Fitting Data to Models

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Abstract

This report presents linear fitting of data and the effect of noise on fitting process.This report assumes two linear models for bessel function of first type.

1 Introduction

1. Bessel function of first type $J_\nu(x)$ for large x can be approximated as,

$$J_\nu(x) \approx \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\nu\pi}{2} - \frac{\pi}{4}) \tag{1}$$

2. The best fit in the least-squares sense minimizes the sum of squared residuals (a residual being: the difference between an observed value, and the fitted value provided by a model).

3. Two linear models taken for bessel function of first type are :

$$A\cos(x) + B\sin(x) \approx J_1(x) \tag{2}$$

$$A\frac{\cos(x)}{\sqrt{x}} + B\frac{\sin(x)}{\sqrt{x}} \approx J_1(x) \tag{3}$$

4. A,B values for modelA(Eqn 2) , modelB(Eqn 3) are estimated in least square sense.

2 Methods

2.1 Get $J_1(x)$ values which is obtained data

1. Generate a vector x of 41 values from 0 to 20 using linspace.

2. Define a function $jv(x)$ to return $J_1(x)$ vector.

3. Below is python code to get $J_1(x)$ vector :

```
#import required packages
from pylab import *
import matplotlib.pyplot as plt
from numpy import *
import scipy.special as sp
#define bessel function
def jv(x):
```

```
        return sp.jv(1,x)
    #define vector x using linspace
    n = 41 #number of observations
    x = linspace(0,20,n)
```

2.2 Estimation of A,B parameters of a Model and νvalues

1. Take an x_0 from 0.5 to 18 . For each x_0 extract a subvector x where $x \geq x_0$ and find vector $J_1(x)$ for that corresponding vector x.

2. for each x_0 fit vector x into models as :

$$\cos(x).A + \sin(x).B = J_1(x)$$

$$\frac{\cos(x)}{\sqrt{x}}.A + \frac{\sin(x)}{\sqrt{x}}.B = J_1(x)$$

3. This reduces to matrix equation of the form $P.\overrightarrow{a} = \overrightarrow{q}$

$$\begin{pmatrix} \cos(x_1) & \sin(x_1) \\ \cos(x_2) & \sin(x_2) \\ \dots & \dots \\ \cos(x_{41}) & \sin(x_{41}) \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} J_1(x_1) \\ J_1(x_2) \\ \dots \\ J_1(x_{41}) \end{pmatrix}$$

$$\begin{pmatrix} \frac{\cos(x_1)}{\sqrt{x_1}} & \frac{\sin(x_1)}{\sqrt{x_1}} \\ \frac{\cos(x_2)}{\sqrt{x_2}} & \frac{\sin(x_2)}{\sqrt{x_2}} \\ \dots & \dots \\ \frac{\cos(x_{41})}{\sqrt{x_{41}}} & \frac{\sin(x_{41})}{\sqrt{x_{41}}} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} J_1(x_1) \\ J_1(x_2) \\ \dots \\ J_1(x_{41}) \end{pmatrix}$$

4. The vector $\overrightarrow{a}$ is estimated by $lstsq(P, q)$ method of python which essentially does $a = inv(P' * P) * P' * q$

5. After getting A,B parameters calculate ϕ from $\cos(\phi) = \frac{A}{\sqrt{A^2+B^2}}$.The value of ν is calculated by equating $\phi = \frac{\nu\pi}{2} + \frac{\pi}{4}$

6. All the above computation is done by calling a function calcnu defined ourselves which takes complete vector x defined under section 2.1 , x_0,model(whether A or B) and returns ν for each x_0

$$nu = calcnu(x, x0, eps, model)$$

7. The eps argument in above calcnu is discussed under section 2.3. For noise less model take eps = 0

8. Append the nu values returned for each x_0 into a list and plot it versus $x_0 range$ ie from 0.5 to 18 .

```
    def calcnu(x,x0,eps,model):
        indices = where(x>=x0)
        #take from x0 to x
        x = x[indices]
        #get bessel function values in q matrix
        q = jv(x)+ eps*randn(size(x))
        #define matrix P
        P = zeros((len(x),2))
        if(model == 'A'):
```

```
        P[:,0] = cos(x)
        P[:,1] = sin(x)
        A,B=lstsq(P,q)[0]
        phi = arccos(A/sqrt(A*A + B*B))
        nu = 2*(phi-pi/4)/pi
        return nu
    if(model == 'B'):
        P[:,0] = cos(x)/sqrt(x)
        P[:,1] = sin(x)/sqrt(x)
        A,B=lstsq(P,q)[0]
        phi = arccos(A/sqrt(A*A + B*B))
        nu = 2*(phi-pi/4)/pi
        return nu

#define vector x using linspace
n = 41 #number of observations
x = linspace(0,20,n)
#for n=41 x0 ranges from 0.5 to 18 in steps of 0.5
x0_range = linspace(0.5,18,36)
nu_listA =[]
nu_listB =[]
nu_listnoiseB = []
for x0 in x0_range:
    nu_A = calcnu(x,x0,0,'A')
    nu_B = calcnu(x,x0,0,'B')
    nu_noiseB = calcnu(x,x0,0.01,'B')
    nu_listA.append(nu_A)
    nu_listB.append(nu_B)
    nu_listnoiseB.append(nu_noiseB)
```

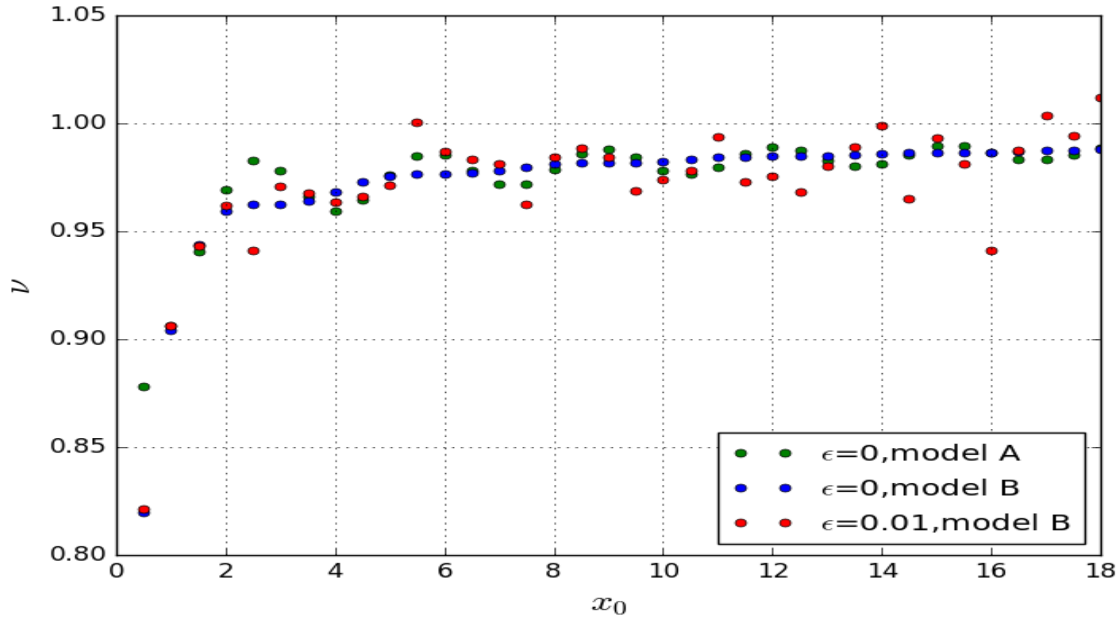
9. The corresponding plots for model A, B are in Figure 1 with blue dots and green dots respectively

2.3 Adding noise to model B

1. The measurements made generally involve noise.
2. To account for noise in model , add $randn(size(x))$ to measured data ie $J_1(x)+eps*randn(size(x))$ where $size(x)$ is number of measurements.
3. This adds normalised noise to measured values with standard deviation of value eps.

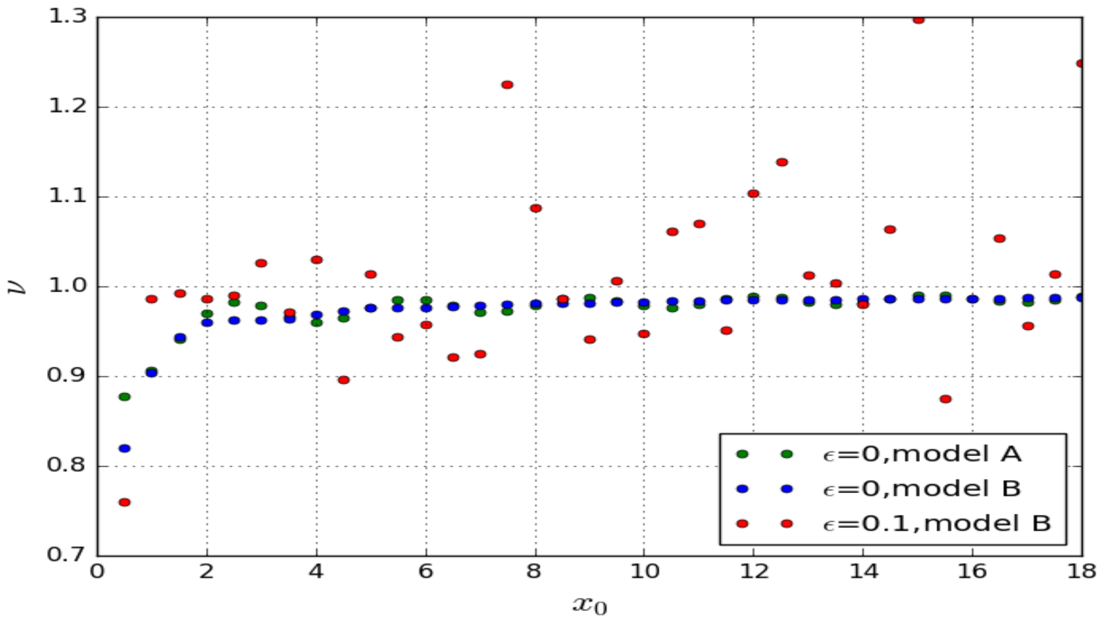
3 Results and discussion

Figure 1: ν vs x_0



1. Model B is better than Model A since it accounts for $\sqrt{x}$ in denominator of amplitude of Eqn 1 . It can be seen from Figure 1.

Figure 2: effect of noise for eps =0.1



2. As noise increases values deviate more from 1 for large x_0 as shown in Figure 2,3.

Figure 3: Effect of noise for eps = 0.05

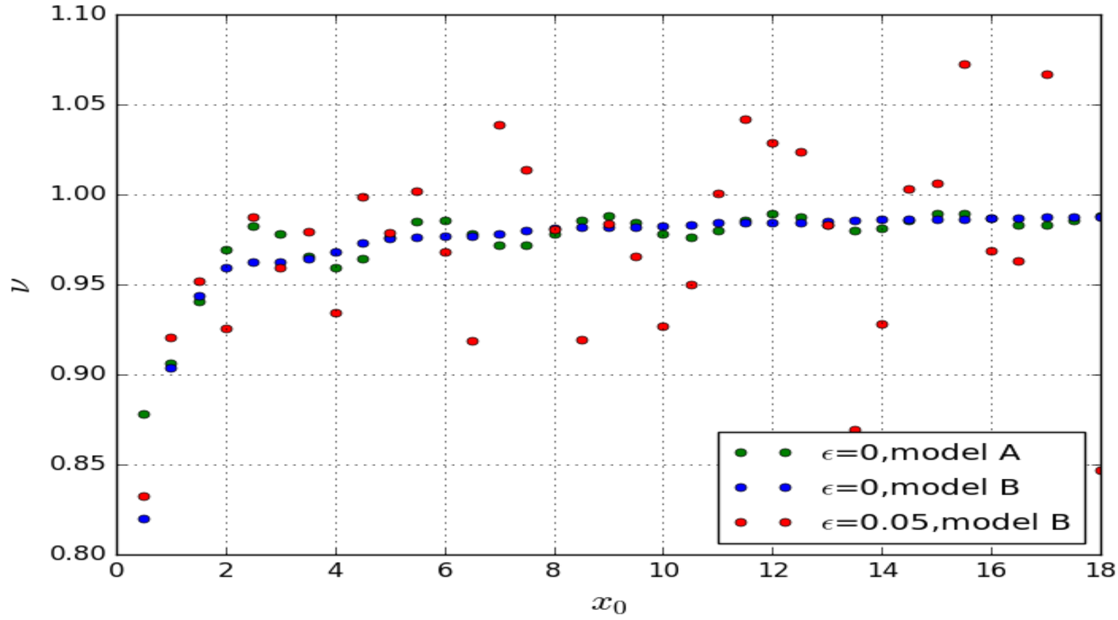


Figure 4: Effect on quality of fit for number of measurements=101

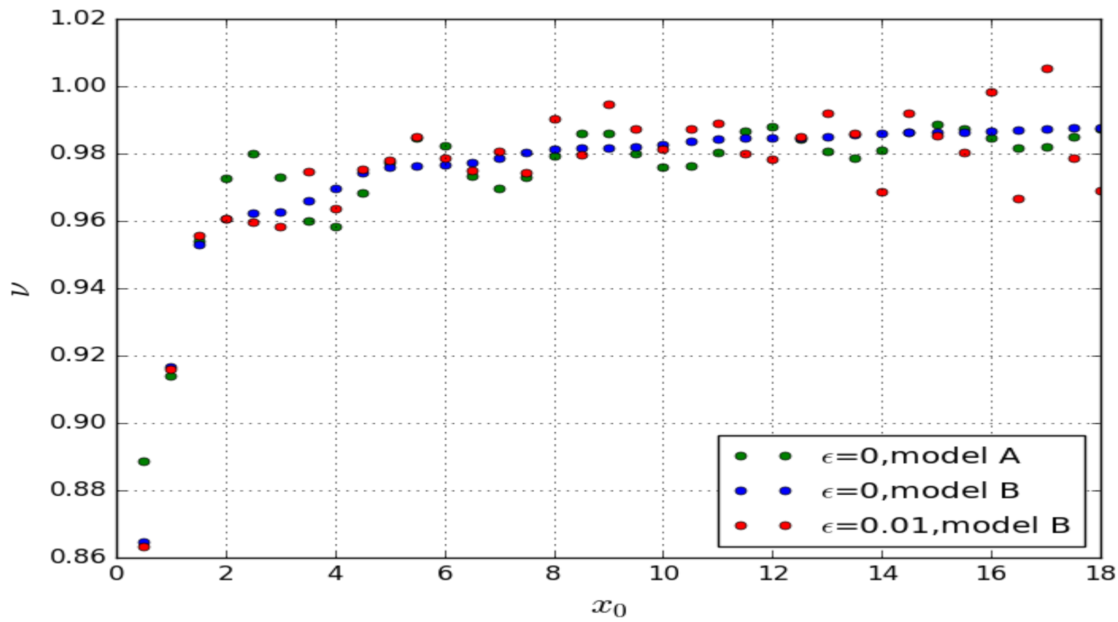


Figure 5: Effect on quality of fit for number of measurements=201

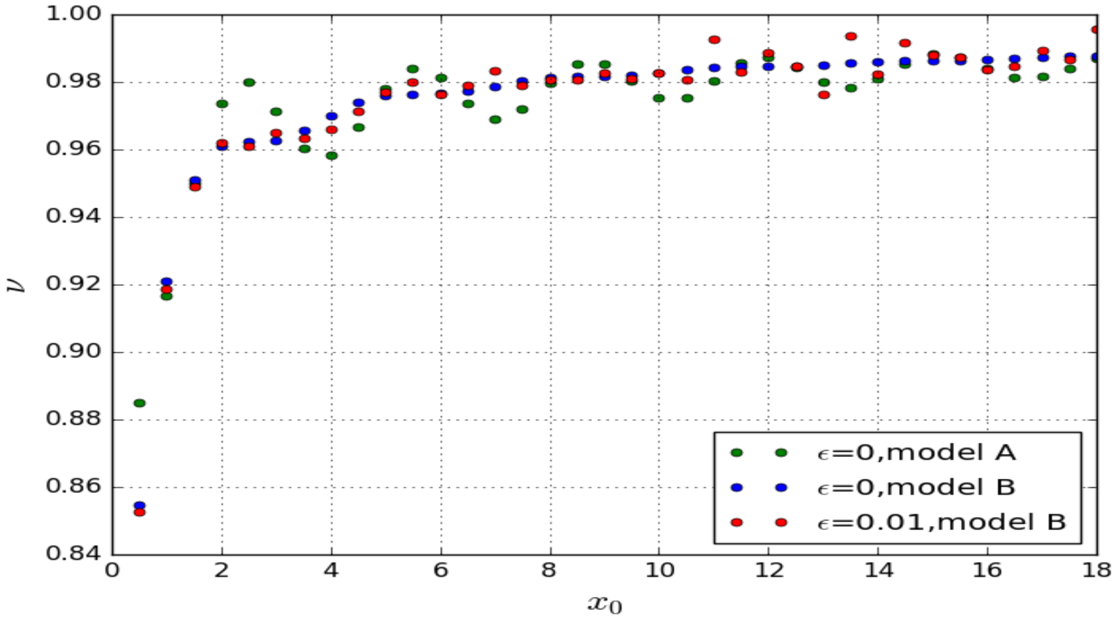


Figure 6: Effect on quality of fit for number of measurements=501

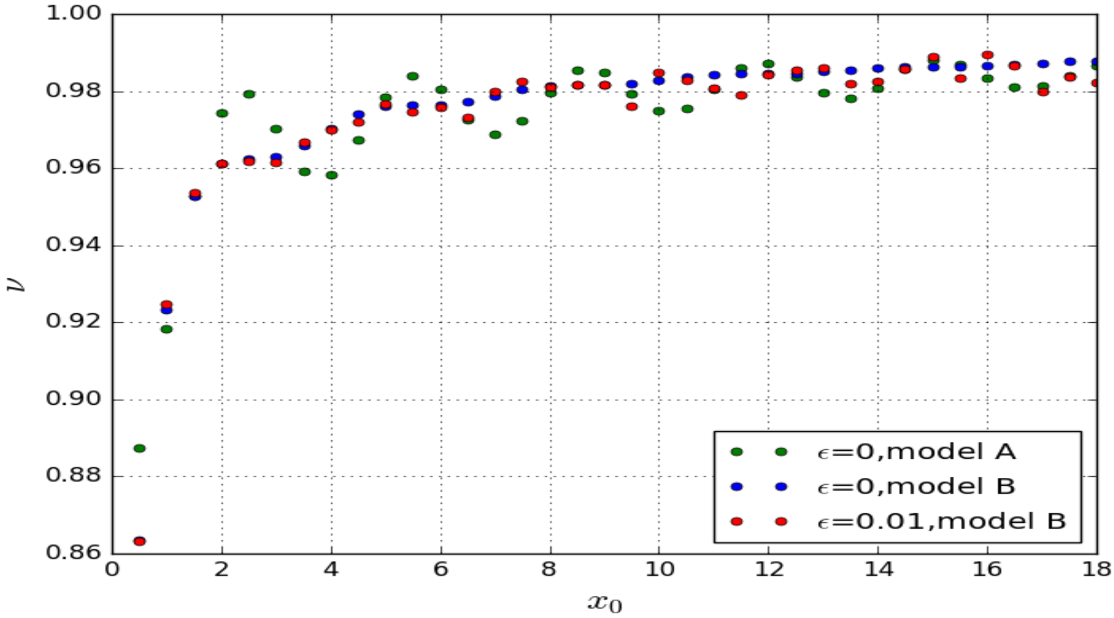
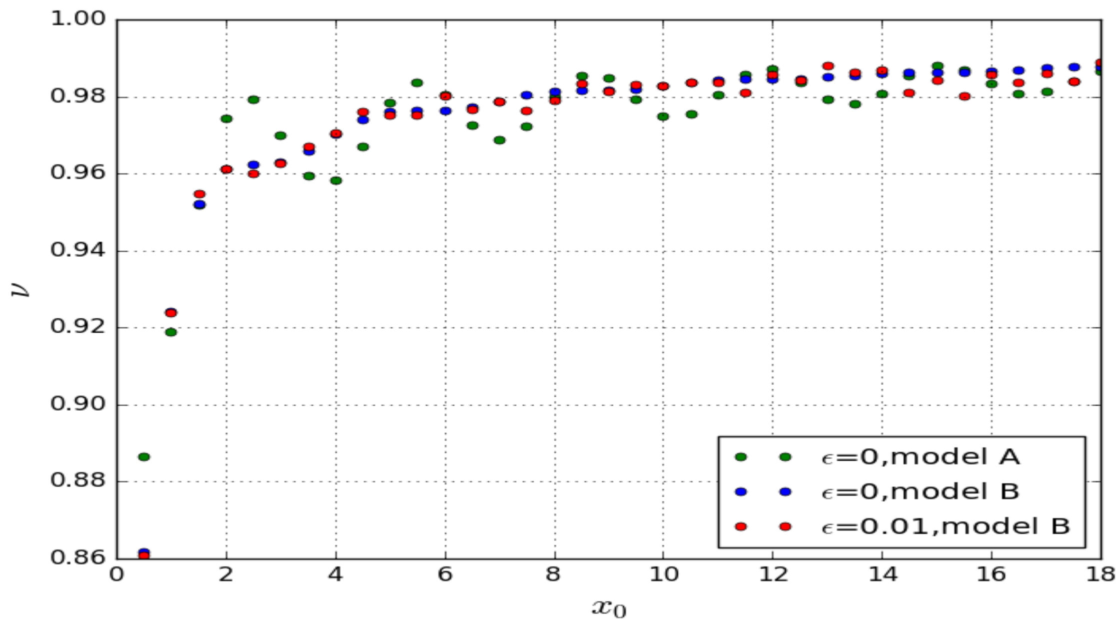


Figure 7: Effect on quality of fit for number of measurements=1001



3. As number of measurements increase from 41 to 1001 for the same range of x, the model with noise matches close to model B without noise as shown in Figure 4,5,6.The models with no noise almost remain the same.

4 Python code :

```
#import required packages
from pylab import *
import matplotlib.pyplot as plt
from numpy import *
import scipy.special as sp
#define bessel function
def jv(x):
    return sp.jv(1,x)
#define calcnu function
def calcnu(x,x0,eps,model):
    indices = where(x>=x0)
    #take from x0 to x
    x = x[indices]
    #get bessel function values in q matrix
    q = jv(x)+ eps*randn(size(x))
    #define matrix P
    P = zeros((len(x),2))
    if(model == 'A'):
        P[:,0] = cos(x)
        P[:,1] = sin(x)
        A,B=lstsq(P,q)[0]
        phi = arccos(A/sqrt(A*A + B*B))
        nu = 2*(phi-pi/4)/pi
        return nu
    if(model == 'B'):
        P[:,0] = cos(x)/sqrt(x)
```

```
        P[:,1] = sin(x)/sqrt(x)
        A,B=lstsq(P,q)[0]
        phi = arccos(A/sqrt(A*A + B*B))
        nu = 2*(phi-pi/4)/pi
        return nu

#define vector x using linspace
n = 41 #number of observations
x = linspace(0,20,n)
#for n=41 x0 ranges from 0.5 to 18 in steps of 0.5
x0_range = linspace(0.5,18,36)
nu_listA =[]
nu_listB =[]
nu_listnoiseB = []
for x0 in x0_range:
    nu_A = calcnu(x,x0,0,'A')
    nu_B = calcnu(x,x0,0,'B')
    nu_noiseB = calcnu(x,x0,0.01,'B')
    nu_listA.append(nu_A)
    nu_listB.append(nu_B)
    nu_listnoiseB.append(nu_noiseB)
plt.xlabel('$x_0$',fontsize = 18)
plt.ylabel(r'$\nu$',fontsize = 18)
plt.plot(x0_range,nu_listA,'go', markersize=5)
plt.plot(x0_range,nu_listB,'bo', markersize=5)
plt.plot(x0_range,nu_listnoiseB,'ro', markersize=5)
plt.legend(['$\epsilon$=0,model A','$\epsilon$=0,model B','$\epsilon$=0.01,model B'],lo
plt.grid(linestyle='dotted')
plt.show()
```