Midterms_Khafaji_2

March 24, 2025

[1]:
```
import numpy as np
import polars as pl
from sympy import *
import scipy.interpolate as spi
import matplotlib.pyplot as plt
```

Construct a clamped cubic spline for the following data

[2]:
```
x_vals = [-1, -0.5, 0, 0.5]
f_vals = [0.86199480, 0.95802009, 1.0986123, 1.2943767]
f_prime_vals = [0.15536240, 0.23269654, 0.33333333, 0.45186776]
```

[3]:
```
vals = {"x": x_vals, "f(x)": f_vals, "f'(x)": f_prime_vals}
pl.DataFrame(vals, strict=False)
```

[3]:
```
shape: (4, 3)

  x      f(x)        f'(x)
  ---    ---         ---
  f64    f64         f64

  -1.0   0.8619948   0.1553624
  -0.5   0.95802     0.232697
  0.0    1.0986123   0.333333
  0.5    1.2943767   0.451868
```

[4]:
```
cs = spi.CubicSpline(x=x_vals,
                     y=f_vals,
                     bc_type=((1, f_prime_vals[0]), (1, f_prime_vals[-1]))
                     )
```

[5]:
```
a = cs.c[3, :]
b = cs.c[2, :]
c = cs.c[1, :]
d = cs.c[0, :]

for i in range(len(x_vals) - 1):
```

```
    print(f"S_{i}(x) = {a[i]} + {b[i]}*(x - {x_vals[i]}) + {c[i]}*(x -␣
↪{x_vals[i]})^2 + {d[i]}*(x - {x_vals[i]})^3")
    print("")
```

```
S_0(x) = 0.8619948 + 0.1553624*(x - -1) + 0.06537407999999989*(x - -1)^2 +
0.0160045600000005*(x - -1)^3

S_1(x) = 0.95802009 + 0.23273990000000028*(x - -0.5) + 0.08938092000000053*(x -
-0.5)^2 + 0.015016239999998682*(x - -0.5)^3

S_2(x) = 1.0986123 + 0.3333829999999998*(x - 0) + 0.11190527999999877*(x - 0)^2
+ 0.008772640000001886*(x - 0)^3
```

[6]:
```
fig, ax = plt.subplots(figsize=(6.5, 4))

x_plot_vals = np.linspace(-1, 0.5)
ax.plot(x_vals, f_vals, 'o', label='data')
ax.plot(x_plot_vals, cs(x_plot_vals), label="S")
ax.set_xlim(-1.5, 1)
ax.legend(loc='lower right')
plt.show()
```