

# Full-Body Compliant Human-Humanoid Interaction: Balancing in the Presence of Unknown External Forces

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**Abstract**—This paper proposes an effective framework of human-humanoid robot physical interaction. Its key component is a new control technique for full-body balancing in the presence of external forces, which is presented and then validated empirically. We have adopted an integrated system approach to develop humanoid robots. Herein, we describe the importance of replicating human-like capabilities and responses during human-robot interaction in this context. Our balancing controller provides gravity compensation, making the robot passive and thereby facilitating safe physical interactions. The method operates by setting an appropriate ground reaction force and transforming these forces into full-body joint torques. It handles an arbitrary number of force interaction points on the robot. It does not require force measurement at interested contact points. It requires neither inverse kinematics nor inverse dynamics. It can adapt to uneven ground surfaces. It operates as a force control process and can therefore accommodate simultaneous control processes using force, velocity or position based control. Forces are distributed over supporting contact points in an optimal manner. Joint redundancy is resolved by damping injection in the context of passivity. We present various force interaction experiments using our full-sized bipedal humanoid platform, in-

cluding compliant balance, even when affected by unknown external forces, which demonstrates the effectiveness of the method.

**Index Terms**—Human-humanoid interaction, Full-body motion control, Biped robot, Balance, Passivity, Compliance, Redundancy, Force control.

## I. INTRODUCTION

**A**DAPTATION of the human environment to suit human sensory and motor capabilities necessitates artificial agents with analogous capabilities that can usefully coexist in our living and working spaces. Humanoid agents can take further advantage of the multipurpose adaptability of the human form itself for performance of a broad range of activities. Such activities might include cooperating with humans themselves, so that humanoid robots not only benefit from suitability of the human form in an individual context but also through its impact on the relationship between humans and robots. For example, the appearance of a human induces an expectation of human-like responses [1][2][3]. Aside from its scientific benefits [4], the most appropriate form for a robot acting as a partner for a human is a humanoid of human proportions [5]. Having similar physical capabilities, a human-like presence and the use of familiar social cues make cooperation natural because it is intuitively easier to relate to the following: experience of interaction with other humans trains people

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This paper has supplementary downloadable material available at <http://ieeexplore.ieee.org>, provided by the authors. This is a single MPEG4 format movie which shows the empirical results. The movie is 10 MB in size. For the details, see “readme.txt” in the same file directory.

to accommodate basic factors such as their partner's balance, range of motion, working space, physical intentions, *etc* [6][7]. Human imitation is therefore of increasing importance: human-like behavior, emotionally and aesthetically pleasing action, and expressive motion are fundamental. In addition, according to the logic that a tool inherently enhances the way we think, humanoid robots increase our understanding of human behavior, and consequently, our intellectual abilities [8][9].

The persistent and increasingly difficult problem for the design of appropriate intelligent systems is therefore *how* to communicate and cooperate with people. Physical interaction is a familiar and reliable method for most humans; it encompasses both physical cooperation and communication through movement. This motivates an exploration of physical interactions between humans and robots. Physical interaction can encourage people to perceive and respond to a robot as a human-like being rather than as a mechanical entity [10], thereby prompting ways of making robot-human interactions more natural.

For the discussions included in this paper, we regard such activities as high-level motion planning that might be conceived of as sub-systems in an integrated view of a humanoid robot system [11][9]. We present rather, a motion sub-system for balancing that exemplifies the interaction and compatibility espoused by the integrated model. The conceptual organization is illustrated in Fig. 1.

### A. Balancing

The need for a balancing system arises from the requirement that the humanoid be capable of performing on a similar level to that of humans. Human-sized robot bipeds are constrained by severe restrictions on force interactions because of the requirements of bipedal

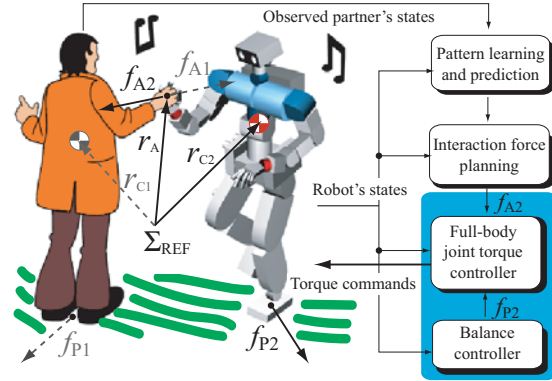


Fig. 1. A possible control flow in force interaction with balance. This paper describes the shaded region: how to generate full-body joint torques for given interaction forces:  $f_{A2}$  and  $f_{P2}$ . The gross motion of the robot  $r_{C2}$  reflects both the control action (joint torque) and the interaction force  $f_{A1}$  applied by the human.

balance. To facilitate continuous interaction with its environment and with people, a bipedal robot must be able to maintain balance in a manner that accommodates external forces. We are interested in a force-controlled balancing system, reasoning that (i) force control is a superset containing velocity-based and position-based control that enables adoption of any of these strategies, thereby allowing for the richest level of physical communication, and (ii) force interaction is a familiar and reliable method for physical communication and cooperation with humans. In addition, the force-controlled system renders our system compliant: interaction with humans is both feasible and safe.

The activity shown in Fig. 1 illustrates the control flow during a force interaction with a human: dancing. In such a force interaction task, two levels are distinct within the control process:

- Designing the interaction forces
- Generating the joint torques

The first process computes the required interaction forces  $f_{A2}$  in accordance with the given task while consid-

Fig. 2. New torque-controllable biped humanoid robot, *CB*, developed by SARCOS

ering the self-balancing force  $f_{P2}$ , which acts on the environment. The latter determines the joint torques for the full body that are necessary to produce the interaction forces. This paper particularly describes issues related to the latter control process, which is indicated as the shaded region in Fig. 1. Specifically, this paper provides a theoretical foundation for bipedal balancing with force interactions. We address the following issues that arise during force interactions between humans and bipedal humanoid robots with multiple degrees of freedom (DoFs):

- (I1) How to maintain self-balance
- (I2) How to cope with external forces
- (I3) How to handle redundant DoF

Historically, issues (I1) and (I2) have mainly been pursued in the field of bipedal walking robots. Several balancing compensators have been developed for position-controlled bipedal humanoids. Some position-based balance compensators have been successful [12][13][14][15][16].

However, position-based controllers cannot adapt quickly to sudden environmental changes, especially to unknown external forces. That insufficiency is an important disadvantage for position-based controllers in terms of *force interaction*. First, the methods require force measurement. Secondly, time delays caused by re-calculation of the joint trajectories are unavoidable. Thirdly, and most problematically, it is difficult to assign appropriate weights in the presence of unknown disturbances. See Section V for the details.

In our prior study [17], to avoid the complex motion

planning that is required for multi-DoF bipedal walking, we applied position-based compliance control so that a robot adapts compliantly to external forces by using a force sensor. Such methods however, can deal only with forces via *virtual* compliance; for that reason, time delays are, in principle, unavoidable. Moreover, the controller requires force sensors at *every* expected contact point. Without them, the robot cannot adapt to unknown external forces. Although it is possible to install force sensors over the whole body, this imparts a heavy computational burden because of the number of contact Jacobians. We therefore have inferred that the best way to treat unknown disturbances and external forces is to transform desired contact (interaction) forces into joint torques directly, even though controlling these joint torques precisely is technically challenging.

### B. Contribution and organization of this paper

The contribution of this paper is the presentation of a simple and practical method of force interaction control addressing issues (I1)–(I3). Specifically, we propose a passivity-based, full-body, contact force controller and apply it to force interaction between humans and dynamically balancing bipedal humanoid robots. Unlike previous methods, the controller can accommodate an arbitrary number of contact forces; it requires no force sensors at the contact points. We experimentally evaluate the controller on our new biped humanoid platform (Fig. 2) developed by SARCOS ([www.sarcos.com](http://www.sarcos.com)). This robot has 50 DoF and torque controllability, which we have been pursuing during hardware and software development over the past several years [4][5]. We highlight our work with various demonstrations of simultaneous force interaction with humans, and bipedal balance maintenance. In particular, we demonstrate that the robot can adapt to *unknown* external forces applied

to *arbitrary* contact points *without sensing* the contact forces.

The paper is organized as follows. Section II describes our passivity-based full-body force control framework [18], where we specifically examine how to compute the commanded joint torque for a given desired contact force. The proposed method is novel with respect to two major points: (1) it is applicable to redundant DoF humanoid robots, (2) it is applicable to multiple contact points, even when they form a closed-loop chain. The former novelty is attributable to passivity-based redundancy resolution, which requires neither a contact-force measurement, nor inverse kinematics/dynamics. The latter novelty comes from an optimization of contact forces in terms of the  $L_2$  norm of the contact forces, avoiding unnecessary internal forces between the contact points. Gravity compensation, which is widely used in position control of manipulator, is highlighted in the context of the full-body-contact force control framework [19]. Gravity compensation is extremely useful for physical interaction between humans and humanoids because the robot's movement can thereby be controlled using very small external forces. Specific methods for force interaction with balance are described in Section III, where simple controllers and simulations are proposed. Section IV describes various balanced force-interaction experiments on our humanoid platform. The experiments include balancing with unknown external forces, adaptation to different contact points. The experimental results are supplemented with a video included with this paper, which is available at <http://ieeexplore.ieee.org>. Section V presents a comparison of the proposed method with other related methods. Finally, Section VI summarizes the results from human-robot interaction perspective and presents discussion of future work.

## II. PASSIVITY-BASED FULL-BODY FORCE CONTROL FRAMEWORK

Bipedal humanoid robots must maintain their balance through contact forces while interacting with the environment. For the study described in this paper, we assume that the central issue for force interaction between humans and bipedal humanoid robots is the control of contact forces. This section presents a new force control framework for humanoids with multiple contact points and multiple force interaction points [18]. First, Section II-A derives *ground applied force* (see the definition therein) from the full nonlinear dynamics of the robot. Then, Section II-B shows how to transform the desired ground applied force to joint torques without using inverse kinematics or inverse dynamics. The key technology is gravity-compensation and passivity-based redundancy resolution. Also, Section II-C explains how to distribute the ground applied force to multiple contact points. Finally, Section II-D summarizes the preceding descriptions and provides the control procedure.

### A. Dynamics of multi-DoF humanoid and the ground applied force

Consider a multi-DoF humanoid robot contact with the ground, as shown in Fig. 3. Let  $r \in \mathcal{R}^3$  in  $\Sigma_W$  be some translational position coordinate (e.g. base position) and  $q \in \mathcal{R}^n$  be the joint angles *and* attitude of the base. Using the generalized coordinates  $\hat{q} = [r, q]^T \in \mathcal{R}^{3+n}$  the exact nonlinear dynamics of humanoids with the constraint can be derived using a standard Lagrangian formulation with constraints [20]:

$$\hat{I}(\hat{q})\ddot{\hat{q}} + \hat{C}(\hat{q}, \dot{\hat{q}}) + \hat{G}(\hat{q}) = u + E(\hat{q})^T \lambda, \quad (1)$$

$$E(q)\dot{\hat{q}} = 0, \quad (2)$$

where  $\hat{I}(\hat{q}) \in \mathcal{R}^{(3+n) \times (3+n)}$  is the inertia matrix,  $\hat{C}(\hat{q}, \dot{\hat{q}}) \in \mathcal{R}^{3+n}$  respectively represent the centrifugal

and Coriolis terms,  $\hat{G}(\hat{q}) \in \mathcal{R}^{3+n}$  is the gravity term,  $u = [0, \tau]^T \in \mathcal{R}^{3+n}$  is the generalized forces,  $E(q) \in \mathcal{R}^{3 \times (3+n)}$  is the constraint Jacobian associated with the contact, and  $\lambda \in \mathcal{R}^3$  is the constraint force. First, we assume that at least one point of the sole is contacting with the ground. For simplicity, we specifically examine the single holonomic constraint. We will discuss the multiple contact case in Section II-C.

Our important control issue described in this paper is balancing. Therefore, it is convenient to address the center of mass (CoM) coordinate. Let  $r_C = [x_C, y_C, z_C]^T \in \mathcal{R}^3$  be the position vector of CoM in the world coordinate frame. Also let  $\Sigma_W$  and  $r_P = [x_P, y_P, z_P]^T \in \mathcal{R}^3$  be the position vector from CoM to the contact point (see Fig. 3). The contact point does not move on the ground surface. For convenience, we introduce a *gross applied force*, or *ground applied force* (GAF)  $f_P = [f_{xP}, f_{yP}, f_{zP}]^T$ , defined as  $f_P := -f_R$ , where  $f_R$  is the ground reaction force (GRF). Herein, GAF represents the force that the robot applies to the environment through the contact point. The control objective of the contact force control is to bring  $f_P$  to

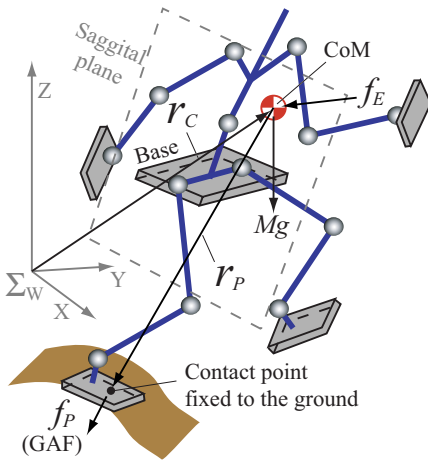


Fig. 3. Coordinate definition

some desired value  $\bar{f}_P$ .

By replacing the generalized coordinates  $\hat{q}$  with  $\hat{q}_C = [r_C, q]^T \in \mathcal{R}^{3+n}$  (1) is converted to the decoupled dynamics:

$$\underbrace{\begin{bmatrix} M & 0 \\ 0 & I(q) \end{bmatrix}}_{\hat{I}} \underbrace{\begin{bmatrix} \ddot{r}_C \\ \ddot{q} \end{bmatrix}}_{\ddot{\hat{q}}_C} + \underbrace{\begin{bmatrix} 0 \\ C(q, \dot{q}) \end{bmatrix}}_{\hat{C}} + \underbrace{\begin{bmatrix} -Mg \\ 0 \end{bmatrix}}_{\hat{G}} = \underbrace{\begin{bmatrix} 0 \\ \tau \end{bmatrix}}_u - \underbrace{\begin{bmatrix} id \\ J_P(q)^T \end{bmatrix}}_{\hat{E}(q)^T} f_P, \quad (3)$$

$$\hat{E}(q)\dot{\hat{q}}_C = 0, \quad (4)$$

wherein  $M = \text{diag}(m, m, m) \in \mathcal{R}^{3 \times 3}$  is the mass matrix ( $m$  is the total mass),  $I(q) \in \mathcal{R}^{n \times n}$  is the inertia matrix,  $C(q, \dot{q}) \in \mathcal{R}^n$  is the centrifugal and Coriolis term,  $id \in \mathcal{R}^{3 \times 3}$  denotes the identity matrix, and  $J_P(q) \in \mathcal{R}^{3 \times n}$  is the Jacobian from CoM to the contact point (derivative of  $r_P$  with respect to  $q$ ). The Jacobian  $J_P(q)$  can be derived easily using a standard kinematics computation (See [21] Section 3.1.3 for example). Note that (4) is the time derivative of the holonomic constraint  $r_C + r_P = \text{const.}$  (see Fig. 3) The upper part of (3) portrays simple linear dynamics.

$$M\ddot{r}_C = Mg - f_P \quad (5)$$

Consequently, we can control  $\ddot{r}_C$  by  $f_P$  as we will show in Section III-A.

Differentiating (4) yields

$$\hat{E}(q)\ddot{\hat{q}}_C + \gamma(q, \dot{q}) = 0, \quad (6)$$

where  $\gamma(q, \dot{q}) = \frac{\partial}{\partial q_C}(\hat{E}\dot{\hat{q}}_C)\dot{\hat{q}}_C$ . Solving (3) and (6) for  $\ddot{\hat{q}}_C$  yields

$$f_P = (\hat{E}\hat{I}^{-1}\hat{E}^T)^{-1} \{ \gamma + \hat{E}\hat{I}^{-1}(u - \hat{C} - \hat{G}) \}. \quad (7)$$

Therefore, for some desired GAF  $\bar{f}_P$  we can calculate the corresponding joint torques  $\tau$  by inverting (7). The torque precisely achieves  $f_P = \bar{f}_P$  as long as the above model is correct (model parameters and sensor measurements). We have considered only one contact point. The single contact point is exactly the center of pressure if no other contact points exist. In this case, the robot is *under-actuated* because the robot cannot generate the moment around the contact point, and the first three entries in  $\tau$  become zero. Inverse dynamics are useful for this case. See [22] for more details of this type of exact GAF control. Usually, inverse dynamics control becomes impractical for robots with many DoFs. In addition, even if the inverse-dynamics have been calculated successfully, we must yet solve the redundancy problem. When there are multiple contact points, it remains unclear how to resolve the contact forces because many possible solutions exist, which is well known as an ill-posedness problem in the field of robotic grasp (see for example, [23][24]). The following two subsections specifically address these issues.

#### B. Calculating commanded joint torque with gravity compensation

The problem here is how to transform the desired GAF into joint torques “effectively.” We intend to avoid exact inverse dynamics calculation because it requires an exact model including the inertia matrix, which becomes impractical for large-DoF systems. Second-ordered velocity terms are especially problematic because the velocity measurement is usually noisy. On the other hand, the gravity term is readily identifiable and a dominant nonlinear term in full dynamics. For that reason, G-comp plays a central role in contact force control.

This point is clarified if we assume that the motion is quasi-static. We assume that the robot foot fully contacts

with the ground. Therefore, the first three entries in  $\tau$  are available, that is, the robot is *fully actuated*. In this case,  $\dot{q} \approx 0$ , therefore  $\gamma \approx 0$ ,  $\hat{C} \approx 0$  hold. For some new force input  $f_u = [f_{ux}, f_{uy}, f_{uz}]^T \in \mathcal{R}^3$ , it is straightforward that the joint torque

$$\tau = J_P^T(f_v + Mg), \quad (8)$$

$$f_v = f_u + (J_P I^{-1} J_P^T)^{-1} M^{-1} f_u \quad (9)$$

renders the closed-loop system satisfying

$$f_P \approx f_u + Mg. \quad (10)$$

See Appendix A for the proof. This is the main idea of G-comp for controlling contact forces. Consequently, we obtain  $M\ddot{r}_C \approx -f_u$ . That is, we can control the CoM movements using  $f_u$ .

In (9), we used the inertial matrix  $I(q)$ , which might be difficult to obtain in some applications. A simpler formula is available by setting

$$\tau = J_P^T(f_u + Mg), \quad (11)$$

that is,  $f_v := f_u$ , instead of (8) and (9). This approximation is valid whenever  $f_u = 0$  (G-comp only) or  $f_u$  is given by some feedback law. We assume this is true for all range of the motion. If the posture is singular this coefficient becomes very large and (10) does not hold unless  $f_u = 0$ .

On the other hand, the static force control for *redundant* manipulators is well known to cause *internal motions* [23]. Some dynamic effects might arise as the internal motions if  $\gamma$  and  $C$  are not small. We must therefore compensate for the internal motion somehow, while simultaneously achieving a desired GAF. One simple way to accomplish these requirements is to assign a simple nonlinear term that compensates  $\gamma$  and  $C$ . Substituting (8) and  $\frac{\partial}{\partial q}(J_P \dot{q})\dot{q} = -J_P \ddot{q}$  into (7), we

can derive a modified version of (8):

$$\tau = J_P^T (f_v + Mg) + \zeta(q, \dot{q}), \quad (12)$$

which yields convergence of the GAF:

$$f_P \rightarrow f_u + Mg \quad (t \rightarrow \infty) \quad (13)$$

provided  $\zeta(q, \dot{q})$  is designed so that

$$I(q)\ddot{q} + C(q, \dot{q}) - \zeta(q, \dot{q}) = 0 \quad (14)$$

is stable. The proof is available in Appendix B. A conservative means to achieve this is simply to set  $\zeta$  as joint-wise damping:

$$\zeta = -D\dot{q}, \quad (15)$$

with a constant diagonal matrix  $D > 0$ . Although we should investigate the structure of the left-hand side of (14), the controller (12) itself can be made very simple, as in this example. See [25][26] for a damping injection strategy for controlling redundant manipulators, where the authors analyzed the convergence of the internal motions and the position error of the end-effector.

The closed loop system, (3) + (12), becomes

$$M\ddot{r}_C = -f_u - f_\zeta + f_E, \quad (16)$$

$$I(q)\ddot{q} + C(q, \dot{q}) = J_P(q)^T f_v + \zeta(q, \dot{q}) \quad (17)$$

where  $f_\zeta$  is the dissipation term associated with the third term of (12). As a result, we obtain the *passivity* [27][28] relationship

$$\begin{aligned} H(q, \dot{q}) &:= \frac{1}{2} \dot{r}_C^T M \dot{r}_C + \frac{1}{2} \dot{q}^T I \dot{q} = \frac{1}{2} \dot{q}^T (J_P^T M J_P + I) \dot{q} \\ &\leq \int f_v^T J_P(q) \dot{q} dt + \int f_E^T J_P(q) \dot{q} dt. \end{aligned} \quad (18)$$

Consequently, we have the following properties:

- CoM tracking is possible with the addition of simple controllers using  $f_u$  as in Section III-A.
- Without external forces  $f_E$ , the CoM is subject to the constant velocity movement,

- Without user forces  $f_u$ , the CoM and *all* the joint movements can be driven externally by  $f_E$ ,

The third one is particularly useful for safe force interaction. For example, humans can escort the robot in an arbitrary direction by pulling its hand. Alternatively, humans can teach dancing movements by holding and moving arbitrary segments of the body if balancing is coordinated automatically by the controller described in Section III-A.

Although G-comp is widely used for position control of manipulators [27], its importance in force interaction has not been well discussed. Realization of G-comp in multi-DoF humanoid robots considering multiple contact points and balancing is not straightforward; no solution has yet been reported. The optimal contact force distribution described in the subsequent section enables us to apply G-comp to various humanoid and mobile robots with various contact situations (e.g., bipedal or quadrupedal, single or double support phase).

### C. Optimal load distribution to multiple contact points

The GAF controller derived in (12) is insufficient for humanoid robots interacting with the environment at multiple contact points. However, it is not easy to solve for the required joint torques because of multiple kinematic loops [24]. For example, if the robot is in the double support phase or quadruped phase, there are many possible combinations of the commanded joint torques which result in the same desired GAF. Our solution is to compute the force closure first explicitly, then transform them to the joint torques using a passivity-based framework, as in the previous section. Therefore, the problem in this section is how to distribute the desired GAF to multiple contact points. To treat force control in a task space in a simple but uniform manner, we first solely consider the translational forces of the

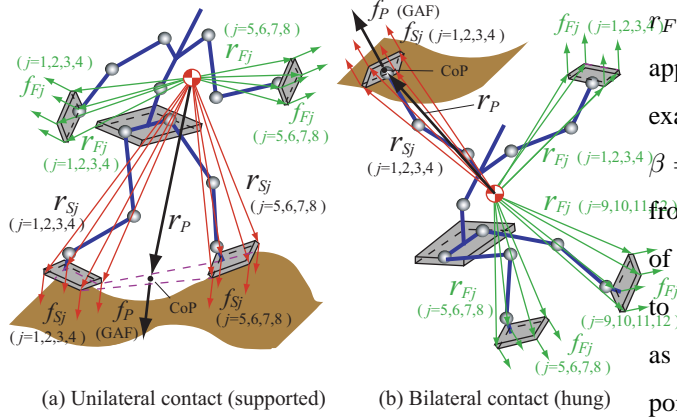


Fig. 4. Definition of GAF, contact points, and forces: each ground contact point  $r_{Sj}$  is assigned the contact force  $f_{Sj}$ , and each non-ground contact point  $r_{Fj}$  is assigned with the interaction force  $f_{Fj}$ . The contact forces  $f_{Sj}$  are determined using a desired GAF  $f_P$  and  $f_{Fj}$ . The CoP  $r_P$  always lies within the supporting convex hull of  $r_{Sj}$ .

active force application points. We do not ignore moment and orientation: they can be controlled by combining multiple translational forces, as demonstrated in Section III-C.

Figure 4 illustrates some contact situations with the force application points, together with the GAF and *center of pressure* (CoP). All of the related position vectors run from the CoM. Assuming that we are interested in a total of  $\alpha$  contact points defined as  $r_S = [r_{S1}, r_{S2}, \dots, r_{S\alpha}]^T \in \mathcal{R}^{3\alpha}$ , where each vector component is  $r_{Si} = [x_{Si}, y_{Si}, z_{Si}]^T \in \mathcal{R}^3$  for each index  $i$ , the associated applied contact forces are defined as  $f_S = [f_{S1}, f_{S2}, \dots, f_{S\alpha}]^T \in \mathcal{R}^{3\alpha}$ , where each vector component is  $f_{Si} = [x_{Si}, y_{Si}, z_{Si}]^T \in \mathcal{R}^3$ . In this description,  $f_S$  is the force that the robot applies to the contact points (care is needed in the force direction). Of course, CoP  $r_P$  must lie within the supporting convex hull of  $r_{Sj}$ . We are also interested in the total of  $\beta$  non-ground contact points or interaction points as

applied forces  $f_F = [f_{F1}, f_{F2}, \dots, f_{F\beta}]^T \in \mathcal{R}^{3\beta}$  and their associated  $r_F = [r_{F1}, r_{F2}, \dots, r_{F\beta}]^T \in \mathcal{R}^{3\beta}$ . For example,  $\alpha = 8$  and  $\beta = 8$  in Fig. 4(a), but  $\alpha = 4$  and  $\beta = 12$  in Fig. 4(b). The discussion here is independent from robot models, and the number and the location of the contact points are arbitrary; they are applicable to one-legged, bipedal, and quadrupedal robots, as long as CoP is definable. It should be noted that the contact points do not necessarily lie in the same plane, as shown in Fig. 4.

Our strategy is distributing the GAF to each ground contact point *through* CoP. From the definition, CoP can be expressed as

$$x_P = \frac{\sum_{j=1}^{\alpha} x_{Sj} f_{zSj}}{\sum_{j=1}^{\alpha} f_{zSj}}, \quad y_P = \frac{\sum_{j=1}^{\alpha} y_{Sj} f_{zSj}}{\sum_{j=1}^{\alpha} f_{zSj}} \quad (19)$$

while the GAF is the sum of the respective contact forces:

$$f_P = \sum_{j=1}^{\alpha} f_{Sj}. \quad (20)$$

In simple notation, these two equations combine to

$$\begin{bmatrix} x_P \\ y_P \\ 1 \end{bmatrix} f_{zP} = \underbrace{\begin{bmatrix} x_{S1} & x_{S2} & \dots & x_{S\alpha} \\ y_{S1} & y_{S2} & \dots & y_{S\alpha} \\ 1 & 1 & \dots & 1 \end{bmatrix}}_{A_z \in \mathcal{R}^{3 \times \alpha}} \begin{bmatrix} f_{zS1} \\ f_{zS2} \\ \dots \\ f_{zS\alpha} \end{bmatrix}, \quad (21)$$

where  $A_z$  represents a *contact-force distribution matrix*.

Given a desired normal GAF  $\bar{f}_{zP}$  and CoP  $\bar{r}_P$  (see Section III-A for the balancing control case), we can calculate the corresponding desired normal contact forces  $\bar{f}_{zS}$ . Specifically, we propose an optimal contact force distribution calculated as

$$\begin{bmatrix} \bar{f}_{zS1} \\ \bar{f}_{zS2} \\ \dots \\ \bar{f}_{zS\alpha} \end{bmatrix} = A_z^{\#} \begin{bmatrix} \bar{x}_P \\ \bar{y}_P \\ 1 \end{bmatrix} \bar{f}_{zP}, \quad (22)$$

with  $A_z^{\#} = A_z^T (A_z A_z^T)^{-1}$ . This solution is optimal in the sense that it minimizes the  $L_2$  norm of the total



contact forces. Because of the convexity of the contact points, the resultant normal contact force  $\bar{f}_{zS}$  cannot be positive (therefore, the unilateral constraint condition is met) as long as  $\bar{f}_{zP}$  is negative definite and the desired CoP  $\bar{r}_P$  is set within the supporting convex hull (they are obvious restrictions).<sup>1</sup>

Similarly, we use

$$\begin{bmatrix} x_P \\ 1 \end{bmatrix} f_{xP} = \underbrace{\begin{bmatrix} x_{S1} & x_{S2} & \cdots & x_{S\alpha} \\ 1 & 1 & \cdots & 1 \end{bmatrix}}_{A_x \in \mathcal{R}^{2 \times \alpha}} \begin{bmatrix} f_{xS1} \\ f_{xS2} \\ \cdots \\ f_{xS\alpha} \end{bmatrix}, \quad (23)$$

$$\begin{bmatrix} y_P \\ 1 \end{bmatrix} f_{yP} = \underbrace{\begin{bmatrix} y_{S1} & y_{S2} & \cdots & y_{S\alpha} \\ 1 & 1 & \cdots & 1 \end{bmatrix}}_{A_y \in \mathcal{R}^{2 \times \alpha}} \begin{bmatrix} f_{yS1} \\ f_{yS2} \\ \cdots \\ f_{yS\alpha} \end{bmatrix} \quad (24)$$

to distribute the horizontal contact forces *in proportion* to the normal contact forces,

$$\begin{bmatrix} \bar{f}_{xS1} \\ \bar{f}_{xS2} \\ \cdots \\ \bar{f}_{xS\alpha} \end{bmatrix} = A_x^\# \begin{bmatrix} \bar{x}_P \\ 1 \end{bmatrix} \bar{f}_{xP}, \quad (25)$$

$$\begin{bmatrix} \bar{f}_{yS1} \\ \bar{f}_{yS2} \\ \cdots \\ \bar{f}_{yS\alpha} \end{bmatrix} = A_y^\# \begin{bmatrix} \bar{y}_P \\ 1 \end{bmatrix} \bar{f}_{yP}, \quad (26)$$

so that the contact points with larger normal contact force receive more ground friction.

As a consequence, a desired GAF might be distributed over the desired contact forces using a simple matrix

<sup>1</sup>As a trivial extension, we can introduce a weight into the optimal criteria as  $\bar{f}_{zS}^T W \bar{f}_{zS}$ , where  $W \in \mathcal{R}^{\alpha \times \alpha}$  is the weighting matrix. The optimal force closure is given as (22), but with  $A_z^\# = W^{-1} A_z^T (A_z W^{-1} A_z^T)^{-1}$ . The weighting might be useful when we need to consider the friction cone of the ground explicitly, or when we need to reduce the load to some limbs because of breakage of some joints.

operation

$$\bar{f}_S = A^\# \begin{bmatrix} \bar{x}_P \bar{f}_{xP} \\ \bar{f}_{xP} \\ \hline \bar{y}_P \bar{f}_{yP} \\ \bar{f}_{yP} \\ \hline \bar{x}_P \bar{f}_{zP} \\ \bar{y}_P \bar{f}_{zP} \\ \bar{f}_{zP} \end{bmatrix}, \quad (27)$$

wherein  $A = \text{diag}(A_x, A_y, A_z)$  is the total contact force distribution matrix.

#### D. Summary

Finally, the commanded joint torques are obtainable as

$$\tau = J_S(q)^T \bar{f}_S + \zeta(q, \dot{q}), \quad (28)$$

where  $J_S(q) \in \mathcal{R}^{3\alpha \times n}$  represents the contact Jacobian from the CoM to the supporting contact points (derivative of  $r_S$  with respect to  $q$ ), where G-comp  $\bar{f}_P = f_v + Mg$  must be included in the calculation of  $\bar{f}_S$ .

The overall control procedure can be summarized as follows:

- (P1) Calculate all forward kinematics.
- (P2) Set the user task force  $f_u$  for given task objectives.
- (P3) Calculate the desired GAF  $\bar{f}_P$  by  $\bar{f}_P = f_u + Mg$ .
- (P4) Determine the contact points  $r_{Sj}$  ( $j \in \{1, 2, \dots, \alpha\}$ ) and active joints  $q_i$  ( $i \in \{1, 2, \dots, n\}$ ); then remove row and column vectors according to inactive joints or contacts.
- (P5) Calculate the desired contact forces  $\bar{f}_S$  by (27).
- (P6) Transform the contact forces  $\bar{f}_S$  to torque commands  $\tau$  for the active joints by (28).

Some notes are in order:

- The attitude of the base, obtained using the gyro sensor, is also included in  $q$ , hence in the Jacobians.
- The proposed method requires no contact force measurement, but it does require kinematic information of the supporting contact points, which are *preset* by the user.
- The contact forces are computed according to the current shape of the supporting convex hull, where respective pre-set contact points do not necessarily lie on the same plane.
- The ground contact points should be defined so that the convex hull includes the *actual* CoP in it.
- Even if some contact point loses its contact with the ground, the controller still *assumes* that its contacts are active. It therefore distributes the contact forces to all the preset contact points.
- We can remove the *row* vector from  $J_S(q)$  if some contact point loses contact with the environment (requires tactile information). However, this is not always a good solution.
- Similarly, if some joints are specified with trajectories or hit the range of motion, we can simply remove the corresponding *column* vector from  $J_S(q)$ .
- We do not consider that defective case in which the task space control objects are too numerous compared to the number of joints. Clearly, we cannot achieve the tasks simultaneously in this case.
- The control torques generated by (28) differ from those of (12) because it uses the null space of  $J_P$  by specifying the contact forces.
- The means to achieve the desired joint torques depends on the hardware (see Section IV-A).

At this point, we have not discussed the balancing tasks or force interaction tasks. We have merely de-

scribed convergence of the ground reaction force and internal motions under gravity compensation control. That is, we have not yet discussed how to give the user task force  $f_u$  ((P2) in the above procedure). These issues are described in the next section.

### III. FORCE INTERACTION CONTROL WITH BALANCE

This section explains simple controllers for force interaction with balancing based on linear feedback. Section III-A shows a simple CoP-constrained balancer. Section III-B shows a simple method for controlling the position and force of hands or swinging legs for force-interaction tasks. Section III-C describes how to generate rotational moments and achieve desired orientations of the base, hands, *etc.*, and how to compensate yaw moment.

#### A. Balance control

The balance control discussed in this section is asymptotic stabilization of the ground projection of CoM to some target position in the supporting convex hull. A simple feedback law can be used:

$$f_u = -K_{PC}(r_C - \bar{r}_C) - K_{DC}(\dot{r}_C - \dot{\bar{r}}_C), \quad (29)$$

with some desired CoM position/velocity  $\bar{r}_C$ ,  $\dot{\bar{r}}_C$  and the task-space PD-gains  $K_{PC}, K_{DC} > 0$ . Of course, we can incorporate feed-forward term  $M\ddot{\bar{r}}_C$  in it. A lower bound is set for  $\bar{f}_{uz}$  to prevent the robot from losing all contact with the ground.

Recall that CoP is, by definition, the representative force application point because it is the weighted sum of the translational ground contact forces as in (20). On the other hand, the *Zero Moment Point (ZMP)* is defined as a point on the ground at which total moments balance (see [29] and references therein), which is denoted as  $r'_P = [x'_P, y'_P, z_P]$ . Since we do not want to calculate

a complex nonlinear term, we use a “simple version” of ZMP. Specifically, we assume the rate change of the angular momentum around CoM is negligible. In this case, ZMP can be written as:

$$\begin{aligned} x'_P &= \frac{z_P m \ddot{x}_C}{m(\ddot{z}_C + g)} = \frac{z_P f_{xP}}{f_{zP}}, \\ y'_P &= \frac{z_P m \ddot{y}_C}{m(\ddot{y}_C + g)} = \frac{z_P f_{yP}}{f_{zP}}. \end{aligned} \quad (30)$$

Therefore, with our G-comp strategy (11) and user force input (29), the desired ZMP  $\bar{r}'_P$  is set to:

$$\bar{x}'_P = \frac{z_P f_{ux}}{mg + f_{uz}}, \quad \bar{y}'_P = \frac{z_P f_{uy}}{mg + f_{uz}}. \quad (31)$$

In those equations,  $z_P = -z_C$ . It should be noted that this ZMP can exit from the supporting region depending on the desired GAF. We restrict ZMP into the supporting convex hull to incorporate the balancing controller with our contact force distribution about CoP (27). We can modify the horizontal GAF ( $f_{ux}$  and  $f_{uy}$ ) and/or normal GAF ( $f_{uz}$ ) to achieve this. In this case, the modified ZMP coincides with CoP because the moment balances at CoP. We use this as the desired CoP for balancing.<sup>2</sup> That is, we set

$$\bar{r}_P = \bar{r}'_P. \quad (32)$$

Finally, the modified GAF  $\bar{f}_P$ , in addition to the desired CoP  $\bar{r}_P$ , are then used for determining the desired contact force closure in (P5) in Section II-D. We call this restricted but simple balance controller a “CoP-constrained balancer.”

The balancing performance is demonstrated in Section IV.

<sup>2</sup>By limiting ZMP into the supporting region, the desired GAF might become small, which might restrict the balancing ability. We should not restrict ZMP if we want to enhance the balancing ability. However, we must revert to full nonlinear dynamics (inverse of (7)) to achieve the desired ZMP. See [22] for example. A simple controller that does not necessitate solving full nonlinear dynamics is under development.

An intuitive interpretation is available by considering a special case:  $\bar{f}_{xP} = \bar{f}_{yP} = 0$  (G-comp only). In this case, the desired ZMP (or CoP) coincides with the ground projection of CoM,  $\bar{x}_P = \bar{y}_P = 0$ , if they are inside the supporting region. Since the gravity is compensated the CoM exhibits constant velocity movement.

### B. Reaching and force-interaction tasks

Using our contact force framework, the motion command for supporting leg joints or swinging leg joints can be handled in a uniform manner. Contact forces at the interested contact points are used for task space force control, and also for position tracking. For example, we can use the interaction forces  $f_{Fj}$  for controlling the position  $r_{Fj}$  of the swinging legs or hands using a simple PD feedback law

$$\begin{aligned} \bar{f}_{Fj} &= -K_{PF}(r_{Fj}^W - \bar{r}_{Fj}^W) - K_{DF}(\dot{r}_{Fj}^W - \dot{\bar{r}}_{Fj}^W), \quad (33) \\ (j &= 1, 2, \dots, \beta) \end{aligned}$$

for given desired positions and velocities  $\bar{r}_{Fj}^W = r_C + \bar{r}_{Fj}$ ,  $\dot{\bar{r}}_{Fj}^W = \dot{r}_C + \dot{\bar{r}}_{Fj}$  in the world coordinate frame and positive gain matrices  $K_P, K_D > 0$ , or by a feed-forward force command. The control torque is given simply as

$$\tau_f = J_F(q)^T \bar{f}_F, \quad (34)$$

where  $J_F(q) = \frac{\partial r_F}{\partial q}$  is the task Jacobian.

Even if the task control force is zero  $\bar{f}_F = 0$ , the swinging legs or hands are gravity-compensated and the internal motions are damped by the main full-body force controller (28). However, we must de-activate the contribution from balancing force input  $f_u$  to those joints  $J_F(q)$  is involved in if we require that the reaching and force interaction task have the same priority as the balancing control. In this case, we can hold the swinging

legs or hands and move them with a slight external force applied to arbitrary contact points.

To achieve good tracking of a desired GAF  $\bar{f}_P$  the interaction forces must be compensated.<sup>3</sup> This can be accomplished by simply subtracting the sum of the interaction forces from the desired GAF *in advance*:

$$\bar{f}_P \xleftarrow{\text{substitute}} \bar{f}_P - \sum_{j=1}^{\beta} \bar{f}_{Fj}. \quad (35)$$

However, this is an approximated compensation. Perfect compensation requires precise forward dynamics (moments of inertia identification and velocity measurement).

### C. Moment generation by contact forces

To this point, we have addressed the translational force, GAF  $f_P$ . However, generation of moment is also crucial for full-body balancing. Our contact force framework is also applicable to the moment generation. As an illustration in this section, we will describe yaw moment control, which is very important for fast bipedal locomotion. Let  $T_z$  be the applied yaw moment around CoP. It is related to the contact forces by

$$T_z = \sum_{j=1}^{\alpha} \begin{bmatrix} x_{Sj} \\ y_{Sj} \end{bmatrix} \times \begin{bmatrix} f'_{xSj} \\ f'_{ySj} \end{bmatrix} \in \mathcal{R}^1, \quad (36)$$

or, in a simple form,

$$T_z = \underbrace{[-y_{S1}, \dots, -y_{S\alpha}, x_{S1}, \dots, x_{S\alpha}]}_{B \in \mathcal{R}^{1 \times 2\alpha}} \begin{bmatrix} f'_{xS1} \\ \dots \\ f'_{xS\alpha} \\ f'_{yS1} \\ \dots \\ f'_{yS\alpha} \end{bmatrix}, \quad (37)$$

<sup>3</sup>If CoM state feedback such as (29) is employed and the motions of legs/hands are sufficiently slow, the force/moment compensation is not necessary.

where  $B$  represents a *contact-moment distribution matrix*. If we calculate the corresponding contact forces by inverting (37) for a desired yaw moment  $\bar{T}_z$ , however, the results might contradict the previous contact forces (25) and (26). Therefore, we combine (23), (38) and (37) to

$$\begin{bmatrix} x_P f_{xP} \\ f_{xP} \\ y_P f_{yP} \\ f_{yP} \\ T_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_x \\ A_y \\ B \end{bmatrix}}_C \begin{bmatrix} f_{xS1} \\ \dots \\ f_{xS\alpha} \\ f_{yS1} \\ \dots \\ f_{yS\alpha} \end{bmatrix}, \quad (38)$$

and invert it to obtain

$$\begin{bmatrix} \bar{f}_{xS1} \\ \dots \\ \bar{f}_{xS\alpha} \\ \bar{f}_{yS1} \\ \dots \\ \bar{f}_{yS\alpha} \end{bmatrix} = C^\# \begin{bmatrix} \bar{x}_P \bar{f}_{xP} \\ \bar{f}_{xP} \\ \bar{y}_P \bar{f}_{yP} \\ \bar{f}_{yP} \\ \bar{T}_z \end{bmatrix}. \quad (39)$$

This paper proposes two important moment controllers. The first one is for controlling the base heading:

$$\bar{T}_{z1} = -K_{PY}(e_3 - \bar{e}_3) - K_{DY}\dot{e}_3, \quad (40)$$

where  $K_{PY}, K_{DY} > 0$  are gains and  $e_3$  is the yaw orientation of the base. The second one is for compensating the yaw moment caused by a swinging leg's (or trunk's or hand's) motion.

$$\bar{T}_{z2} = -\sum_{j=1}^{\beta} \begin{bmatrix} (x_{Fj} - x_P) \\ (y_{Fj} - y_P) \end{bmatrix} \times \begin{bmatrix} \bar{f}_{xFj} \\ \bar{f}_{yFj} \end{bmatrix} \quad (41)$$

The total target moment  $\bar{T}_z = \bar{T}_{z1} + \bar{T}_{z2}$  is then substituted into the RHS of (39) to obtain the desired horizontal contact forces.

Furthermore, orientation control for swinging legs or hands can be implemented similarly to (33); their motions are compensated easily by modifying the original contact forces. Specifically, we subtract the contact forces  $\bar{f}'_{sj}$  caused by the orientation control from the original contact forces, as in (35). That is,

$$\bar{f}_S \xleftarrow{\text{substitute}} \bar{f}_S + \sum_{j=1}^{\alpha} \bar{f}'_{sj}. \quad (42)$$

#### D. Simulations of balancing

Simulation results of balanced squatting with the torso orientation controlled are shown in Fig. 5 and Fig. 6. The simulator is built on a dynamics simulator SD/FAST (www.sdfast.com), where the ground contact is modeled as unilateral virtual springs and dampers. The desired CoP is allowed to travel from  $-0.05$  m to  $+0.1$  m in the  $X$ -direction when both feet are aligned.

The desired horizontal position of CoM is the center of the supporting convex hull, which is not fixed in  $\Sigma_W$ . The desired CoM height  $\bar{z}_C$  and the torso orientation are given simply by sinusoidal trajectories, as shown in Fig. 6. The gains described in (29) are set to  $(K_{PC}, K_{DC}) = (5000, 500)$  for vertical motion and  $(K_{PC}, K_{DC}) = (100, 50)$  for horizontal motion. The joint-wise damping in (15) was set  $D = \text{diag}[d_1, d_2, \dots, d_n]$  with  $d_{legs} = 2$  and  $d_{torso} = 8$ .

A reaching task with balance is shown in Fig. 7. The CoM height  $z_C$  is not controlled, but is largely damped by the derivative gain  $K_{DC}$  in (29). Other parameters are identical to those given previously. The position of the swinging leg in  $\Sigma_W$ ,  $r_F^W$ , is given as a sinusoidal function, as shown in Fig. 8. During the motion, the target equilibrium point, that is, the center of the supporting foot, is moved slightly because of the slip. Nevertheless, the horizontal CoM position  $x_C, y_C$  is stabilized to the *new* equilibrium (the center of the

sole). In both simulations, small tracking errors in CoM are apparent. The errors originate in the uncompensated dynamics and persist as long as the robot is in motion.

Figure 9 shows squatting in our new simulation environment, where the ground contact is modeled in an advanced manner [30]. The contact forces are distributed over the whole body, including the arms (the arm joints are moving).

#### IV. EXPERIMENTS ON FORCE INTERACTION

This section presents some experiments using the proposed method and our new humanoid robot prototype (Fig. 2). First, we briefly describe the robot hardware in Section IV-A. Then we show the results of balancing with unknown external forces in Section IV-B, upper body interaction in Section IV-C, and interaction at different contact points on the lower body in Section IV-D. The robot is supported neither by hydraulic hoses nor by cables. They apply non-negligible disturbances to the robot. All experiments in this paper are performed in the double support phase with an aligned feet configuration. However, the method is applicable to arbitrary shapes of a supporting convex hull without any modification.

##### A. Humanoid platform and joint torque control

The humanoid platform shown in Fig. 2(a) was developed by SARCOS and ATR [5][4]. The robot is called Computational Brain (CB). The hip height is 0.82 m in an upright posture. The robot is attached with the cubic sole that is 0.12-m wide, 0.27-m long and 0.02-m deep. Currently, the total weight is 93.7 kg. The DoF configuration is shown in Fig. 2(b). The arms and legs have 7 DoFs: there are 3 DoFs on the neck; the torso has 3 DoFs. There are 50 DoFs including the fingers (12 DoFs) and the eye cameras (4 DoFs). Most of the DoFs are driven by hydraulic servo actuators. Currently,



Fig. 5. Animation of a balanced squat with the torso swinging for 1–3 s: the desired CoM height and the torso orientation are simply given by sinusoidal trajectories, but no compensation (42) has been applied.

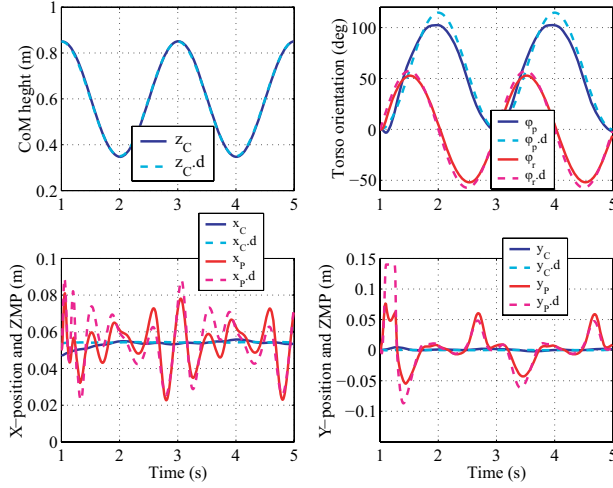


Fig. 6. Time evolution of the balanced squat in Fig. 5: CoM ( $x_C, y_C, z_C$ ) and ZMP ( $x_P, y_P$ ) are shown along with their desired values (indicated by “d”).

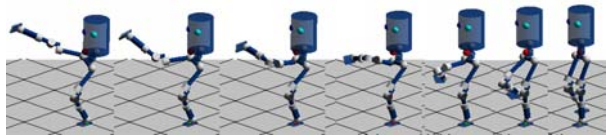


Fig. 7. Animation of a periodic reaching task by a leg with simultaneous balancing

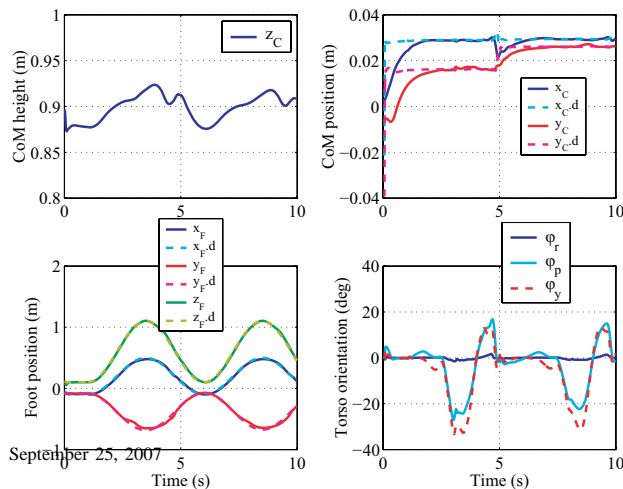


Fig. 8. Simulation result of a reaching task by a leg: the target foot

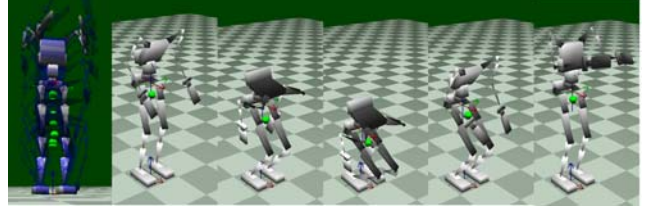


Fig. 9. Squatting on a full-body humanoid simulator

all are operated by 21 MPa supply pressure. The robot is installed with position sensors and force sensors at each joint. A three-axis gyro sensor is attached to the base link. The actuators are controlled by small local controllers which provide low-level position, velocity, and force feedback loop. They are interconnected by a high-speed intra-network [5]. The joint torque control is based solely on the force feedback of the force sensors attached to each joint. See [31] for the force control using hydraulic servo actuators and force sensors. The local controllers compute the feedback signal every 5 kHz and control the current into hydraulic servo-valve using analog circuitry.

However, no joints are collocated with their actuators because the joints are driven by linear hydraulic actuators via linkage mechanisms. Moreover, most joints are not collocated with the force sensors. We therefore implemented local joint torque controllers based on the individual joint kinematics as follows.

- (L1) Convert desired joint torques to the desired actuator forces.
- (L2) Calculate the reaction forces applied to the force sensors.
- (L3) Convert (L2) to the actuator reaction forces.
- (L4) Tracking (L3) to (L1) by high-speed local feedback control.

Kinematics of respective joints are represented using

one-to-one trigonometric functions bounded by the range of motion. The trigonometric functions are approximated by up to third-order polynomials. The transformations described above are also applied to the joint angle controller. That is, the feedback computations are always done in the actuator space. Thereby, we ensure a *flat* actuator response irrespective of the joint configuration. Once the local joint torque control is established, it is straightforward to implement the controller. The control procedure (Section II-D) is simple. Moreover, it can be executed with a low computational cost. All the control processes are complete within 1 ms.

The supplementary video no. 1 shows an experimental result of gravity compensation for the lower body. The legs move as if there is no gravity if no interaction forces exist. This experiment demonstrates not only force interaction without balancing, but also the performance of the joint torque control.

### B. Balancing ability

An experimental result of balancing is shown in Fig. 10 (see also the supplementary video no. 2). Large disturbances were applied to the robot. The top two graphs show positions of the CoM and desired CoP. The limits on the desired CoP are indicated by two dashed lines,  $\pm 0.135$  m. The desired CoP always leads the CoM, which reflects that the controller applies a GAF so that the CoM converges to the desired position (zero in this example). The graphs in the third row depict the desired normal contact forces  $\bar{f}_{zSj}$ , as calculated using (22). The left graph shows the forces applied to the *fore* corners (right sole,  $f_{zS1} + f_{zS2}$ ; left sole,  $f_{zS5} + f_{zS6}$ , and the sum); the right graph shows the contact forces applied to the *rear* corners (right sole,  $f_{zS3} + f_{zS4}$ ; left sole,  $f_{zS7} + f_{zS8}$ , and sum). The indices of the contact points are shown in Fig. 4(a). The dashed line

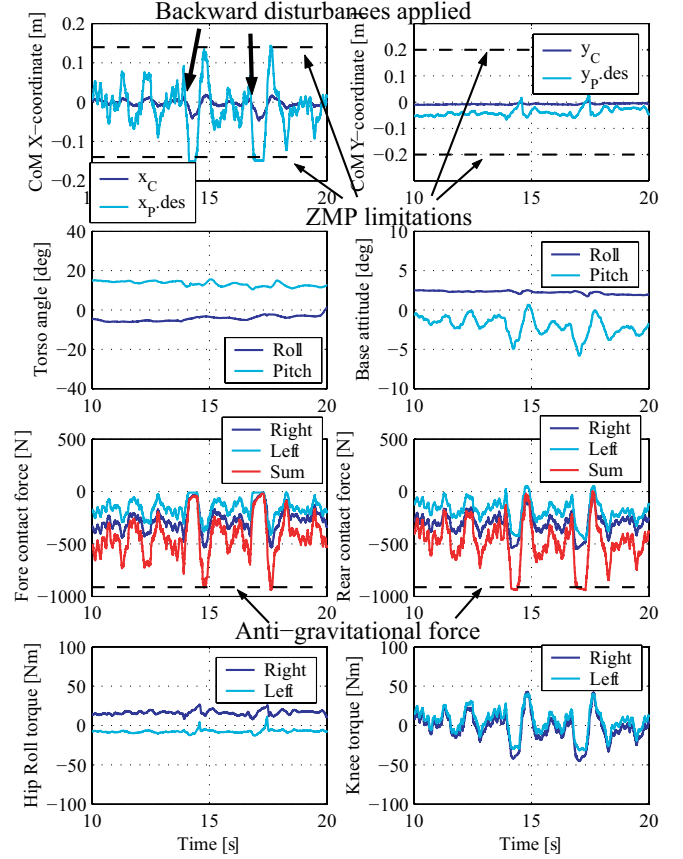


Fig. 10. Experimental data of balancing under disturbances: the disturbances are applied backwards twice. The third row shows the desired normal contact forces.

indicates the total anti-gravitational force:  $-mg$ . The CoM height is not controlled in this example. Therefore,  $\sum_{j=1}^8 f_{zSj} = -mg$  holds. The graphs show that the anti-gravitational force is actually distributed over the normal contact forces appropriately by (22).

A practical issue for model-based balance control is the definition of the origin of  $\Sigma_W$ . We propose a simple solution: defining the origin as the center point between the feet, which means that the origin is always moving according to the current robot configuration. Gyro information is important for both the CoM measurement and the desired GAF direction.



### C. Upper body interaction

Experimental results for upper body interaction with balancing are shown in Fig. 11 (see also the supplementary video no.4). The arms are shaken manually by the operator. In this experiment, the desired ground contact forces are also distributed to the arm joints.

In the experiment, the controller does not know how much external force is applied. However, if the controller knew the external forces *a priori*, it would be able to compensate them in a *anticipatory* manner by simply subtracting the forces to be compensated from the desired GAF in the same way as (35), which provides markedly better performance. For rich humanoid interaction, it will be desirable for the robot to have the ability to predict the partner's action based on observations including external forces, as indicated in Fig. 1.

### D. Force interaction at different contact points on lower body

The location to apply the external force is arbitrary. Figure 12 (see the supplementary video no. 3) shows the typical performance of our balancing controller when a side force is applied. This result nicely illustrates the relationship between the CoM error and the ground-

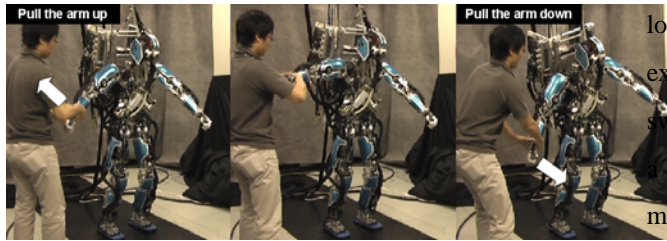


Fig. 11. Experiments on the upper-body interaction with balance. The human shakes the arm. Note that the GAF including the anti-gravitational force is distributed not only to the legs and the torso, but also to the arms. The robot is responding to the external force by the full-body compensatory motion.

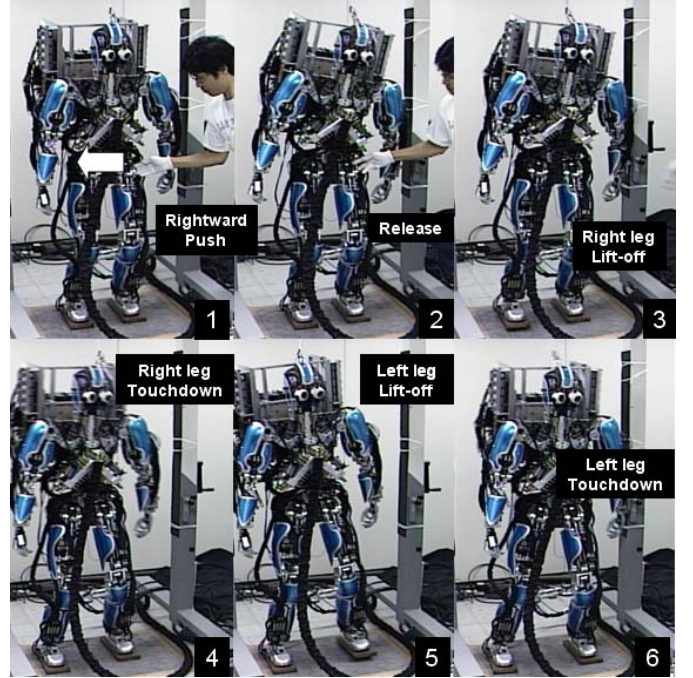


Fig. 12. Our force distribution law reduces some contact forces when a large error in CoM is detected. As a result, a foot might lift off of the ground.

reaction force distribution. In this example, CoM is largely laterally perturbed from the desired equilibrium (zero). The desired CoP is moved onto one edge of the foot while the desired CoM is fixed to the center. When this happens the desired normal contact forces of the other foot become nearly zero. For that reason, one foot loses its contact with the ground and lifts off. In this example, however, we applied no position control for swinging leg. Therefore, the lifted foot moves purely as a result of the balancing control.<sup>4</sup> In this case, the foot moves to the outside. Simultaneously, the CoM moves back to the center of the feet. Then the lifted foot touches the ground. This process repeats once more; finally, the robot balances, but with a different configuration from

<sup>4</sup>In [32] we presented an active stepping control for disturbance rejection against huge external force.



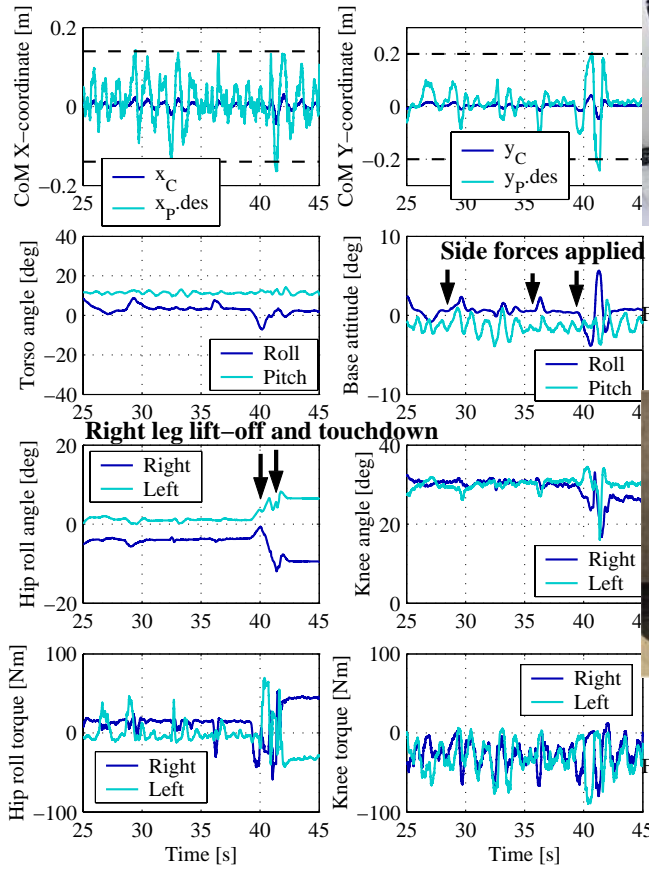


Fig. 13. Experimental data corresponding to Fig. 12

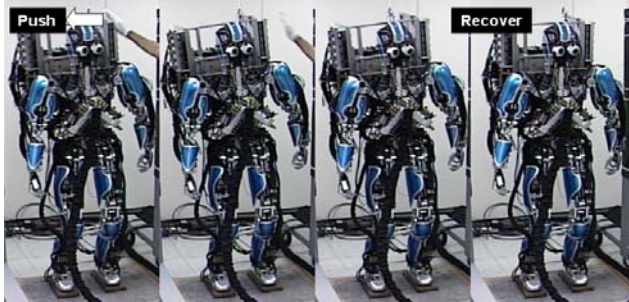


Fig. 14. Compensatory motion for a side push at the back

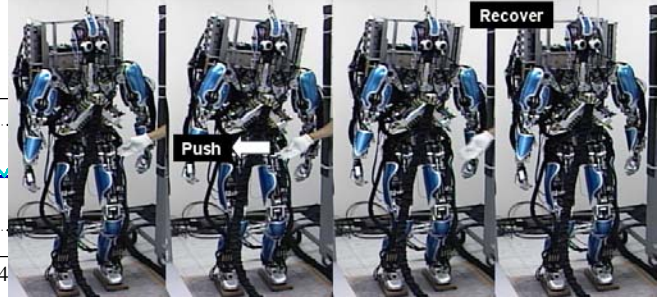


Fig. 15. Compensatory motion for a side push at the hip

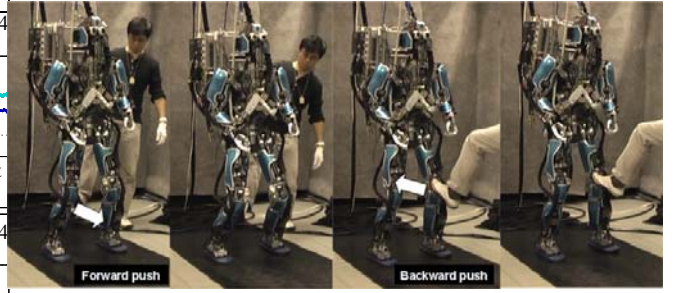


Fig. 16. Compensatory motions for a push at the knee

the initial one. Fig. 13 shows the time evolution of this experiment. The top two graphs show the positions of CoM and the desired CoP. Of course, the lifted foot does not move if we apply some feedback controller to maintain the distance between each foot.

A salient point is that we do not specify which joints should move in compliance with the disturbances. For example, we use no weighting matrix, as in some inverse kinematics approach. The weighting is done automatically by the Jacobian *transpose*  $J_S(q)^T$  in (28), and the compensating contact forces are distributed to the joint torques of the *whole* body. Therefore, if we push the robot torso, the torso compliantly follows first; then the lower body begins to compensate for the CoM error. Similarly, if we push the hip, the hip moves first, as shown in Fig. 14 and Fig. 15. We can even apply external

forces to the leg joints. For example, if we push the knee, the knee joint follows compliantly to the external force; then the remaining joints generate compensatory torques to retain the balance. This can be seen from Fig. 16 (and the supplementary video no. 4). Although the controller does not measure the external force, the response to the external force is *immediate*. The reason is that most joints are commanded to have *zero* torque when the robot has upright posture (equilibrium). The compliance is not only important for natural and safe human-humanoid interaction. It is also crucial to make the robot perform human-like behavior.

## V. DISCUSSION

### A. Comparison with related position-based balancers

We compare our method to some position-based controllers for humanoid robots, in which each joint is controlled by a position servo-controller that requires joint angle command. To our knowledge, [12] is the first report of a biped robot demonstrating balance against large disturbances. Based on a simple linear inverted pendulum, this controller assumes ZMP fixed to some point and calculates the necessary compensatory joint motions in real-time, then applies a local position controller. One method [13] is based on a similar inverted pendulum model, but it allows ZMP to move according to the state of CoM. These methods also differ in the manner in which the disturbances or CoM states are measured: the former measures contact forces directly, whereas the latter measures the ground reaction force. An additional difference is the weight assignment for the compensating DoF: the former uses trunk compensation; the latter uses waist compensation. Mixed weight assignment has also been proposed in [14]. This concept was extended to whole-body motion control in [33][16].

For example, [33] employs the Jacobian inverse

$$\dot{q} = J_P(q)^\# (-\dot{\tilde{r}}_P) \quad (43)$$

for some desired CoM velocity  $-\dot{\tilde{r}}_P$ . Herein,  $J_P^\# = W^{-1} J_P^T (J_P W^{-1} J_P^T)^{-1}$  is the weighted pseudo-inverse with the weighting matrix  $W$ , which should be specified in advance. This type of controller works well if the external forces are measured. Usually, these methods assume GRF measurement; the controller can compute the compensatory motions according to the desired GRF or CoM position.

However, if the external force is applied to some point at which the force sensor is not installed, we encounter a problem. For example, we assume that a disturbance is applied to the waist. The joints are position-controlled. Therefore, the disturbance changes the CoM as well as the ZMP. The method [13] modifies the desired ZMP to recover the balance. Then, to achieve the actual ZMP track to the desired ZMP, the controller accelerates the waist position. That is, if we push the robot waist, the waist immediately counteracts the external force. This behavior is undesirable for humanoid interaction for safety reasons and differs from what a human does. Although the controller (43) can handle weight assignment, we cannot anticipate, *in principle*, which joint must move in compliance to the *unknown* external forces.

### B. Comparison with related force-based controllers

By “force-based controllers,” we mean those controllers that require commanded joint torques rather than the joint angle. In [34] a simple static force control is applied to a 2D bipedal walking robot driven by a novel force-controllable actuator prototype. This work is the first result of force-based walking.

Although not implemented for real robots, [35] recently extended the method [36] for fixed-base robotic

manipulators to full-body humanoid motion control with multi-level prioritization. This method is useful if we have multiple task objectives because it can handle task prioritization explicitly. That control strategy differs from ours because it accompanies the exact cancellation of nonlinear dynamics.

The most closely related work to this paper is [37]. They applied similar passivity-based controller to redundant flexible manipulator. The manipulator is fixed to the ground. Consequently, the balance issue is not addressed. However, they achieved excellent experimental results on gravity compensation and force interaction by their dual manipulator, which is driven by electric servomotors and harmonic gears and installed with torque sensors.

## VI. CONCLUSION

In this paper, we presented our motivation for studying humanoid robots in terms of their suitability to anthropogenic environments, general adaptability, intuitive relationship with humans, and value as experimental tools informing and inspiring our ongoing investigation of ourselves as humans. We explained the importance of the physical interaction between humans and life-size humanoid robots, and described our integrated system paradigm for development of humanoid robot technology. In this context, we therefore clarified the need for a robust and general technique for maintaining balance in bipedal humanoid robots in the form of an integrated sub-system that might accommodate the requirements of high-level motion planning and interpretation.

The primary technical contribution of this paper is a full-body passivity-based force-level balancing control strategy incorporating gravity compensation, thereby effortlessly accommodating an arbitrary number of external force interaction points, and obviating force sensors at the contact points. Using a passivity-based strat-

egy, contact forces are well controlled in a satisfactory dynamic range without computing awkward dynamic terms, whereas internal motions are well suppressed. The method identifies a desired applied force from the robot to the environment, such as anti-gravitational force, which is distributed optimally over the forces acting at arbitrary contact points, and which is then transformed directly into whole-body joint torques. We performed various balancing experiments on a real hardware platform under unknown external forces and demonstrated the effectiveness of the method. In particular, we confirmed the following benefits:

- (i) Measurement of contact forces is not necessary.
- (ii) An arbitrary number of contact points can be handled at little or no additional computational cost.
- (iii) Arbitrary shape of the supporting convex hull is applicable.
- (iv) Solving inverse kinematic and inverse dynamic problems is not necessary, joint trajectories are not necessary, and whole control algorithms are executed in real-time without any recursive computation.
- (v) Joint friction is used to suppress internal motion.
- (vi) Whole-body compliant balancing is possible in the presence of unknown external forces applied to arbitrary joint segments.
- (vii) Joint movements for interaction or reaching can be achieved easily using a naive superposition or approximate compensation.

Our future work will elaborate a model of physical interaction blending high-level motion planning with automatic balance control, as depicted in Fig. 1. Building on the reliable technique we have developed for maintaining balance in a way that can combined with arbitrary

uses of the body while accomplishing tasks.

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## APPENDIX A

### DERIVATION OF STATIC JOINT FORCE CONTROLLER

It is straightforward to prove that (8) and (9) yields (10) when  $\gamma \approx 0$ ,  $\hat{C} \approx 0$ . By substituting (8)

$$\tau = J_P^T(f_v + Mg)$$

into

$$f_P \approx (\hat{E}\hat{I}^{-1}\hat{E}^T)^{-1} \{ \hat{E}\hat{I}^{-1}(u - \hat{G}) \} \quad (44)$$

and expanding the RHS, we obtain

$$\begin{aligned} & \hat{E}\hat{I}^{-1}\hat{E}^T \\ &= \begin{bmatrix} id & | & J_P \end{bmatrix} \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} id \\ J_P^T \end{bmatrix} \\ &= M^{-1} + J_P I^{-1} J_P^T \end{aligned} \quad (45)$$

and

$$\begin{aligned} & \hat{E}\hat{I}^{-1}(u - \hat{G}) \\ &= \hat{E}\hat{I}^{-1} \left( \begin{bmatrix} 0 \\ J_P^T(Mg + f_v) \end{bmatrix} + \begin{bmatrix} Mg \\ 0 \end{bmatrix} \right) \\ &= \hat{E}\hat{I}^{-1}\hat{E}^T Mg + \hat{E}\hat{I}^{-1} \begin{bmatrix} 0 \\ J_P^T f_v \end{bmatrix} \\ &= \hat{E}\hat{I}^{-1}\hat{E}^T Mg + (J_P I^{-1} J_P^T) f_v. \end{aligned} \quad (46)$$

With reference to (45) and (46), (44) becomes

$$f_P \approx Mg + (M^{-1} + J_P I^{-1} J_P^T)^{-1} (J_P I^{-1} J_P^T) f_v. \quad (47)$$

If we give  $f_v$  as (9), that is,

$$\begin{aligned} f_v &= (J_P I^{-1} J_P^T)^{-1} (M^{-1} + J_P I^{-1} J_P^T) f_u \\ &= f_u + (J_P I^{-1} J_P^T)^{-1} M^{-1} f_u, \end{aligned}$$

we recover (10):

$$f_P \approx f_u + Mg.$$

## APPENDIX B

### CONVERGENCE OF THE CONTACT FORCE

It is also straightforward to prove (12)

$$\tau = J_P^T (f_v + Mg) + \zeta(q, \dot{q})$$

achieves (13) when (14) is stable. For illustration, we substitute  $\tau = J_P^T Mg + \zeta$  into (7)

$$f_P = (\hat{E}\hat{I}^{-1}\hat{E}^T)^{-1} \{ \gamma + \hat{E}\hat{I}^{-1}(u - \hat{C} - \hat{G}) \}.$$

The upper part of  $\{ \}$  becomes zero, whereas the lower part becomes

$$\gamma_2 + J_P I^{-1}(\zeta - C) =: w \quad (48)$$

where  $\gamma_2$  is the lower half of  $\gamma = [\gamma_1, \gamma_2]^T$ . On the other hand, the lower part of (6)

$$\hat{E}(q)\ddot{q}_C + \gamma(q, \dot{q}) = 0$$

can be written as

$$J_P \ddot{q} + \gamma_2 = 0. \quad (49)$$

Combining (48) and (49) yields

$$w = -J_P I^{-1} \{ I\ddot{q} + C - \zeta \} \quad (50)$$

Therefore, if we design  $\zeta$  so that  $I\ddot{q} + C - \zeta = 0$  is met (exact nonlinear compensation) or  $I\ddot{q} + C - \zeta$  is asymptotically stable, that is,  $\dot{q} \rightarrow 0$  as  $t \rightarrow \infty$  (e.g. via damping injection), then so  $w \rightarrow 0$  as  $t \rightarrow \infty$ . Combining this with the result from Appendix A we recover (13)

$$f_P \rightarrow f_u + Mg \quad (t \rightarrow \infty).$$