Study on the Stability and Motion Control of a Unicycle*
(3rd Report, Characteristics of Unicycle Robot)

Zaiquan SHENG**, Kazuo YAMAFUJI**
and Sergei V. ULYANOV**

In this paper, the characteristics of a unicycle robot are determined by analyzing a unicycle robot with an asymmetric rotor and closed-link mechanisms. Firstly, the dynamic equations of motion are derived and the robot's postural stability control is analyzed. Then the results are compared with those obtained by experiment. Analytical results for closed-link mechanisms are also obtained and compared with experimental results. It is shown by the analysis that the system's controllability with an asymmetric rotor is better than that with a symmetric rotor, and the robot's postural stability control can be realized using only PD and D controllers when the system has an asymmetric rotor. Longitudinal stability is easily achieved because the closed-link mechanisms can improve the longitudinal stability of the system. Analytical results indicate that the longitudinal stability and lateral stability influence each other and the system is quite nonlinear. It is also found that our improved model simulates a human riding a unicycle more closely than the previous model in terms of control behavior.

Key Words: Unicycle, Postural Stability, Asymmetric Rotor, Closed-Link Mechanism, Nonlinear System, Dynamics

1. Introduction

It is well-known that the unicycle system is an inherently unstable system. Both longitudinal and lateral stability control are needed simultaneously to maintain the unicycle's postural stability ; in fact, this is an unstable problem in three dimensions. However, a rider can achieve postural stability on a unicycle, keep the wheel speed constant and change the unicycle's posture in the yaw direction at will by using his flexible body, good sensory systems, skill and intelligence. In the investigation of this phenomenon and emulation of this system by a robot, we aim to construct a model of human motion dynamics, and evaluate the new methods for the stability control and analysis of unstable system.

From observation of a human riding a unicycle, we know that the rider's posture on a unicycle changes continually, so it is necessary for us to give a definition for the concept of postural stability in this system. Usually, if a system is stabilized, some variables in that system will be controlled to be constant values. However, because the unicycle system is an inherently unstable system, and the postural stability is achieved by means of centrifugal force dynamically created by the rider's actions, so the pitch, roll and yaw angles change continually, it is impossible for the rider to stabilize these three variables at constant values. From observation of a human riding a unicycle, we know that the change of yaw angle is used for the stability control of roll angle, hence if the roll angle changes, the yaw angle will also be changed. The change of pitch and roll angles is often within some range. The upper limit of possible range is 90 degrees ; nevertheless, from observation of a human riding a unicycle, we know that the postural stability is often lost if either pitch angle or roll angle becomes bigger than about 16 degrees. If there is insufficient centrifugal force created by control of this system, the postural stability will be lost in about 1 second, and if there is no control of the system, the postural stability will also be lost in about 1 second. Therefore if the postural stability is maintained for longer than 1

* Received 26th December, 1994.
** Department of Mechanical and Control Engineering, University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182, Japan

second, this indicates the use of a control action on the system.

Based on the characteristics of posture change in a unicycle system, we can give a definition of postural stability of a unicycle as follows.

We say that the posture of a unicycle is stabilized if the following conditions are met :

(1) Neither pitch angle nor roll angle increases indefinitely. Usually, both pitch and roll angles are changeable, but the change is within some range (the upper limit of this range is 90 degrees. Because of the power limit of the motor in a unicycle robot, the range will be much less than 90 degrees).

(2) The posture should be maintained for at least 1 second.

In our first report(1), based on the results of analysis concerning the dynamic characteristics of a human riding a unicycle, a new model was proposed for the emulation of a unicycle system as shown in Fig. 1. In this model, two closed-link mechanisms are used for the emulation of the rider's body, thighs and shanks in the pitch stability control, and a turntable is used for the emulation of the rider's shoulder, torso and arms in roll stability control. In our second report(2) and another paper(3), successful experimental results are reported, and the unicycle robot with stable posture in the experiment is shown in Fig. 2. Based on a preparatory experiment the model was improved by making the turntable asymmetric. In the simulation, a symmetric turntable is considered, and the simulation results indicate that a complicated control method is needed for the postural stability control, but in the experiment, the robot's postural stability is achieved by using simple PD and D controllers as shown in our second report(2).

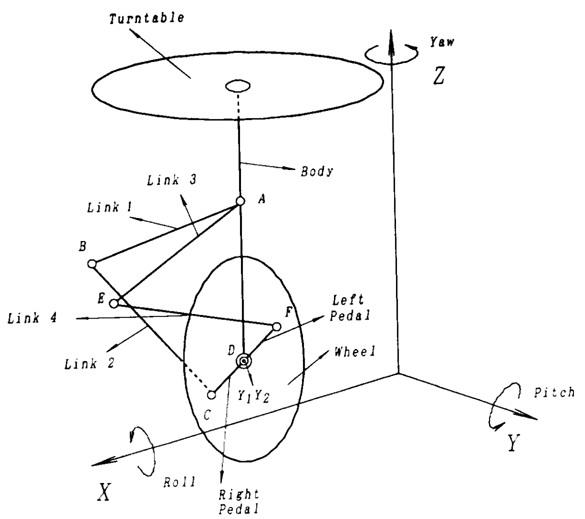


Fig. 1 Robot model emulating human riding unicycle

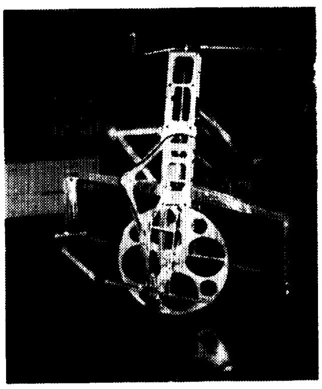


Fig. 2 Robot's stabilized posture

Before our study on the unicycle problem, two typical models were proposed by Ozaka et al(4) and Schoonwinkel(5). In Ozaka's model, the unicycle robot consisted of a wheel, a rigid body and a moveable weight. For that robot, postural stability in the pitch direction was obtained by control of the wheel, and postural stability in the roll direction was achieved by control of the moveable weight that can move in the roll direction. That investigation was conducted in three dimensions, but the experiment was not successful. In Schoonwinkel's model, the unicycle robot is constructed of three rigid bodies which are a wheel, a frame to represent the unicycle frame and the lower part of the rider's body and a rotary turntable to represent the rider's twisting torso and arms. Using this model, experiments were conducted by Schoonwinkel, David and Flotow(6) and Ito(7), but no successful experimental results in three dimensions have been reported. Comparing our model with Ozaka and Schoonwinkel's model, the main difference is in the usage of closed-link mechanisms and an asymmetric turntable (rotor) in our model, so it appears that closed-link mechanisms and asymmetric rotor play an important role in a unicycle robot's postural stability control.

In this paper, the characteristics of a unicycle robot are summarized based on experimental and theoretical results for closed-link mechanisms and an asymmetric rotor.

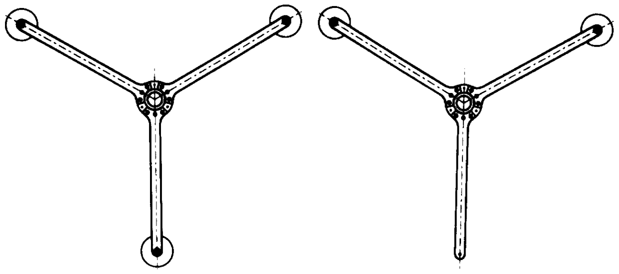
2. Robot with Symmetric Rotor and Asymmetric Rotor

In our first report(1), a unicyle model with symmetric rotor was analyzed theoretically, but a model with an asymmetric rotor was not addressed. In our second report(2), we described a successful experiment conducted using a model with an asymmetric rotor, but this model was not analyzed theoretically. In this section, we investigate the model with an asymmetric rotor theoretically, and compare the results with those for a model with a symmetric rotor obtained by theoretical analysis and experiment.

2.1 Dynamic equations of motion

In the design of a unicycle robot, the rotor is designed as shown in Fig. 3(a), and has a symmetric structure. However, if one of the three weights is taken away, the rotor becomes asymmetric and the model is improved; the asymmetric rotor is shown in Fig. 3(b). To analyze the unicycle robot with an asymmetric rotor, we can treat the two weights as one and derive the link model of the proposed model. We show the parameters that will be used in our computation in Fig. 4. Calculation of the displacement of the center of gravity of every part relative to the global reference coordinate xyz in the model is conducted using the coordinates defined in Fig. 5.

As in Fig. 5 in the 2nd report, coordinates are defined as below: $x_r y_r z_r$ is the global reference coordinate, $x_0 y_0 z_0$ is obtained by translating $x_r y_r z_r$ by the vector $(x_0, y_0, 0)$; rotating $x_0 y_0 z_0$ about z_0 by α, gives $x_1 y_1 z_0$. Rotating $x_1 y_1 z_0$ about x_1 by γ, gives $x_1 y_2 z_2$; $x_3 y_3 z_3$ is obtained by translating $x_1 y_2 z_2$ by the vector $(0, 0, r_\omega)$.



3(a) Symmetric rotor 3(b) Asymmetric rotor

Fig. 3 Two types of overhead rotor

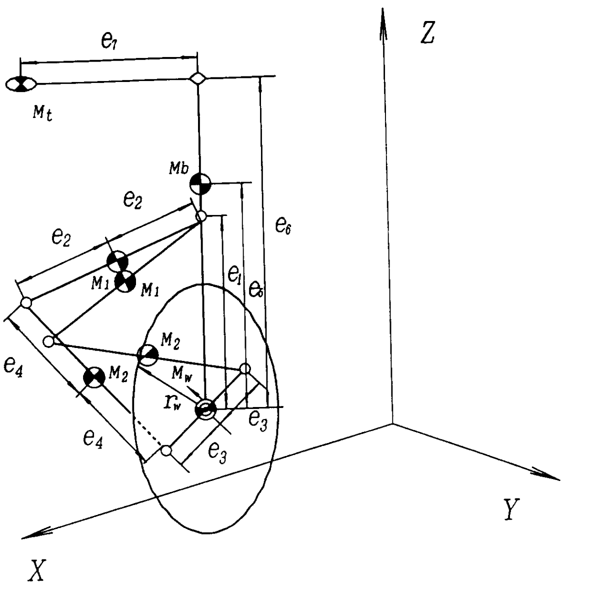


Fig. 4 Link model of a unicycle with an asymmetric rotor and its parameters for simulation

$x_4 y_4 z_4$ is obtained by rotating $x_3 y_3 z_3$ about y_3 by ψ and then translating it by the vector $(e_3, 0, 0)$; $x_5 y_5 z_5$ is obtained by rotating $x_3 y_3 z_3$ about y_3 by ψ and then translating it by the vector $(-e_3, 0, 0)$.

$x_6 y_6 z_6$ is obtained by rotating $x_3 y_3 z_3$ about y_3 by β and then translating it by the vector $(0, 0, e_5)$; $x_7 y_7 z_7$ is obtained by rotating $x_6 y_6 z_6$ about y_6 by θ_1 and then translating it by the vector $(e_2, 0, e_1 - e_5)$.

$x_8 y_8 z_8$ is obtained by rotating $x_6 y_6 z_6$ about y_6 by θ_3 and then translating it by the vector $(e_2, 0, e_1 - e_5)$; $x_9 y_9 z_9$ is obtained by rotating $x_4 y_4 z_4$ about y_4 by θ_2 and then translating it by the vector $(0, 0, e_4)$.

$x_{10} y_{10} z_{10}$ is obtained by rotating $x_5 y_5 z_5$ about y_5 by θ_4 and then translating it by the vector $(0, 0, e_4)$; $x_{11} y_{11} z_{11}$ is obtained by rotating $x_6 y_6 z_6$ about z_6 by η and then translating it by the vector $(0, 0, e_6 - e_5)$. $x_{12} y_{12} z_{12}$ is obtained by translating $x_{11} y_{11} z_{11}$ by the vector $(e_7, 0, 0)$.

Assuming there is rolling motion between the wheel and ground and no slip between them, considering the nonholonomic constraint between the wheel and ground, and using a general method[8] for the computation of the multi-closed-link mechanisms' dynamic motion, we can derive dynamic equations of motion for a unicycle robot with an asymmetric rotor.

$$\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} M(q) & -\dfrac{\partial c}{\partial q} \\ E(q) & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} \tau - B(q)[\dot{q}, \dot{q}] - C(q)[\dot{q}^2] - D(q)[\dot{q}] - G(q) \\ -F(q, \dot{q}) \end{bmatrix}, \tag{1}$$

where $\tau = (\tau_\psi, 0, 0, 0, 0, \tau_{\theta_2}, 0, \tau_{\theta_4}, \tau_7)$: $E(q)$ is a 4×4 coefficient matrix for acceleration, $F(q, \dot{q})$ is a 4-dimensional vector for Coriolis and centrifugal accel-

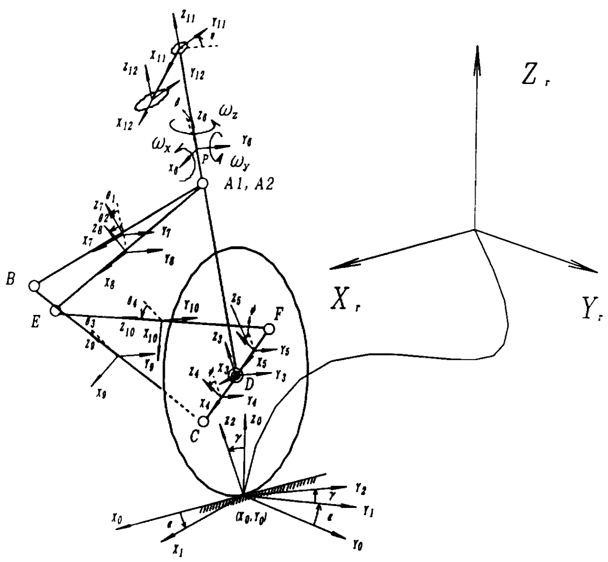


Fig. 5 Description of coordinates and definition of variables

eration, both $E(q)$ and $F(q, \dot{q})$ are determined from constraint equations on a closed-link loop's acceleration; $\frac{\partial c}{\partial q}$ is a 4×4 matrix determined by constraint equations of two closed-link loops, λ is a 4-dimensional vector of Lagrangian multipliers; $M(q)$ is a 9×9 matrix of mass; $B(q)$ is a 9×36 matrix of Coriolis coefficients, $[\dot{q}\dot{q}]$ is a 36×1 vector of velocity products given by $[\dot{q}\dot{q}]=[\dot{\psi}\dot{\alpha}, \dot{\psi}\dot{\gamma}, \cdots, \dot{\theta}_4\dot{\eta}]^T$; $C(q)$ is a 9×9 matrix of centrifugal coefficients, and $[\dot{q}^2]$ is a 9×1 vector given by $(\dot{\psi}^2, \dot{\alpha}^2, \dot{\gamma}^2, \dot{\beta}^2, \dot{\theta}_1^2, \dot{\theta}_2^2, \dot{\theta}_3^2, \dot{\theta}_4^2, \dot{\eta}^2)^T$; $D(q)$ is a 9×9 matrix of friction coefficients and $[\dot{q}]=(\dot{\psi}, \dot{\alpha}, \dot{\gamma}, \dot{\beta}, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4, \dot{\eta})^T$; $G(q)$ is a 9×1 matrix of gravity terms. q is defined as a matrix $(\psi, \alpha, \gamma, \beta, \theta_1, \theta_2, \theta_3, \theta_4, \eta)'$.

Because there are so many variables in this system and the coordinates are defined based on the previous coordinate (except global coordinate $x_\gamma y_\gamma z_\gamma$), the dynamic equation of motion is quite complicated. In order to keep the paper concise, the elements of the equation are not shown in detail here. Comparing this equation with Eq. (18) in the first report[1], in Eq. (1) the variable η appears, but in Eq. (18) in the first report[1], the variable η does not appear due to the symmetric turntable. Equation (1) is much more complicated than Eq. (18) in the first report[1]. When the turntable is asymmetric, the value of the turntable's rotation angle will influence other variables. However, the value of the turntable's rotation angle cannot influence other variables when the turntable is symmetric. This indicates that an asymmetric turntable will be more effective than a symmetric turntable in the robot's postural control.

2.2 Simulation results

From Eq. (1) in subsection 2.1, we know that the system is quite complicated and nonlinear, but the robot's postural stability can be obtained using by simple PD and D controllers. In this subsection, we examine whether this is possible in theory.

As in the experiment, a simple control method is chosen to help the wheel overcome the friction force and control the wheel speed, a PD controller is used to control links 2 and 4, and the rotor is driven by a D controller. The controlling torque is computed using Eqs. (2)-(5) (they are Eqs. (7), (8), (9) and (11) in our second report[2]).

$$\tau_\psi = A \tag{2}$$

$$\tau_{\theta_2} = -kp_1\times\beta - kd_1\times\dot{\beta} \tag{3}$$

$$\tau_{\theta_4} = \tau_{\theta_2} \tag{4}$$

$$\tau_\eta = kd_2\times\dot{\gamma} \tag{5}$$

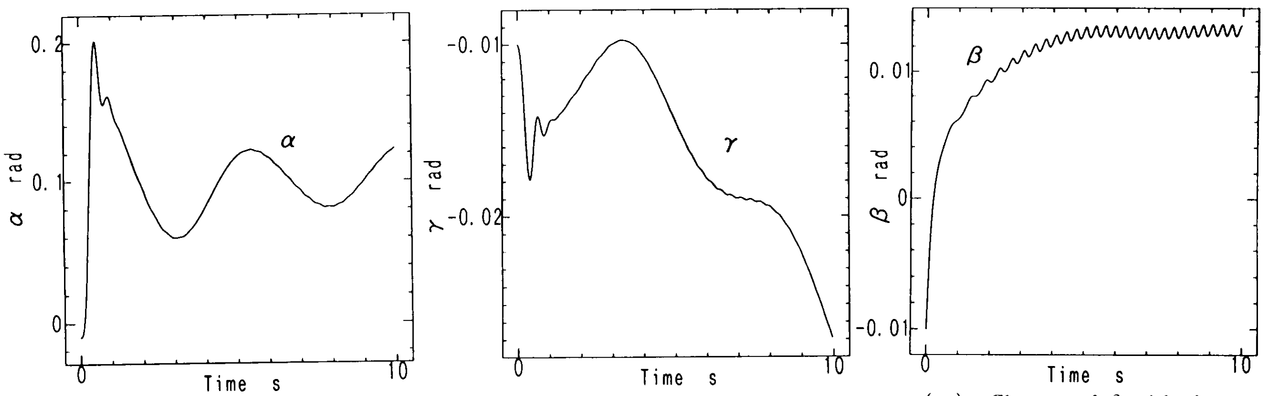
where τ_ψ is the wheel torque; A is a constant. τ_{θ_2}, τ_{θ_4} and τ_η are the torques for link 2, link 4 and the rotor respectively; kp_1, kd_1 and kd_2 are feedback gains.

The measured unicycle parameters are shown in Table 1. By applying the above control methods represented by Eqs. (2), (5) to Eq. (1), computer simulation was carried out using the Runge-Kutta-Gill method. In the simulation, $kp_1=600.0$, $kd_1=80.0$, and $kd_2=30.0$ are used, sampling time is taken as 4 ms and the initial unstable state is taken as $\alpha=-0.01$rad $\beta=-0.01$rad and $\gamma=-0.01$rad.

In Figs. 6 and 7, the simulation results are shown. Figure 6 shows the change of the robot's posture in the pitch, roll and yaw directions. The torques used in the control of the system are given in Fig. 7. From the above results, we know that it is possible to stabilize the robot's posture using PD and D controllers when the rotor is asymmetric.

Table 1 Measured parameters of unicycle robot

m_w	m_b	m_1	m_2	m_4	e_1
1.8kg	4.55kg	0.27kg	1.03kg	2.0kg	0.43m
e_2	e_3	e_4	e_5	e_6	IWX
0.11m	0.11m	0.19m	0.53m	0.66m	0.032kg·m^2
IWY	IWZ	IBX	IBY	IBZ	I1X
0.065kg·m^2	0.032kg·m^2	0.055kg·m^2	0.055kg·m^2	0.002kg·m^2	0kg·m^2
I1Y	I1Z	I2X	I2Y	I2Z	ITX
0.013kg·m^2	0.013kg·m^2	0kg·m^2	0.0075kg·m^2	0.0075kg·m^2	0.00067kg·m^2
ITY	ITZ	f_ψ	f_α	f_β	f_{θ_1}
0.00067kg·m^2	0.00081kg·m^2	0.26	0.26	0.16	0.19
f_{θ_2}	f_{θ_3}	f_{θ_4}	f_t	r_w	r_t
0.18	0.19	0.18	0.2	0.18m	0.29m



(a) Change of α with time (b) Change of γ with time (c) Change of β with time

Fig. 6 Posture change of the robot (Simulation)

When the rotor is symmetric, as analyzed in our first report[1], the unicycle robot's postural stability can be achieved effectively by a gain schedule control method and a PD control method in simulation. However, with that model, because of the rotor's symmetry, the unicycle robot's postural stability could not be achieved using the simple PD and D controllers given by Eqs.(2)-(5) in simulation.

Although the simple PD and D controllers can be effective for a unicycle robot with an asymmetric rotor, they are not effective for postural stability control of a unicycle robot with a symmetric rotor. This means that the system's characteristics are changed if one of the weights is removed from the rotor to make the rotor asymmetric. As shown by simulation results, postural stability control with an asymmetric rotor is easier than that with a symmetric rotor.

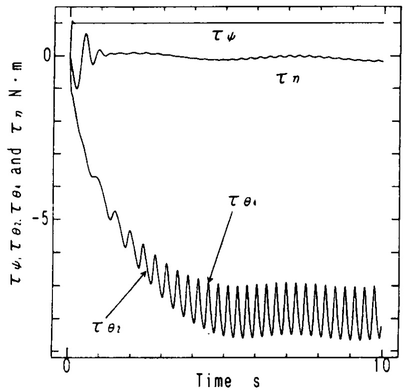
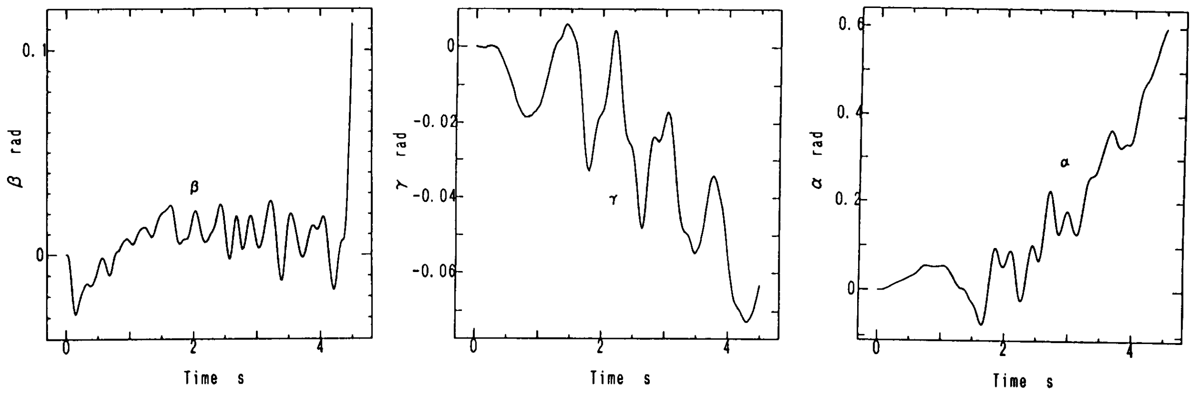


Fig. 7 Torques applied to the robot (Simulation)

2.3 Experimental results

Although we could obtain good results using our model with a symmetric rotor[1] in simulation, we could not obtain similarly good results in experiments using that model. We therefore proposed a gain schedule control method[2]. Good experimental results were obtained for a model with an asymmetric rotor using PD and D controllers[2]. Some of the experimental results are shown in Figs. 8 and 9 (they are Figs. 8 and 9 in our second report[2]), in which $kp_1=6000.0$, $kd_1=120.0$, and $kd_2=2450.0$ were used, and the sampling time was taken as 4 ms. Corresponding to Figs. 6 and 7, Fig. 8 shows the changes of the robot's posture in the pitch, roll and yaw directions. Figure 9 shows the torques applied to the robot. Comparing the simulation results with experimental ones, we find that the

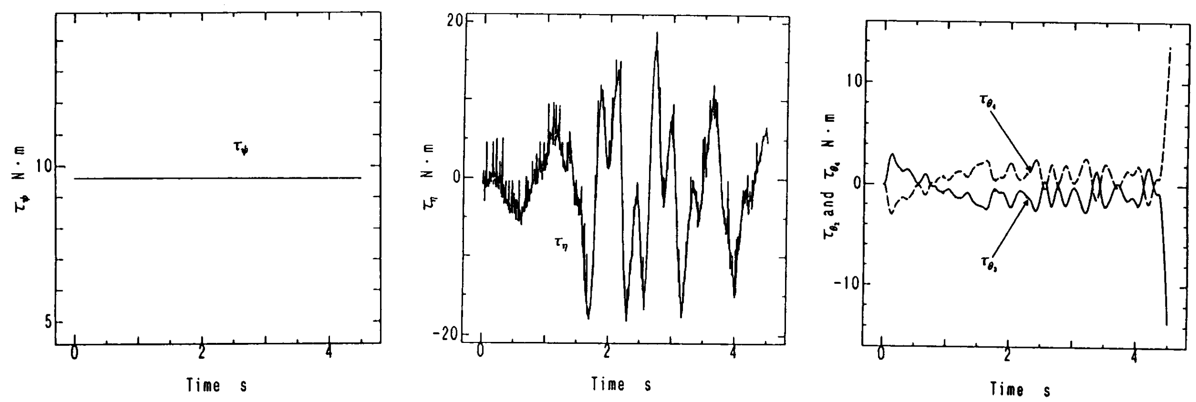


Change of β with time

Change of γ with time

Change of α with time

Fig. 8 Posture changes of the robot (Experiment)



Change of τ_ψ with time

Change of τ_η with time

Change of τ_{θ_2} and τ_{θ_4} with time

Fig. 9 Torques applied to the robot (Experiment) results are similar terms of postural stability. Of course, because the conditions for simulation and experiment are quite different, it is difficult or impossible for us to obtain simulation results which are exactly the same as experimental results.

In the experiment, the robot's initial state is set manually by the operator and this action will have a significant influence on the system. The uneveness of plastic carpet used in the experiments for preventing the wheel's slipping on the ground will also disturb the robot's postural stability control. Furthermore, the change of friction force is not negligible during the experiment, and there is drift due to temperature change and noise in the measurement of angular velocity by the rate-gyro sensors. However, both simulation and experiment show that the PD and D controllers are effective for the robot's postural stability control, and it is proven that the model with an asymmetric rotor is better than that with a symmetric rotor for the emulation of a human riding a unicycle. This result can help us understand why the postures of the rider's torso, shoulders and arms are always asymmetric with respect to the wheel's principal axis.

3. Characteristics of Closed-Link Mechanisms

From previous study[1],[2], we know that the closed-link mechanism plays an important role in a unicycle robot's postural stability control. In this section, the characteristics of the closed-link mechanism will be investigated by several methods.

3.1 Analysis of constraint conditions (Static analysis)

Since longitudinal postural stability of the robot is achieved using the closed-link mechanism as shown in our previous paper[1], the following displacement constraint conditions are always satisfied.

$$e_1 \sin\beta + 2e_2\cos(\theta_1+\beta) - e_3\cos\psi - 2e_4\sin(\psi + \theta_2) = 0 \qquad (6)$$

$$e_1 \cos\beta - 2e_2\sin(\theta_1+\beta) + e_3\sin\psi - 2e_4\cos(\psi + \theta_2) = 0 \qquad (7)$$

$$e_1 \sin\beta + 2e_2\cos(\theta_3+\beta) + e_3\cos\psi - 2e_4\sin(\psi + \theta_4) = 0 \qquad (8)$$

$$e_1 \cos\beta - 2e_2\sin(\theta_3+\beta) - e_3\sin\psi - 2e_4\cos(\psi + \theta_4) = 0 \qquad (9)$$

There are six variables ψ, β, θ_1, θ_2, θ_3 and θ_4 in the above four equations. If the values of two of the six variables are given, the magnitudes of the other variables will be determined uniquely by the above equations. Therefore, to keep β at zero if the wheel's angle is changed in any known form (which means the wheel's angle is given), we can obtain the requirements on θ_1, θ_2, θ_3 and θ_4 from Eqs.(6)-(9). This means that the robot's postural stability in the pitch direction will always be achieved if θ_1, θ_2, θ_3 and θ_4 satisfy Eqs.(10)-(13).

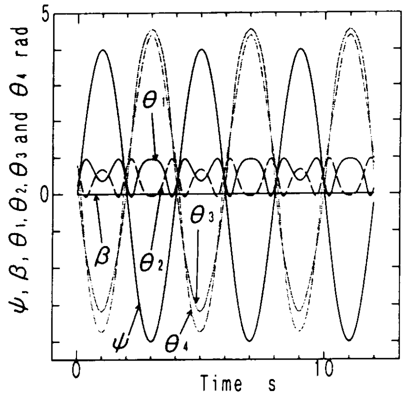


Fig. 10 Change of robot's posture obtained by static analysis

Assuming that the wheel angle ψ is changed in the form of $\sin(\omega t)$ as shown in Fig. 10, if θ_1, θ_2, θ_3 and θ_4 change as shown in Fig. 10, the angle β will remain zero as shown in Fig. 10 due to the displacement constraint conditions of closed-link mechanisms.

$$\theta_1 = \arcsin\left(\frac{e_1^2+4e_2^2+e_3^2-4e_4^2+2e_1e_3\cos\psi}{\sqrt{(4e_2e_3)^2+(4e_1e_2)^2+32e_1e_2^2e_3\sin\psi}}\right) - \arctan\frac{e_3\cos\psi}{e_1+e_3\sin\psi} \qquad (10)$$

$$\theta_2 = \arccos\left(\frac{e_1-2e_2\sin\theta_1+e_3\sin\psi}{2e_4}\right) - \psi \qquad (11)$$

$$\theta_3 = \arcsin\left(\frac{e_1^2+4e_2^2+e_3^2-4e_4^2-2e_1e_3\sin\psi}{\sqrt{(4e_2e_3)^2+(4e_1e_2)^2-32e_1e_2^2e_3\sin\psi}}\right) - \arctan\frac{e_3\cos\psi}{e_3\sin\psi-e_1} \qquad (12)$$

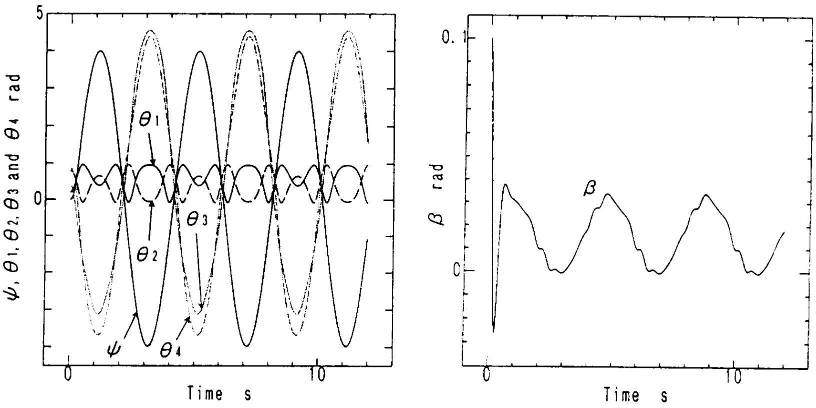
$$\theta_4 = \arccos\left(\frac{e_1-2e_2\sin\theta_3-e_3\sin\psi}{2e_4}\right) - \psi \qquad (13)$$

From the above simple static analysis of the displacement constraint conditions of closed-link mechanisms, we know that this kind of structure is quite special. By using the closed-link mechanisms, it is possible to make the robot's posture in the pitch direction independent of the wheel's movement by applying suitable control to the links.

3.2 Simulation of dynamic performance and experimental results for the closed-link mechanisms

In the last subsection, the static analysis was conducted on the basis of displacement constraint conditions of the closed-link mechanisms. In this subsection, we investigate the dynamic performance of the closed link mechanisms.

In order to investigate the dynamic performance of the closed link mechanism, the initial posture of the robot is set as $\beta=0.1$ rad and $\gamma=0.0$ rad, which means that only the posture in the pitch direction is unstable in the initial state. Under this condition, the wheel is driven to rotate in the form of Amp$\times\sin(\omega t)$, and



Change of ψ, θ_1, θ_2, θ_3 and θ_4 with time

Change of β with time

Fig. 11 Change of robot's posture obtained by dynamic analysis

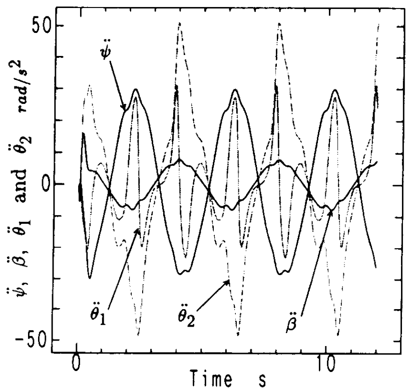


Fig. 12 Change of accelerations with time (Simulation)

both pitch and roll angles are kept at zero. Applying the torques shown in Eqs.(14), (3), (4) and (5) to the control of the wheel, link 2, link 4 and the rotor in the dynamic equation of motion Eq.(1), we conducted computer simulation using the Runge-Kutta-Gill method. From the simulation, we know that the torques applied to the rotor and the roll angle are always zero if the initial state in the roll direction is stable. The simulation results with $kp_3=100$, $kd_3=10$, $kp_1=300$, $kd_1=20$, Amp$=4$ and f$=2.5$ are shown in Figs. 11 and 12.

Figure 11 shows the change of angles ψ, β, θ_1, θ_2, θ_3 and θ_4. Figure 12 illustrates the change of angular accelerations $\ddot{\psi}$, $\ddot{\beta}$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, $\ddot{\theta}_3$ and $\ddot{\theta}_4$. As shown in Fig. 12, although the wheel's angular acceleration changes quickly by a large value, because of the compensation of the acceleration by links 1~4, the angular acceleration $\ddot{\beta}$ in the pitch direction was kept at a small value. In this case, we can obtain a small pitch angle as shown in Fig. 11(b).

$$\tau_\psi = kp_3\times(\mathrm{Amp}\times\sin(\omega t)-\psi)-kd_3\times\dot{\psi}$$

where τ_ψ is the torque applied to the wheel; kp_3 and kd_3 are feedback gains; Amp is amplitude; ω is angular frequency; t is time.

As shown in Figs. 6(a) and 11(b), the pitch angle was kept at a small value effectively in simulation, and in the experiment the pitch angle was also kept at a small value effectively as shown in Fig. 8(a). The analytical and experimental results indicate that the closed-link mechanisms can make the robot's posture in the pitch direction independent of the wheel's movement. In this way, this mechanism can play an important role in longitudinal postural stability control of the robot.

3.3 Response to external disturbance

From observation of a human riding a unicycle, we know that the rider can achieve postural stability control with very small wheel velocity. We assume that the closed-link loop structure formed by the rider's body, thighs and shanks plays an important role in this performance. In this subsection, the robot's longitudinal postural stability control at small wheel speed is studied, and the robot's longitudinal postural stability control under external disturbance is also considered.

From simulation, we know that the robot's longitudinal postural stability can be achieved at a small wheel speed when the initial posture in the roll direction is stable (that means $\gamma=0.0$ rad in the initial state). Furthermore, the robot's longitudinal postural stability can also be achieved under external disturbance and with a small wheel velocity if the initial posture in the roll direction is stable (that means $\gamma=0.0$ rad in the initial state). The torques applied to the system are taken as Eqs.(15), (16), (17) and (5), and the initial state is set as $\beta=0.1$ rad, $\gamma=0.0$ rad and $\alpha=0.0$ rad. The simulation result with $kp_1=300$, $kd_1=20$, $kp_3=100$ and $kd_3=10$ is shown in Fig. 13. From Fig. 13, we know that the closed-link mechanisms are robust against external disturbance and can help the unicycle robot achieve longitudinal postural stability even at a small wheel velocity.

$$\tau_\psi = -kp_3\times\psi-kd_3\times\dot{\psi}$$

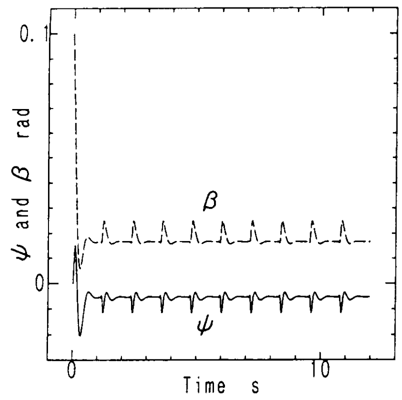


Fig. 13 Response to external disturbance (Simulation)

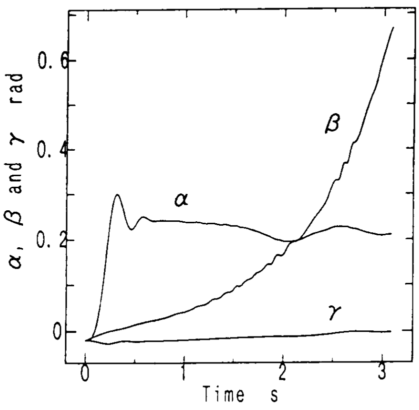


Fig. 14 Robot's posture without control of the closed-link mechanisms (Simulation)

$$\tau_{\theta_2} = -kp_1 \times \beta - kd_1 \times \dot{\beta} + F_1 \tag{15}$$

$$F_1 = \begin{cases} 15\,\mathrm{N \cdot m} & \left(\frac{t}{2} = 0, 1, 2, 3, 4, 5\right) \\ 0\,\mathrm{N \cdot m} & (\text{otherwise}) \end{cases} \tag{16}$$

$$\tau_{\theta_4} = \tau_{\theta_2} \tag{17}$$

where F_1 is an external disturbance force ; t is time ; the other symbols are the same as those used in the previous equations.

3.4 Postural stability control without control of closed-link mechanisms

If no control is applied to the closed-link mechanisms, the robot's postural stability must be achieved using only a wheel and a rotor. For lateral stability control, the robot's movement speed must be controlled by control of the wheel. On the other hand, the robot's longitudinal stability also depends on control of the wheel. In this case, control of the wheel becomes quite difficult. Even if lateral stability is achieved, longitudinal stability may be lost, and vice versa. Without control of the closed-link mechanisms, we have investigated many methods for controlling the robot's postural stability. Simulation results indicate that the robot's postural stability control cannot be achieved by applying torque only to the wheel and rotor. The result of an unsuccessful simulation is shown in Fig. 14 ($A=1$, $kp_1=600$, $kd_1=120$ and $kd_2=60$ with initial state of $\gamma=-0.01$ rad, $\beta=-0.01$ rad and $\alpha=-0.01$ rad). In this computation, the torques applied to the wheel and rotor are given by Eq. (18) and Eq. (5).

$$\tau_4 = A + kp_1 \times \beta + kd_1 \times \dot{\beta} \tag{18}$$

where the symbols are the same as those in the previous equations.

As shown in Fig. 14, the robot's postural stability is lost when angle β reaches a certain value. An experiment without control of closed-link mechanisms was also carried out, but was not successful. The investigation in this subsection shows that control of the closed-link mechanisms is needed for control of the robot's postural stability.

From the analysis results shown in this section, we can conclude that the robot's longitudinal stability can be achieved easily because the closed-link mechanism is robust against disturbance, and it can make longitudinal stability independent of the wheel's movement due to its special structure. Longitudinal stability cannot be achieved if there is no closed-link mechanism which means that the robot's postural stability cannot be achieved without the use of closed-link mechanisms.

4. System Characteristics

Equation (1) shows that this system is quite nonlinear, and we can also observe the following interesting phenomena from the analysis results.

Based on the analysis of the characteristics of the closed-link mechanism in section 3, we find that if the initial posture in the roll direction is stable, (1) the closed-link mechanism is robust against an external disturbance to the robot's postural stability in the pitch direction ; (2) even if the initial unstable value of pitch angle is as large as 0.5 rad, the robot's postural stability in the pitch direction can be achieved ; (3) the postural stability control in the pitch direction is possible with small average wheel velocity.

However, if the initial posture in the roll direction is not stable, the simulation results show that the robot's postural stability can be achieved only when the initial values of pitch angle and roll angle are small. From simulation results, if the initial pitch angle β is less than 0.1 rad and initial roll angle γ is less than 0.05 rad, the robot's postural stability can be achieved ; otherwise, stability is not achieved. This system is nonlinear, which means that the lateral stability and longitudinal stability influence each other. For a nonlinear problem, the present conventional control methods are not sufficient. New control methods should be proposed and evaluated for the

Table 2 Control characteristics of the unicycle robot with symmetric rotor and asymmetric rotor

Model \ Items		Simulation			Experiment		
		Controllability	Stable area	Response to disturbance	Controllability	Stable area	Response to disturbance
Robot with symmetric rotor		Poor	Very small	Very weak	Poor	Very small	Very weak
Robot with asymmetric rotor	Initial state of roll is stable, but initial pitch state is unstable	Very good	Large	Strong	Very good	Large	Strong
	Both roll and pitch are unstable in initial state	Good	Small	Weak	Good	Small	Weak
	Without control of closed link mechanisms	Stability is not obtained					

postural stability control of this system, which can be used as a test case for new control methods. One new control method has been proposed and tested for this system ; the results are reported in our other two papers[9],[10].

Results of our theoretical and experimental investigation of two types of rotor, closed-link mechanisms and the whole system are summarized in Table 2.

5. Conclusions

In this paper, we focused on the investigation of an asymmetric rotor, closed-link mechanisms and the whole system. The conclusions can be summarized as follows.

(1) Both simulation and experiment show that the system's controllability with an asymmetric rotor is better than that with a symmetric rotor, and the asymmetric rotor is useful for lateral postural stability control of the unicycle.

(2) The closed-link mechanisms can make the robot's longitudinal postural stability independent of the movement of the wheel, and the robot's longitudinal postural stability can be efficiently achieved using this special structure.

(3) There exists nonlinearity in the unicycle system, and the robot's longitudinal postural stability and lateral postural stability affect each other due to this nonlinearity.

References

(1) Sheng, Z. and Yamafuji, K., Study on the Stability and Motion Control of a Unicycle (1st Report : Dynamics of a human riding a unicycle and its modeling by link mechanism), JSME Int. J., Ser. C, Vol. 38, No. 2(1995), p. 249.

(2) Sheng, Z. and Yamafuji, K., Study on the Stability and Motion Control of a Unicycle (2nd Report : Design of a unicycle robot and experimental results) (in Japanese), Trans. Jpn. Soc. Mech. Eng., Vol. 61, No. 583, C(1995), p. 1042.

(3) Sheng, Z. and Yamafuji, K., Realization of a Human Riding a Unicycle by a Robot, '95 IEEE Int. Con. on Robotics and Automation, Nagoya (1995), p. 1319.

(4) Ozaka, C., Kano, H. and Masubuchi, M., Stability of a Monocycle-Type Inverted Pendulum, Third Vehicle Automation Symp. of Japan Automation Control Society,(1980), p. 63.

(5) Schoonwinkel, A., Design and Test of a Computer Stabilized Unicycle, Ph. D. Thesis, Stanford University (1987).

(6) D. W. Vos and A. H. von Flotow, Dynamics and Nonlinear Adaptive Control of an Autonomous Unicycle (Theory and Experiment), Proc. of 29 th Conf. on Decision and Control,(1990), p. 182.

(7) Ito, S., Study on the Postural Stability and Locomotion Control of an Inverted Pendulum Type Unicycle (in Japanese), Master's Thesis, University of Electro-Communications (1994).

(8) Sheng, Z. and Yamafuji, K., A General Method for the Direct Dynamic Computation of Closed Link Mechanisms, Journal of Robotics and Mechatronics, Vol. 6, No. 2(1994), p. 169.

(9) Sheng, Z., Yamafuji, K. and Ulyanov, S.V., Study on the Stability and Motion Control of a Unicycle (4th Report, Fuzzy Gain Schedule PD Controller for Managing Nonlinearity of System) JSME Int. J., Ser. C, Vol. 39, No. 3(1996), p. 569.

(10) Sheng, Z., Yamafuji, K. and Ulyanov, S.V., Study on the Stability and Motion Control of a Unicycle (5th Report, Experiment Results by Fuzzy Gain Schedule PD Controller) (to be printed in Journal of Robotics and Mechatronics) (1996).