



01.10.2024

Statistical Methods in AI (CS7.403)

Lecture-15: Introduction to CNN/Deep Learning

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<https://ravika.github.io>

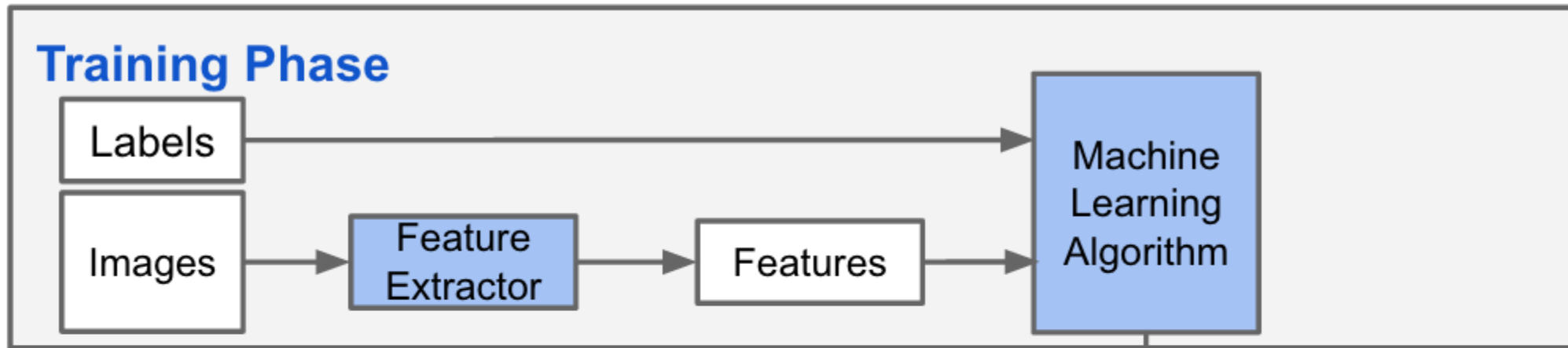
  @vikataravi



Center for Visual Information Technology (CVIT)
IIIT Hyderabad

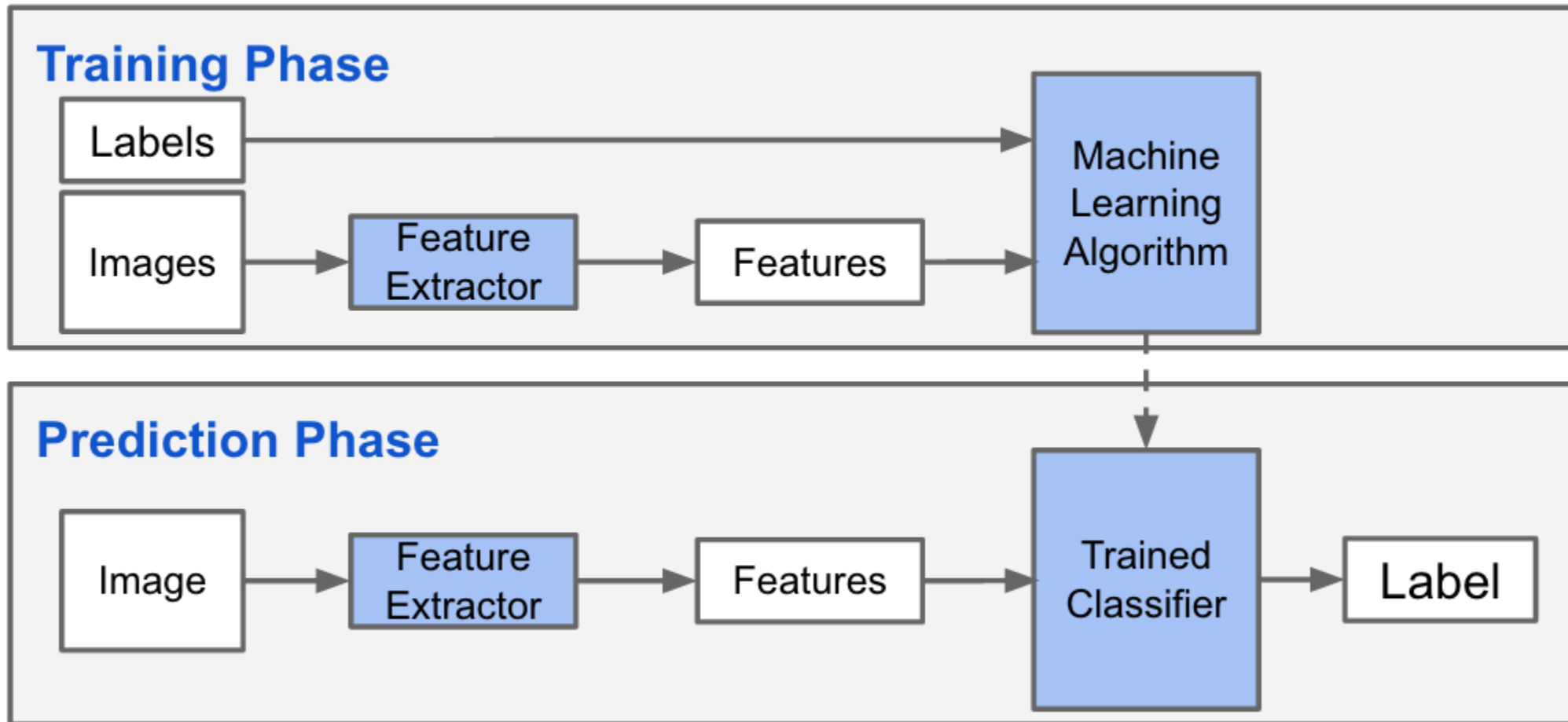


Classification

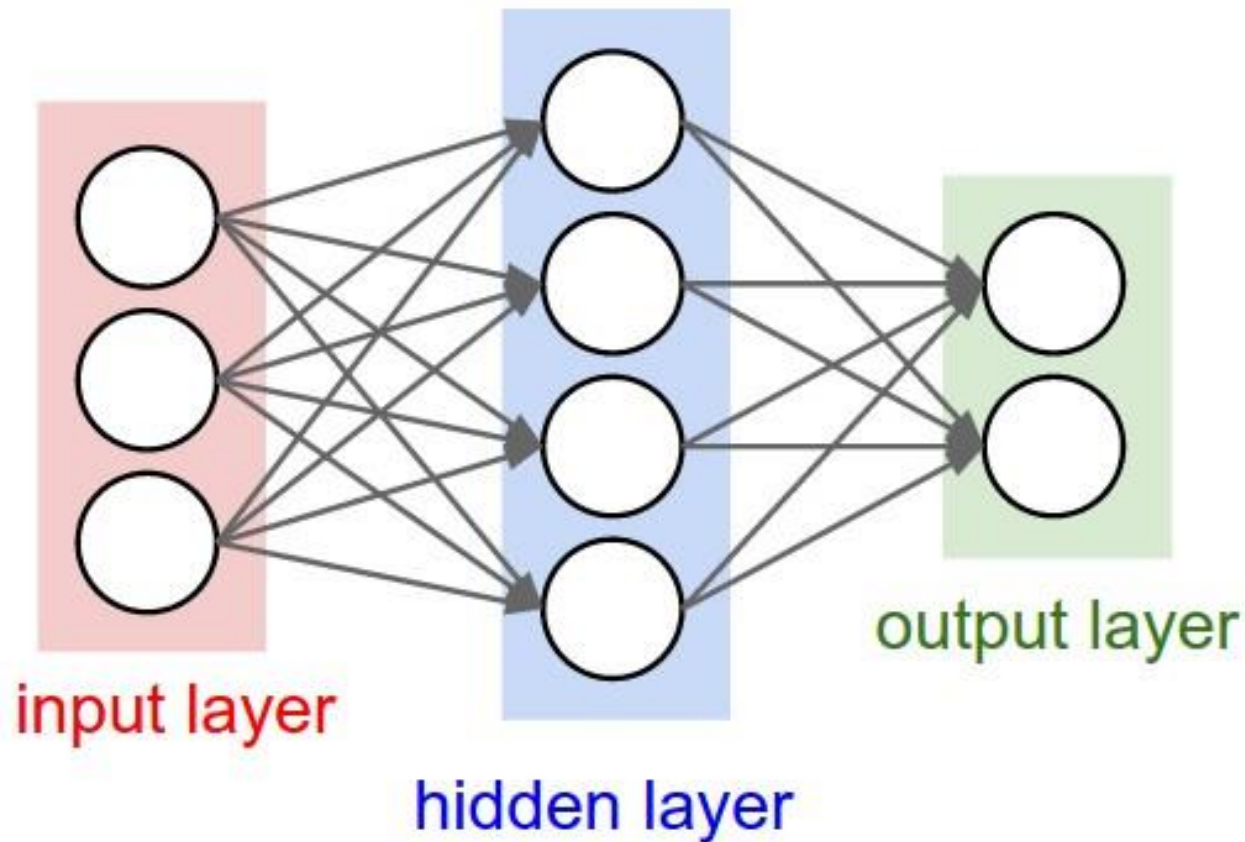




Classification



Traditional approach: Extract features, feed to NN





Wouldn't it be nice if we could feed the image directly ?

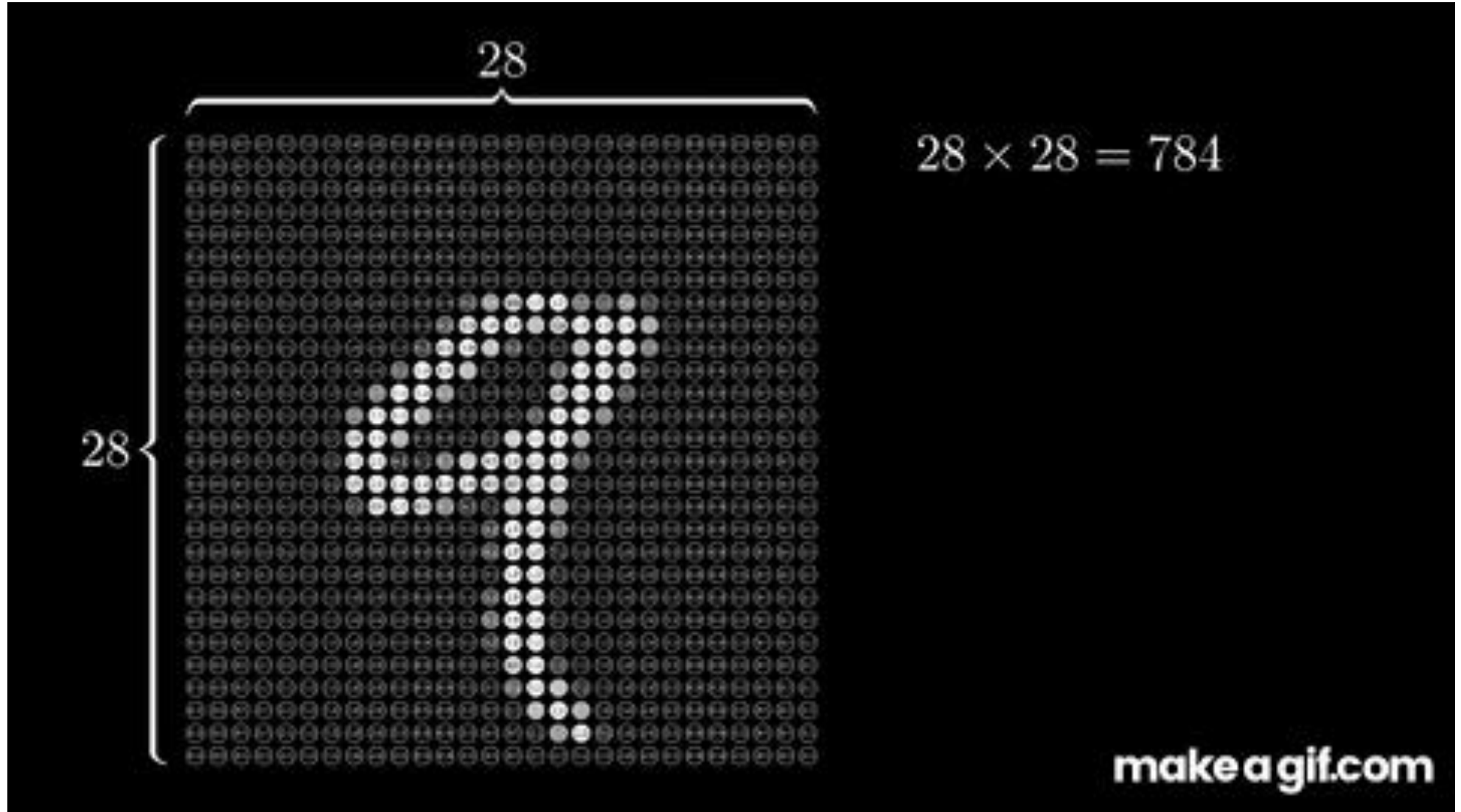
... and let the “learning process” figure out which features to extract ?



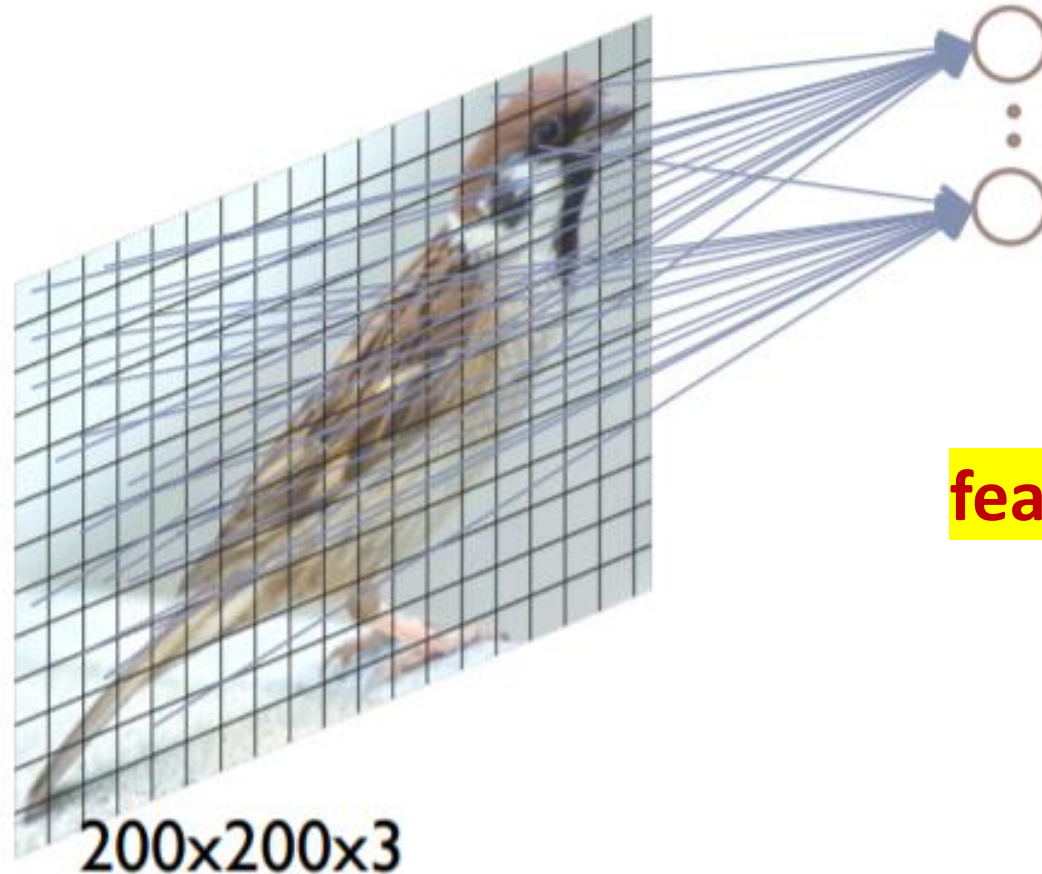
MNIST Handwritten Digits Dataset



Flatten image , Feed to NN



For color images / 2-D structures

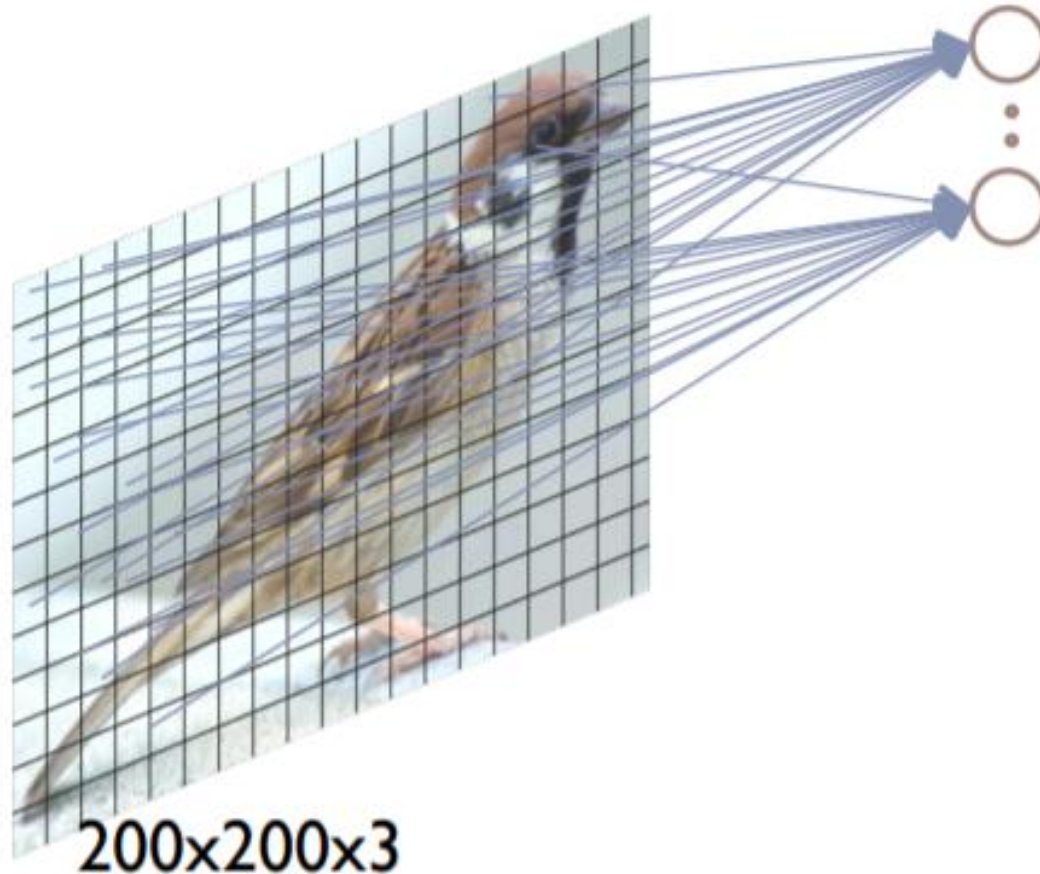


features are i.i.d ?

- #Hidden Units: 120,000
- #Params: 14.4 billion
- Need huge training data to prevent over-fitting!

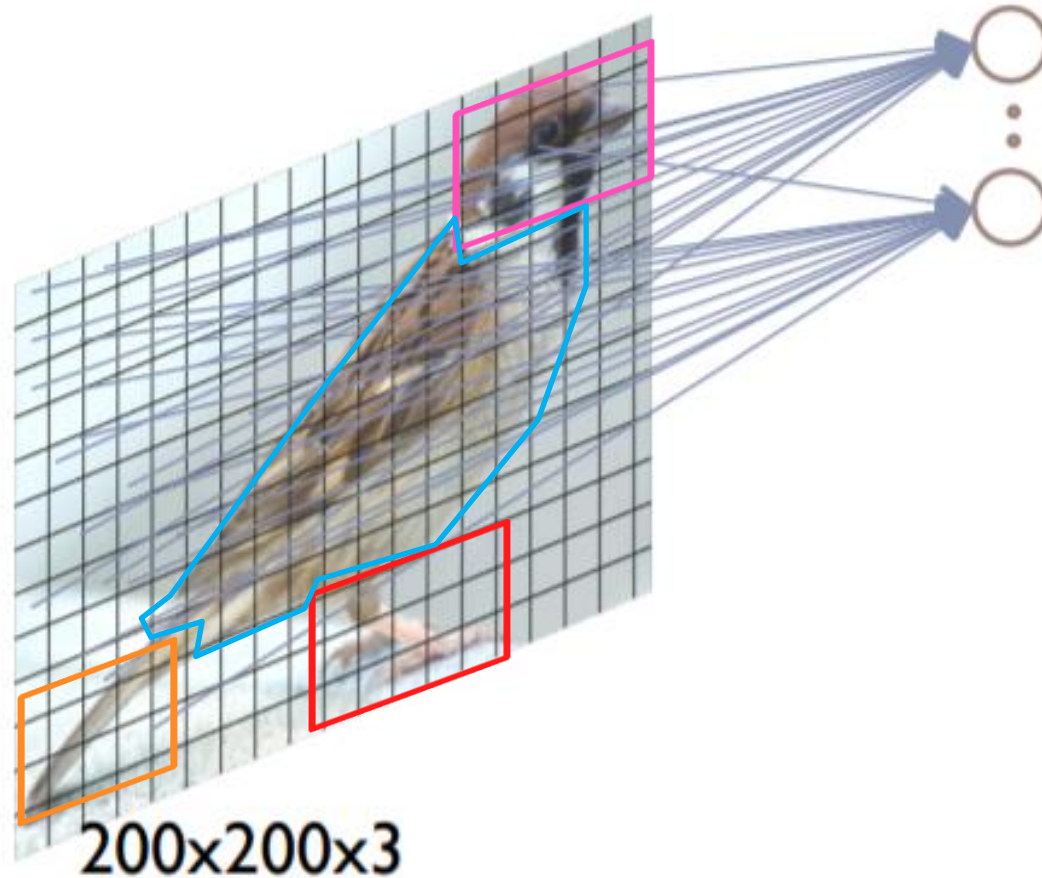


Do we really need full connectivity ?



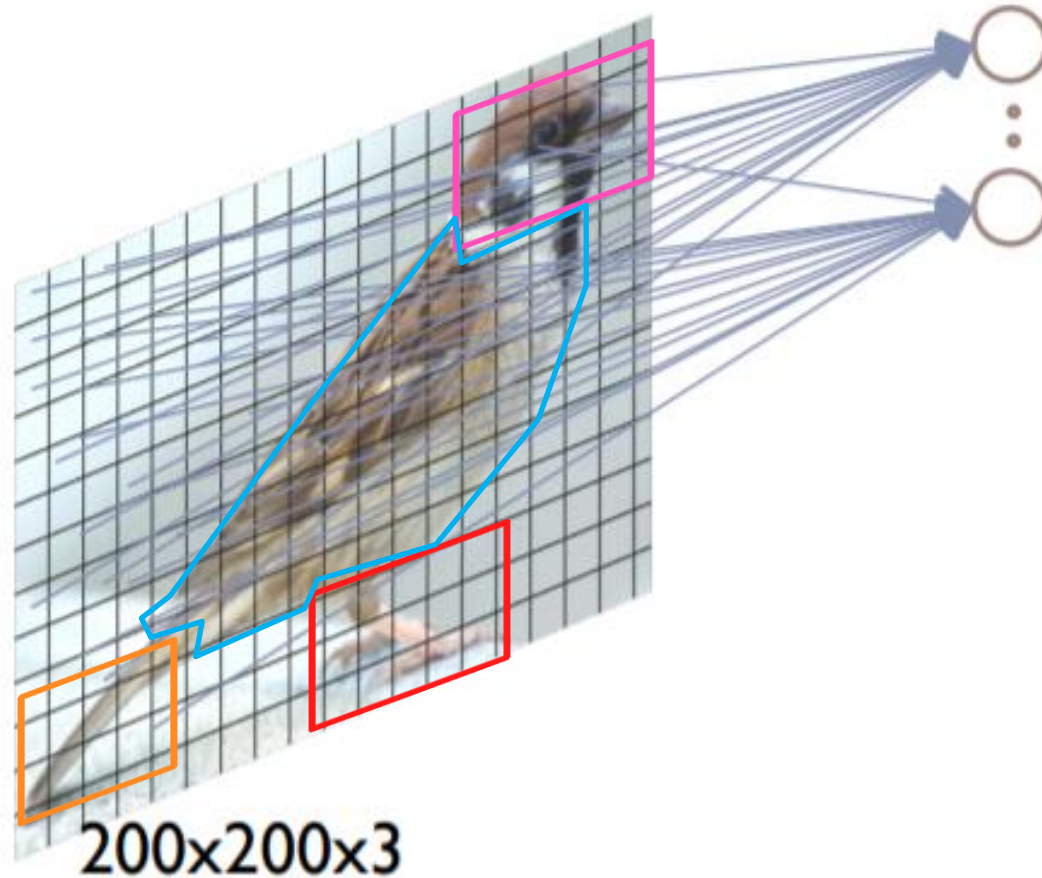
- #Hidden Units: 120,000
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Do we really need full connectivity ?



- Head + Body + Tail + Legs = sparrow

Do we really need full connectivity ?



- Head + Body + Tail + Legs = sparrow





- Images are 2D.
- Assumption: Object image = combination of **2D image patterns**



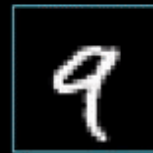


- Images are 2D.
- Assumption: Object image = combination of **2D image patterns**
- Machine Learning Strategy:
 - a) [Pre-final layer] Determine 2D image patterns
 - b) Map 2D image patterns → Target label



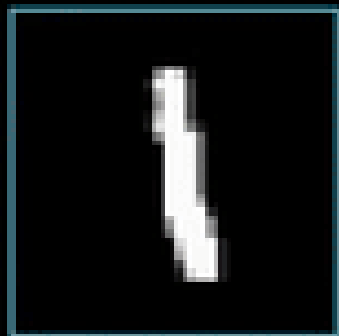
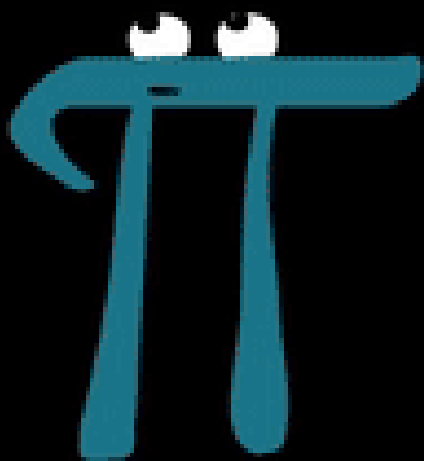
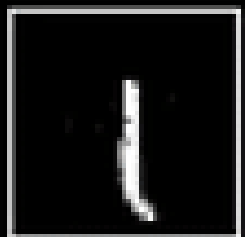
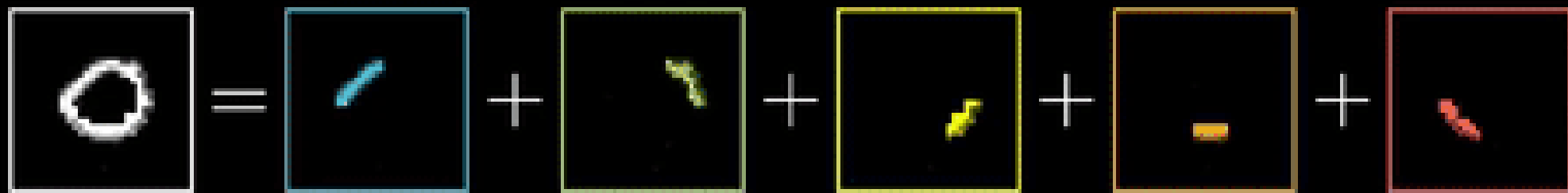


- Images are 2D.
- Assumption: Object image = combination of **2D image patterns**
- Machine Learning Strategy:
 - [Pre-final layer] Determine 2D image patterns
 - Map 2D image patterns → Target label
- NOTE: 2D image patterns are smaller than image





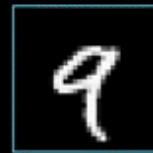
Hierarchical decomposition of input/features



makeagif.com



- Images are 2D.
- Assumption: Object image = **hierarchical** combination of **2D image patterns**
- **Machine Learning Strategy:**
 - [All except pre-final] Determine 2D image patterns
 - [Pre-final layer] Map 2D image patterns → Target label
- NOTE: 2D image patterns are smaller than image





Efficient !

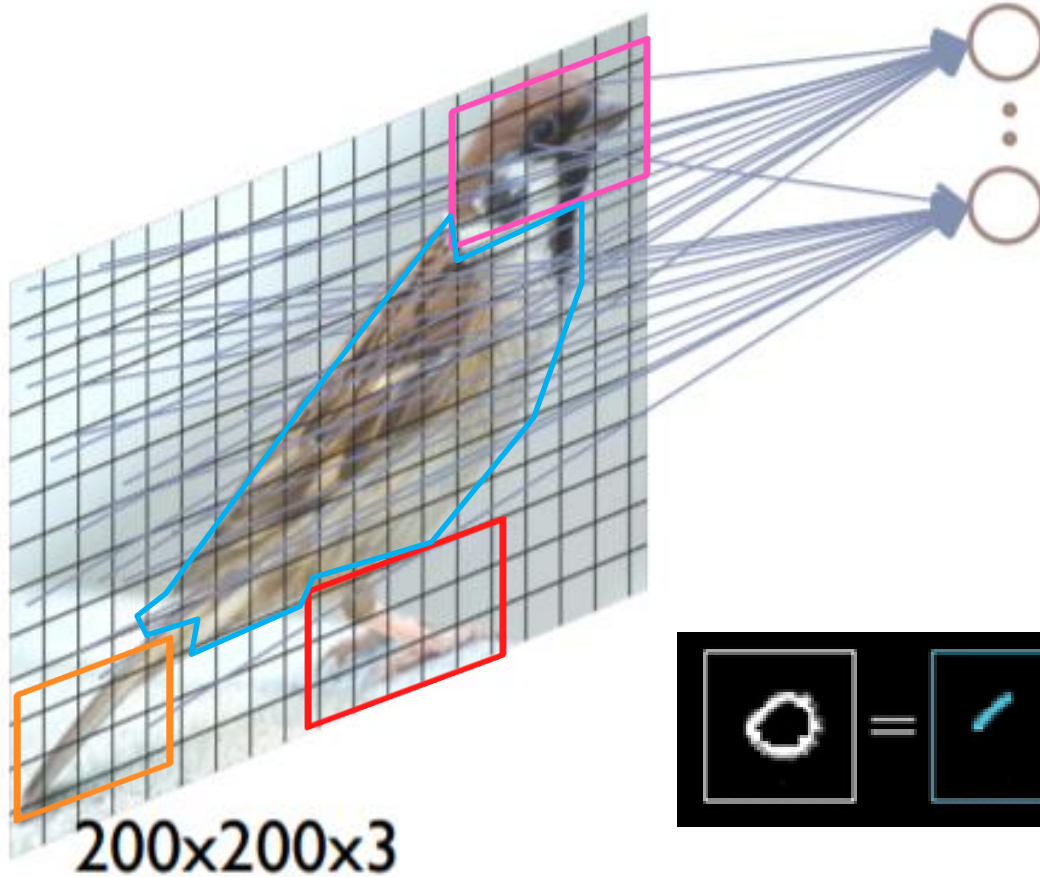
- Images are 2D.
- Assumption: Object image = **hierarchical** combination of **2D image patterns**
- **Machine Learning Strategy:**
 - [All except pre-final] Determine 2D image patterns
 - [Pre-final layer] Map 2D image patterns → Target label
- NOTE: 2D image patterns are smaller than image

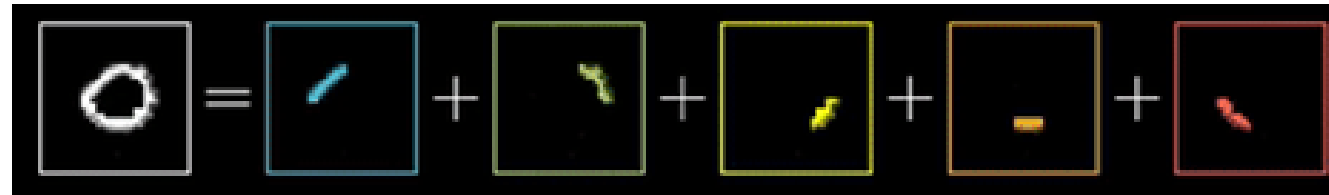




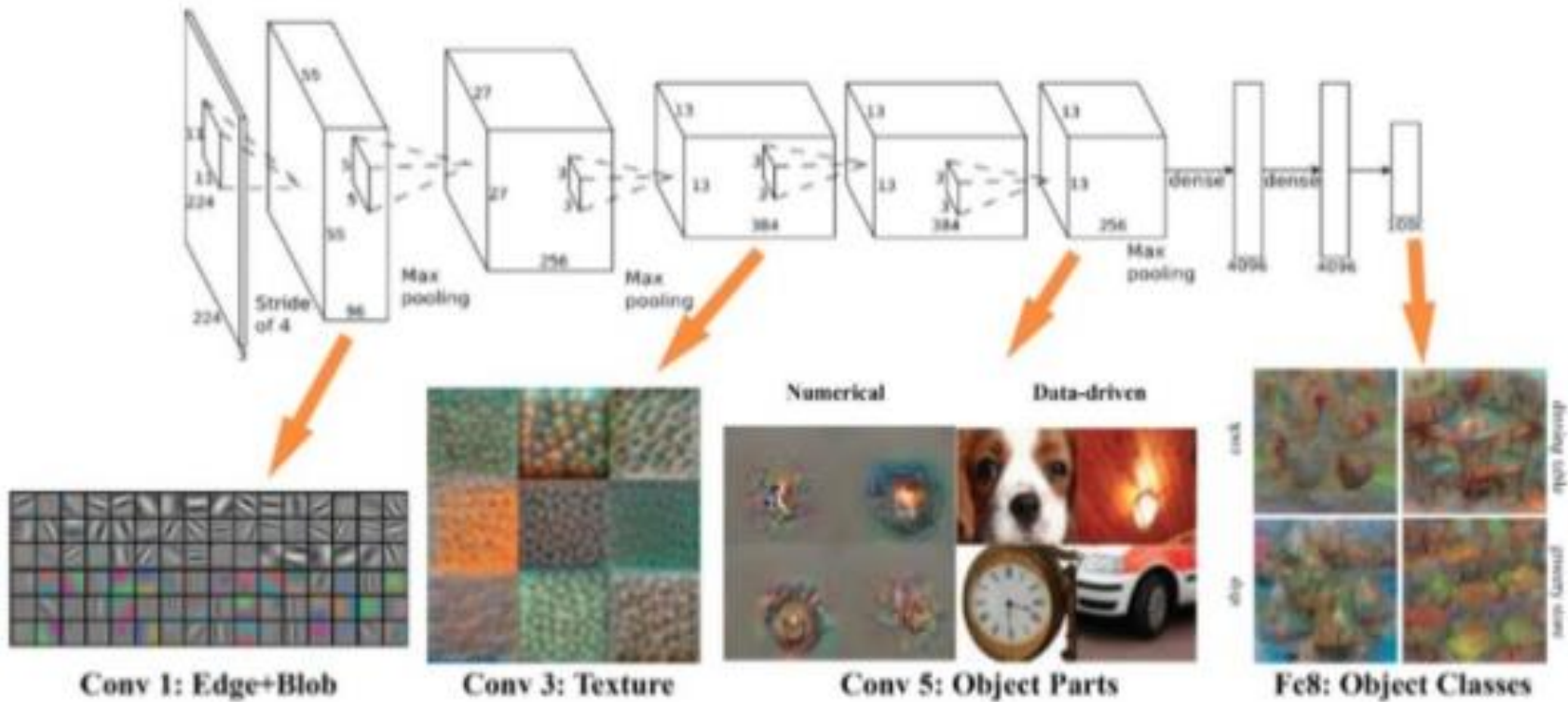
SPOILER ALERT!

Neural Networks don't really
learn such conveniently interpretable
Representations – this is just a toy illustration





$$\text{Circle} = \text{Blue} + \text{Green} + \text{Yellow} + \text{Orange} + \text{Red}$$

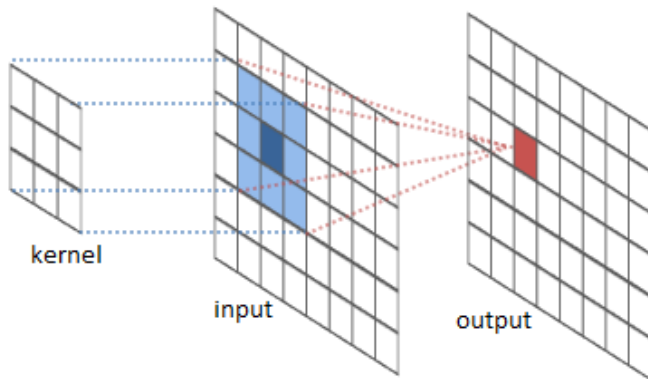


From: [mNeuron: A Matlab Plugin to Visualize Neurons from Deep Models](#), Donglai Wei et. al

- Pattern detectors = Filters
- What should be the size of filters ?
- How many filters do we need at each level ?
- How many levels ?

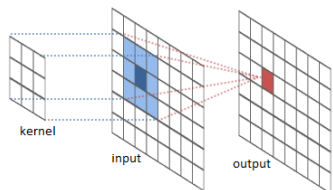
How do filters work ?

- Determine the filter (aka kernel)
- 'Scan' the filter on input image / representation
- Output = A record of all input locations that 'match' the filter



$$\begin{pmatrix} 0 & 1 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

How do filters work ?



0	0	0
0	-1	1
0	0	0



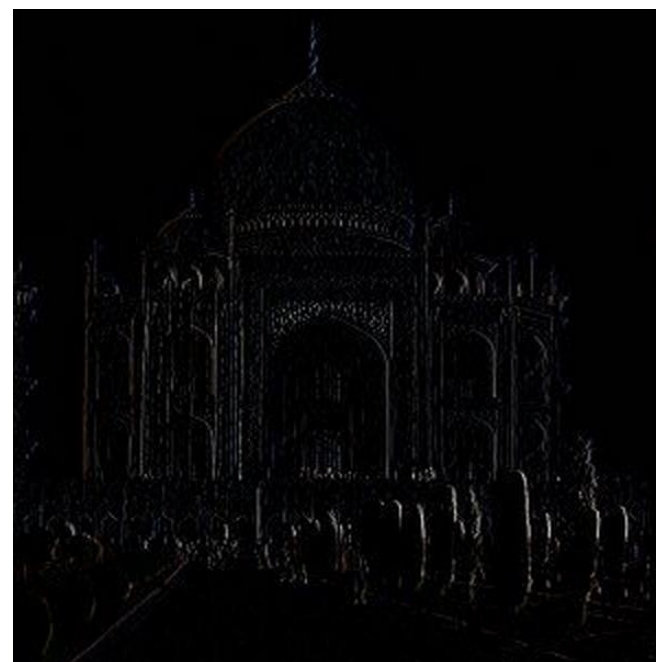
Input



Kernel Convolution

-1	-1	-1
-1	8	-1
-1	-1	-1

Feature Map



Features



Wouldn't it be nice if we could feed the image directly ?

... and let the “learning process” figure out which features to extract ?



Wouldn't it be nice if we could feed the image directly ?

... and let the “learning process” figure out which features to extract ?

(which filters to construct)

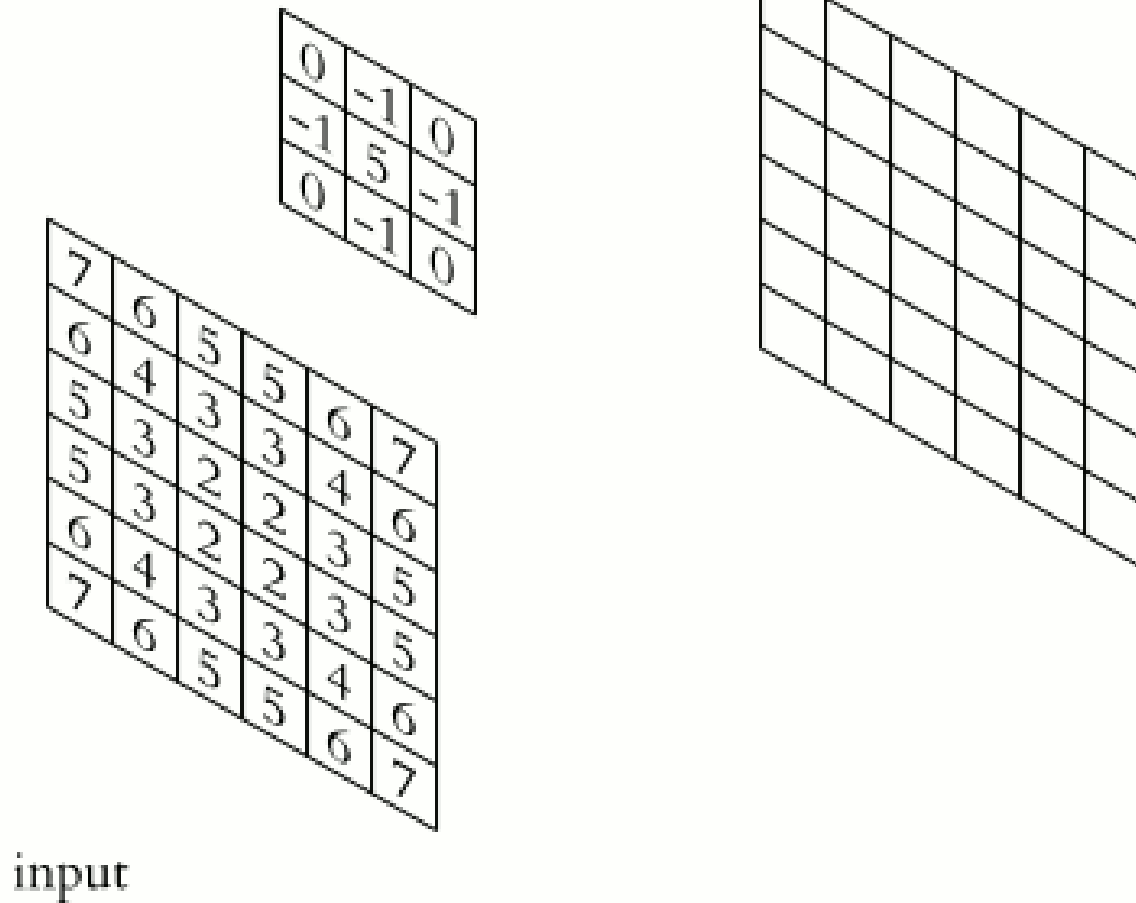


Convolution (in general)

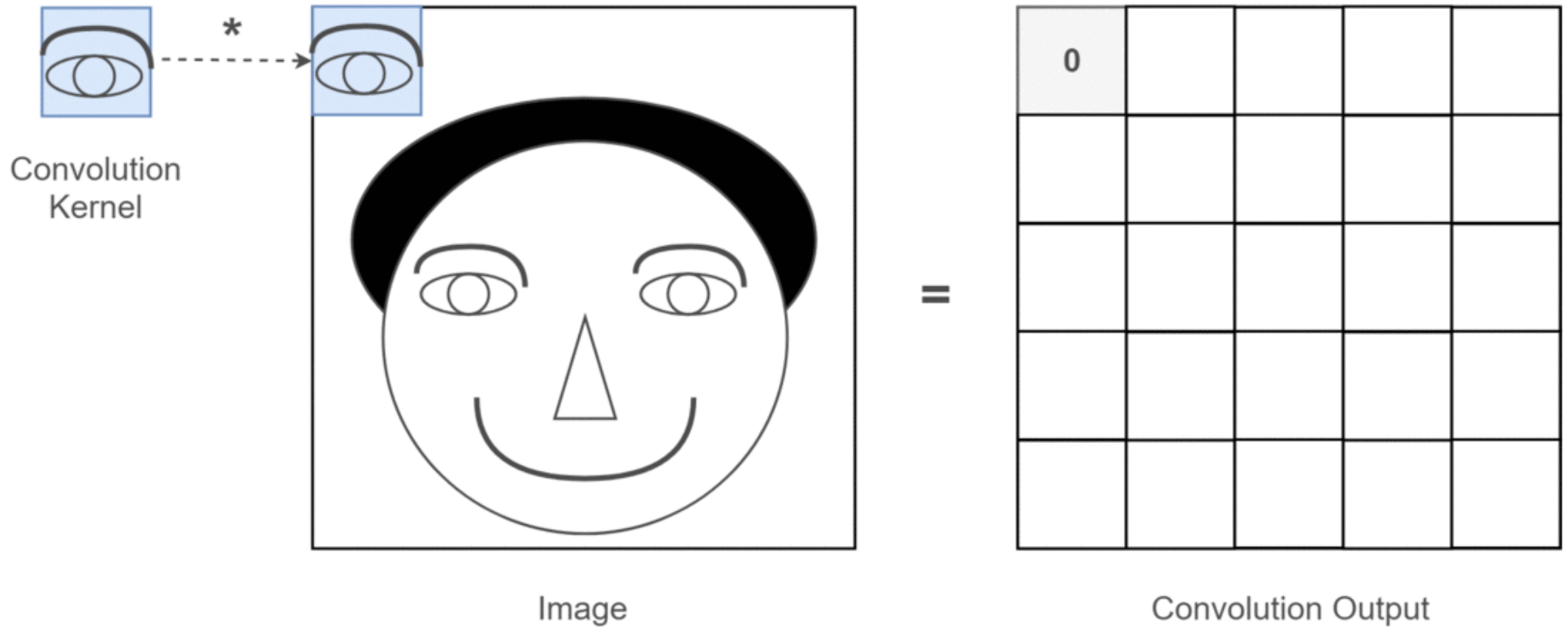
3_0	3_1	2_2	1	0
0_2	0_2	1_0	3	1
3_0	1_1	2_2	2	3
2	0	0	2	2
2	0	0	0	1

12	12	17
10	17	19
9	6	14

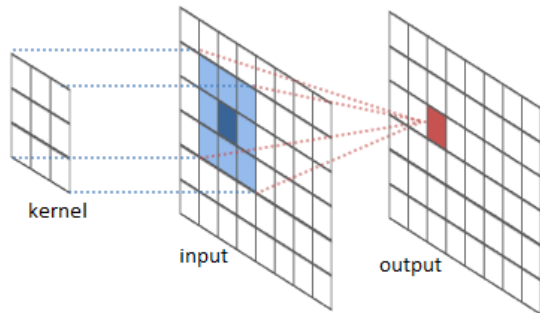
output



Convolution (in general)

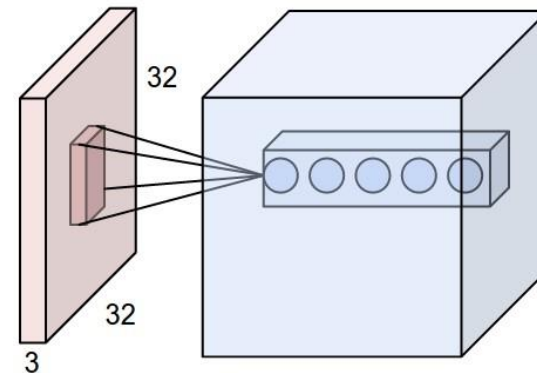
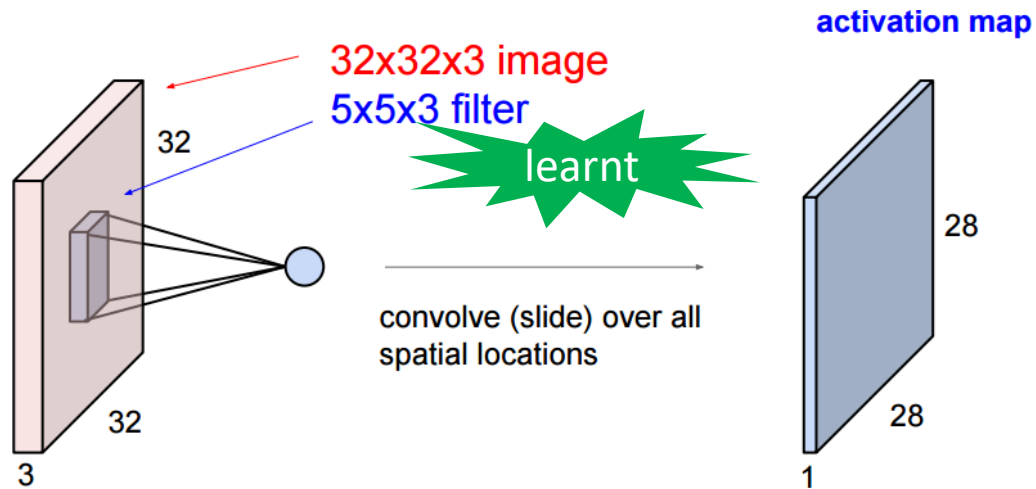


Multiple filters, multiple 'feature maps'

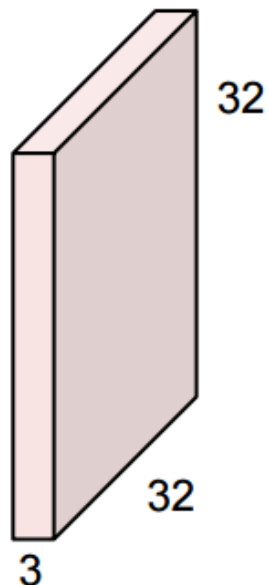




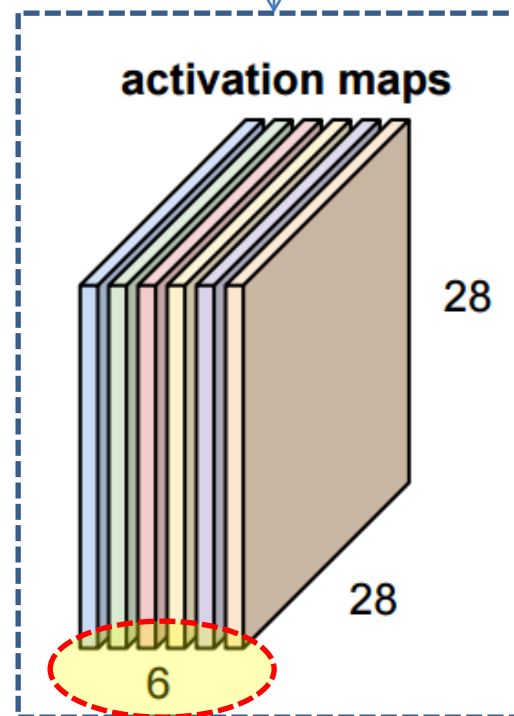
Convolution Layer



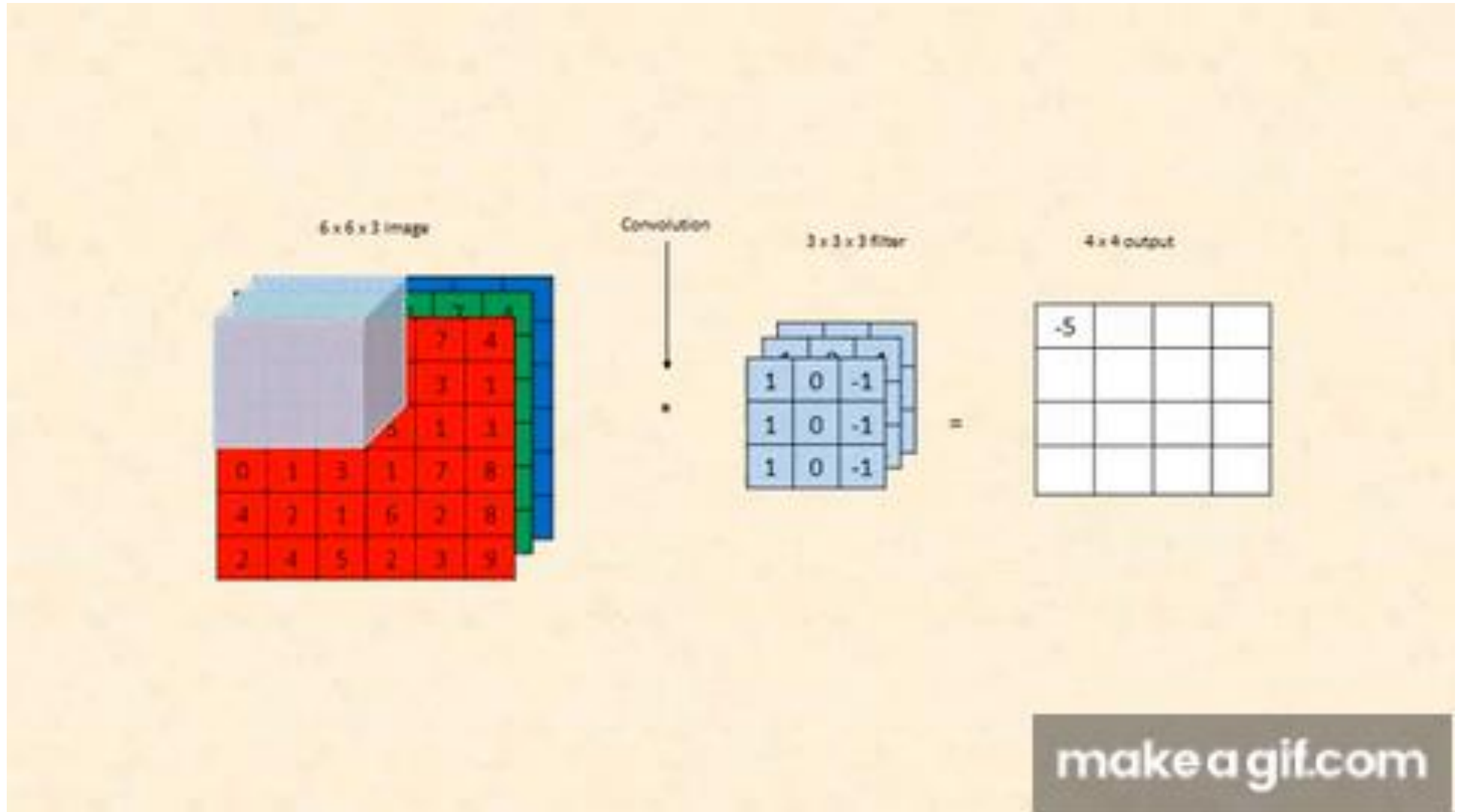
Input to next layer



Convolution Layer

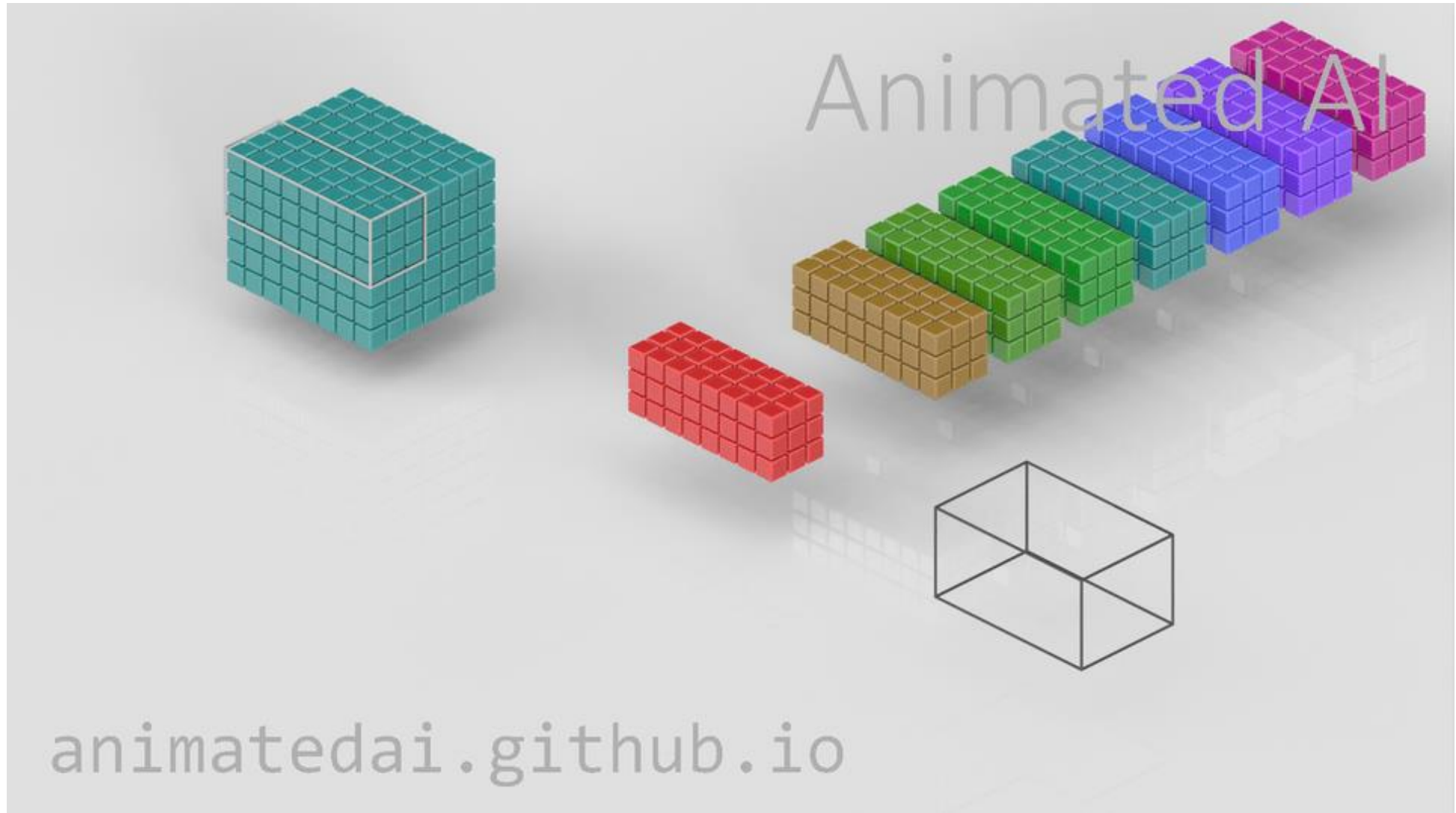


Convolution Layer





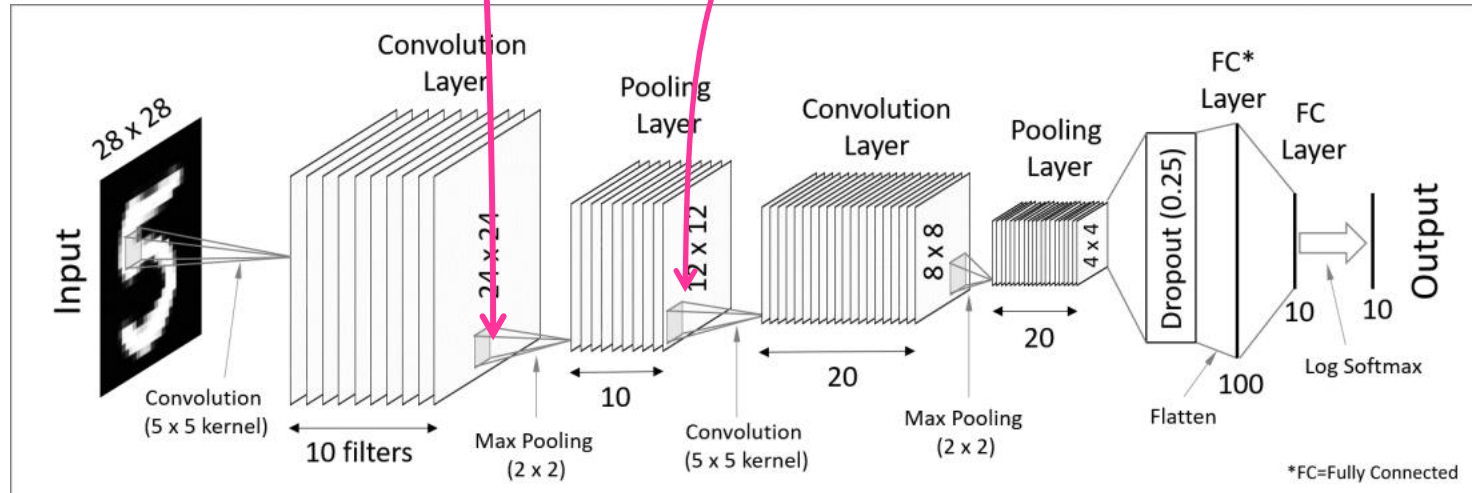
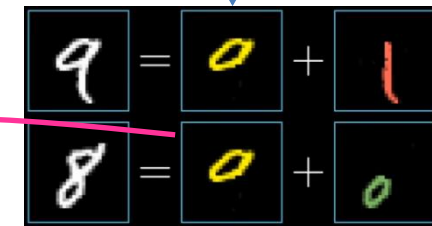
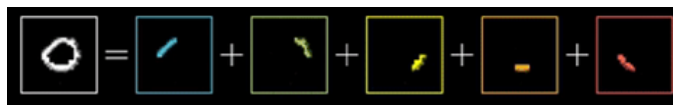
Convolution (in general)





Design choices : Filter size, # of filters

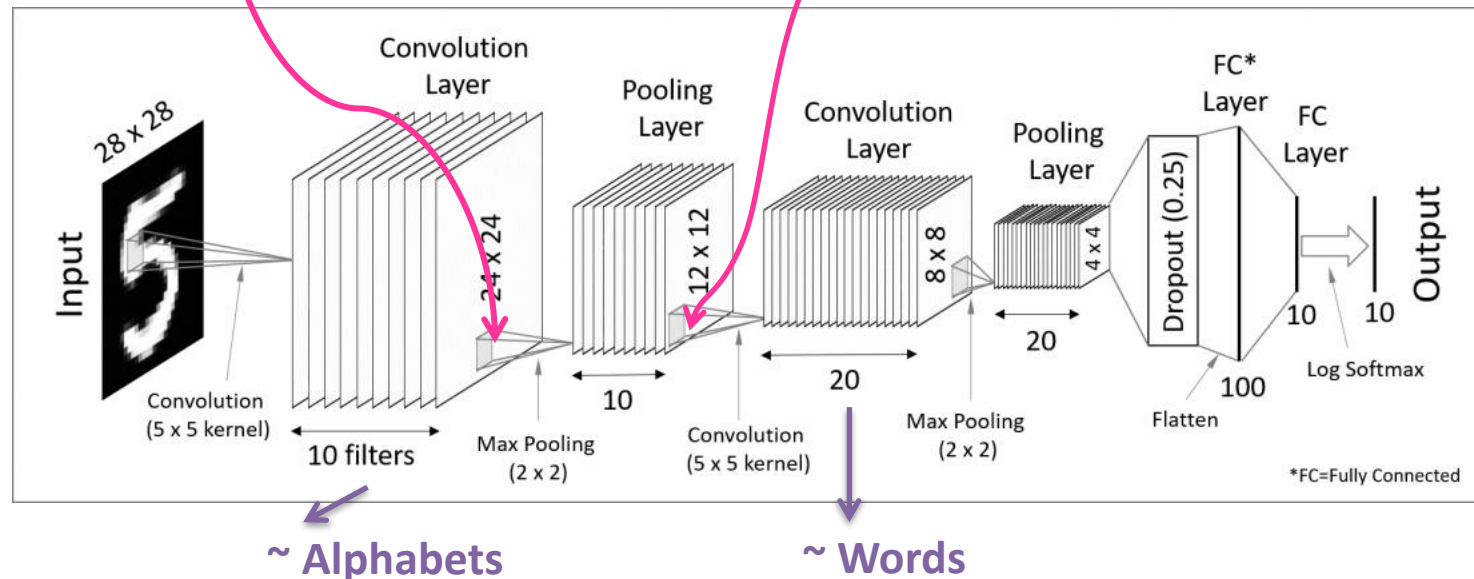
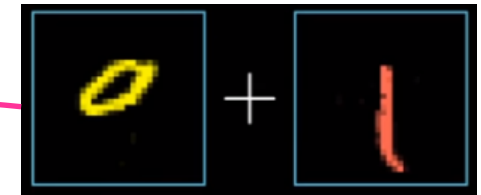
3x3			5x5					7x7						
1	2	1	2	7	12	7	2	1	1	2	2	2	1	1
2	4	2	7	31	52	31	7	1	3	4	5	4	3	1
1	2	1	12	52	127	52	12	2	4	7	8	7	4	2
			7	31	52	31	7	2	5	8	10	8	5	2
			2	7	12	7	2	2	4	7	8	7	4	2
								1	3	4	5	4	3	1
								1	1	2	2	2	1	1



Design choices : Filter size, # of filters

3 ₀	3 ₁	2 ₂	1	0
0 ₂	0 ₂	1 ₀	3	1
3 ₀	1 ₁	2 ₂	2	3
2	0	0	2	2
2	0	0	0	1

12	12	17
10	17	19
9	6	14





Pooling Layer

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

max pool with 2x2 filters
and stride 2



6	8
3	4



Pooling Layer

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

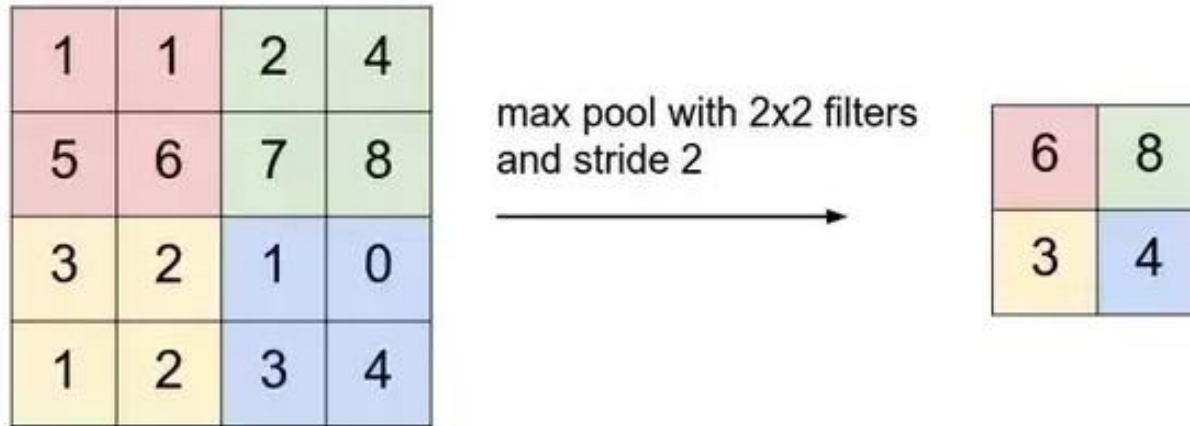
max pool with 2x2 filters
and stride 2



6	8
3	4

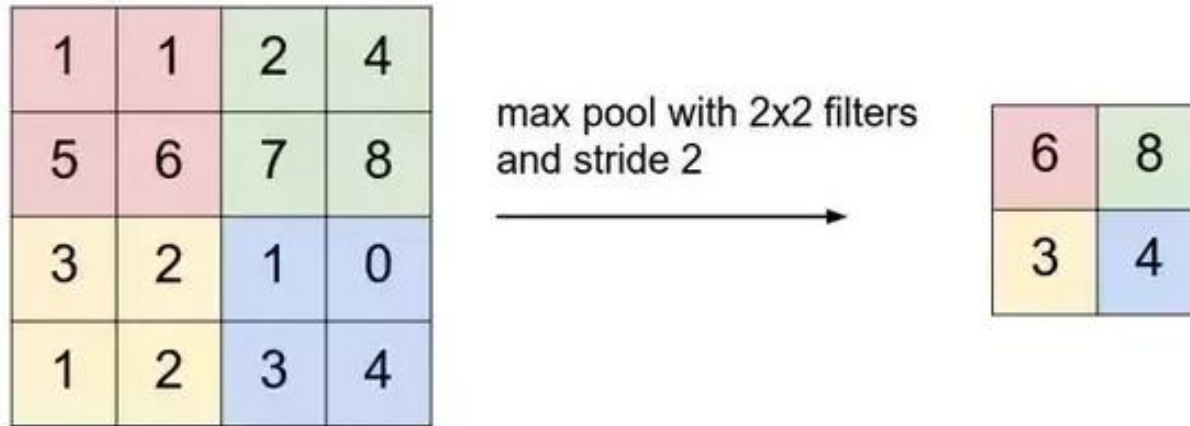


Pooling Layer

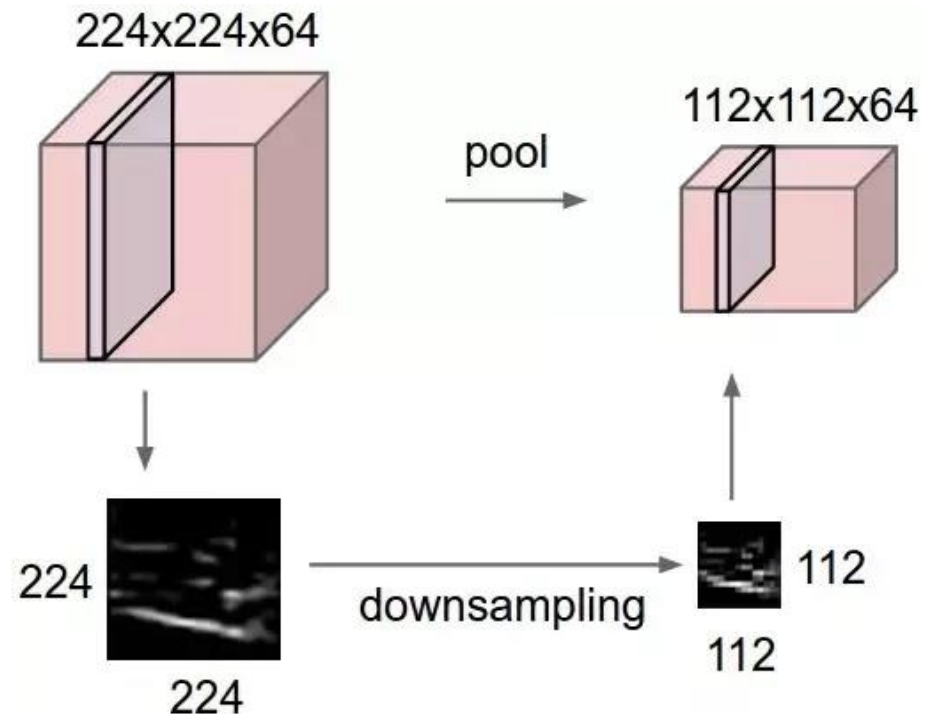


- **Motivation: We care about presence of features, not their exact location !**
- Dimensionality Reduction
- Prevents overfitting

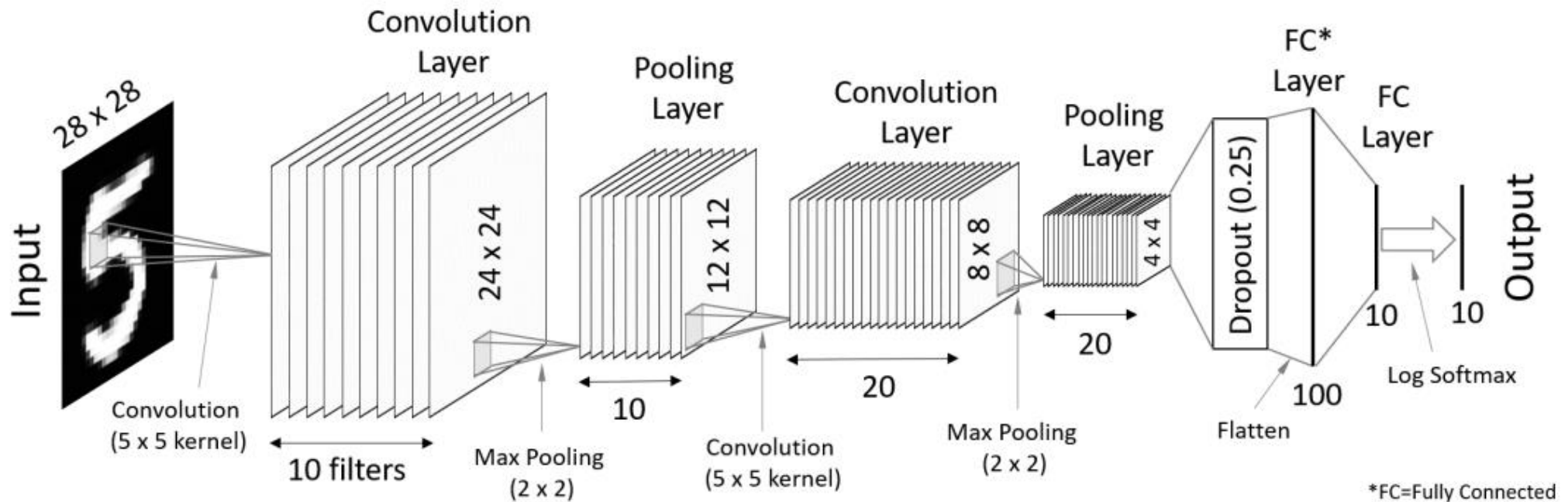
Pooling Layer



- **Motivation: We care about presence of features, not their exact location !**
- Dimensionality Reduction
- Prevents overfitting

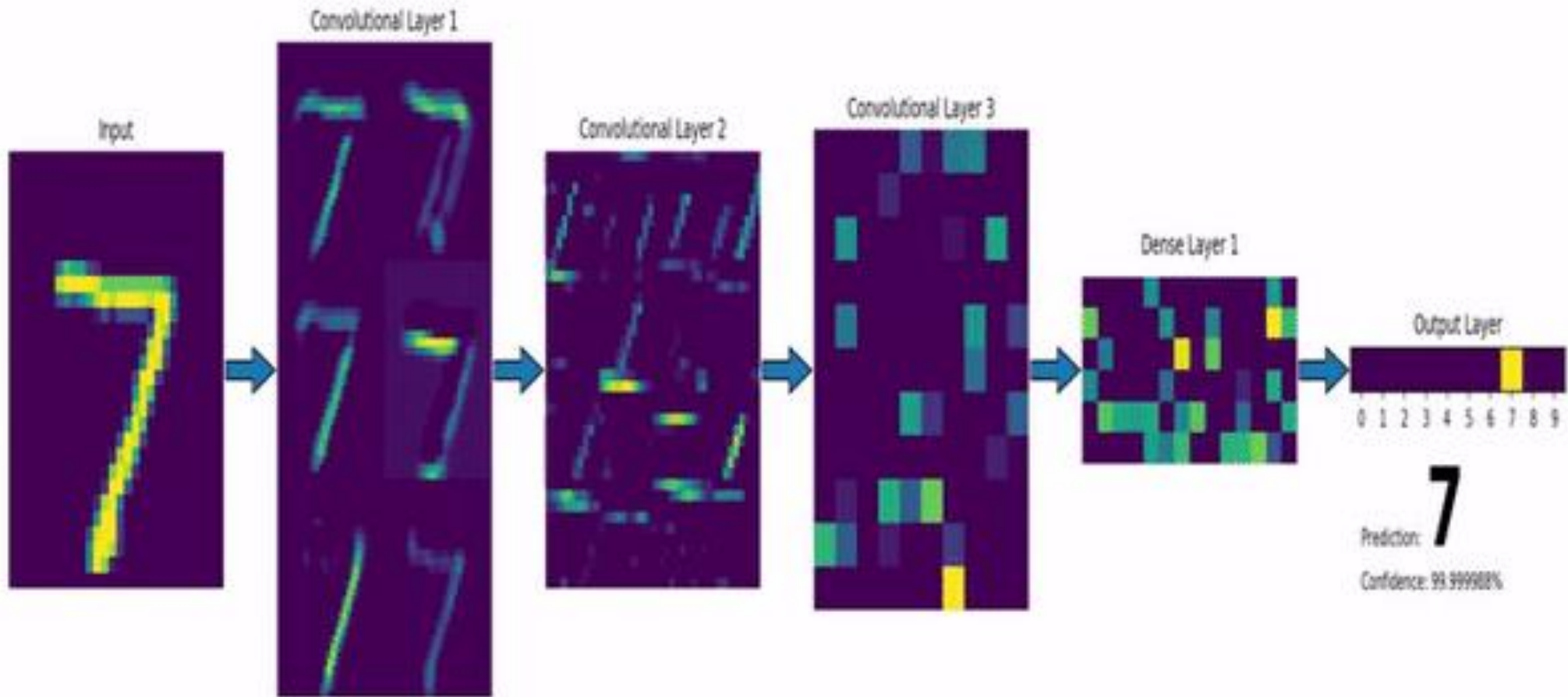


Fully-connected layers

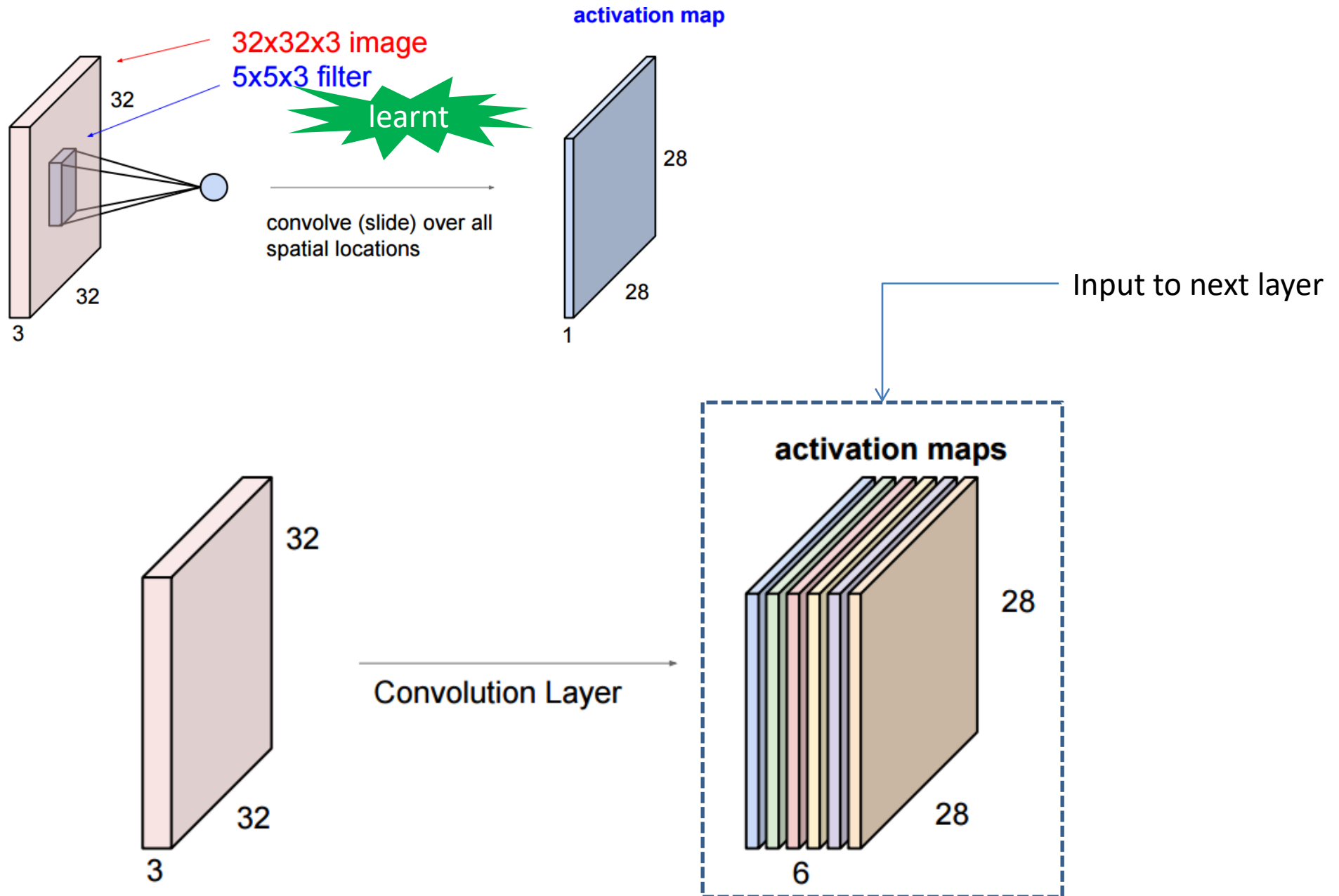




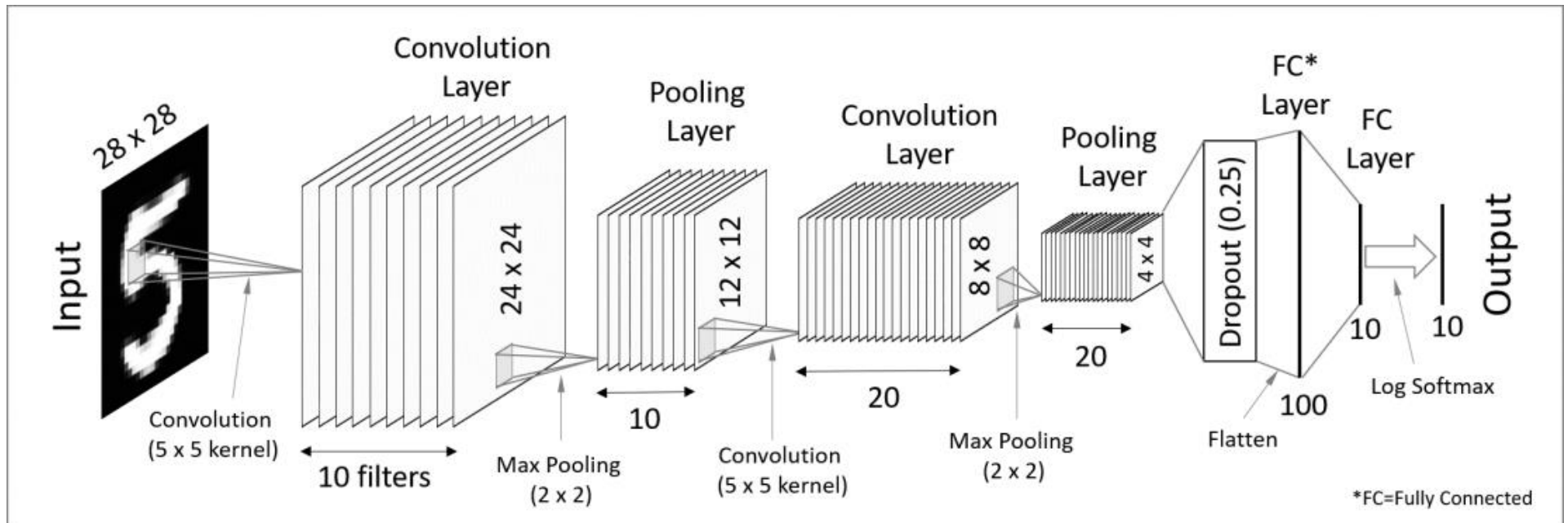
Convolution (in general)



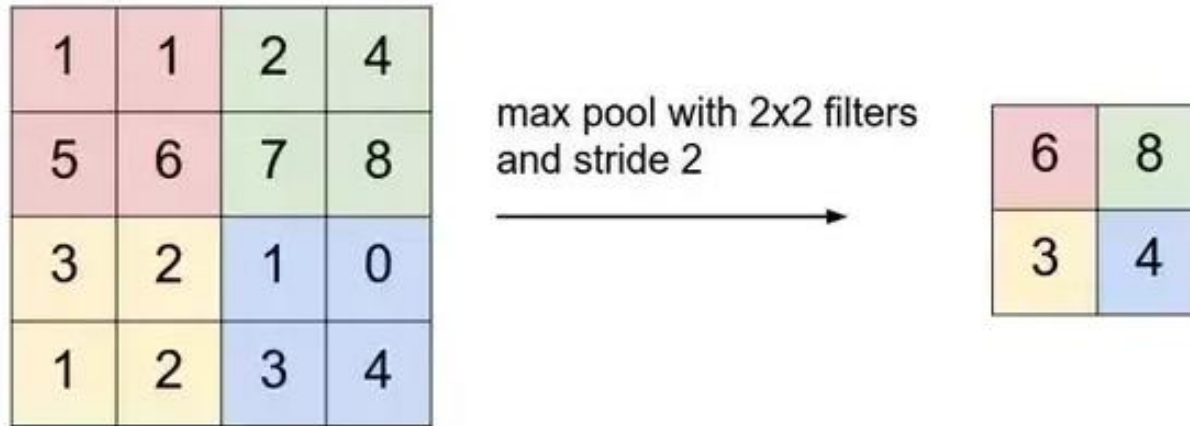
Convolution Layer



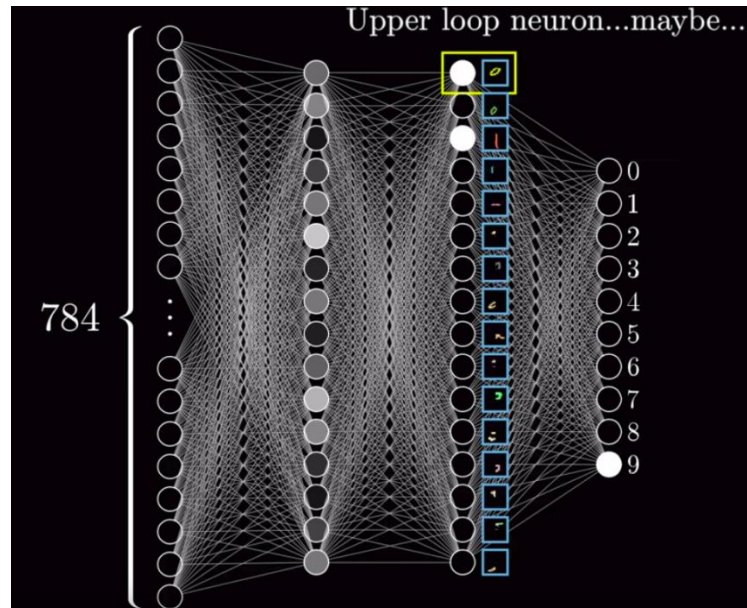
Convolutional Neural Network



Pooling Layer



Fully-connected Layer





Backpropagation in CNNs

NOTE: Filter
stride=2

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Z_1	Z_2
Z_3	Z_4

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{23}$$

$$A * W = Z$$

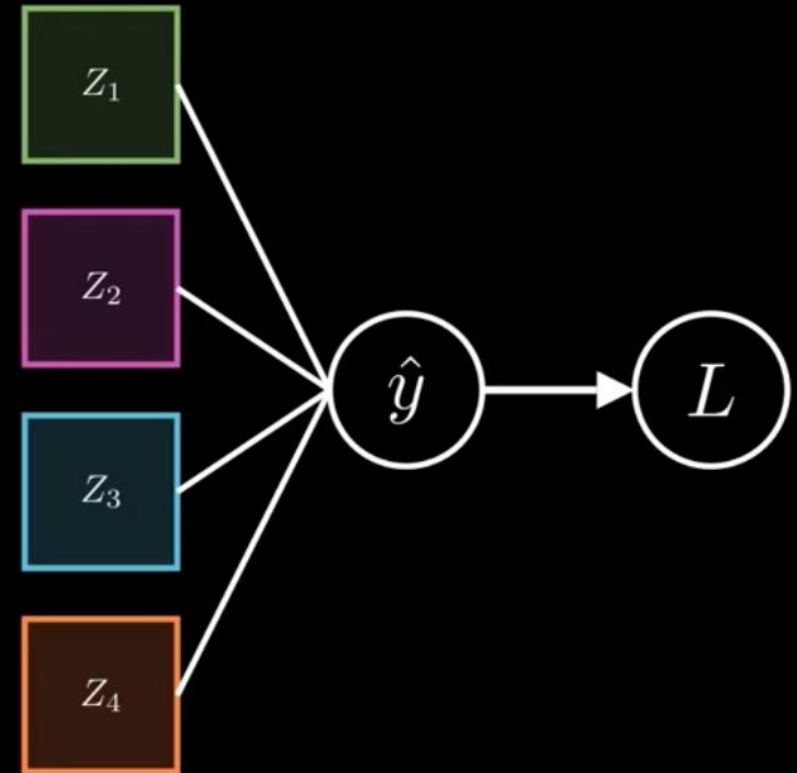


Backpropagation in CNNs

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial w_1}$	$\frac{\partial L}{\partial w_2}$	$\frac{\partial L}{\partial w_3}$
$\frac{\partial L}{\partial w_4}$	$\frac{\partial L}{\partial w_5}$	$\frac{\partial L}{\partial w_6}$
$\frac{\partial L}{\partial w_7}$	$\frac{\partial L}{\partial w_8}$	$\frac{\partial L}{\partial w_9}$

Z_1	Z_2
Z_3	Z_4



$$w_i^* = w_i - \alpha \times \frac{\partial L}{\partial w_i}$$

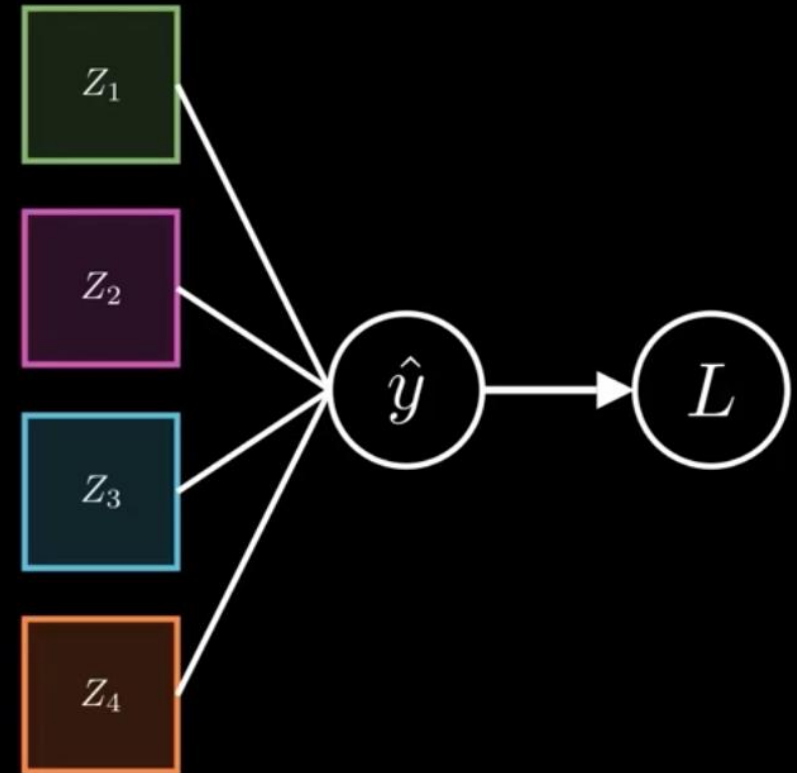


Backpropagation in CNNs

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial w_1}$	$\frac{\partial L}{\partial w_2}$	$\frac{\partial L}{\partial w_3}$
$\frac{\partial L}{\partial w_4}$	$\frac{\partial L}{\partial w_5}$	$\frac{\partial L}{\partial w_6}$
$\frac{\partial L}{\partial w_7}$	$\frac{\partial L}{\partial w_8}$	$\frac{\partial L}{\partial w_9}$

z_1	z_2
z_3	z_4



$$w_i^* = w_i - \alpha \times \frac{\partial L}{\partial w_i}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial z_1}{\partial w_1} \frac{\partial \hat{y}}{\partial z_1} \frac{\partial L}{\partial \hat{y}}$$

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{23}$$

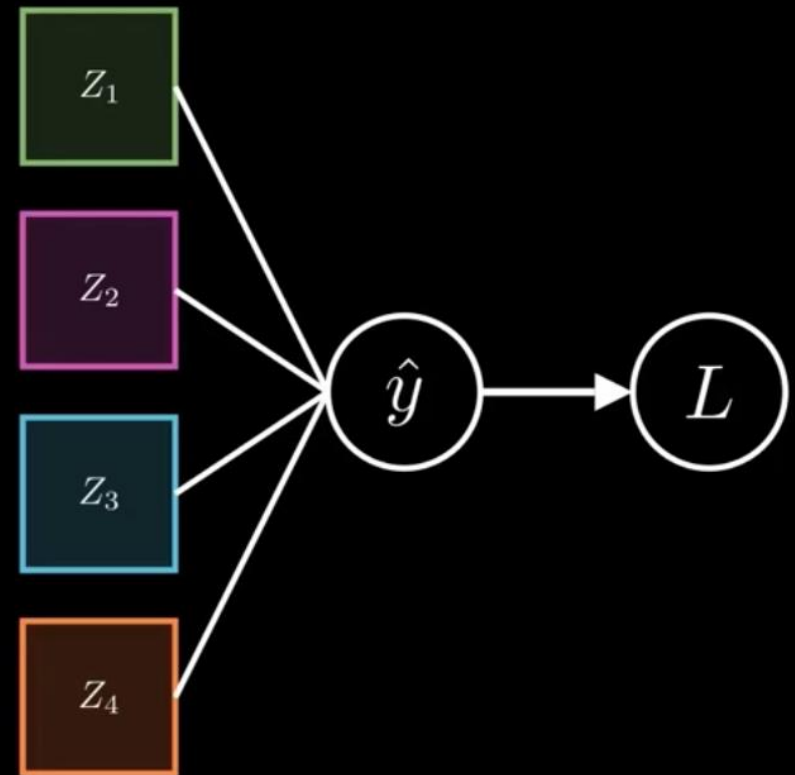


Backpropagation in CNNs

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial w_1}$	$\frac{\partial L}{\partial w_2}$	$\frac{\partial L}{\partial w_3}$
$\frac{\partial L}{\partial w_4}$	$\frac{\partial L}{\partial w_5}$	$\frac{\partial L}{\partial w_6}$
$\frac{\partial L}{\partial w_7}$	$\frac{\partial L}{\partial w_8}$	$\frac{\partial L}{\partial w_9}$

z_1	z_2
z_3	z_4



$$\frac{\partial L}{\partial w_1} = \frac{\partial z_1}{\partial w_1} \frac{\partial \hat{y}}{\partial z_1} \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial z_1}{\partial w_1}} \frac{\partial L}{\partial z_1} + \boxed{\frac{\partial z_2}{\partial w_1}} \frac{\partial L}{\partial z_2} + \boxed{\frac{\partial z_3}{\partial w_1}} \frac{\partial L}{\partial z_3} + \boxed{\frac{\partial z_4}{\partial w_1}} \frac{\partial L}{\partial z_4}$$



Backpropagation in CNNs

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2
z_3	z_4

I like this

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{23}$$

$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial z_1}{\partial w_1}} \frac{\partial L}{\partial z_1} + \boxed{\frac{\partial z_2}{\partial w_1}} \frac{\partial L}{\partial z_2} + \boxed{\frac{\partial z_3}{\partial w_1}} \frac{\partial L}{\partial z_3} + \boxed{\frac{\partial z_4}{\partial w_1}} \frac{\partial L}{\partial z_4}$$

$$z_1 = w_1 \times a_1 + w_2 \times a_2 + w_3 \times a_3 + w_4 \times a_6 \dots w_9 \times a_{13}$$

$$z_2 = w_1 \times a_3 + w_2 \times a_4 + w_3 \times a_5 + w_4 \times a_8 \dots w_9 \times a_{15}$$

$$z_3 = w_1 \times a_{11} + w_2 \times a_{12} + w_3 \times a_{13} + w_4 \times a_{16} \dots w_9 \times a_{23}$$

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{25}$$

$$\frac{\partial L}{\partial w_2} = a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4}$$

Backpropagation in CNNs

$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial z_1}{\partial w_1}} \frac{\partial L}{\partial z_1} + \boxed{\frac{\partial z_2}{\partial w_1}} \frac{\partial L}{\partial z_2} + \boxed{\frac{\partial z_3}{\partial w_1}} \frac{\partial L}{\partial z_3} + \boxed{\frac{\partial z_4}{\partial w_1}} \frac{\partial L}{\partial z_4}$$

$$z_1 = w_1 \times a_1 + w_2 \times a_2 + w_3 \times a_3 + w_4 \times a_6 \dots w_9 \times a_{13}$$

$$z_2 = w_1 \times a_3 + w_2 \times a_4 + w_3 \times a_5 + w_4 \times a_8 \dots w_9 \times a_{15}$$

$$z_3 = w_1 \times a_{11} + w_2 \times a_{12} + w_3 \times a_{13} + w_4 \times a_{16} \dots w_9 \times a_{23}$$

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{25}$$

$$\frac{\partial L}{\partial w_2} = a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4}$$

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2
z_3	z_4

$$\begin{aligned} \frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4} \end{aligned}$$



Backpropagation in CNNs

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

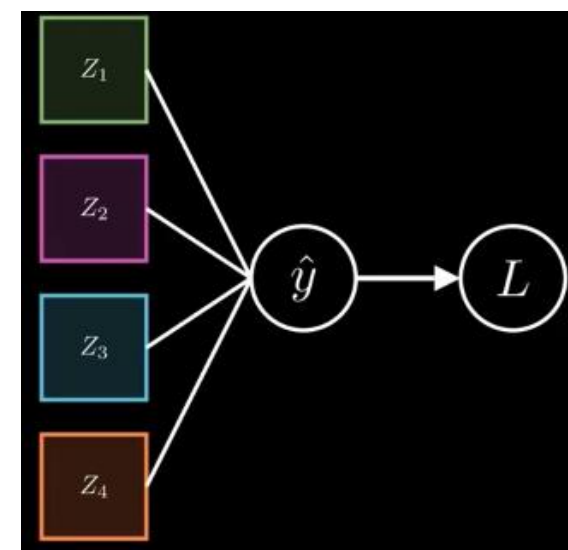
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2
z_3	z_4

$$\begin{aligned}
 \frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_1 \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}
 \end{aligned}$$

$\frac{\partial L}{\partial w_1}$	$\frac{\partial L}{\partial w_2}$	$\frac{\partial L}{\partial w_3}$
$\frac{\partial L}{\partial w_4}$	$\frac{\partial L}{\partial w_5}$	$\frac{\partial L}{\partial w_6}$
$\frac{\partial L}{\partial w_7}$	$\frac{\partial L}{\partial w_8}$	$\frac{\partial L}{\partial w_9}$

$\frac{\partial L}{\partial z_1}$	$\frac{\partial L}{\partial z_2}$
$\frac{\partial L}{\partial z_3}$	$\frac{\partial L}{\partial z_4}$



$$\frac{\partial L}{\partial w_1} = \frac{\partial z_1}{\partial w_1} \frac{\partial \hat{y}}{\partial z_1} \frac{\partial L}{\partial \hat{y}}$$

Backpropagation in CNNs

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial z_1}$	$\frac{\partial L}{\partial z_2}$
$\frac{\partial L}{\partial z_3}$	$\frac{\partial L}{\partial z_4}$

$$\begin{aligned}
 \frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\
 \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}
 \end{aligned}$$

a_1	a_2	a_3
a_6	a_7	a_8
a_{11}	a_{12}	a_{13}

 $\times \frac{\partial L}{\partial z_1}$



Backpropagation in CNNs

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial z_1}$	$\frac{\partial L}{\partial z_2}$
$\frac{\partial L}{\partial z_3}$	$\frac{\partial L}{\partial z_4}$

$$\begin{aligned}\frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}\end{aligned}$$

a_1	a_2	a_3
a_6	a_7	a_8
a_{11}	a_{12}	a_{13}

$\times \frac{\partial L}{\partial z_1} +$

a_3	a_4	a_5
a_8	a_9	a_{10}
a_{13}	a_{14}	a_{15}

$\times \frac{\partial L}{\partial z_2}$



Backpropagation in CNNs

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial z_1}$	$\frac{\partial L}{\partial z_2}$
$\frac{\partial L}{\partial z_3}$	$\frac{\partial L}{\partial z_4}$

$$\begin{aligned}\frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}\end{aligned}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_6 & a_7 & a_8 \\ a_{11} & a_{12} & a_{13} \end{bmatrix} \times \frac{\partial L}{\partial z_1} + \begin{bmatrix} a_3 & a_4 & a_5 \\ a_8 & a_9 & a_{10} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \times \frac{\partial L}{\partial z_2} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{16} & a_{17} & a_{18} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times \frac{\partial L}{\partial z_3} + \begin{bmatrix} a_{13} & a_{14} & a_{15} \\ a_{18} & a_{19} & a_{20} \\ a_{23} & a_{24} & a_{25} \end{bmatrix} \times \frac{\partial L}{\partial z_4} = \begin{bmatrix} \frac{\partial L}{\partial w_1} & \frac{\partial L}{\partial w_2} & \frac{\partial L}{\partial w_3} \\ \frac{\partial L}{\partial w_4} & \frac{\partial L}{\partial w_5} & \frac{\partial L}{\partial w_6} \\ \frac{\partial L}{\partial w_7} & \frac{\partial L}{\partial w_8} & \frac{\partial L}{\partial w_9} \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_2^* & w_3^* \\ w_4^* & w_5^* & w_6^* \\ w_7^* & w_8^* & w_9^* \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} - \alpha \times \begin{bmatrix} \frac{\partial L}{\partial w_1} & \frac{\partial L}{\partial w_2} & \frac{\partial L}{\partial w_3} \\ \frac{\partial L}{\partial w_4} & \frac{\partial L}{\partial w_5} & \frac{\partial L}{\partial w_6} \\ \frac{\partial L}{\partial w_7} & \frac{\partial L}{\partial w_8} & \frac{\partial L}{\partial w_9} \end{bmatrix}$$



CNN – forward & backpropagation

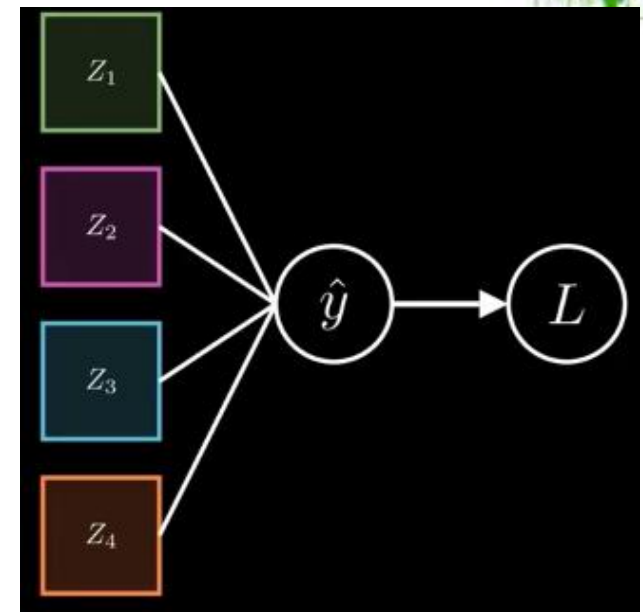


a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2
z_3	z_4

I like this



$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} \dots w_9 \times a_{23}$$

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$\frac{\partial L}{\partial z_1}$	$\frac{\partial L}{\partial z_2}$
$\frac{\partial L}{\partial z_3}$	$\frac{\partial L}{\partial z_4}$

$$\begin{aligned}\frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\ \frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}\end{aligned}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial z_1}{\partial w_1} \frac{\partial \hat{y}}{\partial z_1} \frac{\partial L}{\partial \hat{y}}$$

a_1	a_2	a_3
a_6	a_7	a_8
a_{11}	a_{12}	a_{13}

$$\times \frac{\partial L}{\partial z_1} +$$

a_3	a_4	a_5
a_8	a_9	a_{10}
a_{13}	a_{14}	a_{15}

$$\times \frac{\partial L}{\partial z_2} +$$

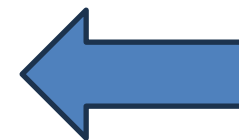
a_{11}	a_{12}	a_{13}
a_{16}	a_{17}	a_{18}
a_{21}	a_{22}	a_{23}

$$\times \frac{\partial L}{\partial z_3} +$$

a_{13}	a_{14}	a_{15}
a_{18}	a_{19}	a_{20}
a_{23}	a_{24}	a_{25}

$$\times \frac{\partial L}{\partial z_4} =$$

$\frac{\partial L}{\partial w_1}$	$\frac{\partial L}{\partial w_2}$	$\frac{\partial L}{\partial w_3}$
$\frac{\partial L}{\partial w_4}$	$\frac{\partial L}{\partial w_5}$	$\frac{\partial L}{\partial w_6}$
$\frac{\partial L}{\partial w_7}$	$\frac{\partial L}{\partial w_8}$	$\frac{\partial L}{\partial w_9}$



Convolution

Multi-Neuron Networks :: Backpropagation

$$J = \sum \frac{1}{2} (y - \hat{y})^2$$

$$\frac{\partial J}{\partial W^{(2)}} = (a^{(2)})^T \delta^{(3)}$$

$$\delta^{(3)} = -(y - \hat{y}) f'(z^{(3)})$$

$$\frac{\partial J}{\partial W^{(1)}} = X^T \delta^{(2)}$$

$$\delta^{(2)} = \delta^{(3)} (W^{(2)})^T f'(z^{(2)})$$

$$\frac{\partial J}{\partial W^{(2)}} = \underbrace{\frac{\partial J}{\partial z^{(3)}}}_{\text{loss from previous layer}} \times \underbrace{\frac{\partial z^{(3)}}{\partial W^{(2)}}}_{\text{loss from local gradient}}$$

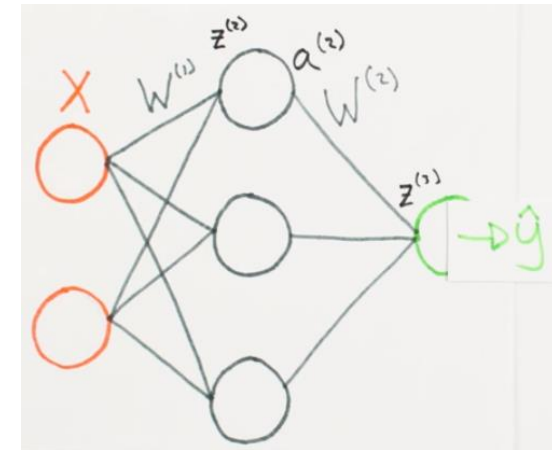
$$\frac{\partial J}{\partial W^{(1)}} = \underbrace{\frac{\partial J}{\partial z^{(2)}}}_{\text{loss from previous layer}} \times \underbrace{\frac{\partial z^{(2)}}{\partial W^{(1)}}}_{\text{loss from local gradient}}$$

$$\begin{aligned} z^{(2)} &= XW^{(1)} & (1) \\ a^{(2)} &= f(z^{(2)}) & (2) \\ z^{(3)} &= a^{(2)}W^{(2)} & (3) \\ \hat{y} &= f(z^{(3)}) & (4) \end{aligned}$$

$$J = \sum \frac{1}{2} (y - f(f(XW^{(1)})W^{(2)}))^2$$

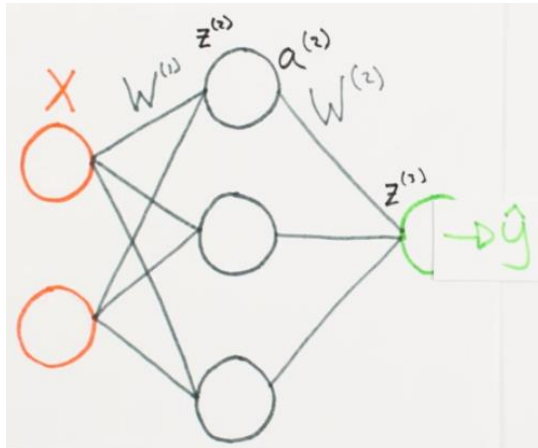
↑ HOW DOES THIS CHANGE IF I CHANGE THESE? ↑

$$\frac{\partial J}{\partial W}$$



$$W^{(1)} = \begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{bmatrix} \quad \frac{\partial J}{\partial W^{(1)}} = \begin{bmatrix} \frac{\partial J}{\partial w_{11}^{(1)}} & \frac{\partial J}{\partial w_{12}^{(1)}} & \frac{\partial J}{\partial w_{13}^{(1)}} \\ \frac{\partial J}{\partial w_{21}^{(1)}} & \frac{\partial J}{\partial w_{22}^{(1)}} & \frac{\partial J}{\partial w_{23}^{(1)}} \end{bmatrix}$$

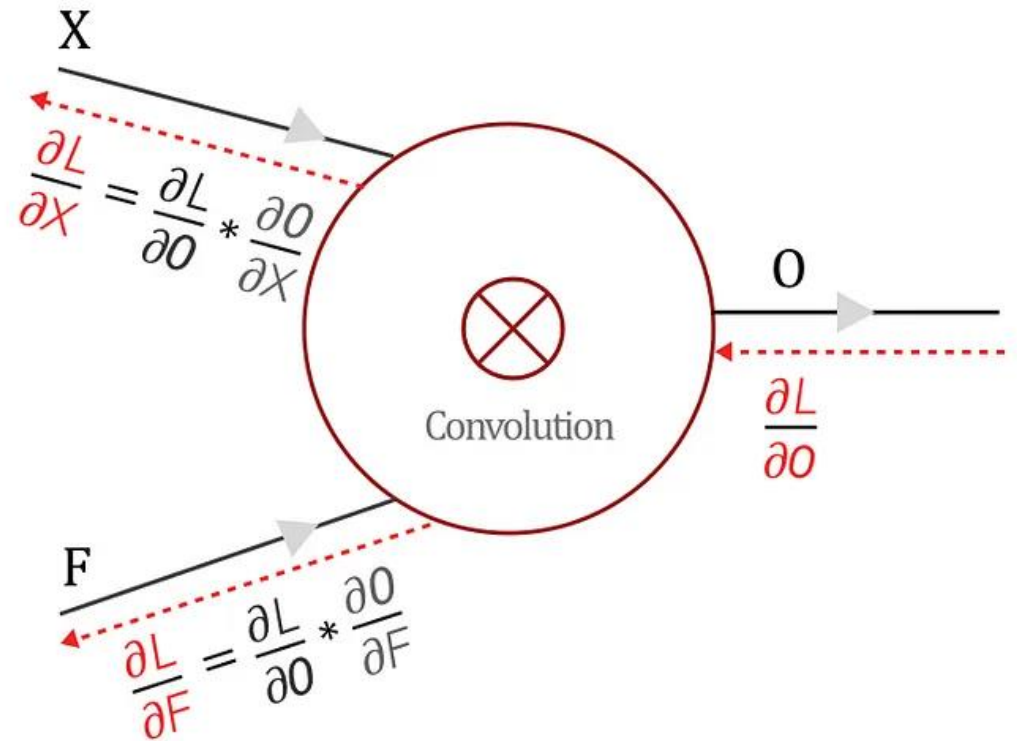
$$W^{(2)} = \begin{bmatrix} w_{11}^{(2)} \\ w_{21}^{(2)} \\ w_{31}^{(2)} \end{bmatrix} \quad \frac{\partial J}{\partial W^{(2)}} = \begin{bmatrix} \frac{\partial J}{\partial w_{11}^{(2)}} \\ \frac{\partial J}{\partial w_{21}^{(2)}} \\ \frac{\partial J}{\partial w_{31}^{(2)}} \end{bmatrix}$$



$$\begin{aligned} z^{(2)} &= XW^{(1)} & (1) \\ a^{(2)} &= f(z^{(2)}) & (2) \\ z^{(3)} &= a^{(2)}W^{(2)} & (3) \\ \hat{y} &= f(z^{(3)}) & (4) \end{aligned}$$

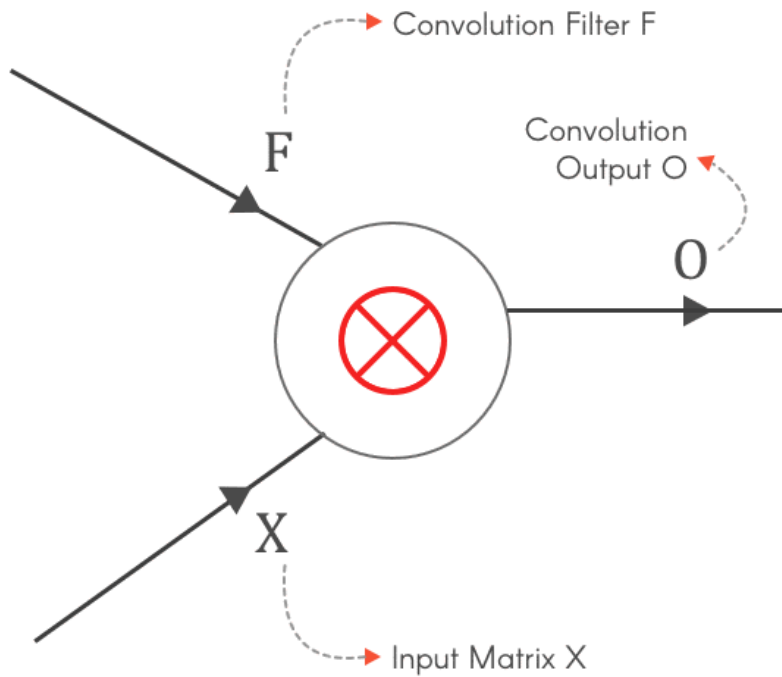
$$\frac{\partial J}{\partial W^{(2)}} = \underbrace{\frac{\partial J}{\partial z^{(3)}}}_{\text{loss from previous layer}} \times \underbrace{\frac{\partial z^{(3)}}{\partial W^{(2)}}}_{\text{loss from local gradient}}$$

$$\frac{\partial J}{\partial W^{(1)}} = \underbrace{\frac{\partial J}{\partial z^{(2)}}}_{\text{loss from previous layer}} \times \underbrace{\frac{\partial z^{(2)}}{\partial W^{(1)}}}_{\text{loss from local gradient}}$$



$$\frac{\partial O}{\partial X} \text{ \& \; } \frac{\partial O}{\partial F} \text{ are local gradients}$$

$\frac{\partial L}{\partial z}$ is the loss from the previous layer which has to be backpropagated to other layers



Backpropagation in a Convolutional Layer of a CNN

Finding the gradients:

$$\frac{\partial L}{\partial F} = \text{Convolution} \left(\text{Input } X, \text{ Loss gradient } \frac{\partial L}{\partial O} \right)$$

$$\frac{\partial L}{\partial X} = \text{Full Convolution} \left(180^\circ \text{rotated Filter } F, \text{ Loss Gradient } \frac{\partial L}{\partial O} \right)$$



Backprop in pooling layer

0.1	0.5	1.2	-0.7
0.8	-0.2	-0.5	0.3
0.4	0.9	-0.1	-0.2
-0.6	0.1	0.5	0.3

Activations

max-pooling

0.8	1.2
0.9	0.5

		X	
X			
	X		
		X	

max locations

Gradients

1.3	0.5
0.4	0.1

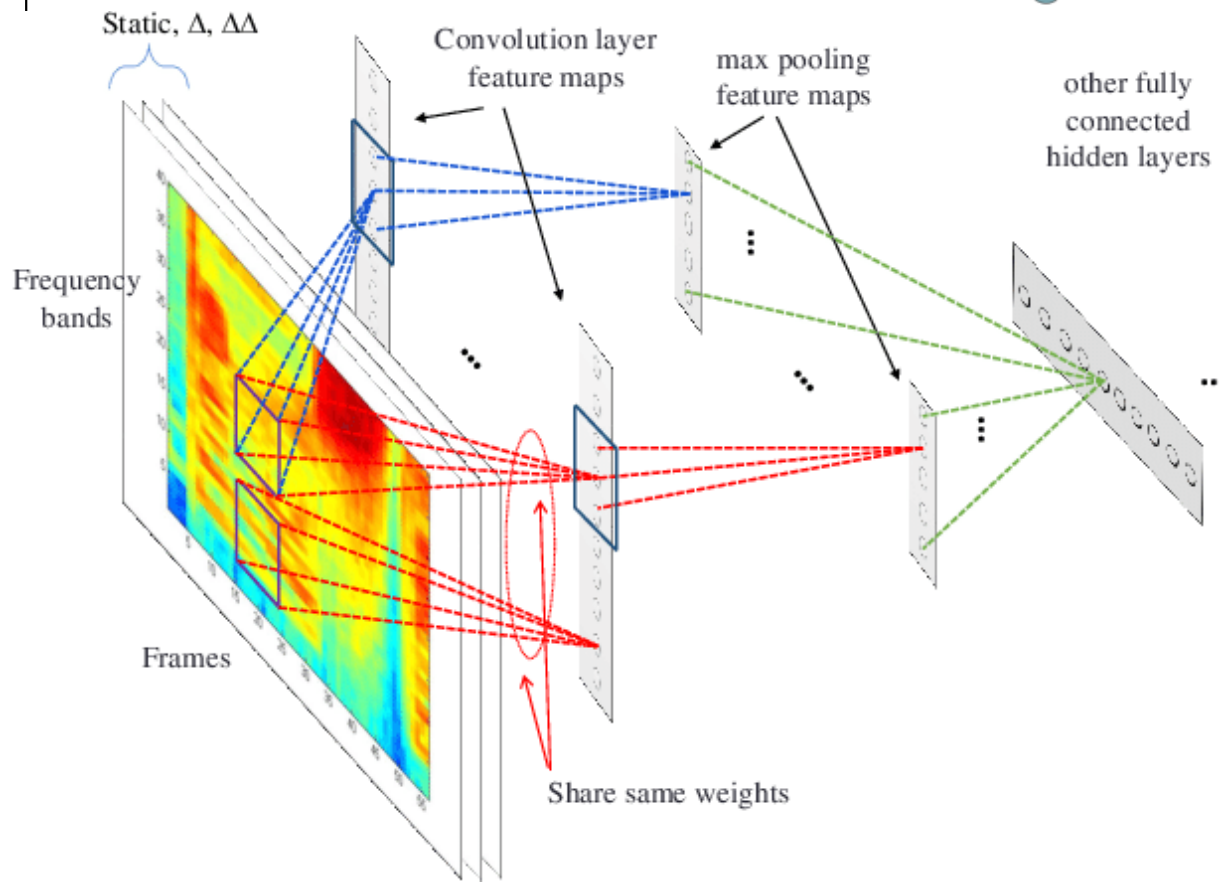
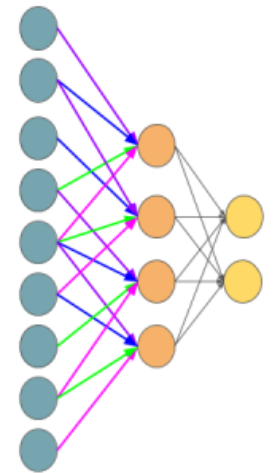
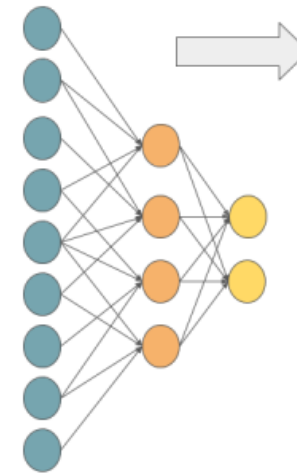
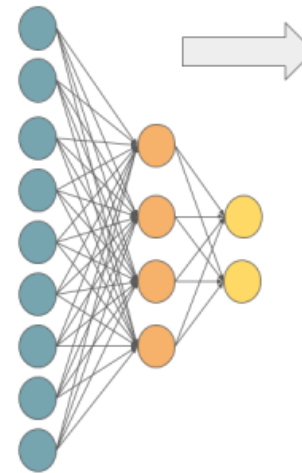
unpooling

0	0	0.5	0
1.3	0	0	0
0	0.4	0	0
0	0	0.1	0



connections cutting

weights sharing

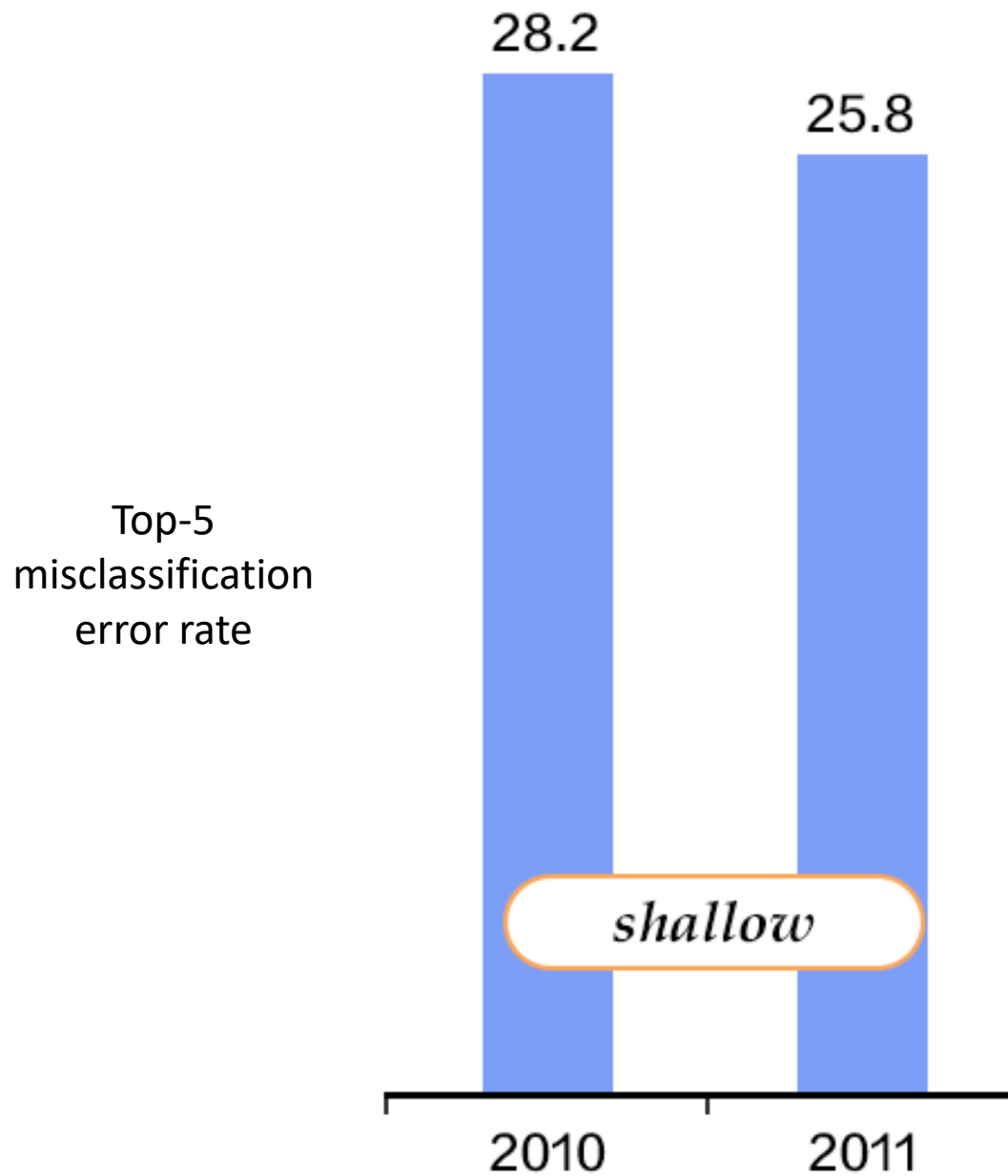


ImageNet Challenge

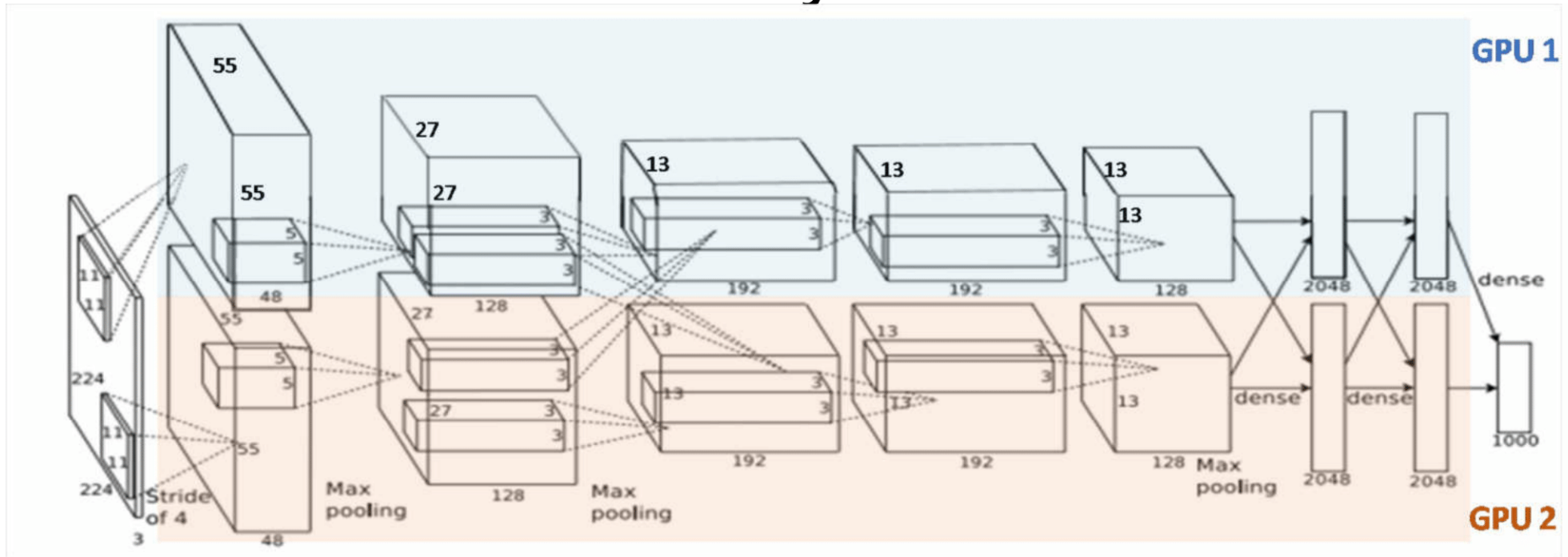
IMAGENET

- 1,000 object classes (categories).
- Images:
 - 1.2 M train
 - 100k test.





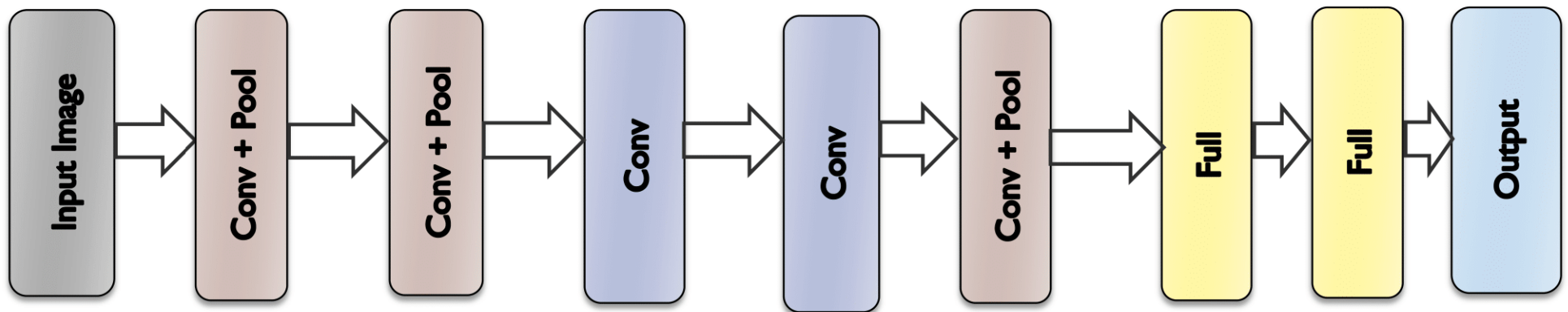
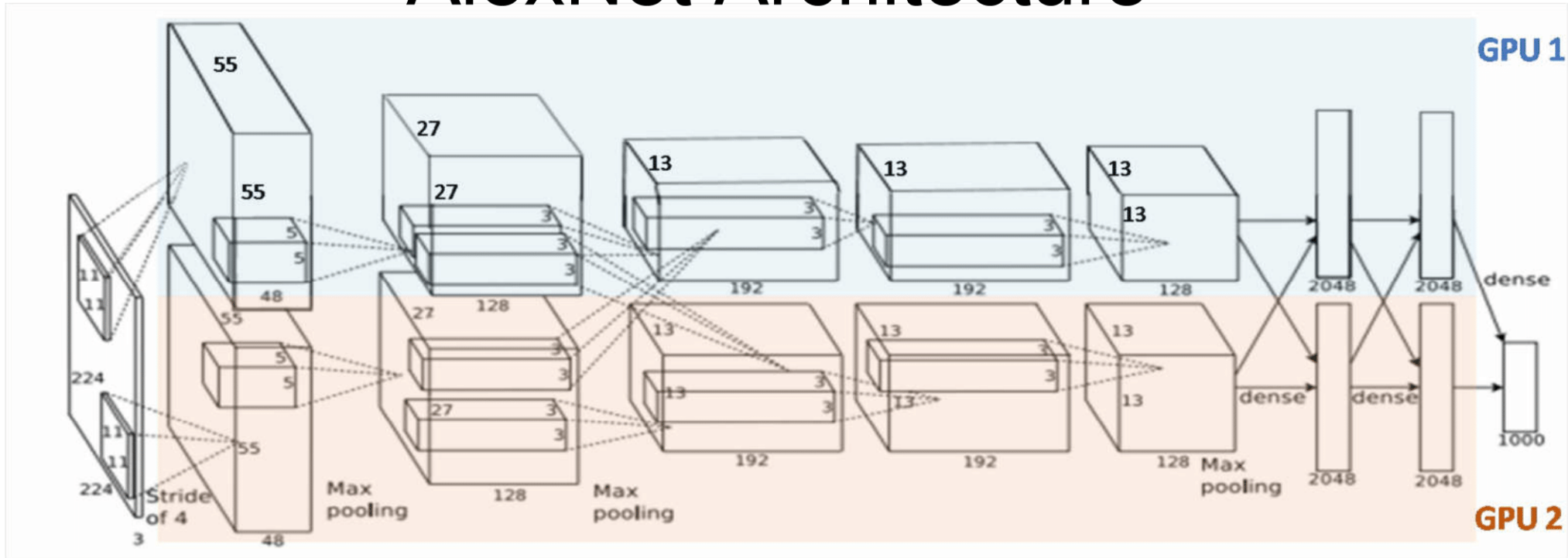
Case Study: AlexNet



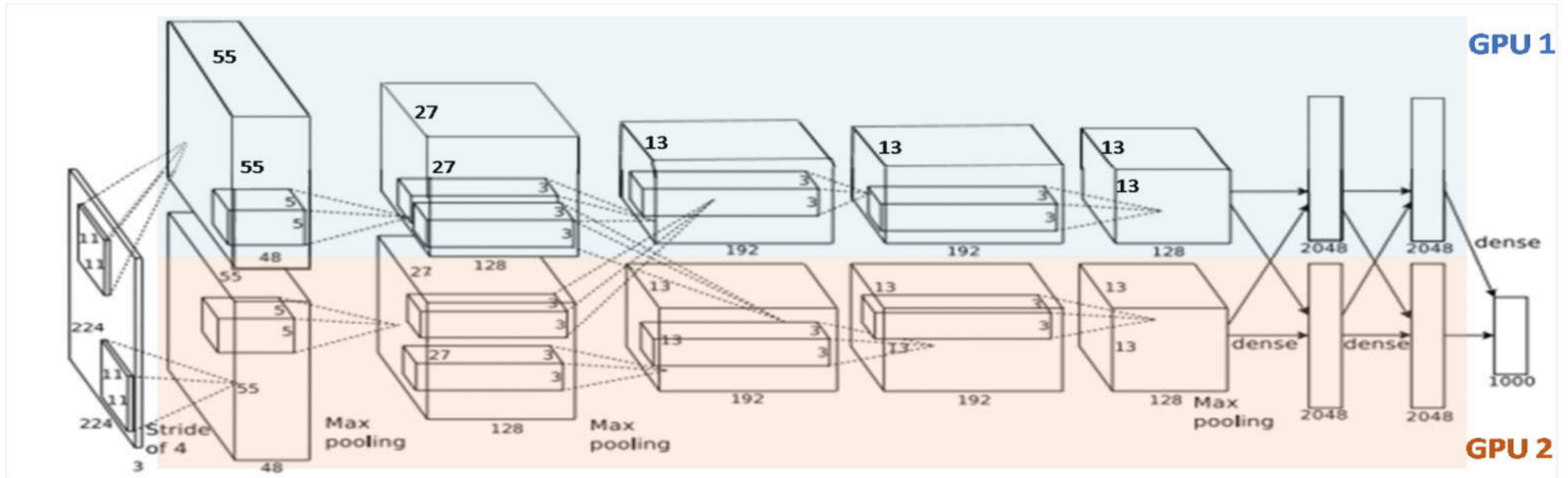
- Winner of ImageNet LSVRC-2010.
- Trained over 1.2M images using SGD with regularization.
- Deep architecture (60M parameters.)
- Optimized GPU implementation (cuda-convnet)

Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *NIPS* 2012.

AlexNet Architecture



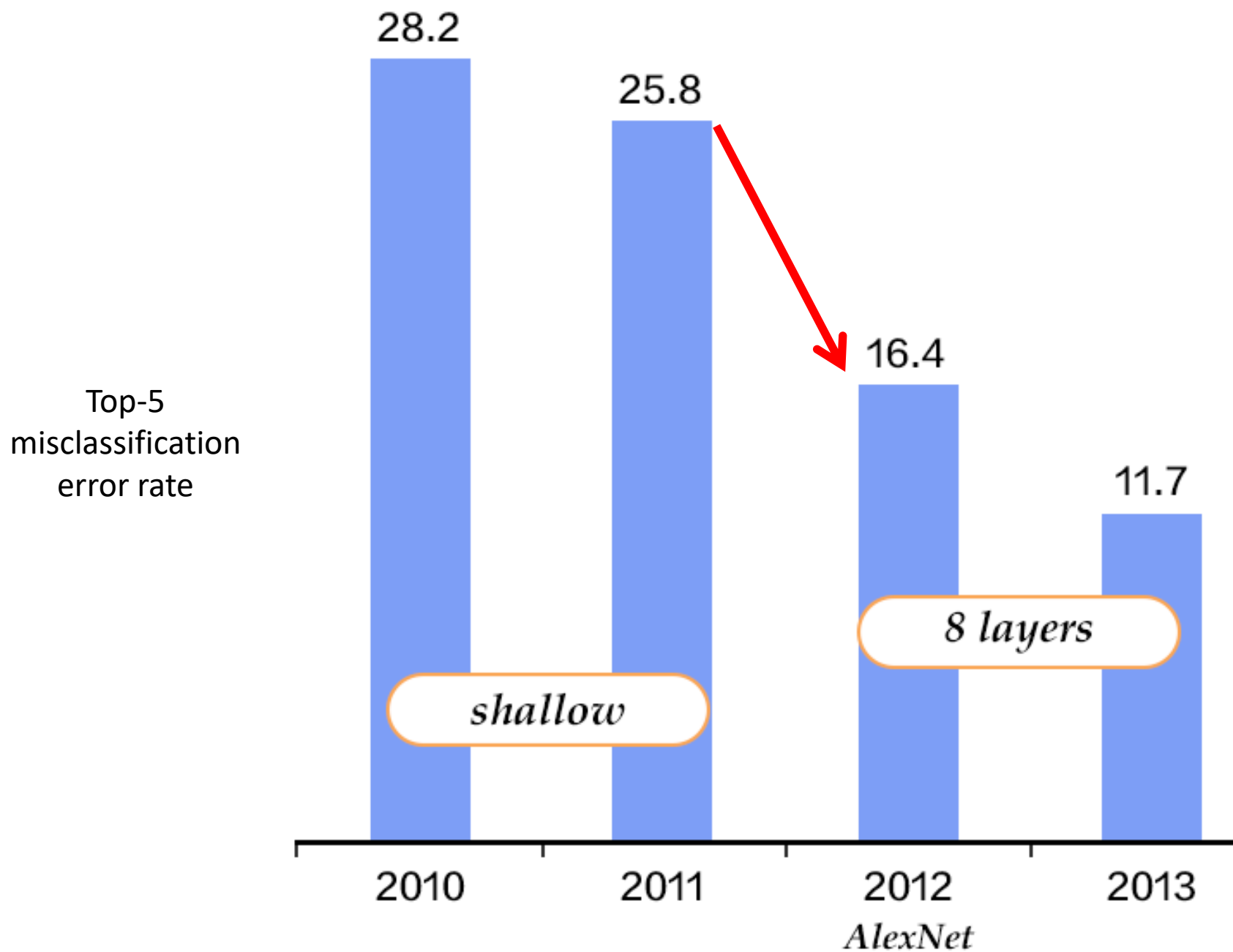
AlexNet Architecture



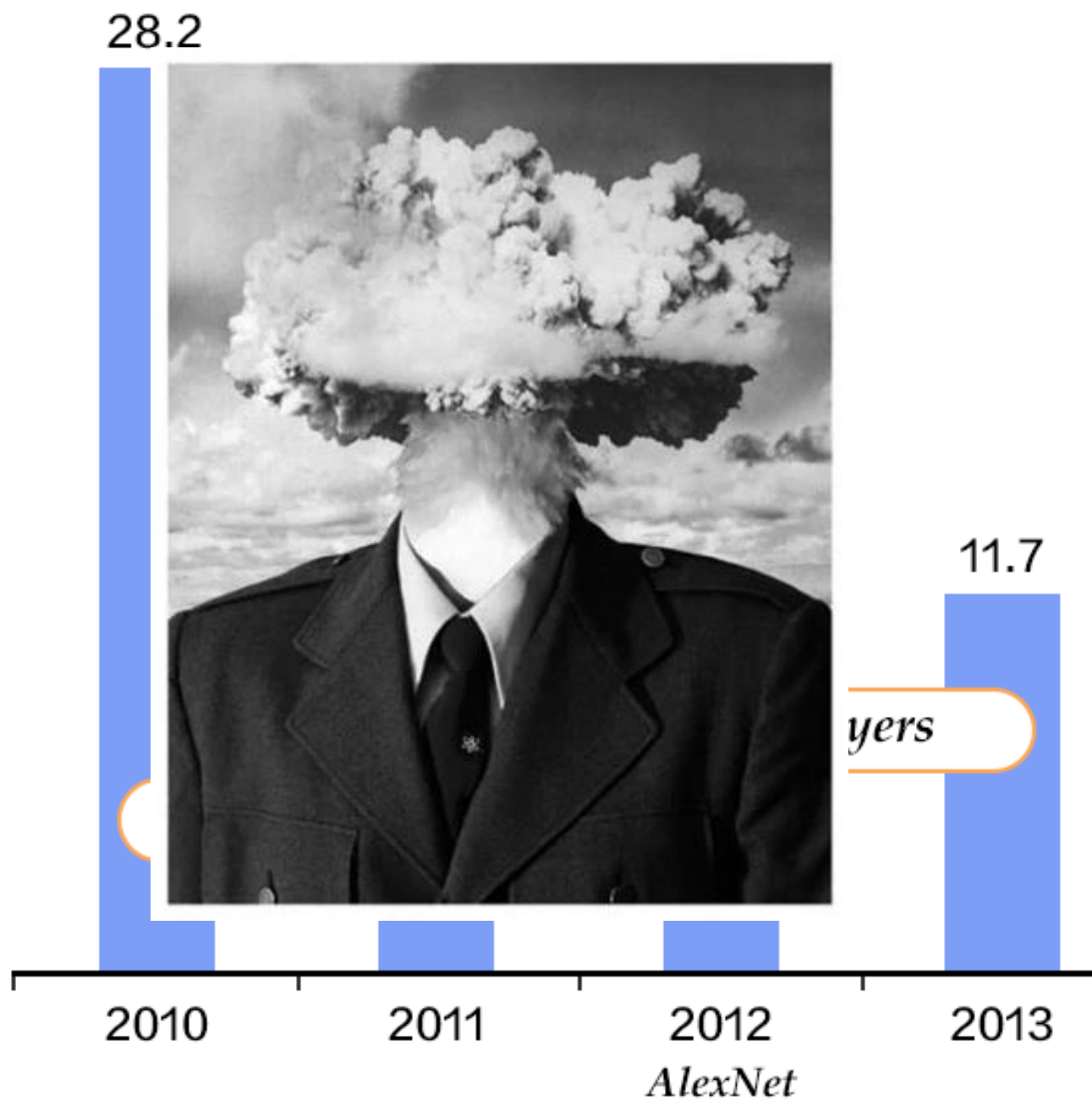
About 57 M parameters are in the fully connected layers

									Total
Parameters :	$[(11 \times 11 \times 3) + 1] \times 96$ = 35 K	$[5 \times 5 \times 48] \times 256$ = 307 K	$[3 \times 3 \times 256] \times 384$ = 884 K	663 K	442 K	37 M	16 M	4 M	60 M
Neurons :	253,440	$27 \times 27 \times 256$ = 186,624	$13 \times 13 \times 384$ = 64,896	$13 \times 13 \times 384$ = 64,896	$13 \times 13 \times 256$ = 43,264	4096	4096	1000	0.63 M

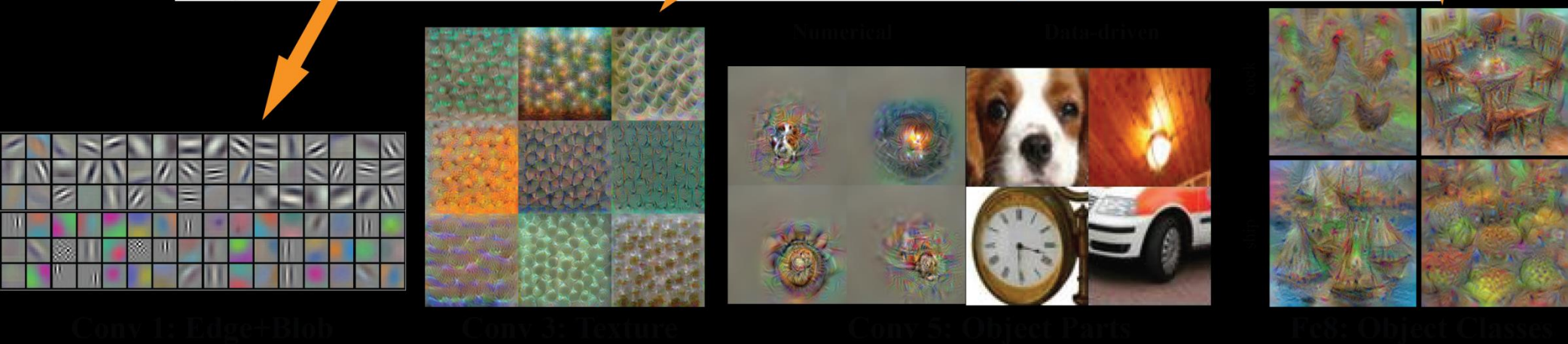
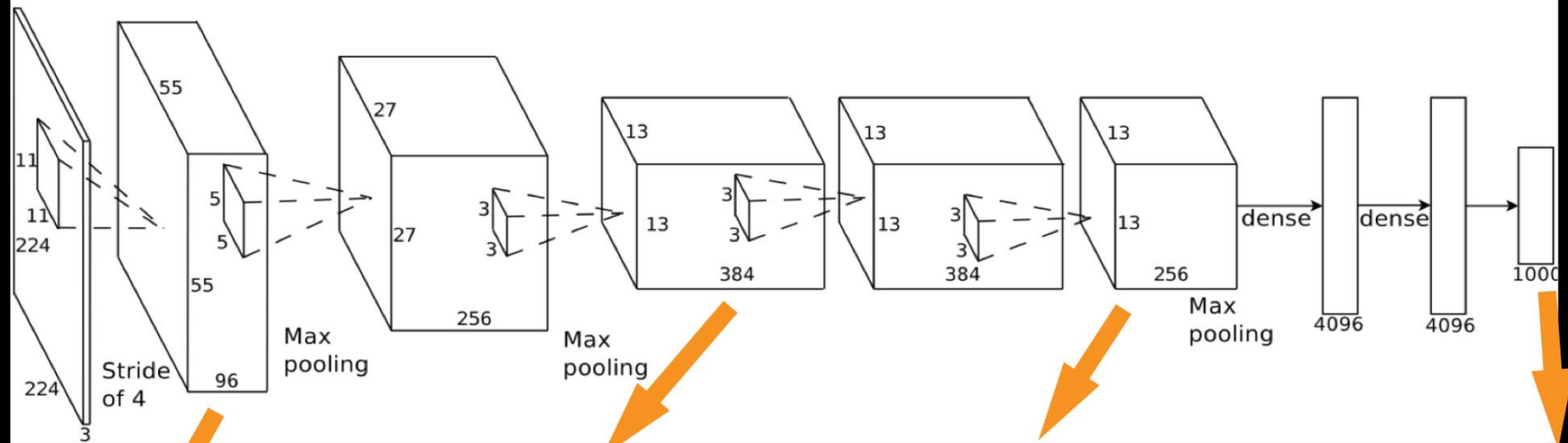
- Convolutional layers cumulatively contain about 90-95% of computation, only about 5% of the parameters
- Fully-connected layers contain about 95% of parameters.



Top-5
misclassification
error rate



Filters learnt by AlexNet





Resources

- What is a convolution: <https://www.youtube.com/watch?v=KuXjwB4LzSA>
- Backprop in CNN: <https://www.youtube.com/@far1din619/videos>
- Local and layer gradients
 - <https://pavisj.medium.com/convolutions-and-backpropagations-46026a8f5d2c>