\dot{q})(\dot{q}_{1d} - \Lambda e) + G(q)\right) \\ &= \underline{H}(x) \end{aligned} \tag{15}$$

where $x = [\ddot{q}_{1d}^{\mathrm{T}} \quad \dot{q}_{1d}^{\mathrm{T}} \quad q_{1d}^{\mathrm{T}} \quad \dot{q}^{\mathrm{T}} \quad q^{\mathrm{T}}]^{\mathrm{T}} \in R^{3\times(n-m)+2\times n}$.

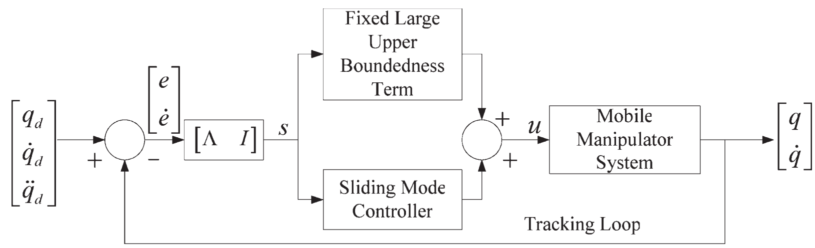

Fig. 3. FLUBSMC scheme.

Because the omnidirectional mobile manipulator moves in the horizontal plane, $R^{\mathrm{T}}(q)G(q) \equiv 0$.

Before we design control law, following Vicente *et al.*'s [27] work, we have the result that the expression $\bar{H}(\ddot{q}_{1r}, \dot{q}_{1r}, \dot{q}, q, t)$ in (13) is upper bounded; and the boundedness is a state-dependent function. Therefore, according to Property 4, the following can be reached:

$$\left\|\bar{H}(\ddot{q}_{1r}, \dot{q}_{1r}, \dot{q}, q, t)\right\| \leq \bar{\beta}_1 \cdot \|\ddot{q}_{1r}\| + \bar{\beta}_2\|\dot{q}_{1r}\| + \bar{\beta}_3\|\dot{q}_{1r}\| \cdot \|\dot{q}\| + \bar{\beta}_4 \quad (16)$$

where $\bar{\beta}_1 = \beta_6^2\beta_1$, $\bar{\beta}_2 = \beta_2\beta_6^2$, $\bar{\beta}_3 = \beta_1\beta_6\beta_7 + \beta_6^2\beta_3$, $\bar{\beta}_4 = \beta_6\beta_5$.

Here, we denote $\rho(t) = \bar{\beta}_1 \cdot \|\ddot{q}_{1r}\| + \bar{\beta}_2\|\dot{q}_{1r}\| + \bar{\beta}_3\|\dot{q}_{1r}\| \cdot \|\dot{q}\| + \bar{\beta}_4$ and call $\rho(t)$ as the upper bounded function, which is obviously a function of the state variables.

Remark 4: From the aforementioned description, we know the upper boundedness of the system dynamics is a positive definite second-order polynomial. What we are concerned with is only the boundedness of $\bar{H}(\ddot{q}_{1r}, \dot{q}_{1r}, \dot{q}, q, t)$, thus the measurement of acceleration signals is not required. This forms the foundation of FLUBSMC.

Assuming that the parameters $\bar{\beta}_i (i = 1, 2, 3, 4)$ of the upper boundedness of the system dynamics are known, we design the FLUBSMC law according to [28]

$$\bar{E}(q)u = -Ks - K_s \mathrm{sgn}(s) - \frac{s \cdot (\rho(t))^2}{\|s\| \cdot \rho(t) + \gamma(t)} \quad (17)$$

where K and K_s are $(n-m) \times (n-m)$ positive definite gain matrixes determined by the designer, $\rho(t)$ is the upper boundedness of the system, $\gamma(t) > 0$ is a time varying function and satisfies $\int_0^t \gamma(\nu) d\nu = a < \infty$.

From (13) and (17), by selecting a large gain K_s, it is easy to obtain:

$$\dot{s}^{\mathrm{T}} s \leq -\sigma|s| \quad (18)$$

where σ is a positive constant. The aforementioned expression shows that any trajectories away from the sliding surface are forced to reach the sliding surface in finite time.

The whole FLUBSMC scheme shown in Fig. 3 consists of the controller design component and the actual mobile manipulator system component. The Tracking Loop provides the feedback signals for the controller designing. Correspondingly, the first two terms of (17) is a sliding mode controller, the third term of (17) is the Fixed Large Upper Boundedness term.

Remark 5: From [28], consider the uncertain dynamic system (6), the FLUBSMC law is chosen as (17). Then, for any $(q(0), \dot{q}(0)) \in \Omega$, the tracking error e and its derivative $\dot{e}$ converge to the sliding surface and are restricted to the surface for all future time. Furthermore, when $t \to \infty$, e and $\dot{e}$ asymptotically converge to zero.

IV. NNSMC Design

Using fixed large boundedness can guarantee good performance, but it is not recommended in practice as large boundedness implies high noise amplification and high control cost. In addition, due to the different dynamic properties of the mobile platform and the manipulator, and highly coupled dynamics between them, the FLUBSMC law (17) which considers the two different dynamic subsystems as a whole system will affect the manipulator's driving outputs obviously. Furthermore, this boundedness cannot be easily obtained in practice. In order to solve the aforementioned problems, a more effective control method NNSMC is first proposed in this section to coordinately control the mobile platform and the mounted manipulator with different dynamics. Because NN has ability to approximate a nonlinear continuous function to arbitrary accuracy, NN in NNSMC is directly used to identify the uncertainty vector of the system dynamics, not to compute the Kronecker product, so the weight matrixes structures are simplified to accelerate the tuning speed. Finally, in order to accelerate the NN learning efficiency and reduce the computing burden, a partitioned NN structure is applied.

During the development of NNSMC, we assume that $M(q)$, $C_1(q, \dot{q})$, and $G(q)$ are completely unknown, then an NN approximator is derived to approximate the integrated uncertain term $\underline{H}(x)$ in (15).

A. Robust NNSMC Scheme Design

Fig. 4 shows the structure of three layers radial basis function NN (RBFNN) adopted in NNSMC, which includes an input layer, a hidden layer, and an output layer. The relationships among these layers can be described as follows.

Let

$$W = \begin{bmatrix} w_{11}, w_{21}, \ldots, w_{h1} \\ w_{12}, w_{22}, \ldots, w_{h2} \\ \vdots \quad \vdots \quad \quad \vdots \\ w_{1k}, w_{2k}, \ldots, w_{hk} \end{bmatrix}, \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_h \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}. \quad (19)$$

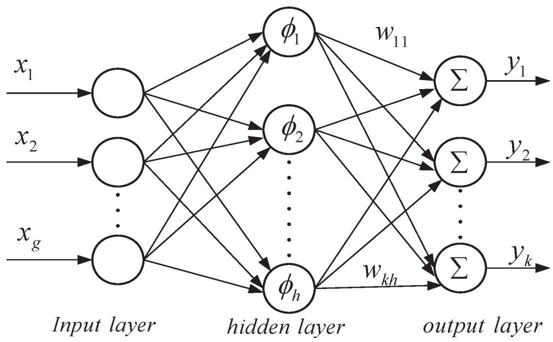

Fig. 4. RBFNN.

In this way, the relationship of the output layer can be written as

$$Y = W^{\mathrm{T}}\phi \tag{20}$$

where W is the weight matrix of the RBFNN, ϕ is the excitation function vector.

For any given real continuous functions on a compact set U, define a smooth function $f(\cdot) : U \to R^n$. If we choose $f(x) \in C^r(U)$, where $C^r(U)$ is the space of the continuous functions, there exists an RBFNN in the form of (20) such that

$$f(x) = W^{\mathrm{T}}\phi(x) + \varepsilon \tag{21}$$

$$\|\varepsilon\| < \varepsilon_N \tag{22}$$

where ε_N is a constant.

This property makes it possible for applying RBFNN to solve almost any nonlinear modeling problems. Because of the universal approximation ability of RBFNN, $\underline{H}(x)$ defined in (15) can be identified using RBFNN with enough number of hidden layer neurons such that

$$\underline{H}(x) = W^{\mathrm{T}}\phi(x) + \varepsilon \tag{23}$$

where $W \in R^{h\times k}$ is assumed to be constant and bounded by

$$\|W\| \le W_B \tag{24}$$

where W_B is one known positive constant. The basis function vector $\phi(x)$ is usually chosen as the *Gaussian* functions defined in [24].

The estimate of the uncertain term $\underline{H}(x)$ is expressed as

$$\hat{\underline{H}}(x) = \hat{W}^{\mathrm{T}}\phi(x) \tag{25}$$

where $\hat{W}$ is the tuning parameter matrix of the network and is adjusted in the learning process.

Assumption 3: In the RBFNN, the input vector x [defined in (15)] is used to identify the uncertain term $\underline{H}(x)$. There exist positive constants c_0 and c_1, such that the following expression holds:

$$\|x(t)\| \le q_B + c_0\,\|s(0)\| + c_1\,\|s(t)\| \tag{26}$$

where s is defined in (10). Assume that (23) holds, according to the definitions in (10) and (11), we know that all x is in the compact set $\Omega_x \equiv \{x|\|x\| < b_x\}$, where $b_x > q_B$. Define the compact set as $\Omega_s \equiv \{s|\|s\| < (b_x - q_B)/(c_0 + c_1)\}$ with $s(0) \in \Omega_s$. Thus, the RBFNN approximation property always holds for all s in the compact set Ω_s.

Consider the uncertain dynamic system (6) with plant uncertainties and external disturbance. Using RBFNN to identify the uncertain term $\underline{H}(x)$. For any $(q(0), \dot{q}(0)) \in \Omega$ and desired trajectory $q_{1d}(t)$, let the NNSMC law be given by

$$\bar{E}(q)u = -Ks - K_s\mathrm{sgn}(s) + \hat{W}^{\mathrm{T}}\phi(x) \tag{27}$$

$$\dot{\hat{W}} = -\alpha\phi s^{\mathrm{T}} - \mu\alpha\|s\|\hat{W} \tag{28}$$

where K and K_s are the same as (6) and (17), α is a positive constant representing the learning rate of the network, μ is a small positive design constant.

From Fig. 5, we can see that the NNSMC scheme is different from the FLUBSMC scheme by adopting NN to identify the uncertain term of system dynamics directly.

Theorem 1: For the uncertain dynamic system (6), there exist appropriate parameters in the NNSMC law (27), and the following performance can be achieved.

1) Because the matrix $R^{\mathrm{T}}(q)B(q)$ is of full rank, the input torque $u(t)$ is bounded for all $t > 0$.
2) With suitable positive gain constants K and K_s, the tracking error e of the uncertain dynamic system will be uniformly ultimately bounded.
3) With sufficient large K_s, the tracking errors e and $\dot{e}$ will asymptotically converge to zero.

Proof:

1) Since all the variables in the right side of (27) are bounded, according to Property 2 and Assumption 1, it is easy to conclude that $u(t)$ is bounded for all $t > 0$.
2) Consider a Lyapunov candidate function as [22]

$$V = \frac{1}{2}s^{\mathrm{T}}\bar{M}s + \frac{1}{2\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\tilde{W}\} \tag{29}$$

where $\tilde{W} = W - \hat{W}$. By substituting (27) and (23) into (13), the time derivative of V leads to

$$\begin{aligned}
\dot{V} &= \frac{1}{2}s^{\mathrm{T}}\dot{\bar{M}}s + s^{\mathrm{T}}\bar{M}\dot{s} + \frac{1}{\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\}\\
&= \frac{1}{2}s^{\mathrm{T}}\dot{\bar{M}}s + s^{\mathrm{T}}\Big(-Ks - K_s\mathrm{sgn}(s) + \hat{W}^{\mathrm{T}}\phi(x) - W^{\mathrm{T}}\phi(x)\\
&\qquad - \varepsilon(x) - R^{\mathrm{T}}(q)d(t) - \bar{C}(q,\dot{q})s\Big)\\
&\quad + \frac{1}{\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\}\\
&= s^{\mathrm{T}}\Big(-Ks - K_s\mathrm{sgn}(s) + \hat{W}^{\mathrm{T}}\phi(x) - W^{\mathrm{T}}\phi(x)\\
&\qquad - \varepsilon(x) - R^{\mathrm{T}}(q)d(t)\Big) + \frac{1}{\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\}\\
&= s^{\mathrm{T}}\Big(-Ks - K_s\mathrm{sgn}(s) - \tilde{W}^{\mathrm{T}}\phi(x) - \varepsilon(x)\\
&\qquad - R^{\mathrm{T}}(q)d(t)\Big) + \frac{1}{\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\}\\
&= s^{\mathrm{T}}\Big(-Ks - K_s\mathrm{sgn}(s) - \tilde{W}^{\mathrm{T}}\phi(x) - \varepsilon(x)\\
&\qquad - R^{\mathrm{T}}(q)d(t)\Big) - \frac{1}{\alpha}\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\dot{\hat{W}}\}
\end{aligned}$$

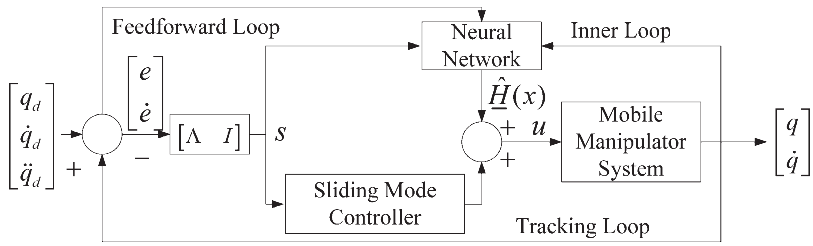

Fig. 5. NNSMC scheme.

$$
\begin{aligned}
&= s^{\mathrm{T}}\Big(-Ks - K_s\mathrm{sgn}(s) - \tilde{W}^{\mathrm{T}}\phi(x) - \varepsilon(x) \\
&\qquad - R^{\mathrm{T}}(q)d(t)\Big) - \frac{1}{\alpha}\mathrm{tr}\left\{\tilde{W}^{\mathrm{T}}\left(-\alpha\phi s^{\mathrm{T}} - \mu\alpha\|s\|\hat{W}\right)\right\} \\
&= s^{\mathrm{T}}\left(-Ks - K_s\mathrm{sgn}(s) - \varepsilon(x) - R^{\mathrm{T}}(q)d(t)\right) \\
&\quad + \mu\|s\|\mathrm{tr}\left\{\tilde{W}^{\mathrm{T}}\hat{W})\right\}.
\end{aligned} \tag{30}
$$

According to the matrix trace, inner product, and Frobenius norm theory, the following property holds [21]:

$$
\begin{aligned}
\mathrm{tr}\{\tilde{W}^{\mathrm{T}}\hat{W}\} &= \mathrm{tr}\left\{\tilde{W}^{\mathrm{T}}(W - \tilde{W})\right\} \\
&= \langle\tilde{W}, W\rangle_F - \|\tilde{W}\|_F^2 \\
&\leq \|\tilde{W}\|_F\|W\|_F - \|\tilde{W}\|_F^2.
\end{aligned} \tag{31}
$$

Thus

$$
\begin{aligned}
\dot{V} &\leq s^{\mathrm{T}}\left(-Ks - K_s\mathrm{sgn}(s) - \varepsilon(x) - R^{\mathrm{T}}(q)d(t)\right) \\
&\quad + \mu\|s\|\left(\|\tilde{W}\|_F\|W\|_F - \|\tilde{W}\|_F^2\right) \\
&\leq -\lambda_{\min}(K)\|s\|^2 - K_s\|s\| + \varepsilon_N\|s\| + \beta_6\beta_5\|s\| \\
&\quad + \mu\|s\|\left(\|\tilde{W}\|_F\|W\|_F - \|\tilde{W}\|_F^2\right) \\
&\leq -\lambda_{\min}(K)\|s\|^2 - K_s\|s\| + \varepsilon_N\|s\| + \beta_6\beta_5\|s\| \\
&\quad + \mu\|s\|\left(\|\tilde{W}\|_F W_B - \|\tilde{W}\|_F^2\right) \\
&= -\|s\|\Bigg(\lambda_{\min}(K)\|s\| + K_s - \varepsilon_N - \beta_6\beta_5 \\
&\qquad + \mu\left(\|\tilde{W}\|_F^2 - \|\tilde{W}\|_F W_B\right)\Bigg) \\
&= -\|s\|\Bigg(\lambda_{\min}(K)\|s\| + K_s - \varepsilon_N - \beta_6\beta_5 \\
&\qquad + \mu\left(\|\tilde{W}\|_F - \frac{W_B}{2}\right)^2 - \mu\frac{W_B^2}{4}\Bigg).
\end{aligned} \tag{32}
$$

From the aforementioned proving process, in order to ensure the approximation effect of RBFNN estimator, we know that if the following expression (33) holds, the sliding variable should be always constrained in the compact set Ω_s:

$$
\|s\| > \frac{\varepsilon_N + \beta_6\beta_5 + \mu\frac{W_B^2}{4} - K_s}{\lambda_{\min}(K)} \equiv b_s. \tag{33}
$$

This can be achieved by choosing suitable positive constants to construct the gain matrix K to satisfy

$$
\lambda_{\min}(K) > \frac{\left(\varepsilon_N + \beta_6\beta_5 + \mu\frac{W_B^2}{4} - K_s\right)(c_0 + c_1)}{b_x - q_B}. \tag{34}
$$

Therefore, the compact set defined by $\|s\| > b_s$ is constrained by Ω_s. As a result, the RBFNN approximation property always holds.

Thus, $\dot{V}$ is guaranteed to be negative outside a compact set. Based on the standard Lyapunov theory and LaSalle theory, this property ensures the uniform ultimate boundedness of the sliding variable s, the NN weights $\tilde{W}$, and the weight estimates $\hat{W}$. Moreover, the norm of the sliding variable $\|s\|$ can be made arbitrarily small by increasing the gain matrix K.

3) Furthermore, we choose a sufficient large K_s such that

$$
\frac{\varepsilon_N + \beta_6\beta_5 + \mu\frac{W_B^2}{4} - K_s}{\lambda_{\min}(K)} \leq 0. \tag{35}
$$

Since $\|s\| \geq 0$ and $\lambda_{\min}(K) > 0$, we know

$$
\dot{V} \leq -\lambda_{\min}(K)\|s\|^2 \leq 0. \tag{36}
$$

Define

$$
V_R = V(t) - \int_0^t \left(\dot{V}(\nu) + \lambda_{\min}(K)\|s(v)\|^2\right) d\nu. \tag{37}
$$

Its time derivatives up to the second order are

$$
\begin{aligned}
\dot{V}_R &= \dot{V}(t) - \dot{V}(t) - \lambda_{\min}(K)\|s\|^2 \\
&= -\lambda_{\min}(K)\|s\|^2
\end{aligned} \tag{38}
$$

$$
\ddot{V}_R = -2\lambda_{\min}(K)s^{\mathrm{T}}\dot{s}. \tag{39}
$$

We can conclude that $|\ddot{V}_R|$ is bounded and $\dot{V}_R$ is continuous. We know that $V(t)$ is bounded from (29) and the second term of V_R is a finite integral. Thus, V_R is bounded. In addition, $\int_0^t \dot{V}_R(\nu)d\nu = V_R(t) - V_R(0)$ is also bounded. Then, according to Barbalat's lemma, $|\dot{V}_R| = \lambda_{\min}(K)\|s\|^2 \rightarrow 0$, when $t \rightarrow \infty$.

According to the aforementioned proof, the sliding variable $s \rightarrow 0$, when $t \rightarrow \infty$. Therefore, based on Remark 5, the tracking error e and $\dot{e}$ asymptotically converge to zero as $t \rightarrow \infty$.

Remark 6: The NNSMC law (27) consists of two components. The first component [i.e., $-Ks - K_s\mathrm{sgn}(s)$] is the sliding mode controller which is used to guarantee the stability of the system and achieve the uniformly ultimately bounded performance, and the second component [i.e., $\hat{W}^{\mathrm{T}}\phi(x)$] is the adaptive RBFNN to approximate the uncertain term. Hence, in practice, the NNSMC law is a hybrid adaptive robust controller.

Remark 7: Most of the conventional adaptive control schemes consider the uncertain dynamics of the holonomic mechanical systems as a product of a regressor matrix and a basis vector. The conventional adaptive control schemes can be employed only to deal with parameter uncertain systems in which the unknown parameters must be assumed to be constant or vary slowly. It is obvious that NNSMC resembles the conventional adaptive method to a great extent. Hence, the analysis and design developed in this paper can also be directly applied to the holonomic constrained mechanical systems with linear parameterization property. Therefore, in essence, the NNSMC law $(-Ks - K_s\mathrm{sgn}(s) + \hat{W}^{\mathrm{T}}\phi(x))$ can be categorized into the indirect adaptive NN controller because NNSMC uses NN to identify the unknown dynamics of the model.

Remark 8: The proposed scheme is nonregressor based and requires no information about the system dynamics. We use NN to estimate the uncertain term $\underline{H}(x)$ and avoid using fixed large boundedness like (17) to guarantee good performance, because large boundedness implies high noise amplification and high control cost.

Remark 9: For the NNSMC law (27), once the control parameters such as K, K_s are determined, the input torque $u(t)$ is bounded for all $t > 0$. If we choose a sufficient large K_s, we can ensure the asymptotical stability, but $u(t)$ may be larger than the maximum output torque of driving motors. This situation is prohibited in practice and need to be considered first in parameters choosing procedure. In addition, large K_s also causes severe chattering. Therefore, when we design a suitable controller, there is a critical tradeoff between the actual input torque and the tracking performance.

B. Partitioned NNs

Given a holonomic constrained mechanical system, it is desired to select a set of basis functions and determine the NN reconstruction error bound so that (23) holds; that is, determine a basis function vector $\phi(x)$, so that the uncertain term defined in (15) can be expressed by (23). However, when we face a complex mechanical system like the omnidirectional mobile manipulator studied in this paper, the inputs and outputs dimensions of NN are large, the computing burden of the learning process is big. These two problems above severely prohibit the application of a single NN in practical system.

From [30], we know that the NN controller allows one to design partitioned NNs to achieve the same performance

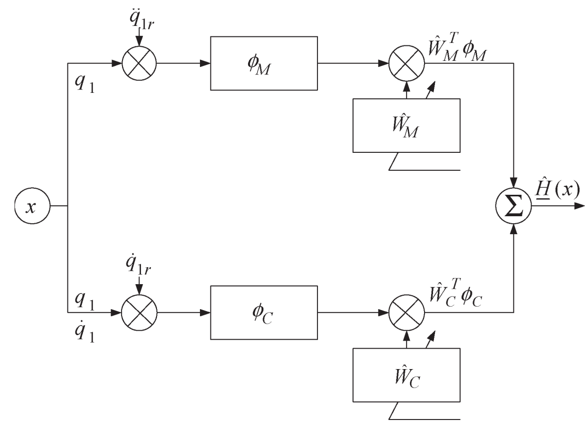

Fig. 6. Partitioned NN structure.

as a single NN. Furthermore, using partitioned structure has following advantages:

1) simplifying the structure design and parameters choosing, so the suitable basis functions for each individual partitioned subsystem can be separately determined;
2) making the network weights tuning procedure faster, so the parameters in each subsystem can be tuned respectively.

For the omnidirectional mobile manipulator system, there is a unique function $\vartheta : R^{n-m} \rightarrow R^m$, such that the holonomic constraint depicted in Section III can always be expressed explicitly as [29]

$$q_2 = \vartheta(q_1). \quad (40)$$

Then, we can easily obtain that the uncertain $\underline{H}(x)$ defined in (15) can be rewritten as the following form:

$$\underline{H}(\bar{x}) = R^{\mathrm{T}}(q_1)M(q_1)R(q_1)\ddot{q}_{1r} + R^{\mathrm{T}}(q_1)C_1(q_1,\dot{q}_1)\dot{q}_{1r} \quad (41)$$

where $\bar{x} = [\ddot{q}_{1r}^{\mathrm{T}} \quad \dot{q}_{1r}^{\mathrm{T}} \quad \dot{q}_1^{\mathrm{T}} \quad q_1^{\mathrm{T}}]^{\mathrm{T}} \in R^{4\times(n-m)}$.

From (41), there are two separated $R^{\mathrm{T}}(q)M(q)R(q)\ddot{q}_{1r}$ and $R^{\mathrm{T}}(q)C_1(q,\dot{q})\dot{q}_{1r}$ in the $\underline{H}(\bar{x})$. Therefore, we can apply two separated NNs to identify them, respectively. Using the similar analysis method as [30] and [31], we can derive that

$$\begin{aligned}\underline{\hat{H}}(\bar{x}) &= \begin{bmatrix}\hat{W}_M^{\mathrm{T}} & \hat{W}_C^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\phi_M \\ \phi_C\end{bmatrix} \\ &= \hat{W}_M^{\mathrm{T}}\phi_M + \hat{W}_C^{\mathrm{T}}\phi_C. \quad (42)\end{aligned}$$

From the partitioned NN structure (42) shown in Fig. 6, it is easy to know that the partitioned decoupled NNs can be tuned, respectively. Then, the learning algorithm (28) can be replaced by

$$\dot{\hat{W}}_M = -\alpha_M\phi_M s^{\mathrm{T}} - \mu_M\alpha_M\|s\|\hat{W}_M \quad (43)$$

$$\dot{\hat{W}}_C = -\alpha_C\phi_C s^{\mathrm{T}} - \mu_C\alpha_C\|s\|\hat{W}_C. \quad (44)$$

According to the theorems in [30], we can conclude that the performance of the partitioned NN structure is equivalent to the single NN.

TABLE I
PARAMETERS FOR THE MOBILE MANIPULATOR

r(m)	D(m)	m_0(kg)	I_0(kg·m^2)	I_{wb}(kg·m^2)	I_{bw}(kg·m^2)
0.10	0.015	60	0.3	0.15	0.05
l_1(m)	l_2(m)	m_1(kg)	I_1(kg·m^2)	m_2(kg)	I_2(kg·m^2)
0.50	0.35	4	0.030	3.5	0.036

Remark 10: In essence, the uncertain term $\underline{H}(x)$ that we pay attention to is a vector, not a matrix. Using the partitioned structure is to simplify the NN design process and accelerate the tuning speed. The partitioned NNs in [30] and partitioned fuzzy logic systems in [31] require Kronecker product to construct new basis function for identifying the uncertain matrix $[R^{\mathrm{T}}(q)M(q)R(q)]$. Compared to directly using NN to identify the uncertain integrated vector $(R^{\mathrm{T}}(q)M(q)R(q)\ddot{q}_{1r})$, employing Kronecker product to estimate the whole uncertain matrix increases the computing burden of tuning weight matrixes obviously, particularly for the complex mechanical systems. Thus, their method only partitions the structure of the NN approximator, but does not simplify the tuning procedure. In this paper, we employ two separated NNs to directly approximate two decoupled $R^{\mathrm{T}}(q)M(q)R(q)\ddot{q}_{1r}$, and $R^{\mathrm{T}}(q)C_1(q,\dot{q})\dot{q}_{1r}$ of the uncertain term $\underline{H}(x)$. This method does not need Kronecker product; therefore, the weight matrixes structures are more compact, and tuning speed is faster.

V. SIMULATION RESULTS

As an example to verify the proposed approach, this section discusses the simulation of the dynamic model and trajectory tracking controller. The physical parameters for the omnidirectional mobile manipulator system are shown in Table I, where m_0, m_1, and m_2 are the masses of the platform, link 1, and link 2, respectively. I_0, I_1, and I_2 are the inertias of the platform, link1, and link2, respectively. I_{wb} and I_{bw} are the rolling and steering inertias for the wheel module.

Let the desired output trajectory q_{1d} be

$$\begin{cases} x_d = 2\cos(t/2) \\ y_d = 2\sin(t/2) \\ \theta_d = 2\sin(t/2) \\ \theta_{1d} = \sin(t) \\ \theta_{2d} = \cos(t). \end{cases} \tag{45}$$

Assume that the disturbances to the system is $d = [20\sin(t) \quad 20\cos(t) \quad 20\cos(t) \quad 3\sin(t) \quad 2\cos(t) \quad 0_{1\times 6}]^{\mathrm{T}}$.

Then, an NNSMC is designed to track the desired trajectory of the omnidirectional mobile manipulator system. Based on the analysis of Section V, a partitioned NN structure is employed to learn the behavior of $\underline{H}(x)$.

First, we design an NN to identify the uncertain term $R^{\mathrm{T}}(q)M(q)R(q)\ddot{q}_{1r}$. This term relates to the exactly known position measurement and the variable $\ddot{q}_{1r}$. Choose the basis function defined in [26] as

$$\phi_{M_j} = \exp\left[-\frac{\|x_M - c_j\|^2}{2\sigma_j^2}\right], \qquad (j = 1, 2, \ldots, 5) \tag{46}$$

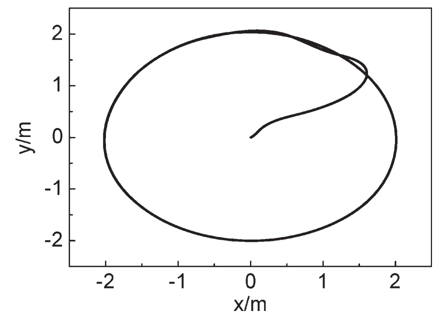

Fig. 7. Trajectory of the mobile platform.

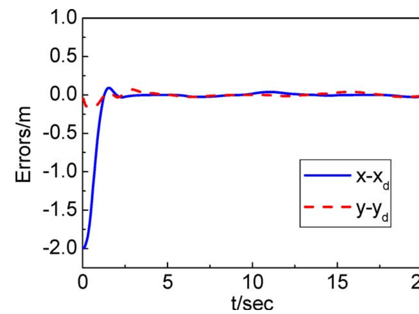

Fig. 8. Position errors of X and Y.

where input state vector $x_M = \begin{bmatrix} q_1 \\ \ddot{q}_{1r} \end{bmatrix}$, the center vector is $[c_1, c_2, \ldots, c_5]^{\mathrm{T}} = [-3, -1.5, 0, 1.5, 3]^{\mathrm{T}}$, and the width vector is $[\sigma_1, \sigma_2, \ldots, \sigma_5]^{\mathrm{T}} = [3, 3, 3, 3, 3]^{\mathrm{T}}$. $\hat{W}_M$ is a 5×11 matrix and all elements are initialized to zero.

Second, we design another NN to approximate the uncertain term $R^{\mathrm{T}}(q)C_1(q,\dot{q})\dot{q}_{1r}$. This term depends on the exactly known position measurement, velocity measurement, and the variable $\dot{q}_{1r}$. The basis function is characterized by the following parameters:

$$\phi_{C_j} = \exp\left[-\frac{\|x_C - c_j\|^2}{2\sigma_j^2}\right], \qquad (j = 1, 2, \ldots, 7) \tag{47}$$

where input state vector $x_C = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ \dot{q}_{1r} \end{bmatrix}$, the center vector is $[c_1, c_2, \ldots, c_7]^{\mathrm{T}} = [-3, -2, -1, 0, 1, 2, 3]^{\mathrm{T}}$, and the width vector is $[\sigma_1, \sigma_2, \ldots, \sigma_7]^{\mathrm{T}} = [2, 2, 2, 2, 2, 2, 2]^{\mathrm{T}}$. $\hat{W}_C$ is a 7×11 matrix, and all elements are initialized to zero.

Now, the estimate of the uncertain term $\underline{H}(x)$ is concretely described by (41). Then, the NNSMC law is modified as

$$\bar{E}(q)u = -Ks + \left(\hat{W}_M^{\mathrm{T}}\phi_M + \hat{W}_C^{\mathrm{T}}\phi_C\right) - K_s\mathrm{sgn}(s). \tag{48}$$

For the convenience of simulations, choose the control parameters as $\Lambda = \mathrm{diag}[2, 2, 2, 2, 2]$, $\alpha_M = 100$, $\mu_M = 0.1$, $\alpha_C = 200$, $\mu_C = 0.1$, $K = \mathrm{diag}[300, 300, 300, 20, 20]$, and $K_s = \mathrm{diag}[10, 10, 10, 5, 5]$, $\rho(t) = 80$, $\gamma(t) = e^{-t}$, respectively.

The proposed NNSMC is applied to the omnidirectional mobile manipulator system and the simulation results are shown in Figs. 7–9. The actual computed trajectory of the platform is shown in Fig. 7. Position tracking errors are shown

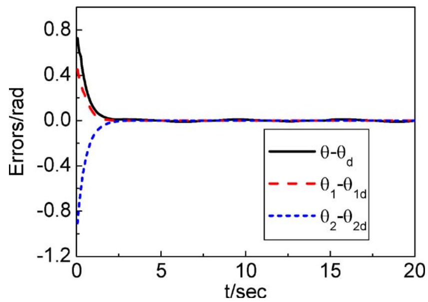

Fig. 9. Position errors of θ, θ_1, and θ_2.

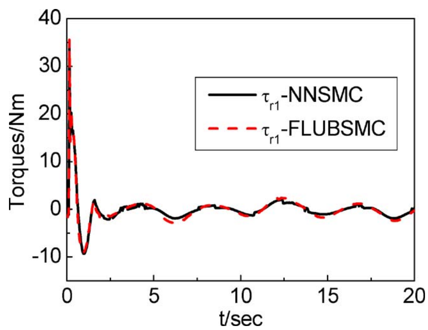

Fig. 10. Output torques of rolling motor.

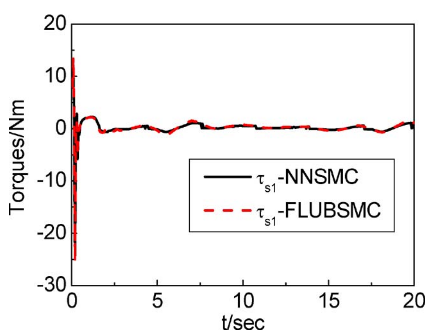

Fig. 11. Output torques of steering motor.

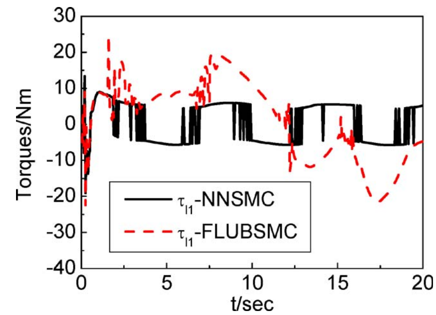

Fig. 12. Comparison of output torques with link 1 motor.

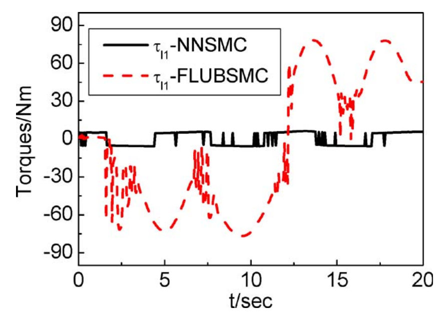

Fig. 13. Comparison of output torques with link 2 motor.

in Figs. 8 and 9. It can be seen that all signals are bounded. From the aforementioned explanations, it is clear that the system converges to the desired trajectory quickly and achieves good tracking performance. Therefore, the proposed control scheme is robust to both the plant uncertainty and time varying external disturbance.

By using the FLUBSMC law and the NNSMC law, the torques exerted on the omnidirectional mobile manipulator system are given in Figs. 10–13. In order to show the output torques of the mobile platform clearly, we only choose the outputs of the wheel module 1 in Figs. 10 and 11, because the outputs of the other two wheel modules are similar to the wheel module 1.

Remark 11: It is well known that the mobile manipulator system has two subsystems: the mobile platform and the mounted manipulator. Usually, these two subsystems have entirely opposite dynamic properties: the mobile platform has large mass and inertia, and responses slowly; the manipulator has small mass and inertia, and responses quickly. The FLUBSMC law considers these two subsystems as an integrated entity and ignores the difference of the dynamic properties. On the contrary, the NNSMC law employs NN to identify the uncertain term, i.e., identifying these two subsystems, respectively. From Figs. 10–13, we can clearly see that the driving torques of the mobile platform in FLUBSMC and NNSMC are similar. However, the driving toques of the manipulator in FLUBSMC are much larger than those in NNSMC. It is not acceptable in practice as large boundedness introduces high noise amplification and high control cost. In addition, much large output driving torques cannot be obtained easily in practical systems. These results adequately illuminate that the FLUBSMC law not only increases the output torques, but also exerts different influence on the mobile platform and the manipulator. Therefore, compared to NNSMC, it is conservative in essence and cannot be applied in practical systems directly. From Figs. 12 and 13, by using NNSMC scheme, the output torques of the motor at joint 1 is reduced up to 80%, and the output torques of the motor at joint 2 is reduced up to 90%.

Remark 12: The values of the center vector and the width vector in the Gaussian functions heavily influence the smoothness of the output surface of RBFNN. How to choose the values is an optimization problem, which is beyond the scope of this paper. By lots of experiments, for this problem, we conclude that the larger the distance between the center elements and the sharper the functions are, the less smooth the output surface is. Therefore, during the NN structure design, these parameters are selected to make the functions smooth so that our proposed control output can also be smooth. In addition, because the SMC in (27) can efficiently compensate the approximation error ε, we can choose finite number basis functions to achieve good tracking performance.

Remark 13: SMC guarantees its robustness in the presence of large disturbance, but the chattering effect is the inherent deficiency. This phenomenon is clearly shown in Figs. 12 and 13. Therefore, if we need smooth control output in practice application, and the tracking effect is satisfying, the NNSMC law (27) can be modified as

$$\bar{E}(q)u = -Ks + \hat{W}^{\mathrm{T}}\phi(x). \tag{49}$$

This is a conventional NN adaptive controller. When the extern disturbances are small or equal to zero, the trajectory tracking performance can be ensured.

Remark 14: NNSMC can take advantage of the strong robustness performance of SMC, and nonlinear mapping properties and the self-learning ability of NN to deal with the unstructured uncertainties. Moreover, NN can alleviate the chattering to some extent and the learning process is online. The best NNSMC parameters, which depend significantly on individual conditions, could be considerably different from these adopted here. In this simulation, the number of hidden layer neurons, the center vector, and the width vector are chosen by trials to make the simulation more perfect. In this paper's research work, although the center vector and the width vector are chosen as equal interval distance, and no optimization method is applied to NN, the system dynamics identification and trajectory tracking performance are effective. However, if there are some standard optimization methods such as gradient descent to be applied in the determination of the weights and center vector of NNSMC, the controller output and the simulation results will be more satisfying.

VI. Conclusion

In this paper, the problems of dynamic model and trajectory tracking are addressed for a redundantly actuated omnidirectional mobile manipulator system with uncertainties and external disturbances. First, FLUBSMC at the dynamic level is developed to solve the trajectory tracking problem. Then, a more effective NNSMC is studied to overcome the inherent deficiency of FLUBSMC, achieving coordinately control the manipulator system effectively. On the one hand, NN in NNSMC is used to identify the system dynamics directly to make the weights matrix structure compact and tuning speed fast. On the other hand, a partitioned NN structure is applied to reduce the computing burden further. The stability and convergence of the whole control system are proved using Lyapunov theory and related lemmas. From the discussion and simulation results, the following conclusions can be reached.

1) NNSMC can take advantage of the strong robustness of SMC and the self-learning and nonlinear mapping properties of NN to deal with both structured and unstructured uncertainties.
2) Taking advantage of SMC and NNC, NNSMC can coordinately control the mobile platform and the mounted manipulator with different dynamics effectively.
3) NN is directly used to identify the uncertain vector of system dynamics in the controller design. No preliminary learning stage is required for the NN weights, the NN learning process is performed online, and its uniform ultimate boundedness is guaranteed by the stability analysis.
4) Compared to FLUBSMC, NNSMC requires no information of the mathematical model and/or the parameterization of the mechanical dynamics, and can overcome the defects of FLUBSMC, which may cause large output torques, particularly for the manipulator.
5) Although the overall structure of NNSMC looks complicated, it is very suitable for real-time application after adopting the partitioned structure, because the partitioned structure developed in this paper simplifies the NN design process and accelerates the tuning speed.

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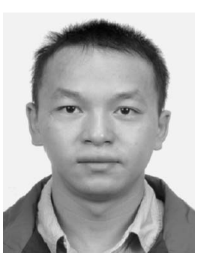

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