

# The Capacitated Vehicle Routing Problem (CVRP):

## General Mathematical Formulation:

### Sets and Parameters:

- $V = \{0, 1, 2, \dots, n\}$ : Set of vertices, where 0 represents the depot.
- $C = \{1, 2, \dots, n\}$ : Set of customers.
- $K = \{1, 2, \dots, m\}$ : Set of vehicles.
- $A$ : Set of arcs, where  $A = \{(i, j) \mid i, j \in V, i \neq j\}$ .
- $q$ : Capacity of each vehicle.
- $d_i$ : Demand of customer  $i$ .
- $c_{ij}$ : Cost (distance) of traveling from node  $i$  to node  $j$ .

### Decision Variables:

- $x_{ijk}$ : Binary variable, where  $x_{ijk} = 1$  if vehicle  $k$  travels directly from node  $i$  to node  $j$ , and 0 otherwise.
- $y_{ik}$ : Binary variable, where  $y_{ik} = 1$  if vehicle  $k$  visits customer  $i$ , and 0 otherwise.
- $q_{ik}$ : Continuous variable representing the load of vehicle  $k$  after servicing customer  $i$ .

### Objective Function:

Minimize the total cost of the routes of all the vehicles: