The Digital Fourier Transform

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Abstract

This report explains how to obtain discrete fourier transform using pylab library and generate spectrum of functions.

1 Introduction

- Python has two commands in pylab library fft() to compute the forward fourier transform,ifft() to compute inverse.

```
numpy.fft.fft()
numpy.fft.ifft()
```

- fftshift command helps us in shifting frequency axis to represent negative frequencies.

$$sin(5t) = \frac{e^{j5t} - e^{-j5t}}{2j}$$

$$Y(f) = \frac{1}{2j}(\delta(f-5) - \delta(f+5))$$

- For the spectrum of $sin(5t)$ the deltas should be at 5 and -5, with amplitudes of 0.5.The phase at 5 should be -$\pi/2$ and phase at -5 should be $\pi/2$. These can be seen in below graph.

- Amplitude modulation can also generate a spectrum. Let us consider $f(t) = (1+0.1cos(t))cos(10t)$ Here we do this using tighther spacing between frequencies.

```
t=linspace(-4*pi,4*pi,513);t=t[:-1]
```

- It generates spikes at 9,10,11 rad/s.

$$0.1cos(t)cos(10t) = 0.05(cos(11t) + cos(9t)) = 0.025(e^{11tj} + e^{9tj} + e^{-11tj} + e^{-9tj})$$

Figure 1: Spectrum of $sin(5t)$

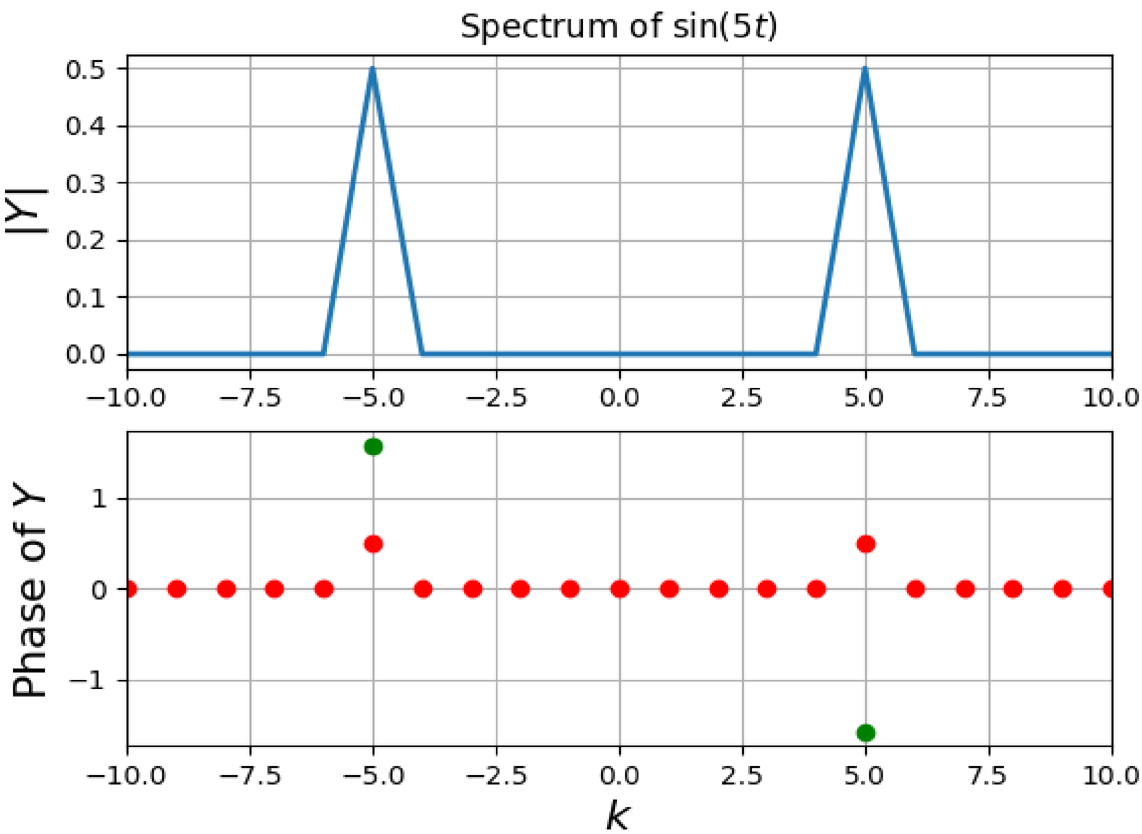
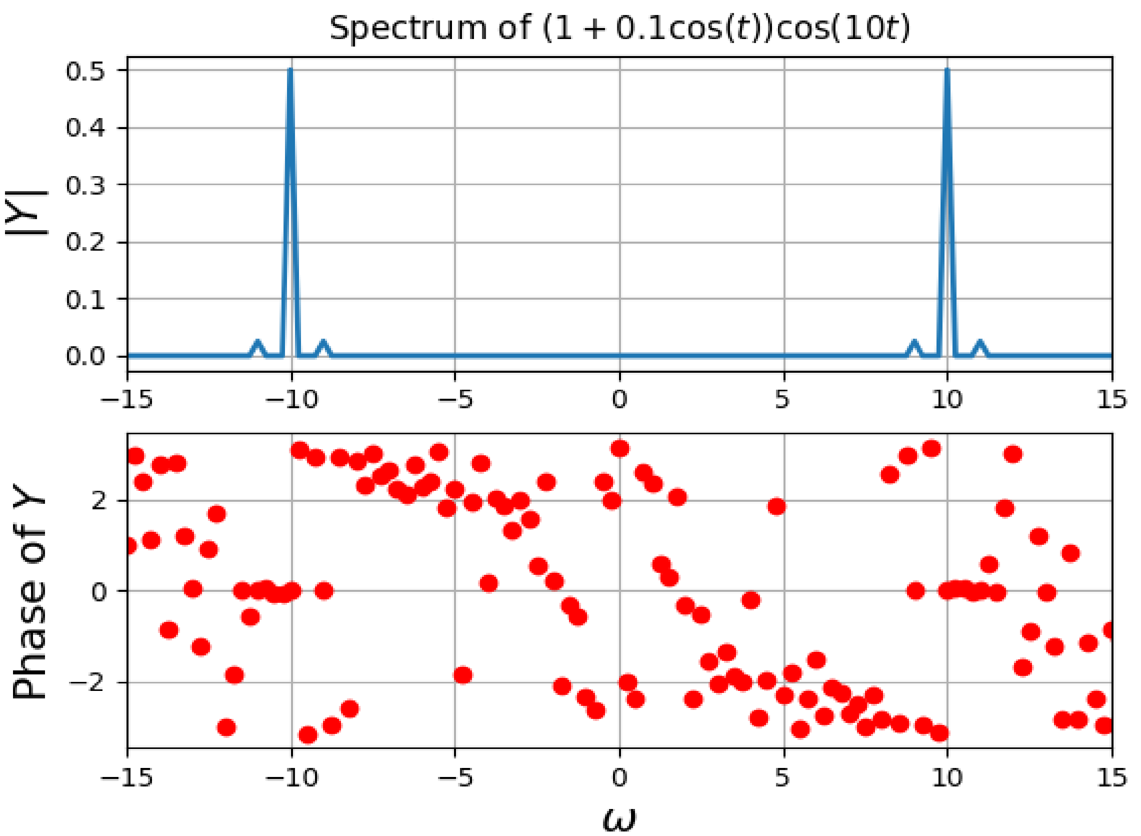


Figure 2: spectrum of $f(t) = (1 + 0.1cos(t))cos(10t)$



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2 Spectrum of sin^3x

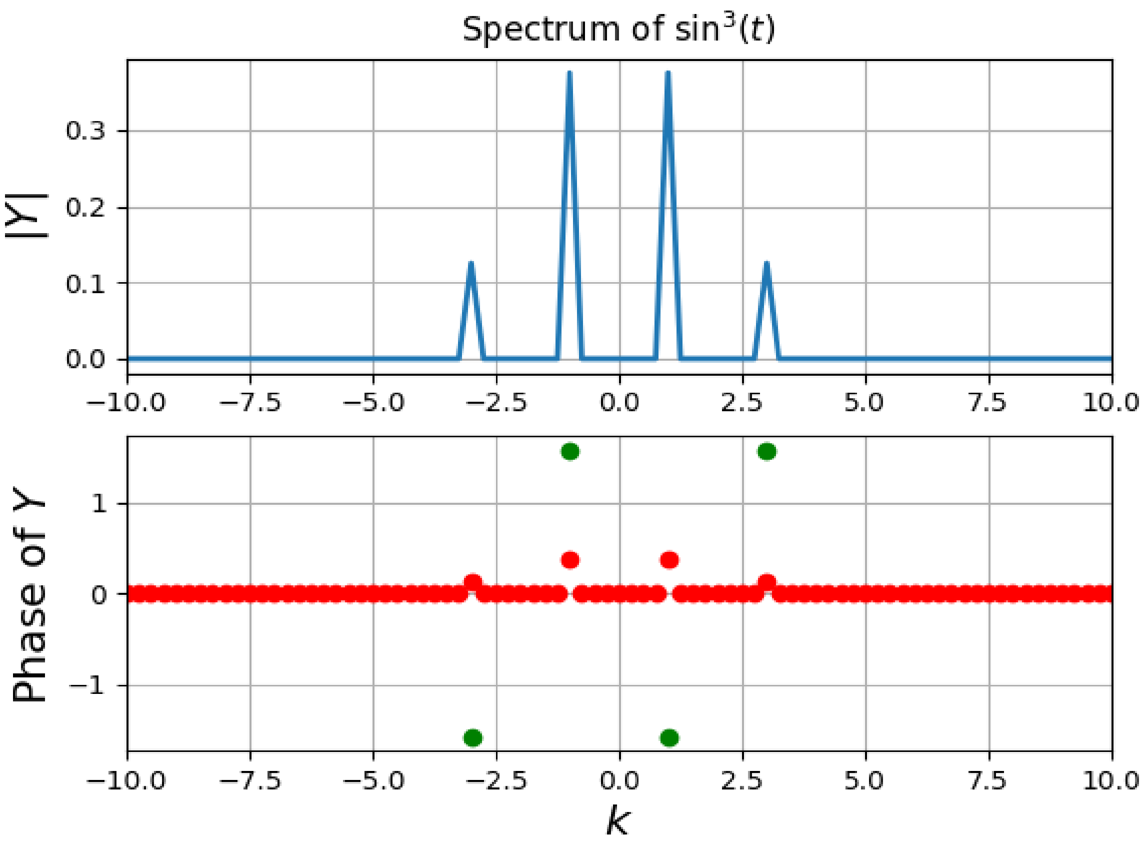
$$y = sin^3(x) = \frac{3sinx - sin3x}{4} = \frac{3e^{jx} - 3e^{-jx} - e^{-3jx} + e^{3jx}}{8j}$$

The expected spectrum is

$$Y(f) = \frac{3}{8j}\left[\delta(f-1) - \delta(f+1)\right] - \frac{1}{8j}\left[\delta(f-3) - \delta(f+3)\right]$$

```
#sin^3t spectrum
x=linspace(0,2*pi,129)
x=x[:-1]
y=pow(sin(x),3)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin^3(t)$")
grid(True)
subplot(2,1,2)
plot(w,abs(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

Figure 3: Spectrum of sin^3t



- From graph the spikes are at 1,-1,3,-3.The height at 1,-1 & 3,-3 are 0.375 and 0.125 respectively.The peaks at 1,-3 has phase of $-\pi/2$ because they have j in the denominator . The peaks at -1,3 has phase of $\pi/2$ because they have -1/j as their multiplying factor.

3 Spectrum of cos^3x

$$y = cos^3(x) = \frac{3cosx + cos3x}{4} = \frac{3e^{jx} + 3e^{-jx} + e^{3jx} + e^{-3jx}}{8j}$$

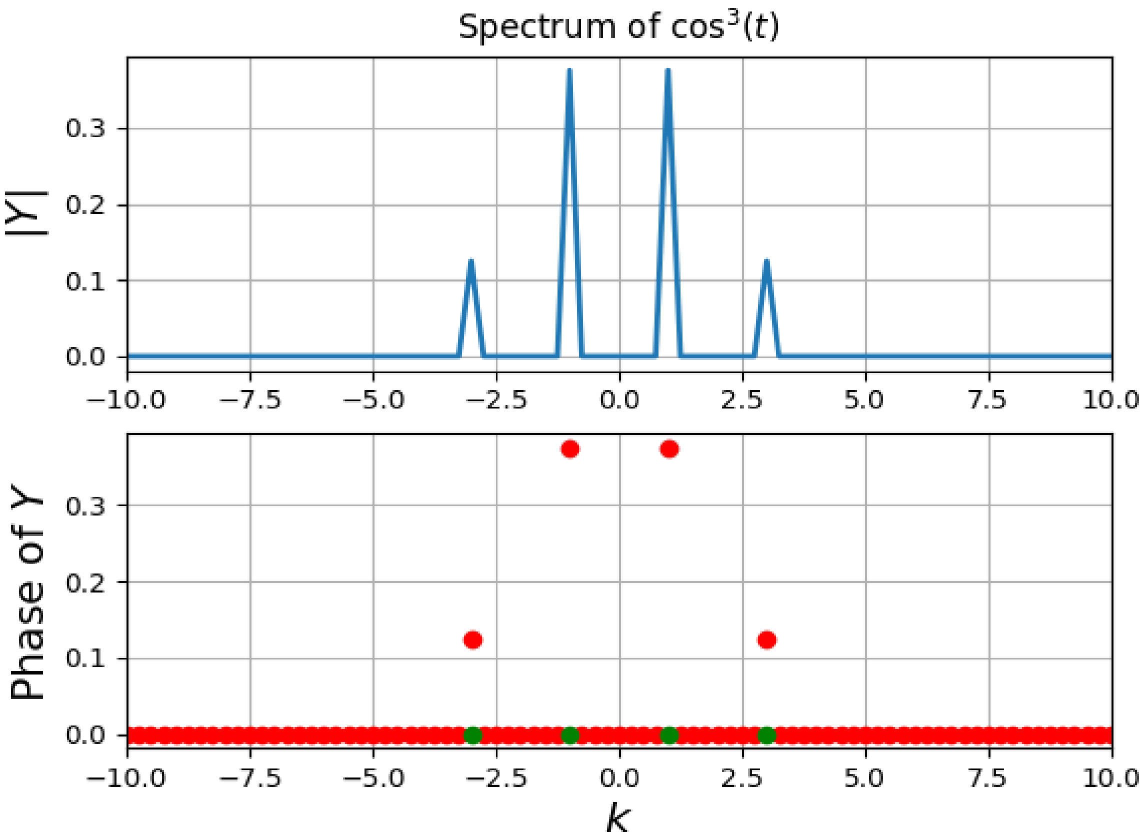
The expected spectrum is

$$Y(f) = \frac{3}{8j}\left[\delta(f-1) + \delta(f+1)\right] + \frac{1}{8j}\left[\delta(f-3) + \delta(f+3)\right]$$

```
# #cos^3t spectrum
x=linspace(0,2*pi,129)
x=x[:-1]
y=pow(cos(x),3)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\cos^3(t)$")
grid(True)
```

```
subplot(2,1,2)
plot(w,abs(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

Figure 4: Spectrum of sin^3t



- From graph the spikes are at 1,-1,3,-3.The height at 1,-1 & 3,-3 are 0.375 and 0.125 respectively.The peaks has $\pi/2$ as phase since they have j in the denominator.

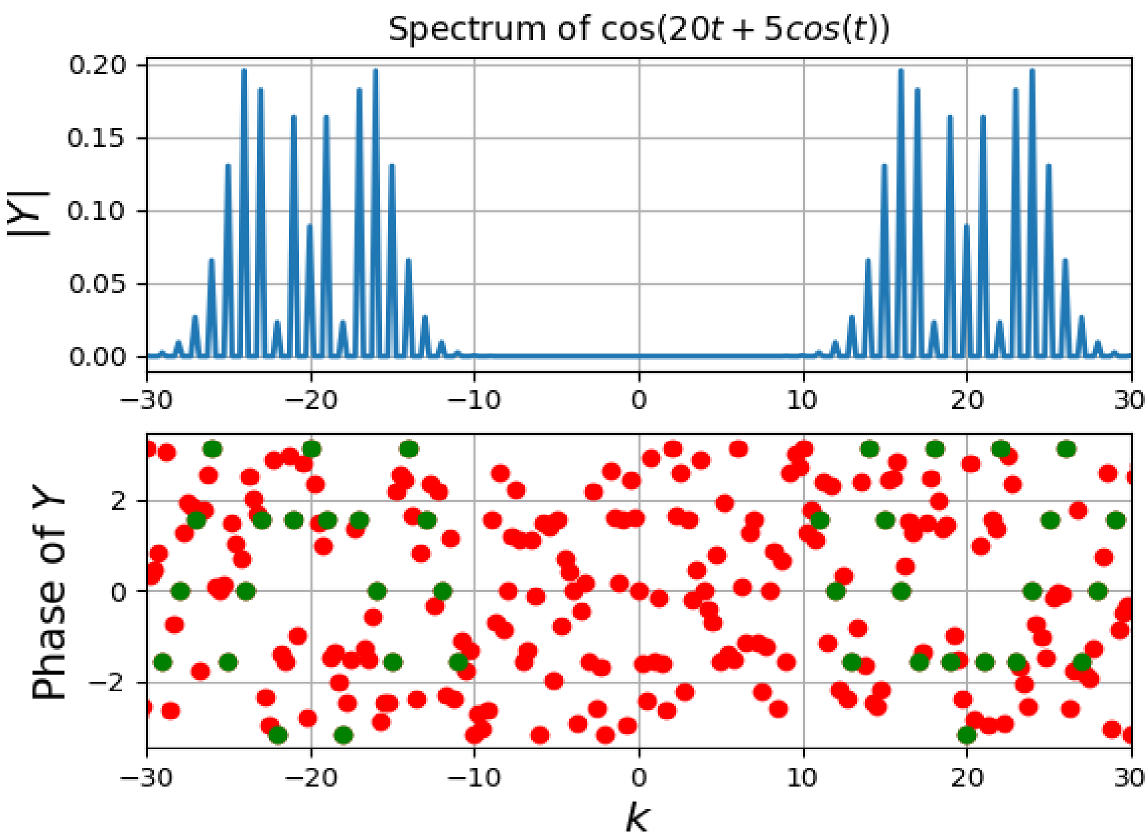
4 Spectrum of $cos(20x + 5cosx)$

- $cos(20x + 5cosx)$ function is peroidic so the spectrum consits of deltas.It is phase modulation of a signal.
- The phase is plot only where magnitude is greater than 10^{-3}.The magnitude plot is symmetric where as phase of those deltas are inverted ie if postive frequencies have phase of ϕthen negative frequencies have phase of $-\phi$.

```
x=linspace(-4*pi,4*pi,513)
x=x[:-1]
y=cos(20*x+5*cos(x))
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513)
```

```
w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-30,30])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\cos(20t+5cos(t))$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-30,30])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

Figure 5: spectrum of $cos(20t + 5cost)$



5 Spectrum of $exp(\frac{-x^2}{2})$

- $exp(\frac{-x^2}{2})$is not a peroidic function. we take fourier inverse of it and take all those frequencies where magnitude of Y(f) is greater than 10^{-6}.It makes signal band limited to frequency of 0.86
- For nyquist criteria to hold we should sample at twice the above frequency. so the t is sampled at $512/8\pi$frequency ,which is higher than 0.86
- fftshift is done for input signal so that it results in phase of zero.

```
x=linspace(-4*pi,4*pi,513)
x=x[:-1]
y=exp(-(x*x)/2.0)
Y=fftshift(fft(fftshift(y)))*8*pi/512.0
w=linspace(-64,64,513)
w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-30,30])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\exp(-t^2/2)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-6)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-30,30])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

Figure 6: Spectrum of $exp(\frac{-x^2}{2})$