The Capacitated Vehicle Routing Problem (CVRP):

General Mathematical Formulation:

Sets and Parameters:

- $V = \{0, 1, 2, \ldots, n\}$: Set of vertices, where 0 represents the depot.
- $C = \{1, 2, \ldots, n\}$: Set of customers.
- $K = \{1, 2, \ldots, m\}$: Set of vehicles.
- A: Set of arcs, where $A = \{(i, j) \mid i, j \in V, i \neq j\}$.
- q: Capacity of each vehicle.
- d_i: Demand of customer i.
- c_{ij}: Cost (distance) of traveling from node i to node j.

Decision Variables:

- x_{ijk}: Binary variable, where $x_{ijk} = 1$ if vehicle k travels directly from node i to node j, and 0 otherwise.
- y_{ik}: Binary variable, where $y_{ik} = 1$ if vehicle k visits customer i, and 0 otherwise.
- q_{ik}: Continuous variable representing the load of vehicle k after servicing customer i.

Objective Function:

Minimize the total cost of the routes of all the vehicles:

$$\text{Minimize} \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk}$$

Constraints:

1. Each customer is visited exactly once:

$$\sum_{k \in K} \sum_{j \in V, j \neq i} x_{ijk} = 1 \quad \forall i \in C$$

$$\sum_{k \in K} \sum_{i \in V, i \neq j} x_{ijk} = 1 \quad \forall j \in C$$

2. Vehicle capacity:

$$\sum_{i \in C} d_i y_{ik} \leq q \quad \forall k \in K$$

3. Flow conservation constraint:

$$\sum_{j \in V, j \neq i} x_{ijk} - \sum_{j \in V, j \neq i} x_{jik} = 0 \quad \forall i \in V, \forall k \in K$$

4. Depot departure and return constraints:

$$x_{0jk} = 1 \quad \forall k \in K$$

$$x_{i0k} = 1 \quad \forall k \in K$$

5. Load constraints:

$$q_{ik} \geq d_i \quad \forall i \in C, \forall k \in K$$

$$q_{jk} \geq q_{ik} + d_j - q(1 - x_{ijk}) \quad \forall i, j \in C, \forall k \in K$$

6.Binary and non-negativity constraints:

$$x_{ijk} \in \{0, 1\} \quad \forall (i, j) \in A, \forall k \in K$$

$$y_{ik} \in \{0, 1\} \quad \forall i \in C, \forall k \in K$$

$$q_{ik} \geq 0 \quad \forall i \in C, \forall k \in K$$