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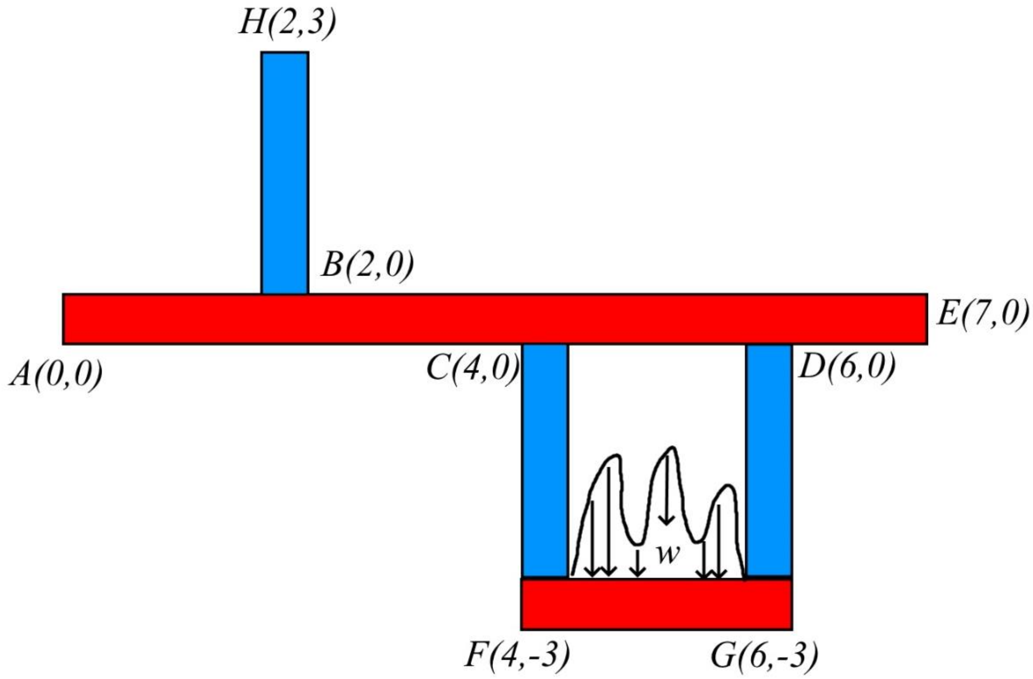
Normal Elasticity Problem 2

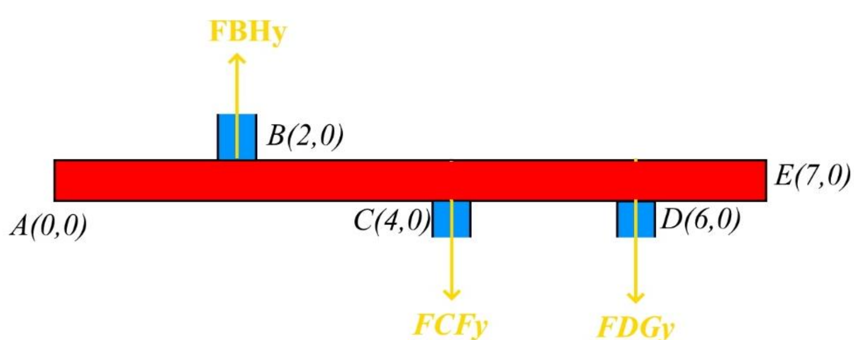
The system below consists of a rigid beam ABCDE suspended to a non-rigid beam BH. Another rigid beam FG is suspended to ABCDE by two non-rigid beams of equal length, CF and DG. All the non-rigid beams are made of the same material and have a young's modulus of E =0.46 GPa, Poisson's ratio of 0.325, width of 0.3 m and thickness of 0.2 m. All beam are massless and a distributive load of $w = 350\sin(x) + 110$ N/m acts on beam FG.

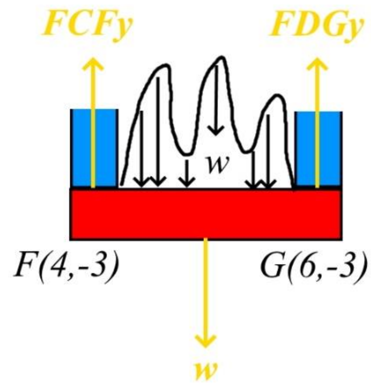


a) Draw an FBD diagram for the rigid beams and determine all reaction forces and moments
b) Draw an internal shear force and bending moment diagram for beam ABCDE
c) Determine the change in length and volume of all the non rigid beams using the properties given
d) Confirm you answer using FlexPDE

Solution:

a)





Since there are no forces acting in the x direction, it is not necessary to account for them in the FBD beceause we know they will be zero. We will now form equations for the sum of forces in the y direction and the moments acting on the beam.

Beam ABCDE:

$$\sum F_y = 0 = F_{BHy} - F_{CFy} - F_{DGy}$$

$$0 = MB - 2 * F_{CFy} + 4 * F_{DGy}$$

Beam FG:

$$\sum F_y = 0 = F_{CFy} + F_{DGy} - \int_0^x w$$

$$0 = 2 * F_{DGy} - \int_0^2 w * x$$

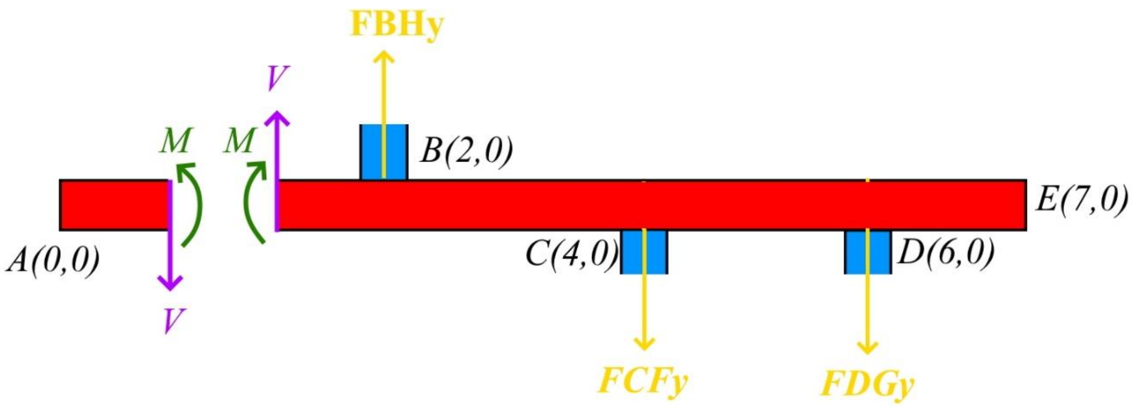
Using Maple, we can solve for reaction forces and moments

Maple Code:

```
restart:
w:=350*sin(x)+110:
solve([
BHy-CFy-DGy,
CFy+DGy-int(w, x=0..2),
DGy*2-int(w*x, x=0..2.0),
MB-2*CFy-4*DGy
]);
```

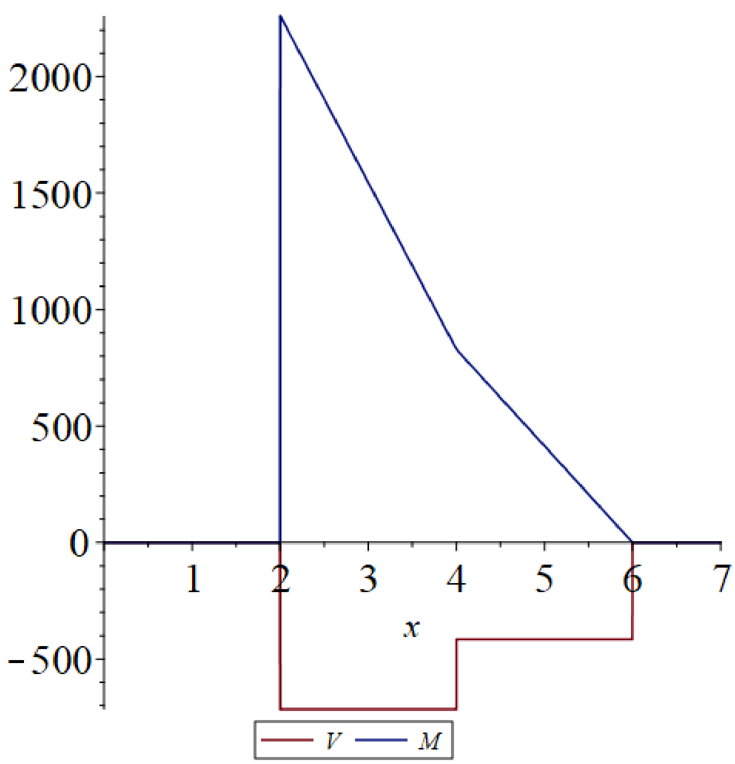
$\{BHy = 715.6513928,\ CFy = 300.8729503,\ DGy = 414.7784425,\ MB = 2260.859671\}$

b) In order to draw the internal moment and shear force diagram for ABCDE we must first draw an FBD depicting the varying shear force and moments as the beam is spit into components. This makes it much easier to write the code correctly on Maple.



Maple Code:

```
V:=piecewise(x>2, -BHy)+piecewise(x>4, CFy)+piecewise(x>6, DGy):
M:=int(V, x=0..x)+piecewise(x>2, MB):
plot([V,M], x=0..7.01, legend=(['V','M']));
```



c) Since we're given all the necessary material properties and we've calculated the forces in all the non-rigid members we can figure out how the beams will elastically deform and ultimately find out their new lengths and dimensions individually.

Beam BH:

Maple Code:

```
#Beam BH
restart:
E:=0.46e9:
nu:=0.325:
Lx:=0.3:
Ly:=3:
Lz:=0.2:
Area:=0.2*0.3:
Fy:=715.65:
S:=1/E*Matrix([[1,-nu,-nu],[-nu,1,-nu],[-nu,-nu,1]]):
Oldlength:=Vector([Lx, Ly, Lz]);
OldVolume:=Lx*Ly*Lz;
Stresses:=Vector([0,Fy/Area,0]):
epsilon:=S.Stresses:
Deltalength:=epsilon*~Oldlength;
NewLength:=Oldlength+Deltalength;
NewVolume:=NewLength[1]*NewLength[2]*NewLength[3];
```

$$Oldlength := \begin{bmatrix} 0.3 \\ 3 \\ 0.2 \end{bmatrix}$$

$$OldVolume := 0.18$$

$$Deltalength := \begin{bmatrix} -2.52811141248729\ 10^{-6} \\ 0.000777880434611475 \\ -1.68540760832486\ 10^{-6} \end{bmatrix}$$

$$NewLength := \begin{bmatrix} 0.299997471888587 \\ 3.00077788043461 \\ 0.199998314592392 \end{bmatrix}$$

$$NewVolume := 0.180001633483033$$

Beam CF:

Maple Code:

```
#Beam CF
restart:
E:=0.46e9:
nu:=0.325:
Lx:=0.3:
Ly:=3:
Lz:=0.2:
Area:=0.2*0.3:
Fy:=300.8729503:
S:=1/E*Matrix([[1,-nu,-nu],[-nu,1,-nu],[-nu,-nu,1]]):
Oldlength:=Vector([Lx, Ly, Lz]);
OldVolume:=Lx*Ly*Lz;
Stresses:=Vector([0,Fy/Area,0]):
epsilon:=S.Stresses:
Deltalength:=epsilon*~Oldlength;
NewLength:=Oldlength+Deltalength;
NewVolume:=NewLength[1]*NewLength[2]*NewLength[3];
```

$$Oldlength := \begin{bmatrix} 0.3 \\ 3 \\ 0.2 \end{bmatrix}$$

$$OldVolume := 0.18$$

$$Deltalength := \begin{bmatrix} -1.06286640035313\ 10^{-6} \\ 0.0000327035815493269 \\ -7.08577600235417\ 10^{-7} \end{bmatrix}$$

$$NewLength := \begin{bmatrix} 0.299998937133600 \\ 3.00003270358155 \\ 0.199999291422400 \end{bmatrix}$$

$$NewVolume := 0.180000686763568$$

Beam DG:

Maple Code:

```
#Beam DG
restart:
E:=0.46e9:
nu:=0.325:
Lx:=0.3:
Ly:=3:
Lz:=0.2:
Area:=0.2*0.3:
Fy:=414.7784425:
S:=1/E*Matrix([[1,-nu,-nu],[-nu,1,-nu],[-nu,-nu,1]]):
Oldlength:=Vector([Lx, Ly, Lz]);
OldVolume:=Lx*Ly*Lz;
Stresses:=Vector([0,Fy/Area,0]):
epsilon:=S.Stresses:
Deltalength:=epsilon*~Oldlength;
NewLength:=Oldlength+Deltalength;
NewVolume:=NewLength[1]*NewLength[2]*NewLength[3];
```

$$Oldlength := \begin{bmatrix} 0.3 \\ 3 \\ 0.2 \end{bmatrix}$$

$$OldVolume := 0.18$$

$$Deltalength := \begin{bmatrix} -1.46524993249286\ 10^{-6} \\ 0.000450846133074727 \\ -9.76833288328575\ 10^{-7} \end{bmatrix}$$

$$NewLength := \begin{bmatrix} 0.299998534750067 \\ 3.00004508461331 \\ 0.199999023166712 \end{bmatrix}$$

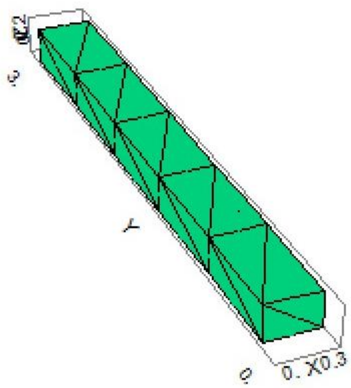
$$NewVolume := 0.180000946754749$$

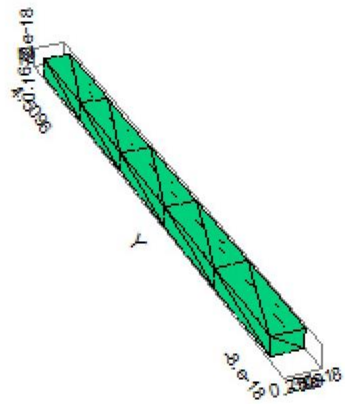
d) We will now use FlexPDE to confirm our answers

FlexPDE Code:

```
TITLE 'H6.2 Assignment hussam43'
COORDINATES cartesian3
 VARIABLES
u
v
w
DEFINITIONS
E=0.46e9
nu=0.325
mag=0.5*globalmax(magnitude(x,y,z))/gl
obalmax(magnitude(u,v,w))
Lx = 0.3
Ly = 3
Lz = 0.2
Fy =715.65
Ay =Lz*Lx
sigmaX = 0
sigmaY = Fy/Ay
sigmaZ = 0
epsilonx = dx(u)
epsilony = dy(v)
epsilonz = dz(w)
C11=E/((1+nu)*(1-2*nu))*(1-nu)
C12=E/((1+nu)*(1-2*nu))*nu
C13=C12
C21=C12
C22=C11
C23=C12
C31=C12
C32=C12
C33=C11
Sx =
C11*epsilonx+C12*epsilony+C13*epsilo
nz
Sy =
C21*epsilonx+C22*epsilony+C23*epsilo
nz
Sz =
C31*epsilonx+C32*epsilony+C33*epsilo
nz
LxNew = val(Lx+u,Lx,Ly,Lz)
LyNew = val(Ly+v,Lx,Ly,Lz)
LzNew = val(Lz+w,Lx,Ly,Lz)
VolOrig=Lx*Ly*Lz
VolNew = LxNew*LyNew*LzNew
VolChange=VolNew-VolOrig
EQUATIONS
u: dx(Sx)=0
v: dy(Sy)=0
w: dz(Sz)=0
EXTRUSION
surface 'bottom' z=0
surface 'top' z=Lz
BOUNDARIES
surface 'bottom'
load(u)=0
load(v)=0
value(w)=0
surface 'top'
load(u)=0
load(v)=0
load(w)=0
REGION 1
    START(0,0)
load(u)=0
value(v)=0
load(w)=0
LINE TO (Lx,0)
 load(u)=0
load(v)=0
load(w)=0
LINE TO (Lx,Ly)
 load(u)=0
load(v)= sigmaY
load(w)=0
LINE TO (0,Ly)
 value(u)=0
load(v)=0
load(w)=0
LINE TO CLOSE
PLOTS
 grid(x,y,z)
grid(x+u*mag,y+v*mag,z+w*mag)
SUMMARY
   report LxNew as 'New x length (m)'
report LyNew as 'New y length (m)'
report LzNew as 'New z length (m)'
report VolNew as 'New Volume (m^3):'
report VolChange as 'Change in volume
(m^3)'
END
```

Beam BH:



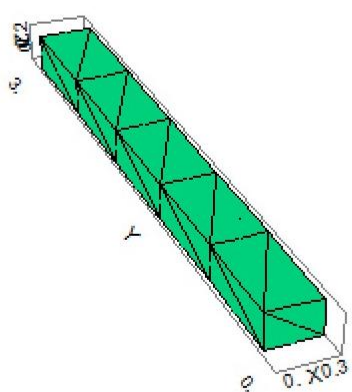


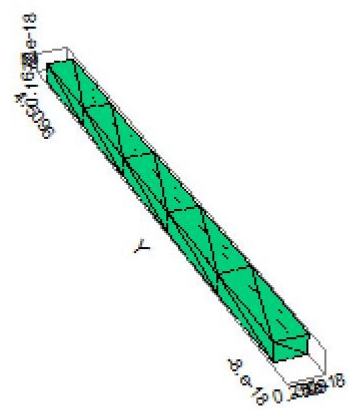
H6.2 Assignment hussam43

SUMMARY

New x length (m)= 0.299997
New y length (m)= 3.000078
New z length (m)= 0.199998
New Volume (m^3):= 0.180002
Change in volume (m^3)= 1.633483e-6

Beam CF:



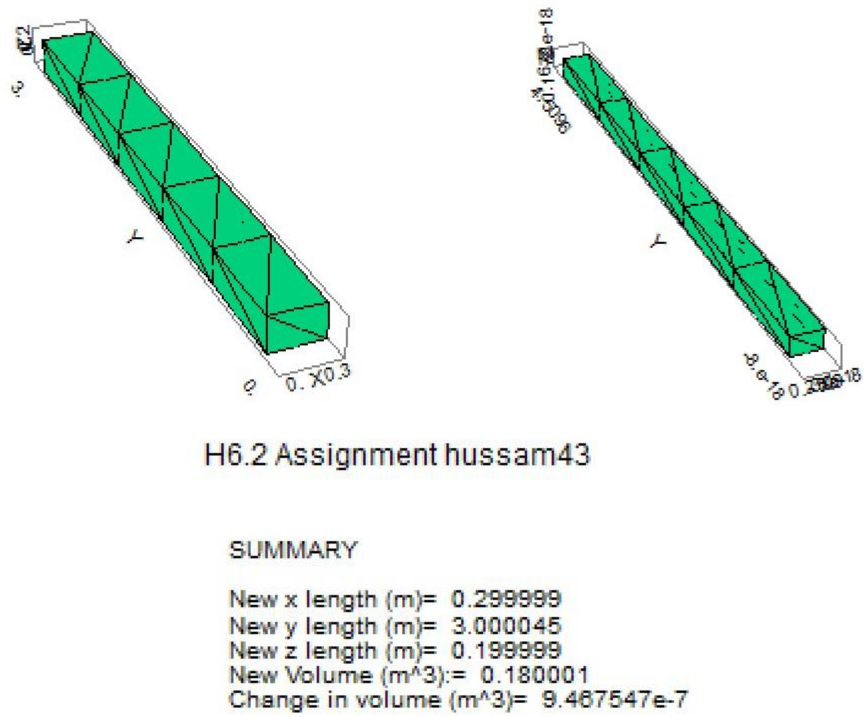


H6.2 Assignment hussam43

SUMMARY

New x length (m)= 0.299999
New y length (m)= 3.000033
New z length (m)= 0.199999
New Volume (m^3):= 0.180001
Change in volume (m^3)= 6.867636e-7

Beam DG:



Evaluation:

As we can see, we obtained the same change in volume and length with FlexPDE which confirms our answers. Overall, this was a fun problem to solve and the only obstacle I faced was understanding and implementing the FlexPDE code. Doing this assignment also helped me review my materials knowledge from last year and actually apply it to a practical problem.