Analysis of Circuits using Laplace Transforms

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Abstract

We discuss symbolic algebra and analysis of circuits using laplace transforms.We get introduced to sympy module and how to convert expressions in sympy to pylab functions.

1 Introduction

- Sympy helps us do symbolic work.These symbols are defined as below.The symbols are not python variables.

$$w = symbols('omega')$$

- It has Matrix function built into it which helps to define a matrix.M = Matrix([1 2],[3 4]) gives $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.The inv function helps us in computing inverse of a matrix .It is done as M.inv().
- Sympy has evalf function to evaluate value for the symbol as expr = sqrt(2) ,expr.evalf() returns 1.414.
- For magnitude response of laplace transform we use lambdify function.

2 Low pass filter

2.1 Writing the nodal equations:

$$\begin{pmatrix} 0 & 0 & 1 & \frac{-1}{G} \\ \frac{-1}{1+sR_2C_2} & 1 & 0 & 0 \\ 0 & -1000 & 1000 & 1 \\ \frac{-1}{R_1} - \frac{1}{R_2} - sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_P \\ V_m \\ V_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{V_i(S)}{R1} \end{pmatrix}$$

- The transfer of the system $\frac{V_o(s)}{V_i(s)}$ comes from solving for x in Ax=b ie x = inv(A).b
- $V_o(s)$ is in x[3].
- For calculating impulse response of system we give $V_i(s) = 1$ and for step response of system we give $V_i(s) = \frac{1}{s}$.
- Sympy helps in defining matrix with symbols. inv function calculates inverse of matrix.The lambdify takes $V_o(s)$ and converts it into a python function.

```
s = symbols('s')
def lowpass(R1,R2,C1,C2,G,Vi):
    A=Matrix([[0,0,1.0,-1.0/G],[-1.0/(1.0+s*R2*C2),1.0,0,0],
    [0,-1000.0,1000.0,1.0],[-1.0/R1-1.0/R2-s*C1,1.0/R2,0,s*C1]])
    b=Matrix([0,0,0,-Vi/R1])
    V=A.inv()*b
    return (A,b,V)
```

2.2 Plot magnitude response

- we define range of w for which magnitude response to be calculated using logspace.
- Since we have transfer function as function of s. For bode plot we put s=jw.The corresponding plot is in figure 1

```
A,b,V=lowpass(10000.0,10000.0,1e-9,1e-9,1.586,1.0)
Vo=V[3]
print simplify(Vo)
w=p.logspace(0,8,801)
ss= 1j*w
hf=lambdify(s,Vo,'numpy')
v=hf(ss)
p.loglog(w,abs(v),lw=2)
p.title('Low pass filter')
p.xlabel('$\omega(rad/s)$')
p.ylabel('$Magnitude$')
p.grid(True)
p.show()
```

2.3 Unit step response of low pass filter.

- For unit step response since we have H(s) , output Y(s) = H(s)X(s). For unit step X(s) = 1/s.Therefore Y(s)= H(s)/s.
- We can find the corresponding coefficients of Y(s) and give as input to sp.lti function which gives y(t).
- Next we plot output y(t) vs t in figure 2

```
#unit step response
H=sp.lti([0.1],[6.31517e-12,8.9455e-7,0.0631517,0])
t,x=sp.impulse(H,None,p.linspace(0,0.0001,1001))
p.title('unit step response of low pass filter')
p.xlabel('t')
p.ylabel('$V_o(t)$')
p.plot(t,x)
p.grid(True)
p.show()
```

2.4 Response for sinusoidal input.

- Given input is $v_i(t) = sin(2000\pi t) + cos(2\times106\pi t)$.The h(t) can be calculted from coefficients of H(s) using sp.lti function.
- sp.lsim gives us output by convolving h(t) with input $v_i(t)$.The plot of response is in figure 3

```
#for sinusoidal input
H=sp.lti([0.1],[6.31517e-12,8.9455e-7,0.0631517])
t=p.linspace(0.0,1e-2,10001)
u=p.sin(2e3*p.pi*t)+p.cos(2e6*p.pi*t)
t,y,svec=sp.lsim(H,u,t)
p.title('response of low pass filter for sinusoidal input')
p.xlabel('t')
p.ylabel('$V_o(t)$')
p.plot(t,y)
p.grid(True)
p.show()
```

2.5 Plots and discussion

Figure 1: Magnitude response of second order low pass filter

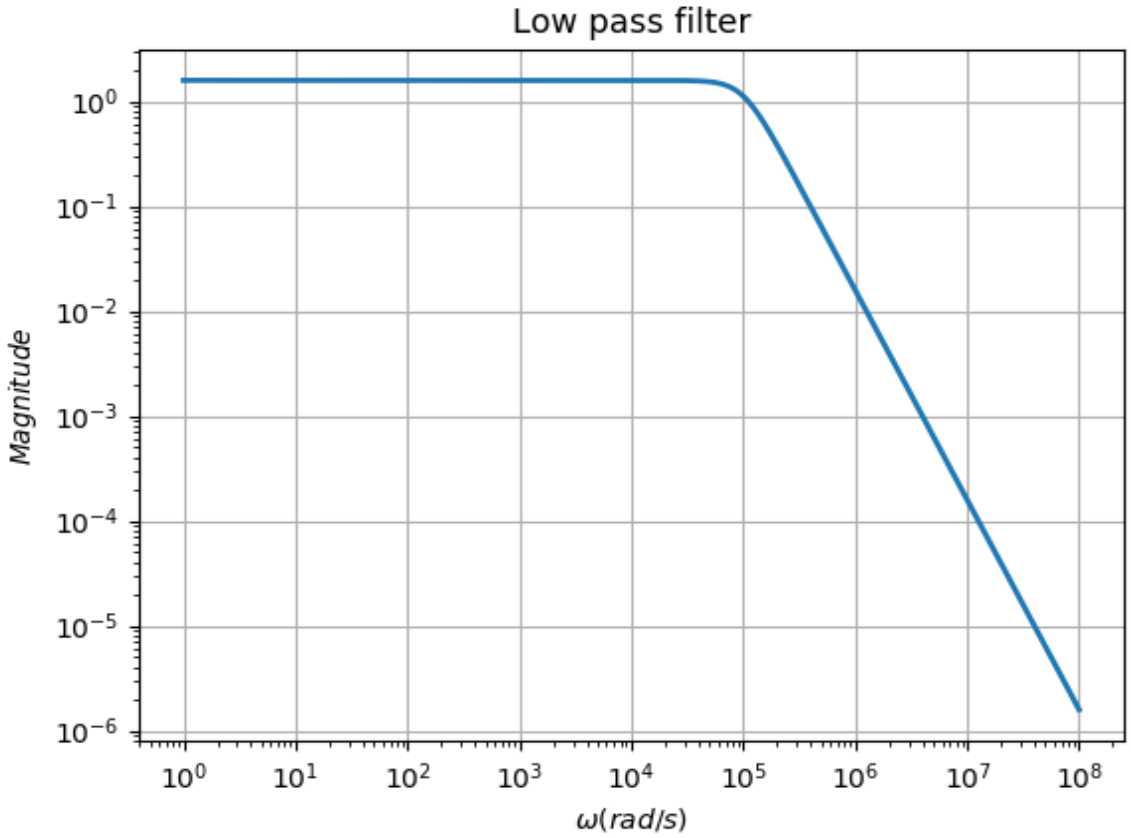


Figure 2: Unit step response of Low pass filter

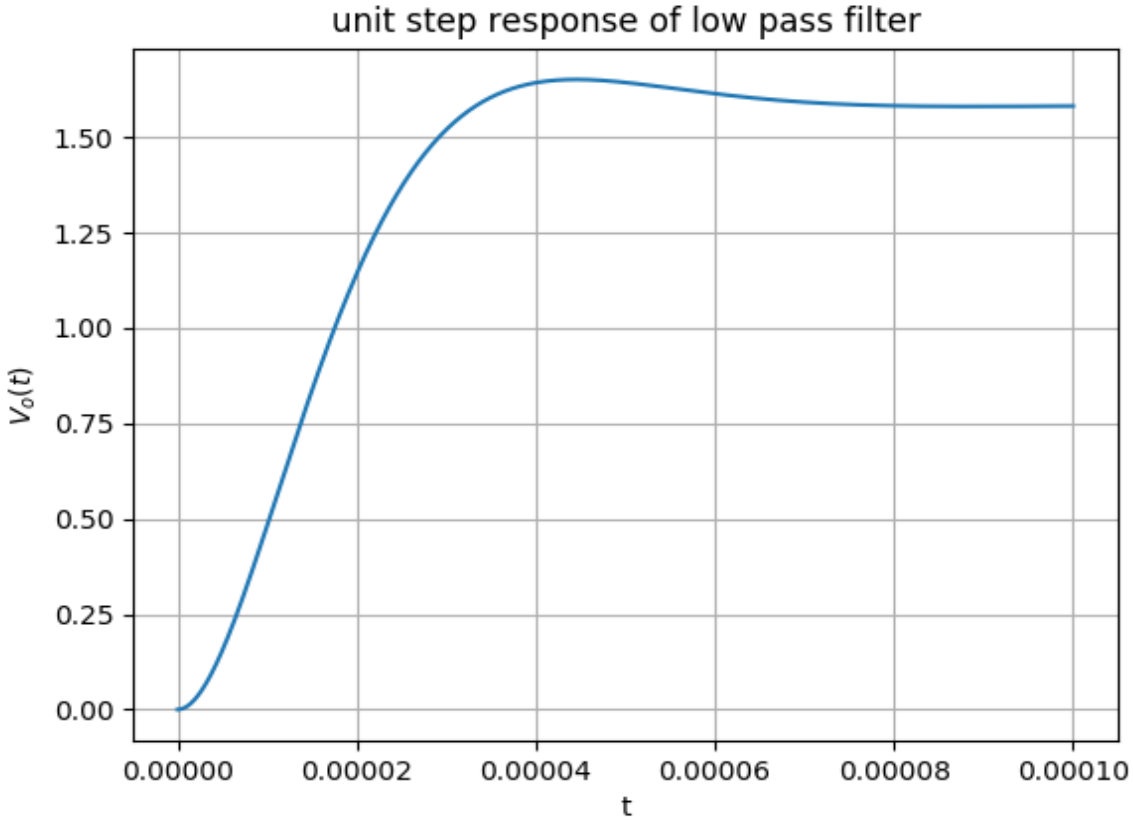
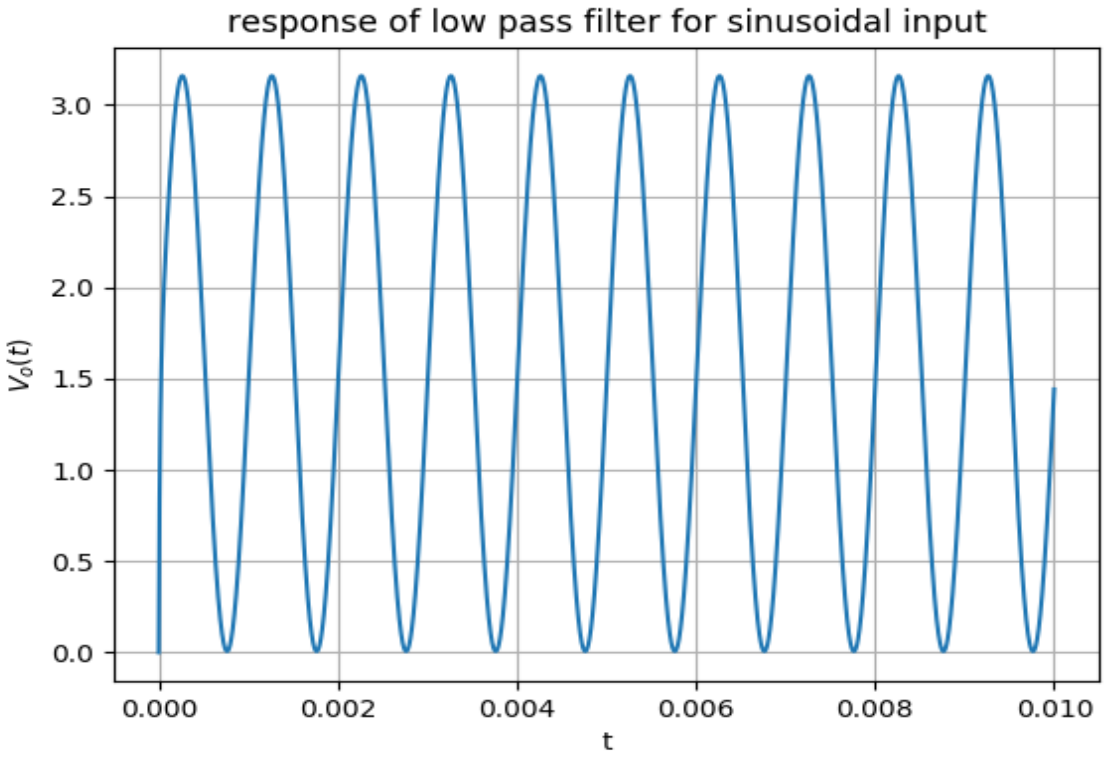


Figure 3: Response of filter for sinusoidal input with two frequencies



- From figure 1 the frequencies greater than 10^5rad/s are attenuated by filter.The lower frequencies are passed with gain of 1.58.The transfer function of this second order filter $\frac{0.1}{6.31517e^{-12}s^2+8.9455e^{-7}s+0.06315}$ clearly dc gain is 1.58 when we put s=0.It is second order low pass filter with cutoff close to

10^5rad/s.These filters are butterworth filters as they are maximally flat with quality factor close to $\frac{1}{\sqrt{2}}$.

- From figure 2 ,since system has damping factor close to $\frac{1}{\sqrt{2}}$,it is underdamped system therefore the response to unit step peaks and then settles to 1.58(dc gain).
- From figure 3 since input has two frequencies $2\pi e^3$and $2\pi e^6$rad/s and our system has cutooff close to 10^5rad/s. The higher frequency is filtered and response will be sinusoid of lower frequency with ripples on it.

3 High pass filter

3.1 Writing the nodal equations:

$$\begin{pmatrix} 0 & 0 & 1 & \frac{-1}{G} \\ \frac{-1}{1+\frac{1}{sC_2R_2}} & 1 & 0 & 0 \\ 0 & 1000 & -1000 & 1 \\ sC_1 - sC_2 - \frac{1}{R_1} & sC_2 & 0 & \frac{1}{R_1} \end{pmatrix} \begin{pmatrix} V_1 \\ V_P \\ V_m \\ V_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)sC_1 \end{pmatrix}$$

- The transfer of the system $\frac{V_o(s)}{V_i(s)}$ comes from solving for x in Ax=b ie x = inv(A).b
- $V_o(s)$ is in x[3].
- For calculating impulse response of system we give $V_i(s) = 1$ and for step response of system we give $V_i(s) = \frac{1}{s}$.
- Sympy helps in defining matrix with symbols. inv function calculates inverse of matrix.The lambdify takes $V_o(s)$ and converts it into a python function.

```
def highpass(R1,R2,C1,C2,G,Vi):
    A=Matrix([[0,0,1.0,-1.0/G],[-1.0/(1.0+1.0/(s*R2*C2)),1.0,0,0],
    [0,1000.0,-1000.0,1.0],[-1.0/R1-s*C1-s*C2,s*C2,0,1.0/R1]])
    b=Matrix([0,0,0,-Vi*s*C1])
    V=A.inv()*b
    return (A,b,V)
```

3.2 Plot magnitude response

- we define range of w for which magnitude response to be calculated using logspace.
- Since we have transfer function as function of s. For bode plot we put s=jw.The corresponding plot is in figure 4

```
A,b,V=highpass(10000.0,10000.0,1e-9,1e-9,1.586,1.0)
Vo=V[3]
print simplify(Vo)
w=p.logspace(0,8,801)
ss= 1j*w
hf=lambdify(s,Vo,'numpy')
v=hf(ss)
p.loglog(w,abs(v),lw=2)
p.title('High pass filter')
p.xlabel('$\omega(rad/s)$')
p.ylabel('$Magnitude$')
p.grid(True)
p.show()
```

3.3 Unit step response of low pass filter.

- For unit step response since we have H(s) , output Y(s) = H(s)X(s). For unit step X(s) = 1/s.Therefore Y(s)= H(s)/s.
- We can find the corresponding coefficients of Y(s) and give as input to sp.lti function which gives y(t).
- Next we plot output y(t) vs t in figure 5

```
#unit step response
H=sp.lti([1e-9,0],[6.2951e-10,8.885e-5,6.29517])
t,x=sp.impulse(H,None,p.linspace(0,0.0001,1001))
p.title('unit step response of high pass filter')
p.xlabel('t')
p.ylabel('$V_o(t)$')
p.plot(t,x)
p.grid(True)
p.show()
```

3.4 Response for sinusoidal input.

- Let input be $V_i(t) = exp(-1e^4t)cos(2e^5\pi t)$.The h(t) can be calculted from coefficients of H(s) using sp.lti function.
- sp.lsim gives us output by convolving h(t) with input $v_i(t)$.The plot of response is in figure 6

```
#for sinusoidal input
H=sp.lti([1e-9,0,0],[6.2951e-10,8.885e-5,6.29517])
t=p.linspace(0,5e-4,1001)
u=p.exp(-1e4*t)*p.cos(2e5*p.pi*t)
t,y,svec=sp.lsim(H,u,t)
p.title('response of high pass filter for damped sinusoidal input')
p.xlabel('t')
p.ylabel('$V_o(t)$')
p.plot(t,y)
p.grid(True)
p.show()
```

3.5 Plots and discussion

Figure 4: Magnitude response of High Pass filter

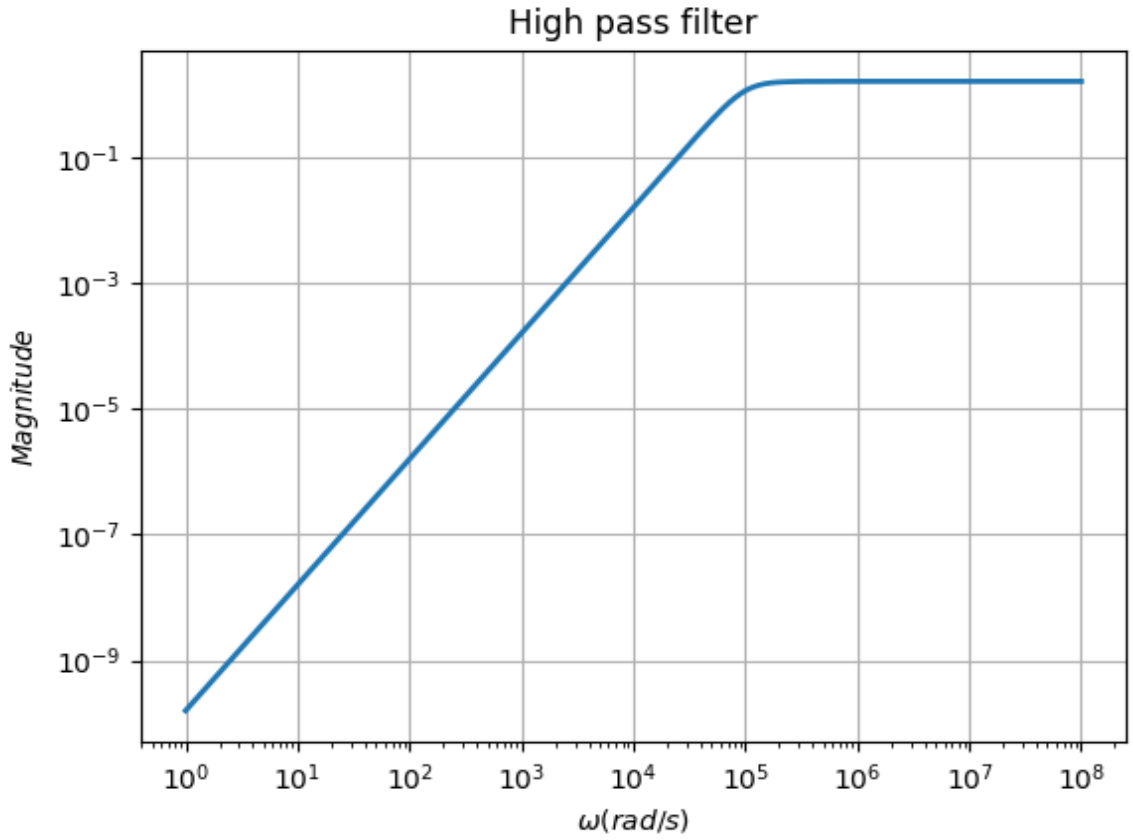


Figure 5: Unit step response of High pass filter

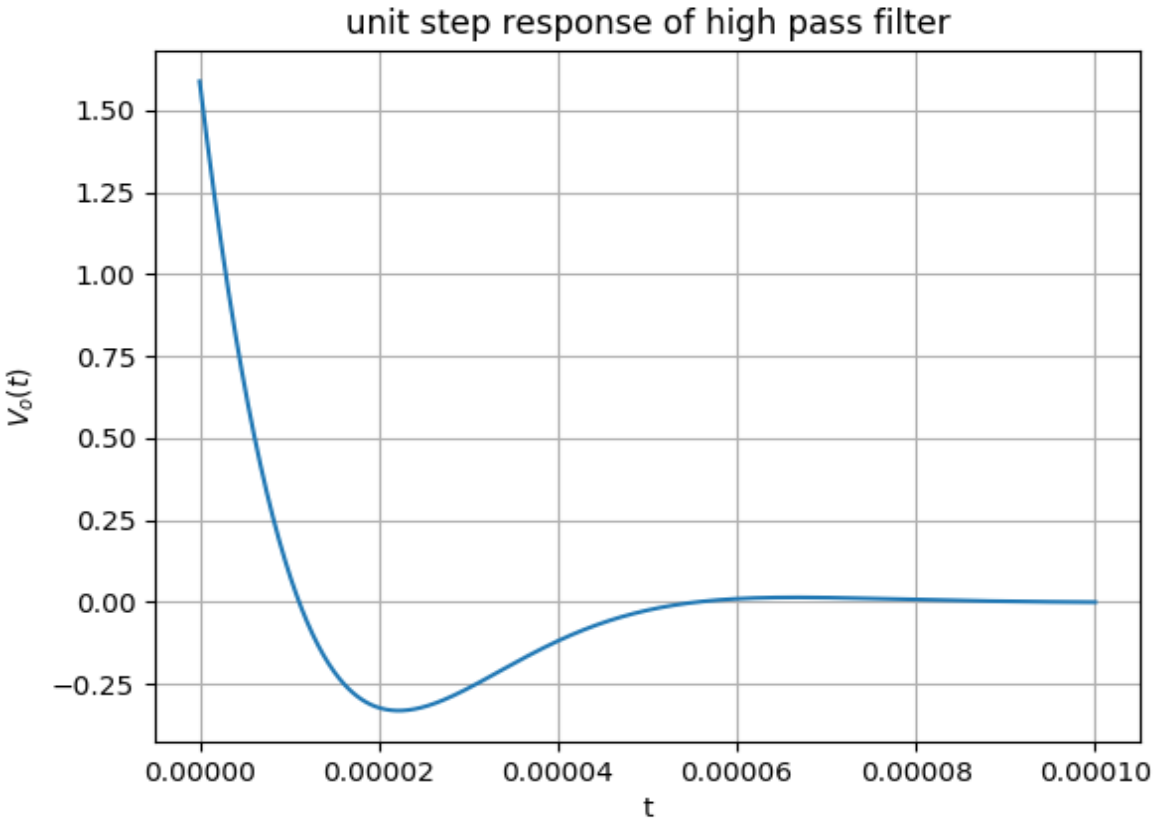
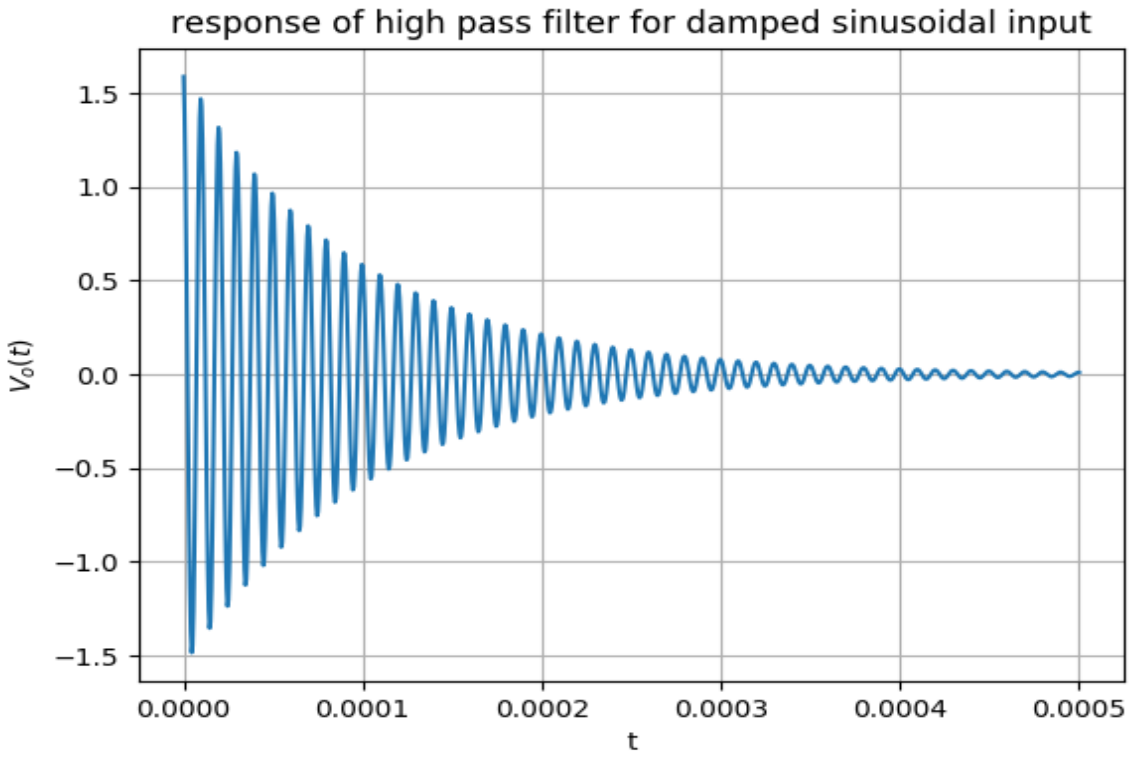


Figure 6: Response of filter for damped sinusoid input $V_i(t) = exp(-1e^4 t)cos(2e^5 \pi t)$



- From figure 4, it is high pass since it filters out low frequencies. It allow frequencies greater than 10^5 with gain of 1.58.The transfer function of second order filter is $\frac{1e-9s^2}{6.2951e^{-10}s^2+8.885e^{-5}s+6.29517}$.
- From figure 5, The output to step response is exactly inverted to that of low pass filter.The output starts from high value and decays to zero with a minimum peak as t increases which is expected because in steady state since input is dc and high pass filters out dc, output should be zero.
- From figure 6, The filter passes out input damped sinusoid as cutoff is less than input frequency but since it decays with time . The output is decaying sinusoid.