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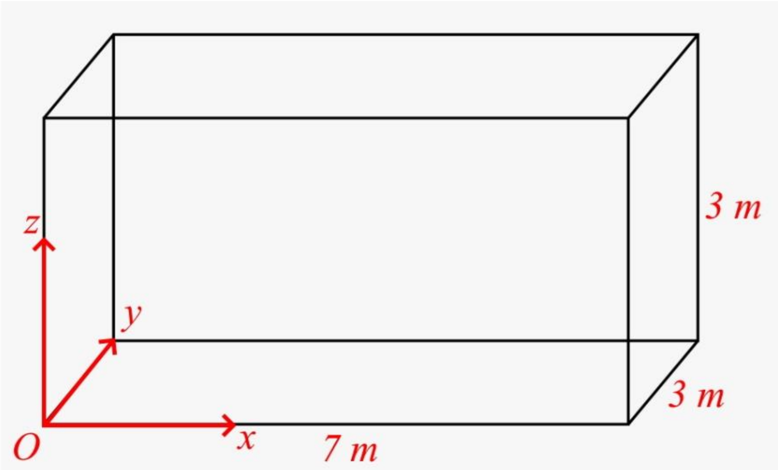
Shear Elasticity Problem

Question:

A rectangular prism located entirely in the positive (x, y, z) octant has dimensions of (7, 3, 3) m and is made of Polytetrafluoroethylene (PTFE) with E = 400 MPa and v = 0.42. It is placed into pure shear by applying a large shear stress of τ_{zx} = 150 MPa on it's +z surface in the +x direction along with corresponding shear stresses on its +x and x = 0 faces. The bottom, z = 0 face is fixed to the surface.



a) Determine the shear stresses applied to the +x and x = 0 faces and explained why these are necessary in order to keep the prism in pure shear equilibrium.
b) Using Maple, calculate the displacement of the +z surface in the +x direction as a result of this shear stress.
c) Using FlexPDE, confirm your answer for the +x displacement of the +z surface and also calculate the y, z displacement of this surface if any.
d) Redo the problem and determine the displacements of the +z surface if there are no additional shear stresses on its x = 0 and +x surfaces and the prism is no longer in pure shear.
e) Now, the bottom face, z = 0 is not fixed to the surface but the prism is still in pure shear with additional shear stresses on it's sides. Determine the displacement of the +z surface.

Solution:

a) In order to keep the prism in pure shear equilibrium, additional stresses of 150 MPa on the +x face in the +z direction and 150 MPa on the x=0 face in the -z direction must be applied. This is necessary to prevent the prism from deforming incorrectly. If these stresses were not to be applied, the prism would deform differently than predicted by pure shear. There would also be a shear stress of 150 MPa on the z = 0 face (fixed face) in the -x direction due to reaction force.

b) We are given $\tau_{zx} = 150$ MPa and since we have the young's modulus and poison's ratio, we can calculate the shear modulus, G. We can then calculate our shear strain using τ_{zx} and G. Once we have our shear strain, we can calculate the change in x by multiplying it with the height which is the z coordinate of the +z surface

Maple Code:

```
restart:
E:=400e6:
nu:=0.42:
tau_zx:=150e6:
Lz:=3:
G:=E/(2*(1+nu));
g_zx:=tau_zx/G;
DeltaX:=g_zx*Lz;
```

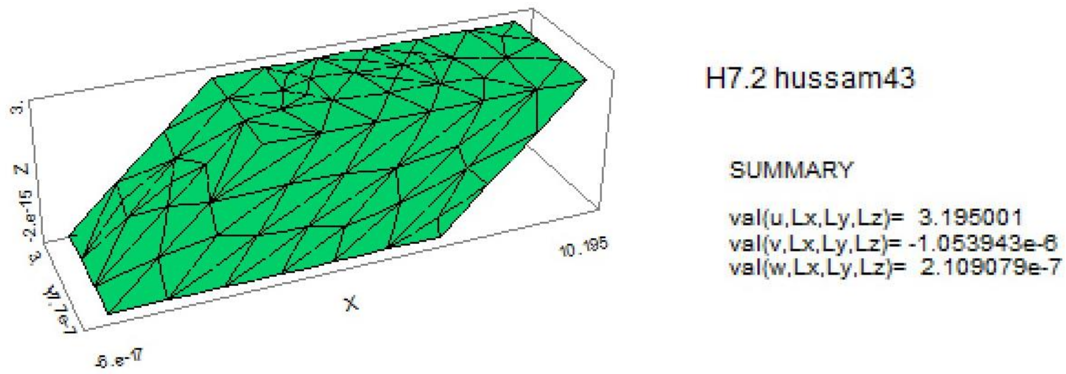
$$G := 1.408450704\ 10^{8}$$
$$g\_zx := 1.065000000$$
$$DeltaX := 3.195000000$$

We see that the +z surface is displaced in the +x direction by 3.195 m.

c) We shall now confirm this answer with FlexPDE while also depicting our deformation and calculating the y and z displacements of the +z face which by prediction should be negligible.

FlexPDE Code:

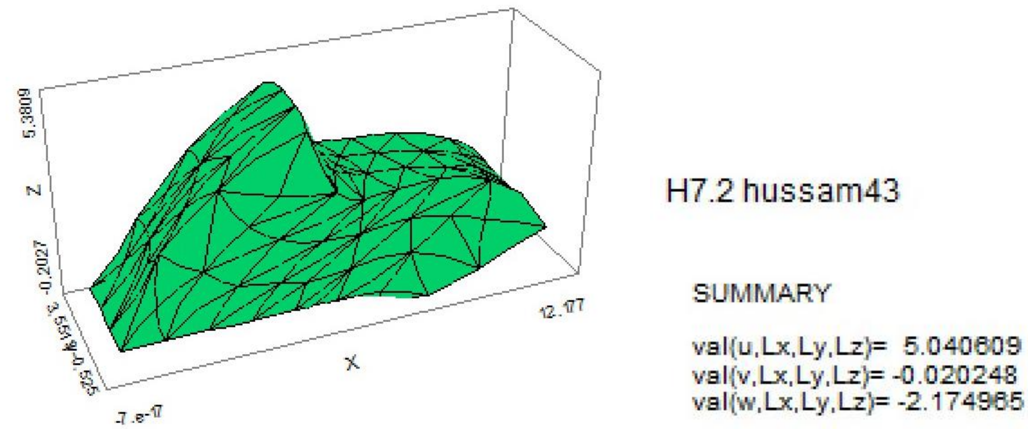
```
TITLE
'H7.2 hussam43'
SELECT
errlim=1e-4
ngrid=6
spectral_colors
COORDINATES
cartesian3

VARIABLES
u          !Displacement in x
v !Displacement in y
w !Displacement in z

DEFINITIONS
Lx=7
Ly=3
Lz=3
nu=0.42
E=400e6
G=E/(2*(1+nu))

s_applied = 150e6 !applied
stress


C11 =E*(1-nu)/(1+nu)/(1-2*nu)
C22 = C11
C33 = C11

C12 = E*nu/(1+nu)/(1-2*nu)
C13 = C12
C21 = C12
C23 = C12
C31 = C12
C32 = C12

!! Strain
!Axial Strain
ex=dx(u)
ey=dy(v)
ez=dz(w)
!Engineering Shear Strain
gxy=(dx(v)+dy(u))
gyz=(dy(w)+dz(v))
gxz=(dz(u)+dx(w))

!!Stress via Hooke's law
!Axial Stress
sx = C11*ex+C12*ey+C13*ez
sy = C21*ex+C22*ey+C23*ez
sz = C31*ex+C32*ey+C33*ez

!Shear stress
sxy=G*gxy
sxz=G*gxz
syz=G*gyz
EQUATIONS
!FNet = 0
u:
          dx(sx)+dy(sxy)+dz(
sxz)=0
v:
          dx(sxy)+dy(sy)+dz(
syz)=0
w:
          dx(sxz)+dy(syz)+dz
(sz)=0

EXTRUSION
surface 'bottom' z=0
surface 'top' z=Lz

BOUNDARIES
surface 'bottom'
value(u)=0
value(v)=0
value(w)=0
surface 'top'
load(u)=s_applied
load(v)=0
load(w)=0

          REGION 1
          START(0,0) !y=0
surface:

          load(u)=0

          load(v)=0

          load(w)=0
          LINE TO (Lx,0)
!x=Lx surface

          load(u)=0

          load(v)=0

          load(w)=s_applied
                    LINE TO
(Lx,Ly) !y=Ly surface

          load(u)=0

          load(v)=0

          load(w)=0
                    LINE TO
(0,Ly) !x=0 surface

          load(u)=0

          load(v)=0

          load(w)=-s_applied
                    LINE TO
CLOSE

MONITORS
contour(u) painted on x=0
contour(v) painted on x=0
contour(w) painted on x=0
grid(x+u,y+v,z+w)

PLOTS
          grid(x+u, y+v,
z+w)
          contour(u) on
surface z=0
          elevation(sx,sy,sz
) from (0,0,0) to (0,0,Lz)

          summary
                    report
val(u,Lx,Ly,Lz)
                    report
val(v,Lx,Ly,Lz)
                    report
val(w,Lx,Ly,Lz)
end
```



As we can see, what once was a simple rectangular prism now looks like a parallelogram prism. We have also confirmed our answers for the displacement of the +z surface in the +direction of 3.195 m. As predicted, the y and z displacement is nearly negligible

d) We will now redo the problem while assuming that there are no additional stresses applied to the sides and the prism is no longer in pure shear to observe how our deformation differs from what pure shear predicts. The bottom surface (z = 0) however is fixed. The prism is now not in pure sheer equilibrium but is in static equilbrium. Before even solving this part, we can predict that the prism will not topple over since it's bottom is fixed but will instead bend in the direction of the shear force. In order to do so, we make load(w) = 0 on the x=1 surface and load(w) = 0 on the -x surface.

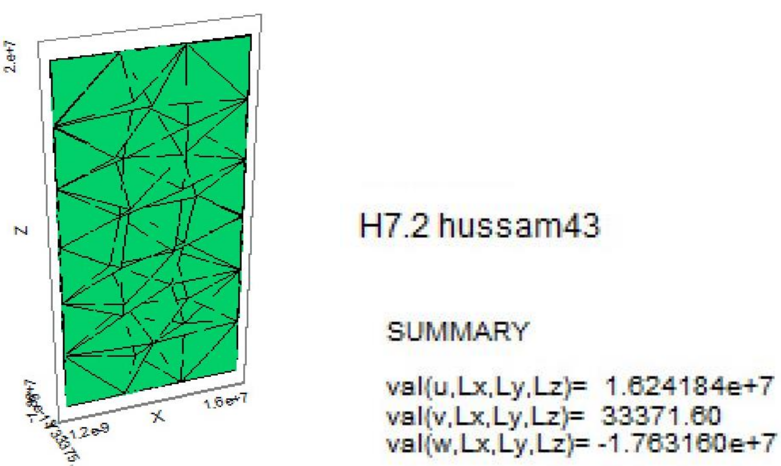
```
REGION 1
      START(0,0) !y=0 surface:
            load(u)=0
            load(v)=0
            load(w)=0
      LINE TO (Lx,0) !x=Lx surface
            load(u)=0
            load(v)=0
            load(w)=0
            LINE TO (Lx,Ly) !y=Ly surface
            load(u)=0
            load(v)=0
            load(w)=0
            LINE TO (0,Ly) !x=0 surface
            load(u)=0
            load(v)=0
            load(w)=0
            LINE TO CLOSE
```



As predicted, the prism deformed irregularly and morphed into a completely new shape even though only one force acted on it on the top surface. The +z face of the prism was displaced greatly in +x direction as it shifted forwards 5.040609 m and also in the -z direction as the height of the prism decreased by 2.174965 m. This is obviously due to the bending movement of the prism causing it to bow down which could not have been prevented as the prism was not in pure shear equilibrium. Surprisingly, there was also a shift of 0.02 m in the -y direction.

e) For this part we want to observe what will happen if the bottom surface, z = 0 of the prism isn't fixed. In order to do so, we add load(w) = 0 to our code while defining our bottom boundary.

```
BOUNDARIES
surface 'bottom'
value(u)=0
value(v)=0
load(w)=0
surface 'top'
load(u)=s_applied
load(v)=0
load(w)=0
```



The depiction of the deformation is quite unclear and difficult to make out. From observing the summary, it seems as if the prism has been completely flattened out in a rather unpractical manner. The height of the prism (z component) which was initially 3 m has decreased by a large value of $1.763160 * 10^7$ m which seems quite impossible. I therefore assume that the height is now 0 m and the prism is completely flat. The top surface shifted in the x direction by a whopping $1.623184 * 10^7$ m which also seems quite impossible to me. There might be something in my code which I cannot account for that is causing this drastic change. But overall it seems that the flat prism has elongated and also widened in the +y direction.