FEEDBACK CONTROL SYSTEMS LAB (EE 361L) PROJECT

BALL ON A BEAM

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ABSTRACT

The ball and a beam experiment is a classic example of a control system in which a ball is controlled to follow a specific path and settle itself on a desired setpoint. This project aims to study the behavior of a control system and understand the principles of feedback control. The experiment involves a beam that is sloped at an angle, and a ball is placed on the beam. A DC motor is used to move the beam, and an IR sensor is used to measure the ball's position on the beam. The experiment's goal is to control a nonlinear system that is unstable in an open-loop configuration and tune it in a closed-loop design to keep the ball in a specific position despite any disturbances that may be present. The experiment results can be used to demonstrate the effectiveness of feedback control systems using different control setups in maintaining stability and precision. This experiment is a valuable tool for teaching control systems theory and developing practical control engineering skills.

INTRODUCTION

Inverted Pendulum

Background Work

The inverted pendulum is a classic example in control theory, and it is commonly used as a benchmark problem to test and evaluate control algorithms. The inverted pendulum consists of a pole that is balanced in an inverted position on a moving cart. The goal of the control system is to stabilize the pendulum in the upright position by using sensors and encoders to control the cart's motion. This problem is challenging because the system is unstable, and small disturbances can cause the pendulum to fall

Main Motivation.

The main motivation for doing an inverted pendulum experiment using control systems is to learn and develop control techniques that can be applied to other systems with similar characteristics, such as robotics, aerospace, and automotive control.

Ball on a Beam

Background Work

Ball on a beam experiment is a famous example in the field of control systems. The experiment involves using a control system involving an IR sensor and potentiometer to balance a ball on a beam in a stable position. This experiment has gained popularity over the years due to its simplicity and effectiveness in demonstrating the concepts of control systems.

Main Motivation,

The primary motivation for doing a ball on a beam project is to provide an understanding of how control systems work in real-life applications such as robots, airplanes, and spacecraft.

Additionally, the ball on a beam project can be used to explore advanced control systems techniques, such as non-linear control and adaptive control, which are crucial in modern-day engineering applications.

We have chosen the “Ball on a Beam” project because we are more interested in gathering data from two different sensors and calibrating them in a way such as controlling the ball's position on the beam most efficiently.

METHODOLOGY

Calibration\of Sensors

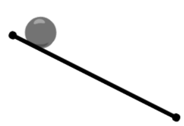
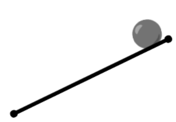
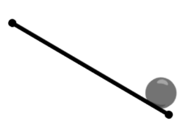
The setup contains a proximity sensor to track the ball's position along the beam. Similarly, it includes a potentiometer to follow the angle of the beam along its point of contact on the stand. The setpoints are determined from the readings obtained at the analog input (Pins A1 and A15) of the Arduino microcontroller. (*Note: The values vary with each setup*). A low-pass filter block removes the unwanted noise from the sensor inputs. We aim to achieve these two desired points to ensure the ball is balanced on the center of the beam.

Restriction on the Motor rotation

An upper and lower bound for the motion of the beam is set to restrict the motion of the beam within $\pm 20^{o}$ of the full rotation achievable by the DC motor. This ensures the safety of the apparatus and linearizes the setup as much as possible. Otherwise, the oscillations experienced by the ball at the endpoints is too much to make it settle at the desired position.

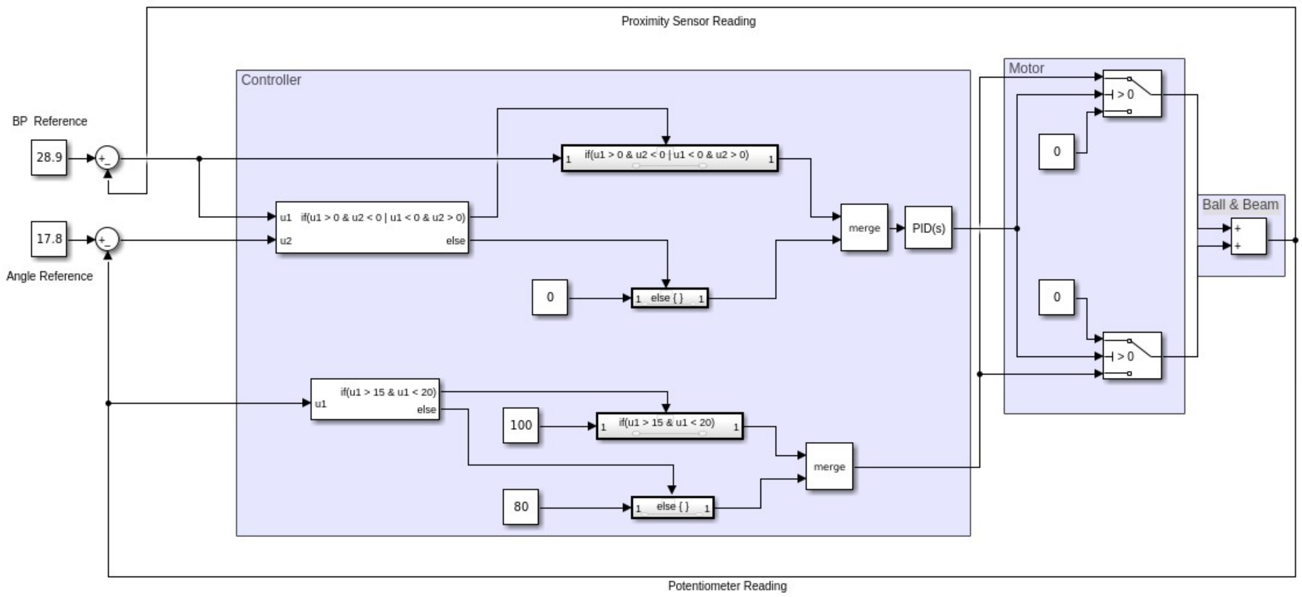
Scenarios requiring actuation from the Motor

There will be four different scenarios (not including the desired scenario) at any point during the experiment. Based on the error recorded after passing through the comparator block, the PID controller generates a control response. Refer to the table below. The magnitude (gain) of the control signal is inversely proportional to the ball's distance from the center point. This ensures minimizing the ball's unwanted oscillations around the center point. Also, note that the motor has to do nothing (set off) in case scenarios II and III because the ball will eventually move to the center by the gravitational force.

Scenarios	θ error	Ball position error	Controller command
	Negative	Positive	Rotate Clockwise
	Positive	Positive	Do Nothing
	Negative	Negative	Do Nothing
	Positive	Negative	Rotate Anticlockwise

To handle these multiple conditions, an if-else action block is used in Simulink. Our implementation works well with a proportional controller.

BLOCK DIAGRAM

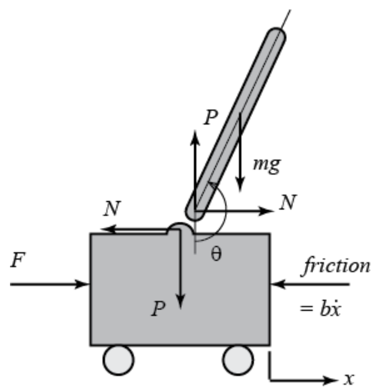


MATHEMATICAL MODEL

Inverted Pendulum

System Parameters:
M - mass of the cart
m - mass of the pendulum
b - coefficient of friction of cart
l - length to pendulum center of mass
I - mass moment of inertia of the pendulum
F - force applied to the cart
x - cart position coordinate
θ - pendulum angle from vertical (down)

Derivation of System equations:
The free-body diagrams of the two systems (inverted pendulum and motorized cart) of the inverted pendulum system is as follows:



Summing the forces in the free-body diagram of the cart in the horizontal direction:

$$F = M\ddot{x} + bx \dot{+} N$$

Similarly, summing the forces in the free-body diagram of the pendulum in the horizontal direction, the following result for reaction force “N” is obtained:

$$N = m\ddot{x} + \mathrm{ml}\ddot{\theta}\cos\theta - \mathrm{ml}\theta^2\sin\theta$$

Substituting this into the first equation:

$$F = (M + m)\ddot{x} + b\dot{x} + \mathrm{ml}\ddot{\theta}\cos\theta - \mathrm{ml}\ddot{\theta}^2\sin\theta$$

The second equation for motion is obtained by summing the forces perpendicular to the pendulum:

$$P\sin\theta + N\cos\theta - \mathrm{mg}\sin\theta = \mathrm{ml}\,\ddot{\theta} + m\ddot{x}\cos\theta$$

Summing the moments about the centroid of the pendulum we get the following equation:

$$-P\,l\sin\theta - N\,l\cos\theta = I\,\ddot{\theta}$$

Combining these last two equations we get:

$$(I + ml^2)\,\ddot{\theta} \; + \mathrm{mgl}\sin\theta = -\mathrm{ml}\ddot{x}\cos\theta$$

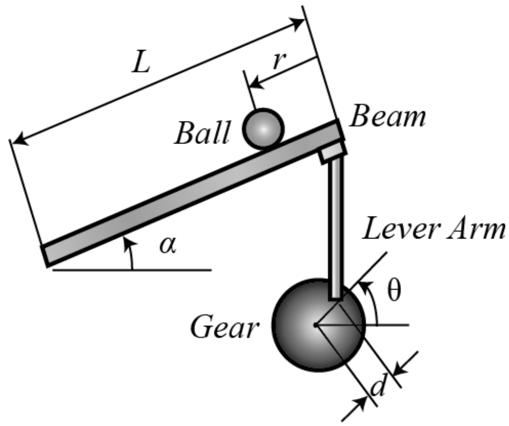
We need to linearize these second order nonlinear differential equations around the equilibrium position $\theta = \pi$. Let ϕ represent a small deviation from the equilibrium. Using small angle approximations of the nonlinear functions:

$$\cos\theta = \cos(\pi + \phi) \approx -1$$
$$\sin\theta = \sin(\pi + \phi) \approx -\phi$$
$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0$$

After substituting the above approximations into the nonlinear governing equations, we arrive at the two linearized equations of the motion:

$$(I + m\,l^2)\ddot{\phi} - \mathrm{mgl}\phi = \mathrm{ml}\ddot{x}$$
$$F = \; (M + m)\ddot{x} + b\dot{x} - \mathrm{ml}\ddot{\phi}$$

Ball on a Beam:



System Parameters:
m - mass of the ball
R - radius of the ball
d - lever arm offset
g - gravitational acceleration
L - length of the beam
J - ball's moment of inertia
r - ball position coordinate
a - beam angle coordinate
θ - servo gear angle

Derivation of Systems equations:
The second derivative of the input angle *a actually affects the second derivative of r.* However, we will ignore this contribution. The Langrangian equation of motion for the ball is then given by the following:

$$0 = \left(\frac{J}{R^2} + m\right)\ddot{r} + m\,g\sin\alpha - m\,r\,\dot{\alpha}^2$$

Linearization of the equation around the beam angle, *a* = 0, gives us the following linear approximation of the system:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -m\,g\,\alpha$$

The equation which relates the beam angle to the angle of the gear can be approximated as linear by the equation below:

$$\alpha = \frac{d}{L}\theta$$

Substituting this in the previous equation gives:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -m\,g\,\frac{d}{L}\theta$$

DETAIL OF COMPONENTS

Sr. No.	Description	Specification	Quantity
1	Base	Wooden	1
2	Proximity Sensor	GPD12	1
3	Beam	Plastic	1
4	Ball	Rubber	1
5	Variable Resistor	10 KΩ	1
6	DC Motor	12 V	1
7	Coupling Shaft	2 Inch	1
8	H-Bridge	-	1
9	Arduino	Mega 2560	1

DELIVERABLES

The outcome of the set-up is shown in two different videos. One without a disturbance and one with a disturbance. As seen in the video, the setup can achieve the desired setpoint in a fairly reasonable amount of time.

PROBLEMS FACED DURING COMPLETION OF PROJECT AND THE LAB AS A WHOLE

- Too many fluctuations in the sensor readings
- Sensor readings too noisy, had to apply physical + digital low pass filters
- Proximity sensor detached frequently from its original position, a discrepancy in readings
- Uneven potentiometer readings
- Too many wires to handle. The circuit short-circuited several times.

SUGGESTIONS FOR IMPROVEMENT IN PROJECT HARDWARE AND THE LAB AS A WHOLE

- Use designated connectors for different connections in the circuit
- Better insulation of the hardware
- Use up-to-date sensors with fewer fluctuations
- Need to update the whole set-up as a whole

References:

Ball on a Beam System Modelling:
https://ctms.engin.umich.edu/CTMS/index.php?example=BallBeam&section=SystemModeling
Inverted Pendulum System Modelling:
https://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=SystemModeling