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Torsional Elasticity Problem

Question:

A solid PVC pipe (not hollow) is centered on the z-axis with one end fixed on the z = 0 surface and Lz = 5 m. The young's modulus of the PVC is E = 4.1 GPa and the poisson's ratio is v = 0.320. The pipe has a circular cross section with a diameter of 1 m. If a torsional shear stress is applied on it's z = Lz surface given by $\begin{cases} \tau_{zx} = -y\frac{2\text{ MPa}}{m} \\ \tau_{zy} = x\frac{2\text{ MPa}}{m} \end{cases}$,

a) Find the resulting bending moment acting on the +z surface due to these individual stresses
b) Determine the resulting twist angle of the pipe
c) Using FlexPDE, determine how the top face of the pipe changes and if it increases or decreases in area while also confirming your answer to part b)

Solution:

a) Since the pipe is symmetrical and is centered on the z-axis, we know that it extends the same distance in all directions (+ve and -ve) and therefore, the stress distribution creates no net shear force. We utilize the fact that the distribution of shear forces acting on the +z surface create a torque about (0,0) on that face given by:

$$\boldsymbol{r} \times d\boldsymbol{F} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \times \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ 0 \end{bmatrix} dA = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & 0 \\ \tau_{zx} & \tau_{zy} & 0 \end{vmatrix} dA = \begin{bmatrix} 0 \\ 0 \\ x\tau_{zy} - y\tau_{zx} \end{bmatrix} dA$$

We can then use this to calculate our resultant bending moment with Maple.

Maple Code:

```
restart:
E:=4.1e9:
nu:=.320:
G:=E/(2*(1+nu)):
Lz:=5:
tauzx:=-y*2e6:
tauzy:=x*2e6:
R:=1/2:
Mz:=int(int(y*(-tauzx)+x*tauzy, y=-sqrt(R^2-x^2)..sqrt(R^2-x^2)), x=-R..R);
```

$$Mz := 196349.5409$$

b) In order to find the angle the pipe twists due to the resulting moment we first need to find the torsion constant JT using the formula given in the notes. Since the cross section is a circle, this will be found by $\frac{\pi}{2} * r^4$. We can then find the torsional rigidity by using the shear modulus (JT*G) and we will have enough information to find the twisting angle using Maple.

Maple Code:

```
restart:
E:=4.1e9:
nu:=.320:
G:=E/(2*(1+nu)):
Lz:=5:
tauzx:=-y*2e6:
tauzy:=x*2e6:
R:=1/2:
Mz:=int(int(y*(-tauzx)+x*tauzy, y=-sqrt(R^2-x^2)..sqrt(R^2-x^2)), x=-R..R);
JT:=3.14159/2*R^4; #Torsion constant
thetaTip:=Mz*Lz/(G*JT);
```

$$Mz := 196349.5409$$

$$JT := 0.09817468750$$

$$thetaTip := 0.006439029830$$

c) We will now use FlexPDE to figure out how the stress subjected surface, z = Lz changes as a result and also confirm if out twisting angle obtained in part b) is correct.

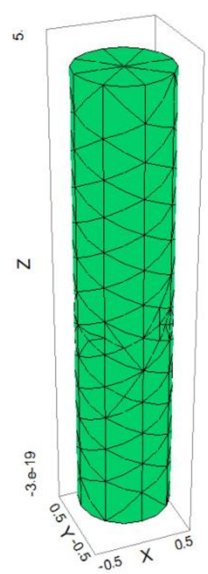
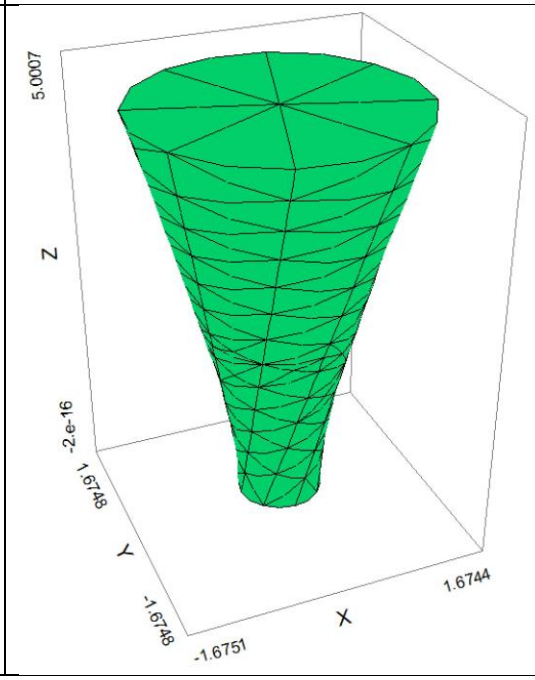
FlexPDE Code:

```
TITLE
'H9.2 hussam43'
SELECT
errlim=1e-8
ngrid=12
spectral_colors
COORDINATES
cartesian3

VARIABLES
u      !Displacement in x
v !Displacement in y
w !Displacement in z

DEFINITIONS
Lz=5
diam=1 !diameter
nu=0.320
E=4.1e9
G=E/(2*(1+nu))

r = sqrt(x^2+y^2)
phi=atan2(y,x)
!Calculate the twist angle in
flexPDE directly:
thetatest = atan2(y+v,x+u)-phi
!twist angle, but possibly
outside the range -Pi to Pi
theta = if(thetatest<-pi) then
thetatest+2*pi else if
(thetatest>pi) then thetatest-
2*pi else thetatest !twist
angle


mag = 500 !magnification

C11 =E*(1-nu)/((1+nu)*(1-
2*nu))
C22 = C11
C33 = C11

C12 = E*nu/((1+nu)*(1-2*nu))
C13 = C12
```

```
C21 = C12
C23 = C12
C31 = C12
C32 = C12

!! Strain
!Axial Strain
ex=dx(u)
ey=dy(v)
ez=dz(w)
!Engineering Shear Strain
gxy=(dx(v)+dy(u))
gyz=(dy(w)+dz(v))
gxz=(dz(u)+dx(w))

!!Stress via Hooke's law
!Axial Stress
sx = C11*ex+C12*ey+C13*ez
sy = C21*ex+C22*ey+C23*ez
sz = C31*ex+C32*ey+C33*ez
!Shear stress
sxy=G*gxy
sxz=G*gxz
syz=G*gyz


EQUATIONS
!FNet = 0
u:
      dx(sx)+dy(sxy)+dz(sxz)=0
v:    dx(sxy)+dy(sy)+dz(syz)=0
w:    dx(sxz)+dy(syz)+dz(sz)=0

EXTRUSION
surface 'bottom' z=0
surface 'top' z=Lz

BOUNDARIES
surface 'bottom'
value(u)=0
value(v)=0
value(w)=0
surface 'top'
load(u)=-(y)*2e6
```

```
load(v)=(x)*2e6
load(w)=0

      REGION 1
      START(-diam/2,0)
            load(u)=0
            load(v)=0
            load(w)=0
            arc(center=0,0)
angle=360

            TO CLOSE


PLOTS
      grid(x+u, y+v, z+w)
      grid(x+u*mag, y+v*mag,
z+w*mag)
      contour(sxz) painted on
surface z=Lz
      contour(syz) painted on
surface z=Lz
      contour(sz) painted on
surface z=Lz
      contour(u) painted on
surface z=Lz
      contour(theta) painted
on surface z=Lz
      elevation(u,v,w) from
(0,0,0) to (0,0,Lz)
      elevation(theta) from
(diam/4,0,0) to (diam/4,0,Lz)

      summary
            report
val(u,diam/2,0,Lz)
            report
val(v,diam/2,0,Lz)
            report
val(w,diam/2,0,Lz)
            report
val(theta,diam/2,0,Lz)
end
```

Results below:

Original	After
	

H9.2 hussam43

SUMMARY

val(u,diam/2,0,Lz)= -7.503528e-7
val(v,diam/2,0,Lz)= 3.213517e-3
val(w,diam/2,0,Lz)= 7.790231e-8
val(theta,diam/2,0,Lz)= 6.426955e-3

At first glance, we see our value for theta and see that it's the same as calculated in maple which confirms our answer. The z = Lz face clearly increased in area extending in the +-x and +y direction. The height of the pipe remains approximately the same.