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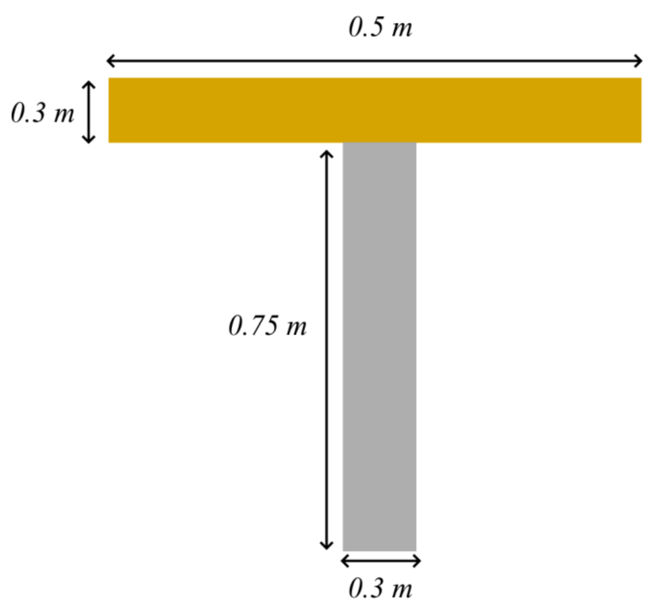
Flexural Elasticity Problem

Question:

A T-beam is fixed to the wall at one end while the rest of it is left unconstrained. The beam is 2.5 m long with the horizontal part being 0.5 m wide and 0.3 m high and the vertical part being 0.3 m wide and 0.75 m high. A bending moment of 12 kN-m is applied at the end of the beam in the downward direction and gravity also acts with respect to the beam's weight in the downward direction. Both should cause the beam to curl downwards. The horizontal portion of the beam is made of gold (HIGHLY UNPRACTICAL!) with (E = 74 GPa, v = 4.15, ρ = 19.32 g/cm^3) and the vertical portion is made of molybdenum with (E = 329 GPa, v = 0.29, ρ = 10.22 g/cm^3). A cross section of the x-y plane of the beam is shown below.



a) Draw a shear force and bending moment diagram for the beam in the length direction.
b) Determine the tip displacement analytically using Maple.
c) Determine the tip displacement using FlexPDE.

Solution:

a) We start off our question by defining the centre junction of both the vertical and horizontal components as (0,0,0) which should aid us in solving the problem with less confusion. We define our length and then define the rest of our varying properties with piecewise functions including the width which changes when y is more and less than 0. We use the height varying densities to solve for linear mass density which will be used to find the reaction force and reaction moment at the fixed end. This in turn will aid us in finding the length varying shear force and bending moment to make our diagram.

Maple Code:

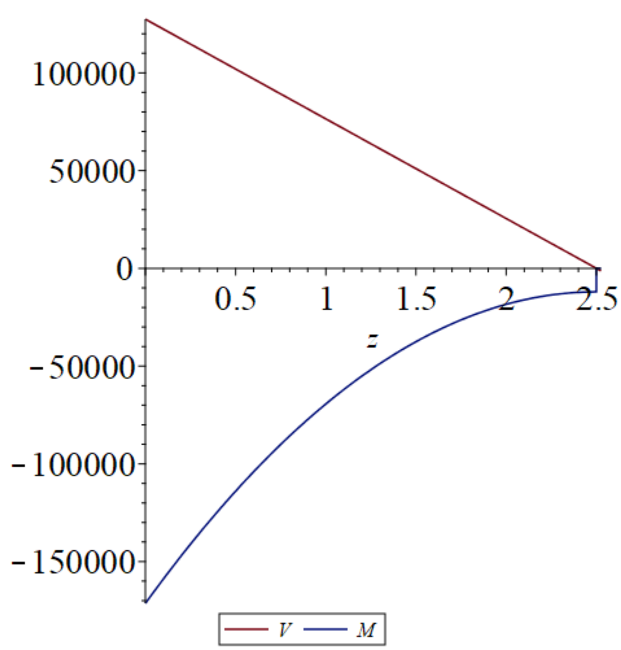
```
restart:
Lz:=2.5:
width:=piecewise(y<0,.3, y>0,.5):
LyM:=.75: LyG:=.3:
E:=piecewise(y<0,329e9, y>0, 74e9):
nu:=piecewise(y<0,0.29, y>0, 0.415):
rho:=piecewise(y<0,10220, y>0, 19320):
Mend:=12e3:
mu:=int(rho*width,y=-LyM..LyG); #LinearMassDensity;
solve([
Ry-int(mu*9.81,z=0..Lz),
-RM+int(mu*9.81*z,z=0..Lz)+Mend]); assign(%):
V:=Ry-mu*9.81*z;
M:=-RM+int(V,z=0..z)+piecewise(z>Lz,Mend);
plot([V,M],z=0..Lz*1.01 , legend=(['V','M']));
```

$$\mu := 5197.500000$$

$$\{RM = 171335.8594,\ Ry = 127468.6875\}$$

$$V := 127468.6875 - 50987.47500\, z$$

$$M := -171335.8594 + 127468.6875\, z - 25493.73750\, z^2 + \left(\begin{cases} 12000. & 2.5 < z \\ 0 & otherwise \end{cases}\right)$$



b) The displacement of the tip is a result of the shear force and the flexure. The differential displacement per unit length is the ratio of negative shear force and cross sectional area divided by the combined shear modulus of the entire beam. We then solve for flexural rigidity (EI) and use the ratio of the bending moment found earlier over EI to find the curvature. We can then find the displacement due to flexure at any point and evaluate for the displacement of the tip.

Maple Code:

```
G:=E/(2*(1+nu)):
A:=int(width,y=-LyM..LyG);
GCombined:=int(G*width,y=-LyM..LyG)/A;
tauzy:=-V/A;
gammazy:=tauzy/GCombined;
vShearTip:=int(gammazy,z=0..Lz);
Ybar:=int(width*E*y,y=-LyM..LyG)/int(width*E,y=-LyM..LyG);
EI:=int(E*(y-Ybar)^2*width,y=-LyM..LyG);
M:=-RM+int(V,z=0..z):
c:=M/EI;
dsolve([diff(vFlexure(z),z,z)=c,vFlexure(0)=0,D(vFlexure)(0)=0]); #fixed support
vFlexure:=evalf(rhs(%)):
vFlexureTip:=subs(z=Lz, vFlexure);
vTotalTip:=vShearTip+vFlexureTip;
```

$$A := 0.3750000000$$

$$GCombined := 8.697099187\ 10^{10}$$

$$tauzy := -339916.5000 + 135966.6000\ z$$

$$gammazy := -3.908389367\ 10^{-6} + 1.563355747\ 10^{-6}\ z$$

$$vShearTip := -4.885486708\ 10^{-6}$$

$$Ybar := -0.3065418502$$

$$EI := 6.213669507\ 10^{9}$$

$$c := -0.00002757402196 + 0.00002051423678\ z - 4.102847356\ 10^{-6}\ z^2$$

$$vFlexure(z) = -\frac{1025711839}{3000000000000000}\ z^4 + \frac{1025711839}{300000000000000}\ z^3 - \frac{689350549}{50000000000000}\ z^2$$

$$vFlexureTip := -0.00004610194991$$

$$vTotalTip := -0.00005098743662$$

c) In order to solve for the tips displacement in FlexPDE, we first define our height varying material properties. We also account for the weight as a volumetric force in our equation for force in the y direction. After defining our beams dimensions we also have to account for the equivalent axial z-stress that needs to be put on the tip to create the required moment. We first solve for this value in Maple

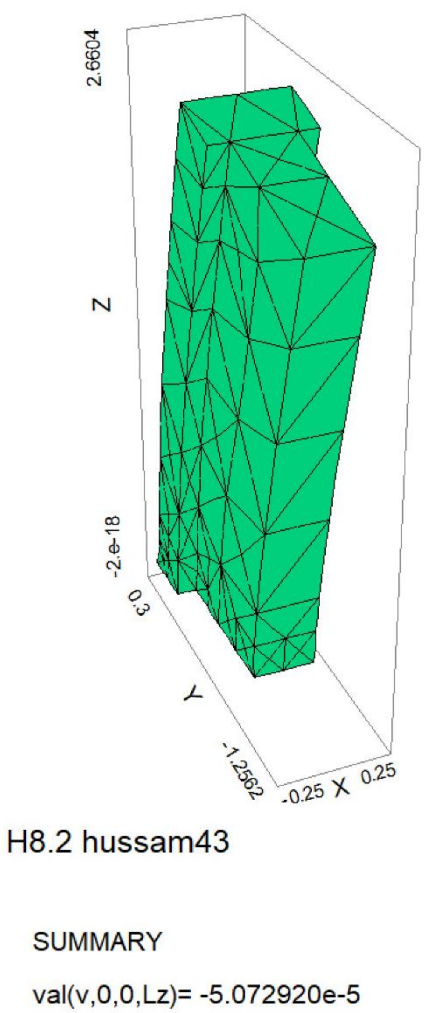
Maple Code:

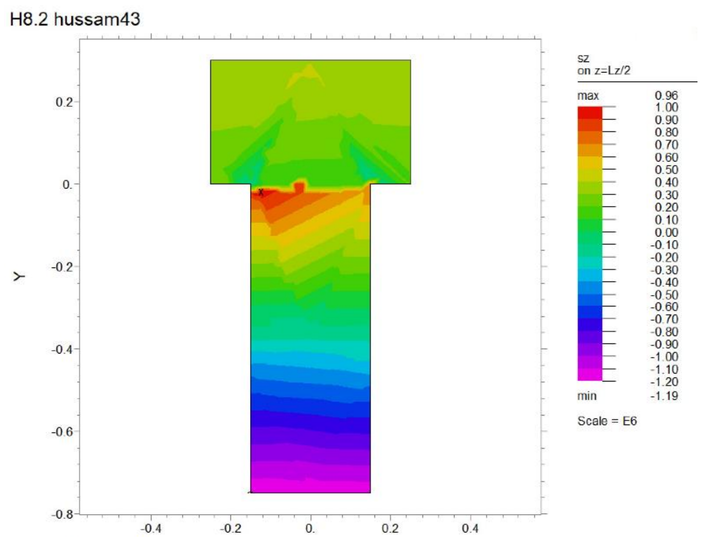
```
ybar:=int(width*y,y=-LyM..LyG)/int(width,y=-LyM..LyG);
sigzTip:=Mend*(y-ybar)/int(width*(y-ybar)^2,y=-LyM..LyG);
```

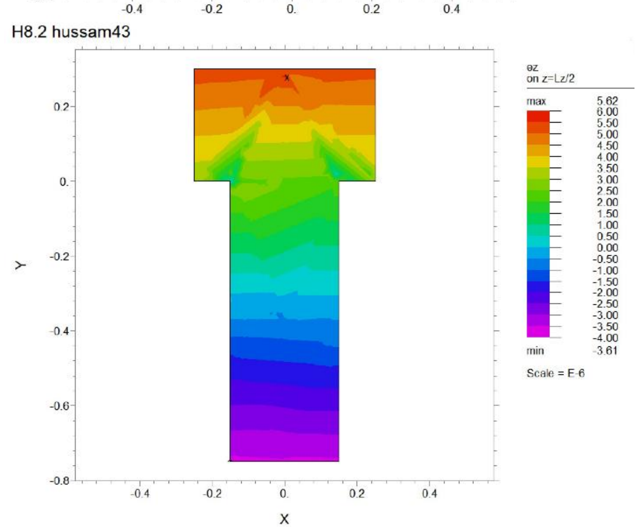
$$ybar := -0.1650000000$$

$$sigzTip := 328964.2765\, y + 54279.10562$$

Once we've obtained this value we can solve in FlexPDE to confirm if our tip displacement obtained in Maple is correct.







As we can see, we obtain the same displacement of -0.000051 m from FlexPDE hence confirming our answer. We can also observe the stress and strain across the junction while seeing a depiction of how the beam bends in real time.

FlexPDE Code:

```
TITLE 'H8.2 hussam43'
COORDINATES cartesian3
VARIABLES        { system variables }
        u
        v
        w
SELECT         { method controls }
ngrid=5
DEFINITIONS    { parameter definitions }
mag=1e4

grav=9.81

wM = .3
hM = .75
wG = .5
hG = .3
Lz = 2.5

E= if y<0 then 329e9 else 74e9
nu= if y<0 then 0.29 else 0.415
rho= if y<0 then 10220 else 19320

G=E/(2*(1+nu))

C11 =E*(1-nu)/((1+nu)*(1-2*nu))
C22 = C11
C33 = C11

C12 = E*nu/((1+nu)*(1-2*nu))
C13 = C12
C21 = C12
C23 = C12
C31 = C12
C32 = C12

sigzTip =328964.2765*y + 54279.10562

!!Strain
!Axial Strain
ex=dx(u)
ey=dy(v)
ez=dz(w)
!Engineering Shear Strain
gxy=dx(v)+dy(u)
gyz=dy(w)+dz(v)
gxz=dz(u)+dx(w)
!Stress via Hooke's Laws
!Axial Stress
sx = C11*ex + C12*ey + C13*ez
sy = C21*ex + C22*ey + C23*ez
sz = C31*ex + C32*ey + C33*ez
!Shear Stress
syz= G*gyz
sxz= G*gxz
sxy= G*gxy

EQUATIONS        { PDE's, one for each variable }
!Fnet = 0
u:               dx(sx)+dy(sxy)+dz(sxz)=0
v:               dx(sxy)+dy(sy)+dz(syz)-rho*grav=0
w:       dx(sxz)+dy(syz)+dz(sz)=0

EXTRUSION
surface 'bottom' z=0
surface 'top' z=Lz

BOUNDARIES       { The domain definition }
surface 'bottom'
        value(u)=0
        value(v)=0
        value(w)=0
surface 'top'
        load(u)=0
        load(v)=0
        load(w)=sigzTip

  REGION 1       { For each material region }
    START(-wM/2,-hM)
        load(u)=0
        load(v)=0
        load(w)=0
        LINE TO(wM/2,-hM) TO(wM/2,0) TO(wG/2,0)
TO(wG/2,hG)TO(-wG/2,hG)TO(-wG/2,0)TO(-wM/2,0)TO
CLOSE
PLOTS
        grid(x+mag*u,y+mag*v,z+mag*w)
        contour(sz) painted on z=Lz/2
        contour(ez) painted on z=Lz/2
        elevation(ex,ey,ez) from (0,0,0) to
(0,0,Lz)
SUMMARY
        report val(v,0,0,Lz)
END
```

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What I learned:

Solving this problem while understanding all the concepts in depth was actually quite fun. Most of the confusions I had during H8.1 were cleared and I was able to get a better grasp on the math involved too. I look forward to applying what I learned in the weekly topics that will follow and eventually in a real life problem.