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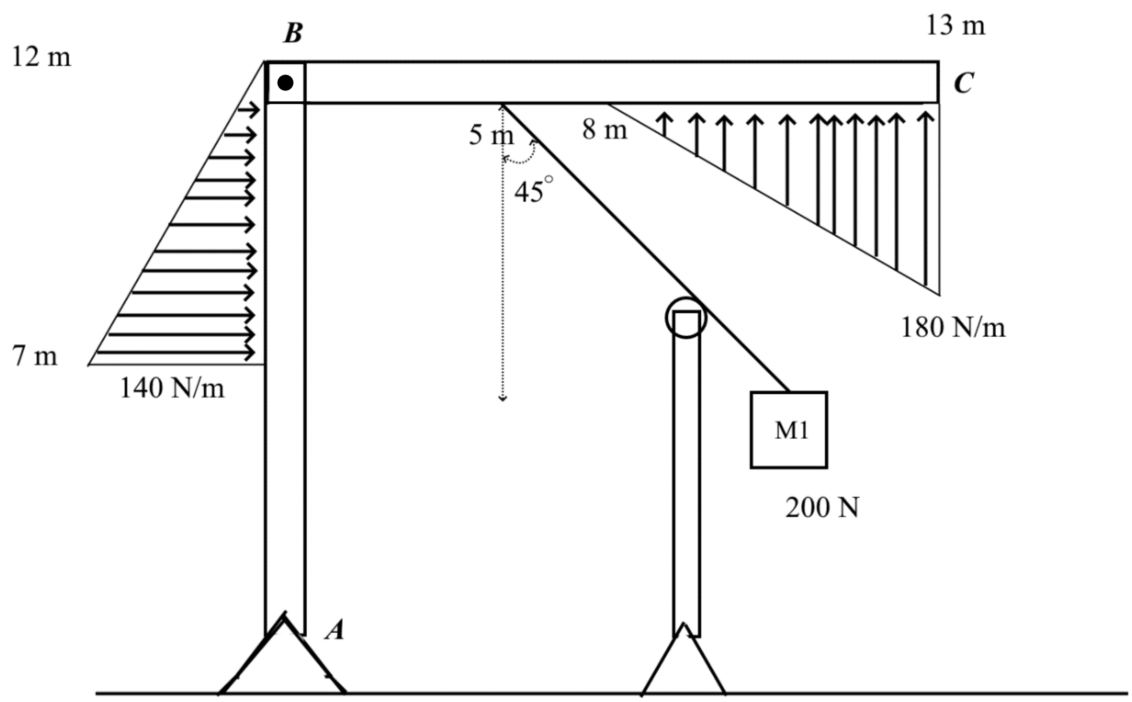
Internal Forces and Moments Problem

Question:

The structure below which consists of two ‘L’ forming beams is fixed to the ground at Point A. A distributive load is acting on both beams along with a weight M_1 hanging from the horizontal beam at an angle of 45^o. Disregard the thickness of the beams for this question.



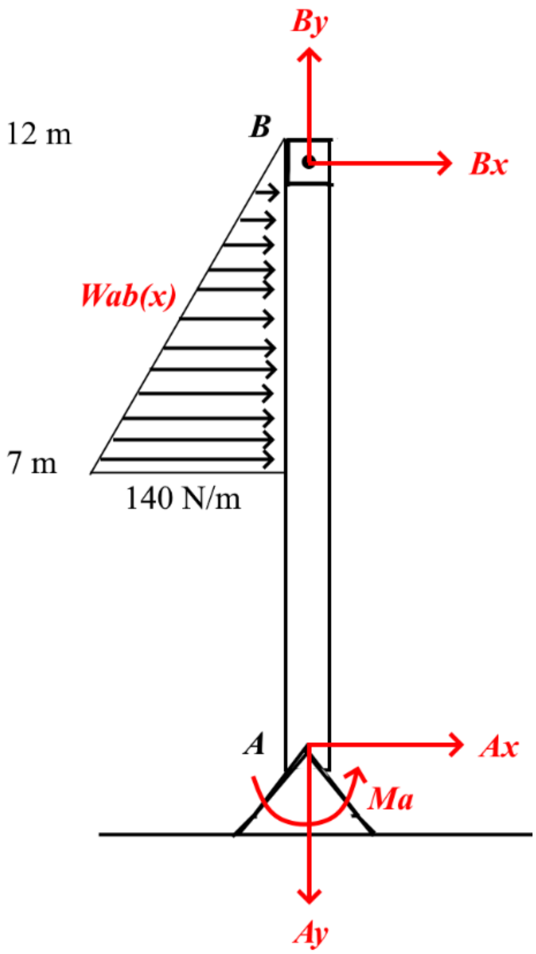
a) Draw FBD’s for the forces and moments acting on the beams
b) Generate force and torque equations for the structure considering it’s in equilibrium
c) Solve for reaction forces and torques about point A and B
d) Draw a free body diagram of the internal forces on the horizontal beam
e) Draw an internal shear force, normal force and bending moment diagram for the horizontal beam

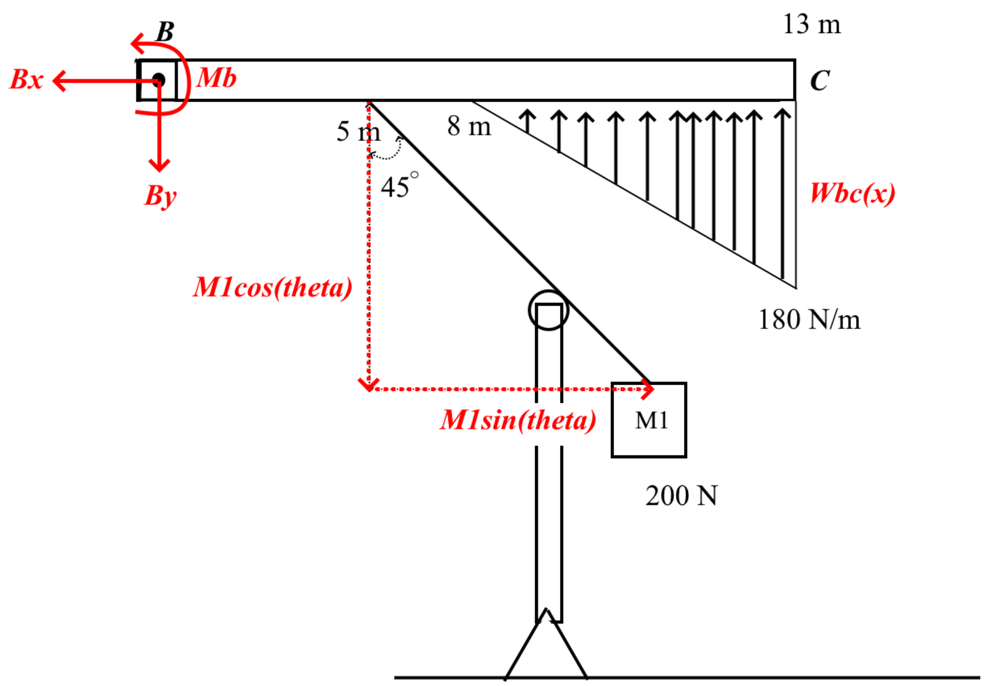
Solutions :

a)

Beam AB:

Beam BC:





b)

Forces acting on Beam AB:

$$\sum F_x = 0 = A_x + B_x + \int_{L_1}^{L_2} W_{AB} dx$$

$$\sum F_y = 0 = -A_y + B_y$$

Torque about Point A (Beam AB):

$$\sum \tau_A = 0 = M_A - \int_{L_1}^{L_2} W_{AB} * x dx - 12 * B_x$$

Forces acting on Beam BC:

$$\sum F_x = 0 = -B_x + M_1 \sin(\theta)$$

$$\sum F_y = 0 = -B_y - M_1 cos\theta + \int_{L_1}^{L_2} W_{BC} dx$$

Torque about Point B (Beam BC):

$$\sum \tau_B = 0 = M_B - 5 * M_1 cos\theta + \int_{L_1}^{L_2} W_{BC} * x dx$$

Where, $M_1 = 200, \theta = 45°, W_{AB}(x) = 140\left(1 - \frac{x}{5}\right), W_{BC}(x) = \frac{180x}{5}$

c) Maple Code:

```
restart:
M1:= 200:
theta:=45*3.14159/180:
wab:=140*(1-x/5):
wbc:=(180*x)/5:
solve([
Ax+Bx+int(wab, x=7..12),
-Ay+By,
MA-int(wab*x, x=7..12)-12*Bx,
-Bx+M1*sin(theta),
-By-M1*cos(theta)+int(wbc, x=8..13),
MB-5*M1*cos(theta)+int(wbc*x, x=8..13)]);
```

Forces (N) and Moments (N.m):

{Ax = 488.5787376, Ay = 1748.578550, Bx = 141.4212624,

By = 1748.578550, MA = -4579.611518, MB = -19512.89275}

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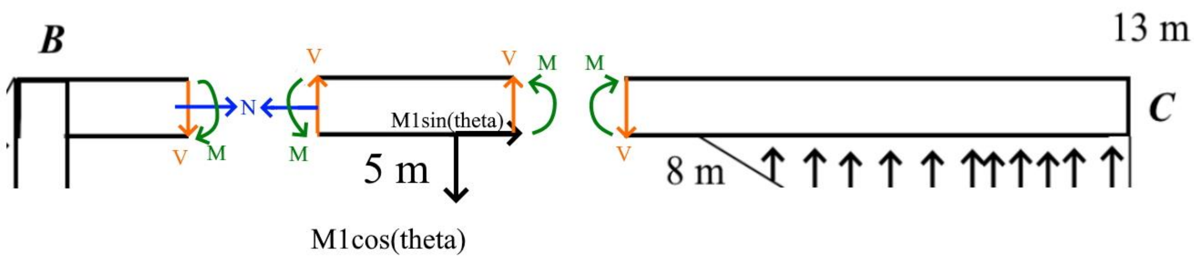
d) In order to accurately describe the internal forces acting on the beam, the beam was sliced at a point before the acting forces and then at a point after the first force.



e) Maple Code:

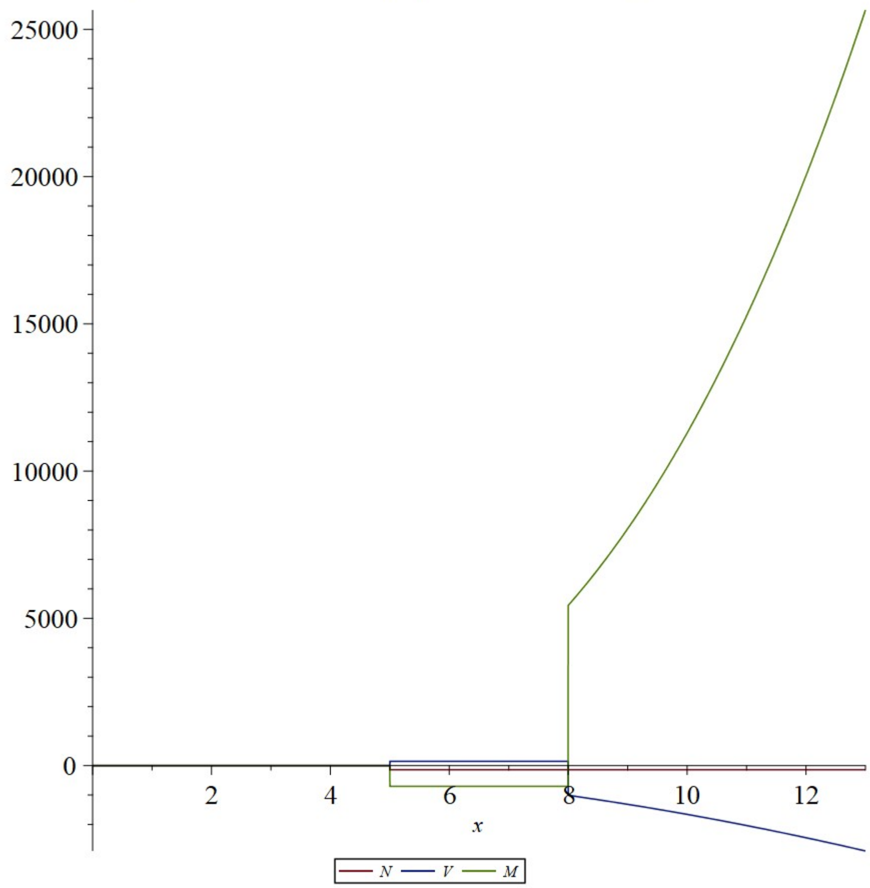
```
restart:
M1:= 200:
theta:=45*3.14159/180:
wab:=140*(1-x/5):
wbc:=(180*x)/5:
solve([
Ax+Bx+int(wab, x=7..12),
-Ay+By,
MA-int(wab*x, x=7..12)-12*Bx,
-Bx+M1*sin(theta),
-By-M1*cos(theta)+int(wbc, x=8..13),
MB-5*M1*cos(theta)+int(wbc*x, x=8..13)]);
N:=piecewise(x>5, -M1*sin(theta));
V:=piecewise(x>5, M1*cos(theta))+piecewise(x>8, int(-wbc, x=0..x));
M:=piecewise(x>5, -M1*cos(theta)*5)+piecewise(x>8, int(wbc*x, x=0..x));
plot([N, V, M], x=0..13);
```

$$\{Ax = 488.5787376,\ Ay = 1748.578550,\ Bx = 141.4212624,\ By = 1748.578550,\ MA = -4579.611518,\ MB = -19512.89275\}$$

$$N := \begin{cases} -141.4212624 & 5 < x \\ 0 & \textit{otherwise} \end{cases}$$

$$V := \left(\begin{cases} 141.4214501 & 5 < x \\ 0 & \textit{otherwise} \end{cases} \right) + \left(\begin{cases} -18\,x^2 & 8 < x \\ 0 & \textit{otherwise} \end{cases} \right)$$

$$M := \left(\begin{cases} -707.1072505 & 5 < x \\ 0 & \textit{otherwise} \end{cases} \right) + \left(\begin{cases} 12\,x^3 & 8 < x \\ 0 & \textit{otherwise} \end{cases} \right)$$



Observing the graph of varying internal forces, we see that the normal force in the beam is zero until x=5 which is where a horizontal force acts. The normal force increases in the opposite direction here to counter act that force and remains constant throughout as there are no other x-component forces. The internal shear force increases slightly in the upwards(positive) direction after x=5 where a relatively small downward force acts on the beam and then begins to increase significantly in the downwards direction after x=8 which is where the distributive force begins to act on the beam. The internal bending moment force also does the same thing but in opposite directions. After x=8, M increases in the counter clockwise direction drastically. The results were as expected and hence we can conclude that we have done everything correctly.