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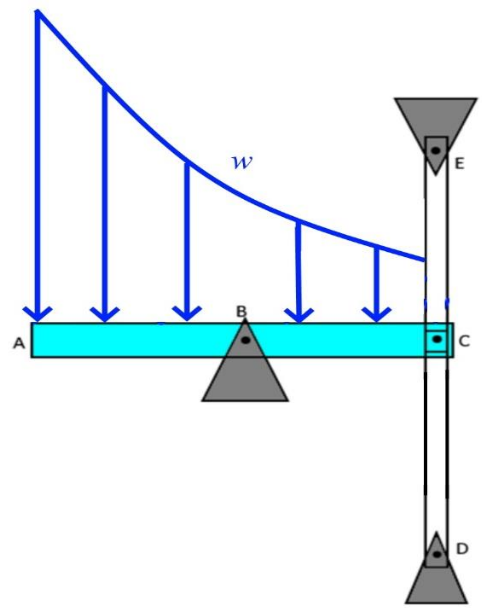
Normal Elasticity Problem 1

Altered Problem:



Member ABC is rigid and subjected to a downward distributed load of $w(x) = -250\sqrt{x} + 500$ in N/m, where *x* is in m. All members are light and you can ignore their weight.
Coordinates are (in m): A(0,0), B(1,0), C(2,0), D(2,-1.5), E(2, 1).

Members CD and CE are not rigid, but are made of steel with EY = 200 GPa and Poisson's ratio of 0.4, and have a rectangular cross section with thickness 1 cm (out of the page) by 5 cm (in the *x*-direction).

1. a) Draw an FBD for member ABC.
   b) Determine the equilibrium displacement of point A, the tension in CD and CE, and the reaction forces at B.
   c) Create an internal shear force and bending moment diagram for ABC (include an FBD showing the split you used to de fine the internal shear force and bending moment directions).

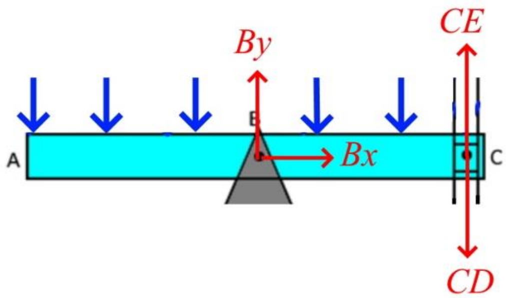
2. a) Determine the stiffness and compliance matrices for the steel of member CD.
   b) Calculate the change in dimensions of member CD (assume the stress in each dimension is uniform; i.e., ignore edge effects from the pins)
   c) Determine the actual volume change of member CD.
   d) Modification: also model beam CD in FlexPDE and confirm the dimension change and volume change.

Alterations:

Unlike the original question where the distributed load was relatively the same over the length of member ABC, I altered it to be more dominant to the left of support point B and significantly less towards the right to observe how the changes effect the non-rigid members CD and CE. Furthermore, I changed the lengths of member CD to make it longer then member CE.

Solution:

1a) FBD for member ABC



b) Maple Code:

```
restart: w:=-250*sqrt(x)+500;
EY:=200e9: Area:=.01*.05:
solve([
Bx,
By+CE-CD-int(w,x=0..2),
1*CE-1*CD+int((x-1)*(-w), x=0..2.0),
sigmayCD=CD/Area,
sigmayCD = EY*epsilonyCD,
epsilonyCD=vC/1.5,
sigmayCE = CE/Area,
sigmayCE = EY*epsilonyCE,
epsilonyCE = -vC/1
]); assign(%):
vA:=-vC;
```

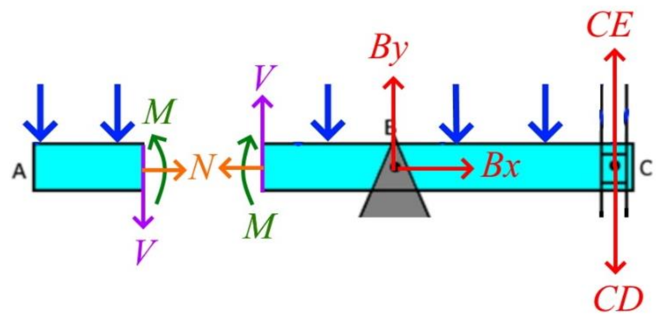
$$w := -250\sqrt{x} + 500$$

$$\{Bx = 0.,\ By = 622.8763834,\ CD = 37.71236167,\ CE = -56.56854249,\ vC = 5.656854249\ 10^{-7},\ epsilonyCD = 3.771236167\ 10^{-7},\ epsilonyCE = -5.656854249\ 10^{-7},\ sigmayCD = 75424.72333,\ sigmayCE = -113137.0850\}$$

$$vA := -5.656854249\ 10^{-7}$$

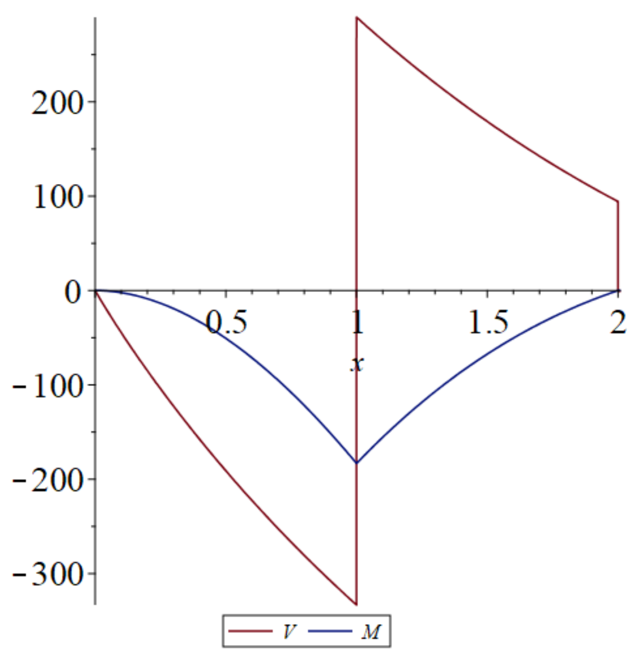
The equilibrium displacement of point A is -5.657×10^{-7}m which is significantly less than the original answer. After playing around with my distributive load equation and reducing it's incline to give a relatively similar distributive load as the original question, I observed the displacement stayed the same. I can therefore conclude that the change in length of the members to make CD longer than CE (opposite of original question) was the sole cause of this reduced displacement as no other factor was altered. The strain on both members decreased and the stress on both members increased.

c) Internal shear force and bending moment diagram for ABC



Maple code:

```
N:=piecewise(x>1, -Bx):
V:=-int(w,x=0..x) + piecewise(x>1, By) +piecewise(x>2, CE-CD) assuming(x>0):
M:=int(V, x=0..x):
plot([V,M], x=0..2.01, legend=(['V','M']));
```



2a) Stiffness and compliance matrices for the steel of member CD

Maple Code:

```
with(LinearAlgebra): E:=200*10^9: nu:=4/10:
S:=1/E*Matrix([[1,-nu,-nu], [-nu, 1, -nu], [-nu, -nu, 1]]);
C:=MatrixInverse(S);
```

$$S := \begin{bmatrix} \frac{1}{200000000000} & -\frac{1}{500000000000} & -\frac{1}{500000000000} \\ -\frac{1}{500000000000} & \frac{1}{200000000000} & -\frac{1}{500000000000} \\ -\frac{1}{500000000000} & -\frac{1}{500000000000} & \frac{1}{200000000000} \end{bmatrix}$$

$$C := \begin{bmatrix} \frac{3000000000000}{7} & \frac{2000000000000}{7} & \frac{2000000000000}{7} \\ \frac{2000000000000}{7} & \frac{3000000000000}{7} & \frac{2000000000000}{7} \\ \frac{2000000000000}{7} & \frac{2000000000000}{7} & \frac{3000000000000}{7} \end{bmatrix}$$

b+c) Change in dimensions and volume of member CD

```
eps:=<ex,ey,ez>: sig:=<sx,sy,sz>:
sy:=sigmayCD: sx:=0: sz:=0:
eps = S.sig:
StrainEqn:=eps = S.sig:
seq(lhs(StrainEqn)[n]=rhs(StrainEqn)[n], n=1..3):
solve([%]); assign(%);
dimsOrig:=<.05, 1.5, .01>;
dimsNew:=<seq(dimsOrig[n]*(1+eps[n]),n=1..3)>;
dimsChange:=dimsNew-dimsOrig;
VolOrig:=product([seq(dimsOrig[n],n=1..3)][n],n=1..3);
VolNew:=product([seq(dimsNew[n],n=1..3)][n],n=1..3);
VolChange:=VolNew-VolOrig;
```

$$dimsOrig := \begin{bmatrix} 0.05 \\ 1.5 \\ 0.01 \end{bmatrix}$$

$$dimsNew := \begin{bmatrix} 0.04999999246 \\ 1.500000566 \\ 0.009999998492 \end{bmatrix}$$

$$dimsChange := \begin{bmatrix} -7.53999999936195\ 10^{-9} \\ 5.65999999979638\ 10^{-7} \\ -1.50800000091322\ 10^{-9} \end{bmatrix}$$

$$VolOrig := 0.00075$$

$$VolNew := 0.0007500000568$$

$$VolChange := 5.68\ 10^{-11}$$

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d) Check using FlexPDE

Flexcode:

```
TITLE 'H6.1 hussam43'     { the problem identification }
COORDINATES cartesian3  { coordinate system, 1D,2D,3D, etc }
VARIABLES        { system variables }
u
v
w
! SELECT         { method controls }
DEFINITIONS    { parameter definitions }
mag = 100

Lx = .05
Ly = 1.5
Lz = .01

E = 200e9
nu = .4

ex = dx(u)
ey = dy(v)
ez = dz(w)

C11 = E/((1+nu)*(1-2*nu))*(1-nu)
C12 = E/((1+nu)*(1-2*nu))*nu
C13 =C12
C21 =C12
C22 = C11
C23 = C12
C31 = C12
C32 = C12
C33 = C11

sx = C11*ex + C12*ey+C13*ez
sy = C21*ex + C22*ey+C23*ez
sz = C31*ex + C32*ey+C33*ez

LxNew = val(Lx+u,Lx,Ly,Lz)
LyNew = val(Ly+v,Lx,Ly,Lz)
LzNew = val(Lz+w,Lx*.2,Ly/2,Lz)
VolNew = LxNew*LyNew*LzNew
VolChange = VolNew-Lx*Ly*Lz
LxChange = LxNew-Lx
LyChange = LyNew-Ly
LzChange = LzNew-Lz
! INITIAL VALUES
EQUATIONS        { PDE's, one for each variable }
u: dx(sx) = 0
v: dy(sy) = 0
w: dz(sz) = 0

EXTRUSION
surface 'bottom' z = 0
surface 'top' z = Lz
```

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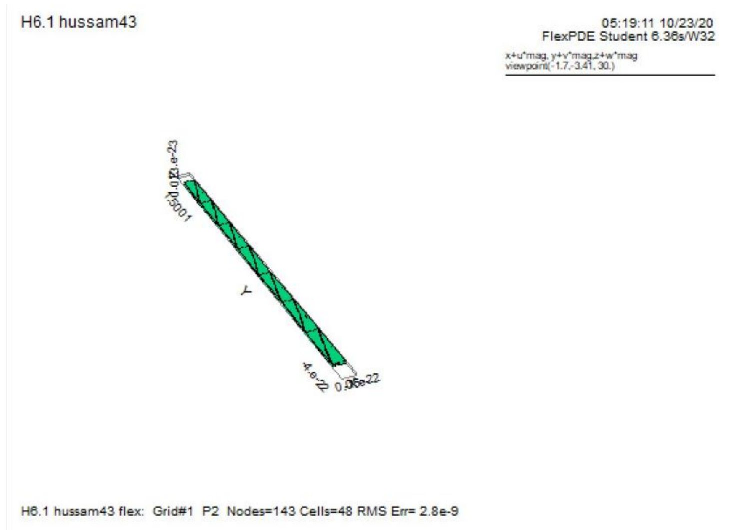
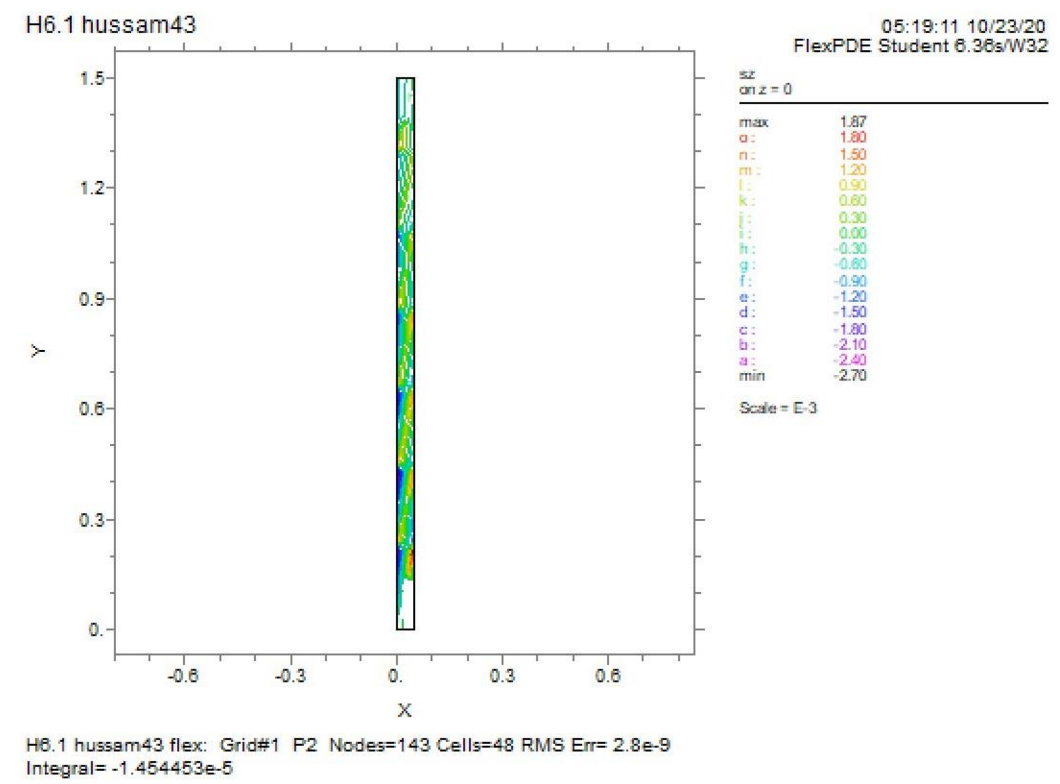
```
BOUNDARIES      { The domain definition }
surface 'bottom'
value(w)=0
surface 'top'


  REGION 1       { For each material region }
    START(0,0)   { Walk the domain boundary }
            value(v) = 0                LINE TO (Lx,0)  !y=0
            load(v) = 0              LINE TO (Lx,Ly)  !x=Lx
            load(v) = 75425       LINE TO (0,Ly) !y=Ly
            load(v) = 0  value(u) = 0          LINE TO CLOSE !x=0
! TIME 0 TO 1    { if time dependent }
MONITORS        { show progress }
PLOTS           { save result displays }
            grid(x+u*mag, y+v*mag,z+w*mag)
  CONTOUR(sz) on z = 0
SUMMARY
report val(ex,.5*Lx,.5*Ly,.5*Lz)
report val(ey,.5*Lx,.5*Ly,.5*Lz)
report val(ez,.5*Lx,.5*Ly,.5*Lz)
report val(u,Lx,Ly,Lz)
report val(u,0,Ly,Lz)
report val(Lx+u,Lx,Ly,Lz)
report val(w,Lx,Ly,Lz)
report val(w,0,Ly,0)
report val(v,Lx,Ly,Lz)
report LxChange
report LyChange
report LzChange
report VolNew
report VolChange
END
```





SUMMARY

H6.1 hussam43

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FlexPDE Student 6.36s/W32

```
SUMMARY

val(ex,.5*Lx,.5*Ly,.5*Lz)= -1.508500e-7
val(ey,.5*Lx,.5*Ly,.5*Lz)=  3.771250e-7
val(ez,.5*Lx,.5*Ly,.5*Lz)= -1.508500e-7
val(u,Lx,Ly,Lz)= -7.542500e-9
val(u,0,Ly,Lz)=  0.000000
val(Lx+u,Lx,Ly,Lz)=  0.050000
val(w,Lx,Ly,Lz)= -1.508500e-9
val(w,0,Ly,0)=  0.000000
val(v,Lx,Ly,Lz)=  5.656875e-7
LxChange= -7.542500e-9
LyChange=  5.656875e-7
LzChange= -1.508500e-9
VolNew=  7.500001e-4
VolChange=  5.656867e-11
```

The change in dimensions and volume is exactly the same as obtained from Maple.

What I learned:

Doing this problem allowed me to gain a deeper understanding of how practical systems are modelled in the real world. While doing previous week's problems, I figured that calculating the effect of point forces and distributive loads alone weren't sufficient in accurately modelling systems such as bridges but this weeks content gave me an insight on the many other factors which need to considered, especially material properties and their responses to forces. I also got to see how FlexPDE is extremely useful in depicting how stress and strain act on a member through contour maps which was definitely very interesting. The coding involved was significantly more difficult than before but Dr Minnick's video guidance gave a good walkthrough. More practice is still required however to be prepared for H6.2.