

# A Humanoid Robot Carrying a Heavy Object

Kensuke Harada, Shuuji Kajita, Hajime Saito, Mitsuharu Morisawa,  
Fumio Kanehiro, Kiyoshi Fujiwara, Kenji Kaneko, and Hirohisa Hirukawa  
*Humanoid Research Group, Intelligent Systems Research Institute  
National Institute of Advanced Industrial Science and Technology  
Tsukuba 305-8568, JAPAN*

**Abstract**—This paper studies the balance of a humanoid robot carrying a heavy object. Without knowing the mass and the position of the center of gravity of the object, the humanoid robot carries a heavy object stably by using the force sensor information attached at the wrists and the ankles. We first show how to generate the motion of a humanoid robot by taking the force sensor information into consideration. We also show the method for generating the gait pattern by taking the dynamics of the carried object. The effectiveness of the proposed method is shown by experiments.

**Index Terms**—Humanoid robot, Identification, Pick and Place.

## I. INTRODUCTION

A humanoid robot is expected to work instead of a human in our daily life. While the manipulation is necessary for a humanoid robot to perform a task in the real environment, the number of research on manipulation by a humanoid robot is limited. On the other hand, this research focuses on the manipulation by the whole body motion of a humanoid robot.

As an example of manipulation by a humanoid robot, let us consider picking up a large object placed on the floor and putting it at the desired position. Fig. 1 shows an image of the pick and place task by a humanoid robot where the robot first squats down, then captures an object, stands up, and begins to walk forward. The pick and place tasks are quite common for a human, and a human can usually perform such tasks without difficulty. However, the pick and place of a large object often becomes difficult for a humanoid robot. Let us consider the situation where the humanoid robot does not know the amount of external force applied at the hands. In such a case, the humanoid robot has to predict the amount of external force, and the humanoid robot may fall down if the difference between the actually applied external force and its prediction becomes large. If the weight of the object is much heavier than expected, the robot may fall down forward, and vice versa. Thus, when a humanoid robot picks up a large object, the hand reaction forces should be appropriately taken into consideration. While we can expect that such pick and place tasks will become quite common also for a humanoid robot if a humanoid robot begins to work in the real environment, there has been no research focusing on the pick and place of a large object by a humanoid robot so far.

In this paper, assuming that the 6-axis force/torque sensors are attached at the wrists and the ankles of a humanoid robot, we first consider modifying the posture of a

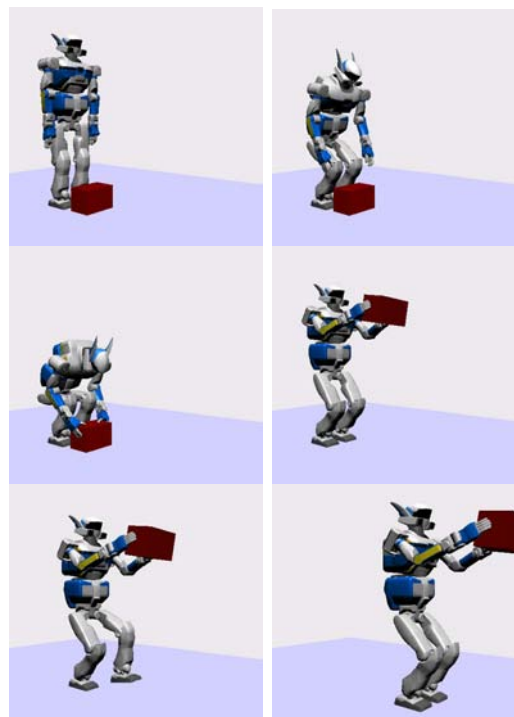


Fig. 1. Pick and place by a humanoid robot

humanoid robot generated without assuming the carried object based on the measured information of the force/torque at the wrists. Then, we propose a method for generating the gait of a humanoid robot taking the dynamics of the carried object into consideration. Lastly, we show simulation and experimental results by a humanoid robot HRP-2 carrying a heavy object.

## II. RELATED WORKS

There have been many researches on the humanoid robot without considering the interaction between the hand and the environment. For example, Kaneko et al.[1] developed the humanoid robot named HRP2 and realized several whole body motions[2]. Nishiwaki et al.[3] developed the motion controller for the humanoid H7. Fujita et al.[4] developed the small humanoid robot named SDR-4X.

Recently, focusing on the function of the arms, the research on manipulation by a humanoid robot has begun. Inoue et al.[9] and Nishihara et al.[10] determined the foot position maximizing the manipulability of the arms. Yokoyama et al.[8] realized by the humanoid robot HRP-

2 carrying a panel with a human. The authors proposed the position control based[5] and the force control based[6] pushing manipulation. The authors[7] also studied the ZMP during the manipulation of an object. Hwang et al.[11] also studied the static relationship between the hand reaction force and the ZMP position. Yokokohji et al.[12] studied the posture of a humanoid robot whose hands contact with an environment. However, there has been no research on carrying a heavy object by a humanoid robot taking the reaction force of the hands into consideration.

### III. FORMULATION

#### A. Definition of variables

Fig. 2 shows the model of a humanoid robot carrying an object used in this research. We assume that 6-axis force/torque sensors are attached at the wrists and the ankles. Let  $\mathbf{p}_{Hi} = [x_{Hi} \ y_{Hi} \ z_{Hi}]^T$  be the position vector of the wrists ( $i = 1, 2$ ) where the force  $\mathbf{f}_{Hi} \in R^3$  and the torque  $\boldsymbol{\tau}_{Hi} \in R^3$  are applied. Also, let  $\mathbf{p}_{Fi} = [x_{Fi} \ y_{Fi} \ z_{Fi}]^T$  be the position vector of the ankles ( $i = 1, 2$ ) where the force  $\mathbf{f}_{Fi} \in R^3$  and the torque  $\boldsymbol{\tau}_{Fi} \in R^3$  are applied. Focusing on the motion of the robot, we define  $\mathcal{P} = M\dot{\mathbf{p}}_G$  and  $\mathcal{L}$  as the linear and the angular momenta, respectively, with respect to the COG of the robot. Also, focusing on the motion of the object, we define  $\mathcal{P}_o = m_o\dot{\mathbf{p}}_o$  and  $\mathcal{L}_o$  be the linear and the angular momenta, respectively, with respect to the COG of the carried object. Let  $\mathbf{p}_Z = [z_Z \ y_Z \ x_Z]^T$  be the position vector of the zero moment point(ZMP)[13] where  $\mathbf{f}_Z \in R^3$  and  $\boldsymbol{\tau}_Z \in R^3$  denote the ground reaction force/torque applied at the ZMP.

#### B. ZMP when carrying an object

For a humanoid robot to carry an object stably, we can use either the hand reaction force/torque or the dynamics of the object as occasion requires. To keep balance of the robot when carrying an object, let us focus on the position of the ZMP. When a humanoid robot carries an object, the position of the ZMP can be defined by using either the hand reaction force or the dynamics of the object.

First, by taking the hand reaction force/torque into consideration, the ground reaction force/torque are generated by the linear/angular momenta of the robot, the gravity force of the robot, and the force/torque applied at the wrists. Here, the ground reaction force/torque are expressed as

$$\mathbf{f}_Z = M(\ddot{\mathbf{p}}_G - \mathbf{g}) - \sum_{j=1}^2 \mathbf{f}_{Hj}, \quad (1)$$

$$\boldsymbol{\tau}_Z = M(\mathbf{p}_G - \mathbf{p}_Z) \times (\ddot{\mathbf{p}}_G - \mathbf{g}) + \dot{\mathcal{L}} - \sum_{j=1}^2 \{(\mathbf{p}_{Hj} - \tilde{\mathbf{p}}_Z) \times \mathbf{f}_{Hj} + \boldsymbol{\tau}_{Hj}\}, \quad (2)$$

where  $\mathbf{g} = [0 \ 0 \ -g]$  denotes the gravity force vector.

Second, by taking the dynamics of the carried object into consideration, the ground reaction force/torque are generated by the linear/angular momenta of the robot, the gravity force of the robot, and those of the carried object.

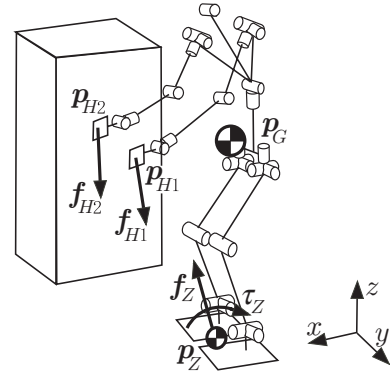


Fig. 2. Model of the system

In this interpretation, the ground reaction force/torque are also expressed as

$$\mathbf{f}_Z = M(\ddot{\mathbf{p}}_G - \mathbf{g}) + m_o(\ddot{\mathbf{p}}_o - \mathbf{g}), \quad (3)$$

$$\boldsymbol{\tau}_Z = M(\mathbf{p}_G - \mathbf{p}_Z) \times (\ddot{\mathbf{p}}_G - \mathbf{g}) + \dot{\mathcal{L}} + m_o(\mathbf{p}_o - \mathbf{p}_Z) \times (\ddot{\mathbf{p}}_o - \mathbf{g}) + \dot{\mathcal{L}}_o \quad (4)$$

Since the ZMP is defined to be a point on the ground at which the horizontal component of the ground reaction torque becomes zero, we can obtain the position of ZMP in two ways by solving eqs.(2) and (4):

$$x_Z = \frac{-\dot{\mathcal{L}}^{(y)} + Mx_G(\ddot{z}_G + g) - M(z_G - z_Z)\ddot{x}_G}{M(\ddot{z}_G + g) - \sum_{j=1}^2 f_{Hj}^{(z)}} - \frac{\sum_{j=1}^2 \{x_{Hj}f_{Hj}^{(z)} - (z_{Hj} - z_Z)f_{Hj}^{(x)}\}}{M(\ddot{z}_G + g) - \sum_{j=1}^2 f_{Hj}^{(z)}}, \quad (5)$$

$$= \frac{-\dot{\mathcal{L}}^{(y)} + Mx_G(\ddot{z}_G + g) - M(z_G - z_Z)\ddot{x}_G}{M(\ddot{z}_G + g) + m_o(\ddot{z}_o + g)} + \frac{-\dot{\mathcal{L}}_o^{(y)} + m_o x_o(\ddot{z}_o + g) - m_o(z_o - z_Z)\ddot{x}_o}{M(\ddot{z}_G + g) + m_o(\ddot{z}_o + g)}, \quad (6)$$

$$y_Z = \frac{\dot{\mathcal{L}}^{(x)} + My_G(\ddot{z}_G + g) - M(z_G - z_Z)\ddot{y}_G}{M(\ddot{z}_G + g) - \sum_{j=1}^2 f_{Hj}^{(z)}} - \frac{\sum_{j=1}^2 \{y_{Hj}f_{Hj}^{(z)} - (z_{Hj} - z_Z)f_{Hj}^{(y)}\}}{M(\ddot{z}_G + g) - \sum_{j=1}^2 f_{Hj}^{(z)}}, \quad (7)$$

$$= \frac{\dot{\mathcal{L}}^{(x)} + My_G(\ddot{z}_G + g) - M(z_G - z_Z)\ddot{y}_G}{M(\ddot{z}_G + g) + m_o(\ddot{z}_o + g)} + \frac{\dot{\mathcal{L}}_o^{(x)} + m_o y_o(\ddot{z}_o + g) - m_o(z_o - z_Z)\ddot{y}_o}{M(\ddot{z}_G + g) + m_o(\ddot{z}_o + g)}, \quad (8)$$

where  $\mathbf{p}_G = [x_G \ y_G \ z_G]^T$ ,  $\mathbf{p}_o = [x_o \ y_o \ z_o]^T$ ,  $\mathcal{L} = [\mathcal{L}^{(x)} \ \mathcal{L}^{(y)} \ \mathcal{L}^{(z)}]^T$ ,  $\mathcal{L}_o = [\mathcal{L}_o^{(x)} \ \mathcal{L}_o^{(y)} \ \mathcal{L}_o^{(z)}]^T$ , and  $\mathbf{f}_{Hi} = [f_{Hi}^{(x)} \ f_{Hi}^{(y)} \ f_{Hi}^{(z)}]^T$ . Here, we note that the ZMP using the hand reaction force/torque is introduced in the following subsection while the ZMP using the dynamics of the object will be introduced in the next section to generate the walking pattern when carrying an object. We also note that, since the dynamics of the object is not usually known in advance, it is identified through experiments.

### C. Compensation of robot's motion

In this section, we assume that the motion of a humanoid robot has been generated without taking the carried object into consideration. Under such situation, we consider the method for modifying the motion of the robot when carrying a heavy object. First, let  $\bar{p}_Z = [\bar{x}_Z \ \bar{y}_Z \ \bar{z}_Z]^T$  be the position of the ZMP without taking the reaction force/torque at the wrists into consideration. By using eqs.(5) and (7), the relationship between  $p_Z$  and  $\bar{p}_Z$  are given by

$$x_Z - \bar{x}_Z = \sum_{j=1}^2 \frac{(z_{Hj} - z_Z) f_{Hj}^{(x)} + (x_Z - x_{Hj}) f_{Hj}^{(z)}}{M(\ddot{z}_G + g)}, \quad (9)$$

$$y_Z - \bar{y}_Z = \sum_{j=1}^2 \frac{(z_{Hj} - z_Z) f_{Hj}^{(y)} + (y_Z - y_{Hj}) f_{Hj}^{(z)}}{M(\ddot{z}_G + g)} \quad (10)$$

Under the static conditions, let us consider the physical interpretation of eqs.(9) and (10). Fig.3 shows the change of the ZMP position of a humanoid robot when carrying an object. As shown in Fig.3(a), when a humanoid robot does not carry an object,  $p_Z = \bar{p}_Z$  is satisfied. On the other hand, when pushing a large object as shown in Fig.3, the position of the ZMP will shift to the back of the robot. In this case, the displacement of the ZMP position will increase as the hand reaction force in the horizontal direction  $f_{Hj}^{(x)}$  and the height of the hand position  $(z_{Hj} - z_Z)$  becomes larger. Also, when carrying an object as shown in Fig.3(b), the position of the ZMP will shift to the front of the robot. In this case, the displacement of the ZMP position will increase as the magnitude of the hand reaction force in the vertical direction  $f_{Hj}^{(z)}$  becomes larger and the amount of displacement in the horizontal direction between the ZMP and the hand becomes smaller. Let  $l_t$  and  $l_h$  be the distance between the current ZMP and the toe and between the current ZMP and the heel, respectively(Fig.3(d)). For given position of the ZMP, the maximum amount of the hand reaction force where the humanoid robot can regist is given by

$$f_{\max}^{(z)} = \frac{Mgl_t}{x_{H1} - x_Z}, \quad (11)$$

where we assume  $x_{H1} = x_{H2}$ .

To avoid the humanoid robot falling down when carrying an object, we consider modifying the position of the waist according to the amount of force applied at the hands. When the acceleration generated by the robot is small, the amount of modification of the waist position is obtained from eqs.(9) and (10):

$$\Delta x_B = - \sum_{j=1}^2 \frac{(x_Z - x_{Hj}) f_{Hj}^{(z)}}{Mg}, \quad (12)$$

$$\Delta y_B = - \sum_{j=1}^2 \frac{(y_Z - y_{Hj}) f_{Hj}^{(z)}}{Mg}. \quad (13)$$

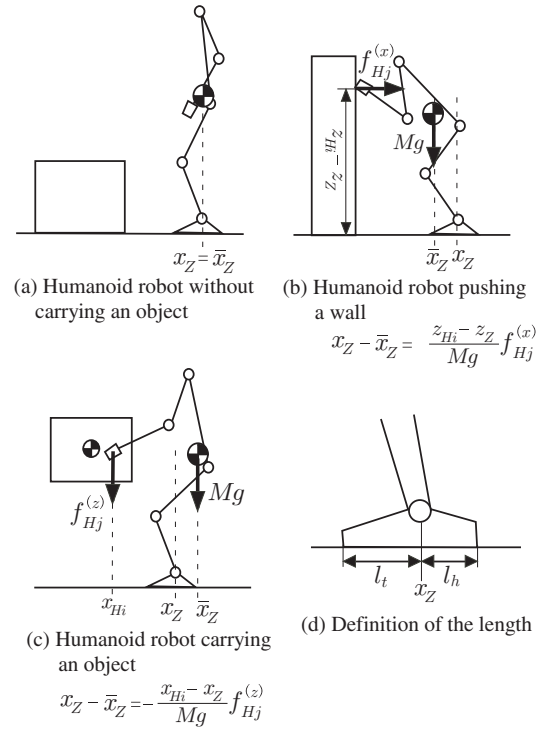


Fig. 3. Compensation of the waist position

Here, we note that, recently, research has been done on the pushing manipulation by a humanoid robot such as [5], [6], [11] where the reaction force in the horizontal direction becomes significant. However, in case of the lifting motion considered in this reseach, the hand reaction force in the vertical direction becomes significant, and the position of the ZMP cahnges mainly due to the reaction force in the vertical direction.

### IV. GAIT PATTERN GENERATION

After lifting up, a humanoid robot walks with carrying an object to place the object at the desired position. Here, if the walking pattern is generated without taking the carried object into consideration, the humanoid robot may fall down since the walking pattern without considering the effect of the carried object is different from the one with considering such effect. In this section, we outlines how the walking pattern changes if the robot carries an object.

Before the robot carries an object, since the robot usually does not know the physical parameters of the object such as the weight and the position of the center of gravity, they have to be identified. While the difficulty of the identification of the physical parameters is shown in the appendix, the position of the COG of the object within the horizontal plane can be identified by the following equation if the acceleration applied on the robot is slow enough.

$$\bar{x}_o = \frac{\sum_{j=1}^k x_Z^{(j)} + M(x_Z^{(j)} - x_G)/\bar{m}_o}{k}, \quad (14)$$

$$\bar{y}_o = \frac{\sum_{j=1}^k y_Z^{(j)} + M(y_Z^{(j)} - y_G)/\bar{m}_o}{k}, \quad (15)$$

where  $x_Z^{(j)}$  and  $y_Z^{(j)}$  denote the  $j$ -th measurement of the position of ZMP.

By using the result of identification of the object's physical parameters, the walking pattern is generated. Without loss of generality, let us consider the motion of the robot within the sagittal plane. Assuming that the angular acceleration about the COG is small enough in eq.(6), the position of the ZMP caused by the horizontal motion of the COG satisfies

$$x_Z = \tilde{x}_G - \frac{(\tilde{z}_G - z_Z)\ddot{\tilde{x}}_G}{g}, \quad (16)$$

where  $\mathbf{p}_G = (M\mathbf{p}_G + \bar{m}_o\mathbf{p}_o)/(M + \bar{m}_o)$  and  $\mathbf{p}_G = [\tilde{x}_G \ \tilde{y}_G \ \tilde{z}_G]^T$ . Also,  $\bar{m}_o$  denotes the result of identification of the object's mass. Let us also assume that the desired ZMP trajectory is given as a spline function:

$$x_Z^{(j)} = \sum_{i=0}^n a_i^{(j)}(t - t_{j-1})^i, \quad (17)$$

$$t_{j-1} \leq t \leq t_j, \quad j = 1, \dots, m,$$

where  $x_Z^{(j)}$  denotes the ZMP trajectory included in the  $j$ -th segment of time. The trajectory of robot's COG in the  $j$ -th segment of time can be obtained by substituting eq.(17) into eq.(16):

$$x_G^{(j)} = \left\{ V^{(j)} \cosh(T_c(t - t_{j-1})) + W^{(j)} \sinh(T_c(t - t_{j-1})) + \sum_{i=0}^n A_i^{(j)}(t - t_{j-1})^i \right\} \frac{M + \bar{m}_o}{M} - \tilde{x}_o \frac{\bar{m}_o}{M}, \quad (18)$$

$$j = 1, \dots, m$$

$$a_i^{(j)} = A_i^{(j)} - \frac{1}{T_c^2}(i+1)(i+2)A_{i+2}^{(j)}, \quad i = 0, \dots, n-2 \quad (19)$$

$$a_i^{(j)} = A_i^{(j)}, \quad i = n-1, n, \quad (20)$$

where  $T_c = \sqrt{g/(\tilde{z}_G - z_Z)}$ , and  $V^{(j)}$  and  $W^{(j)}$  ( $j = 1, \dots, m$ ) denote the scalar coefficients which can be obtained by using the two-point boundary value problem[14]. When  $\bar{m}_o = 0$  and  $\mathbf{p}_G = \tilde{\mathbf{p}}_G$ , eq.(18) becomes same as that of a humanoid robot without carrying an object. Therefore, eq.(18) shows an extension of the walking pattern generation to the case where a humanoid robot carries an object.

## V. EXPERIMENT

We performed experiments of the lifting-up motion by using the humanoid robot HRP-2[1]. The total dof of HRP-2 is 30. Also, the height and the weight of the HRP-2 are  $h = 1.539$ [m] and  $M = 58$ [kg], respectively. The overview of the lifting-up controller is shown in Fig.4. As shown in the figure, first, the whole body motion satisfying the statical balance of the robot is generated without considering the hand reaction force/torque. Then, the waist position is modified based on the force/torque sensor informations attached at the wrists. The humanoid

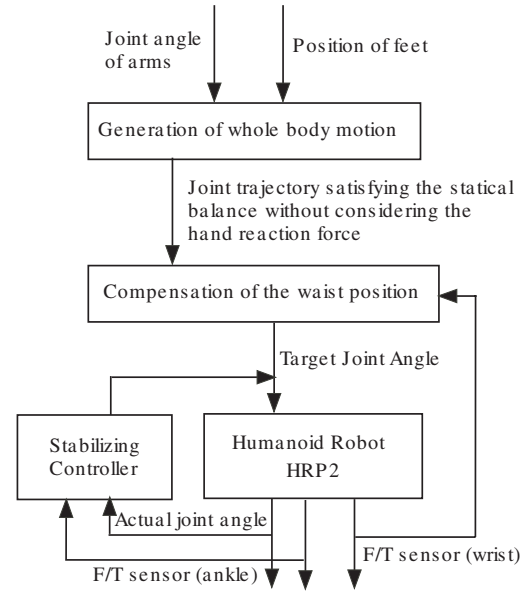


Fig. 4. Block diagram of the controller

robot HRP-2 is driven according to the desired joint data along with the stabilizing controller regulating the actual ZMP position to the desired one.

Fig.5 shows the snap-shot of the experiment. The robot first squats down(Fig.5(a) and (b)). Then, the robot captures an object(Fig.5(c), (d) and (e)). The robot further lifts up the object (Fig.5(f),(g) and (h)). In the experiment, the weight of the carried object was set as  $m_o = 8.5$ [kg].

Fig.6 shows the output of the force applied at the wrists. Here, the  $z$  direction coincides with the vertical direction. The force in the  $y$  direction corresponds to the internal force to grasp the object stably where the amount of the internal force is about  $f_i = 6$  [N]. Also, Fig.7 shows the position of the ZMP during the lifting-up motion. While the position of the ZMP shifts about 0.04[m] during the lifting-up motion (between 8 and 10 seconds), the position of the ZMP does not shift out of the foot supporting area and the robot keeps balance.

On the other hand, Fig.8 shows the simulation result without compensating the waist position according to the hand reaction force. As shown in the figure, the robot cannot keep balance and falls down.

Fig. 9 shows a simulation result of walking with carrying an object. We can see that the humanoid robot carries an object without falling down.

## VI. CONCLUSIONS

In this paper, we discussed the problem of the lifting-up motion by a humanoid robot. By considering the information of the force/torque sensors attached at the wrists of the robot, the position of the waist was modified. Also, the walking pattern generation when carrying an object was discussed. The effectiveness of the proposed method was confirmed by simulation and experiments.

Now we are performing the walking experiment with carrying an object. Measurement of the position of the object



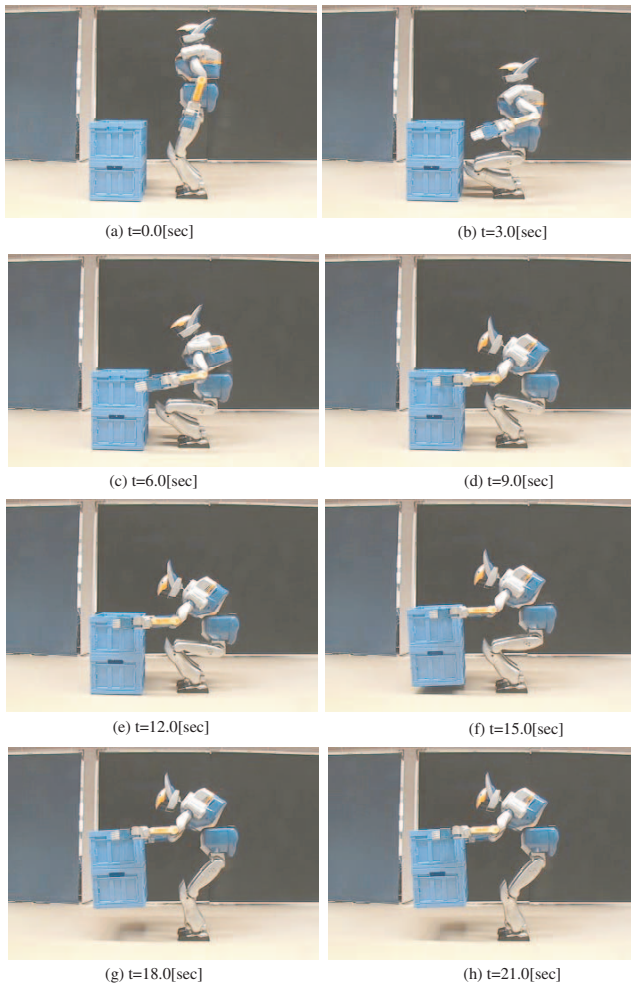


Fig. 5. Snap-shot of Experiment

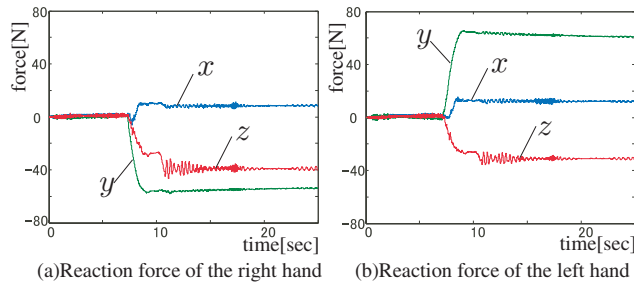


Fig. 6. Experimental result of the force sensor information at the wrists

by using camera system is considered to be our future research topic.

#### REFERENCES

- [1] K. Kaneko et al.: "The Humanoid Robot HRP2", Proc. of IEEE Int. Conf. on Robotics and Automation, 2004.
- [2] S. Kajita et al.: "Resolved Momentum Control: Humanoid Motion Planning based on the Linear and Angular Momentum", Proc. of IEEE/RSJ Int. Conf. Intelligent Robots and Systems, pp. 1644-1650, 2003.
- [3] K. Nishiwaki, S. Kagami, Y. Kuniyoshi, M. Inaba, and H. Inoue: "Online Generation of Humanoid Walking Motion based on a Fast Generation Method of Motion Pattern that Follows Desired ZMP", Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 2684-2689, 2002.

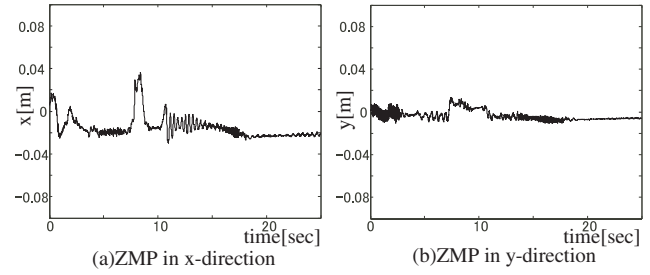


Fig. 7. Experimental results of the ZMP position

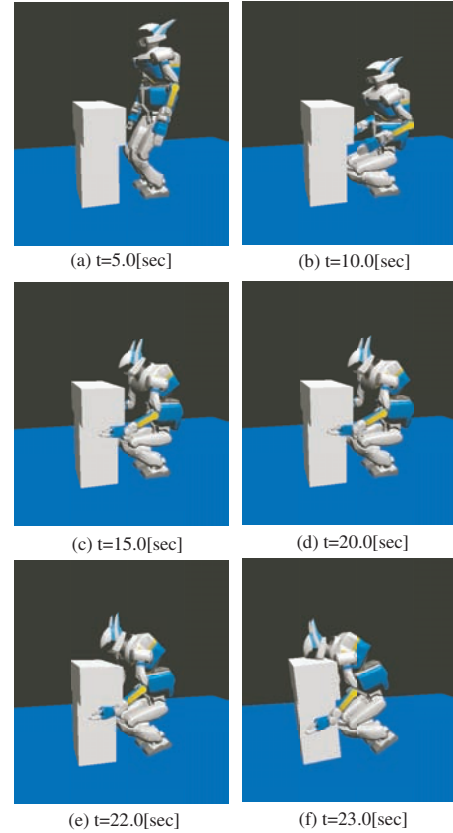


Fig. 8. Simulation results without compensating the waist position

- [4] M. Fujita, Y. Kuroki, T. Ishida, and T. Doi: "Autonomous Behavior Control Architecture of Entertainment Humanoid Robot SDR-4X", Proc. of IEEE/RSJ Int. Conf. Intelligent Robots and Systems, pp. 960-967, 2003.
- [5] K. Harada, S. Kajita, K. Kaneko, and H. Hirukawa: "Pushing Manipulation by Humanoid considering Two-Kinds of ZMPs", Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1627-1632, 2003.
- [6] K. Harada, S. Kajita, F. Kanehiro, K. Fujiwara, K. Kaneko, K. Yokoi, and H. Hirukawa: "Real-Time Planning of Humanoid Robot's Gait for Force Controlled Manipulation", Proc. of 2004 IEEE Int. Conf. on Robotics and Automation, pp. 616-622, 2004.
- [7] K. Harada, S. Kajita, K. Kaneko, and H. Hirukawa: "ZMP Analysis for Arm/Leg Coordination", Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 73-79, 2003.
- [8] K. Yokoyama, H. Handa, T. Isozumi, Y. Fukase, K. Kaneko, F. Kanehiro, Y. Kawai, F. Tomita, and H. Hirukawa: "Cooperative Works by a Human and a Humanoid Robot", Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 2985-2991, 2003.
- [9] K. Inoue, H. Yoshida, T. Arai, and Y. Mae: "Mobile Manipulation of Humanoids -Real-Time Control Based on Manipulability and Stability-", Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 2217-2222, 2000.

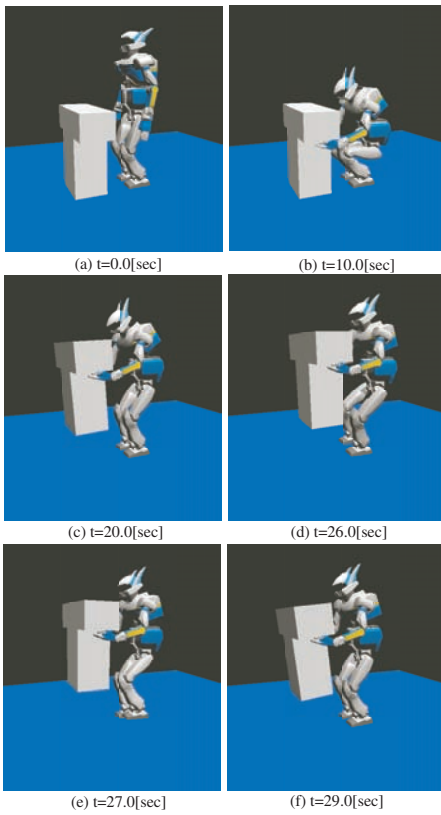


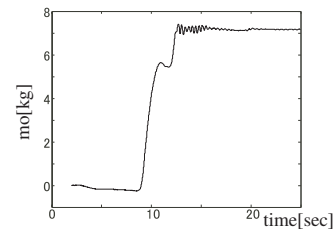
Fig. 9. Simulation results of stepping with carrying an object

- [10] Y. Nishihara, K. Inoue, T. Arai, and Y. Mae: “*Mobile Manipulation of Humanoid Robots –Control method for Accurate Manipulation–*”, Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 1914-1919, 2003.
- [11] Y. Hwang, A. Konno, and M. Uchiyama: “*Whole Body Cooperative Tasks and Static Stability Evaluations for a Humanoid Robot*”, Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 1901-1906, 2003.
- [12] Y. Yokokohji, S. Nomoto, and T. Yoshikawa: “*Static Evaluation of Humanoid Robot Postures Constrained to the Surrounding Environment through their Limbs*”, Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1856-1863, 2002.
- [13] M. Vukobratovic and D. Juricic: “*Contribution to the Synthesis of Biped Gait*”, IEEE Trans. on Bio-Med. Eng., vol. BME-16, no. 1, pp. 1-6, 1969.
- [14] K. Harada, S. Kajita, M. Morisawa, F. Kanehiro, K. Fujiwara, K. Kaneko, and H. Hirukawa: “*Analytical Approach on Real-time Gait Planning by a Humanoid Robot*”, Proc. of 2004 IEEE-RAS Int. Conf. on Humanoid Robotics, 2004.
- [15] T. Arai and Y. Nakamura: “*Estimation of Mass and Center of Gravity for Unknown Object using 6 Axis F/T Sensor Data*”, Proc. of Int. Conf. on Advanced Robotics, pp. 547-552, 1995.

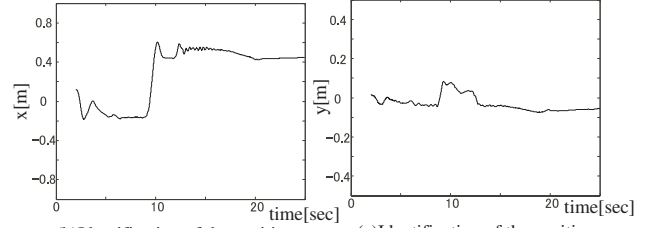
#### APPENDIX

This section discusses on the identification of the physical parameters of the carried object based on the force/torque sensors attached at the wrists and the ankles. First, the mass of the object can be identified if the force/torque sensors are attached at the wrists. By using the least squares method, the mass of the carried object can be identified by

$$\bar{m}_o = \frac{\sum_{j=1}^k \sum_{i=1}^2 \mathbf{g}^T \mathbf{f}_{Hi}^{(j)}}{k \mathbf{g}^T \mathbf{g}}, \quad (21)$$



(a) Identification of the mass



(b) Identification of the position object's COG in x-direction (c) Identification of the position object's COG in y-direction

Fig. 10. Identification of the object's physical parameters

where  $k$  and  $\mathbf{f}_{Hi}^{(j)}$  denote the number of measured data and the  $j$ -th measurement of the hand reaction force, respectively.

On the other hand, the identification of the COG of the carried object is not straightforward. When the object does not move, the relationship between the measured force/torque at the wrists and the COG of the object are expressed as:

$$\sum_{i=1}^2 \mathbf{f}_{Hi} = m_o \mathbf{g}, \quad (22)$$

$$\sum_{i=1}^2 (\mathbf{p}_{Hi} \times \mathbf{f}_{Hi} + \boldsymbol{\tau}_{Hi}) = m_o \mathbf{p}_o \times \mathbf{g}. \quad (23)$$

By solving the above equations, we find that the position of the COG of the object cannot be uniquely obtained[15]:

$$\mathbf{p}_o = \frac{\sum_{i=1}^2 (\mathbf{p}_{Hi} \times \mathbf{f}_{Hi} + \boldsymbol{\tau}_{Hi}) \times \mathbf{g}}{\mathbf{g}^T \mathbf{g}} + \mathbf{g} \phi, \quad (24)$$

where  $\phi$  denotes an arbitrary scalar parameter. Eq.(24) shows that the horizontal component of  $\mathbf{p}_o$  can be uniquely determined while the vertical component cannot. Therefore, to identify the position of the COG, the force/torque has to be measured at some different postures of the object[15].

Fig.10 shows the result of identification of the physical parameters of the carried object during the experiment shown in Fig.4. Since the orientation of the object does not change, it is difficult to identify the position of the COG in the vertical direction. Thus, as for the COG of the object, we show only the horizontal components. As shown in the figures, we can see that the physical parameters are converged to the desired ones as the humanoid robot lifts up the object.