DP3: Spacecraft with Star Tracker

Li, Jason * and Veranga, Joshua †
University of Illinois at Urbana-Champaign, Champaign, IL 61801, USA

We design, implement, and validate a combined estimation-and-control architecture that keeps a reaction-wheel spacecraft correctly oriented for safe Space Cat docking. Four wheels are arranged in a configuration for full torque authority and a star-tracker monitors three carefully placed guide stars. Linearizing the rigid-body dynamics and measurement model about the docking attitude yields a controllable and observable state-space system. An LQR controller and Luenberger observer are tuned by scaling the attitude-error weights with η in the cost matrix Q; a sweep over $\eta \in \{10^0, \ldots, 10^5\}$ identifies $\eta = 10^4$ as optimal. In 100 Monte-Carlo simulations with sensor noise, 2Nm actuator saturation, and random debris impacts, the closed-loop system achieves a $\sim 85\%$ docking success rate and maintains estimator errors an order of magnitude smaller, satisfying the desired safety criterion: success rate$> 80\%$.

I. Nomenclature

α, β	=	Spherical azimuth and elevation angles, rad
$s_i^{(n)}$	=	Body-frame star unit vector (after rotation)
$y_k^{(n)}$	=	Image-plane coordinates of star n
R_{ij}	=	Rotation matrix (inertial to body)
ψ, θ, ϕ	=	Euler angles (yaw, pitch, roll), rad
ω_i	=	Angular velocity in body frame $\frac{\text{rad}}{\text{s}}$
τ_i	=	Control torque applied by reaction wheels, N·m
J_{ij}	=	Moment of inertia tensor (body frame), kg·m^2
ϵ_{ijk}	=	Levi-Civita permutation symbol
x_i	=	State vector component
u_ζ	=	Control input vector component
f_i	=	Nonlinear system dynamics function
$T_i j$	=	Translation matrix (angular velocity to Euler angle rates)
$A_{ij}, B_{i\zeta}, C_{\beta j}$	=	Linearized state-space matrices
$W_{i\sigma}$	=	Controllability matrix
$Q_{ij}, R_{\zeta\lambda}$	=	LQR weighting matrices
$K_{\zeta j}$	=	State feedback gain matrix
$\hat{x}_i$	=	Estimated state vector component
$L_{i\beta}$	=	Observer gain matrix
$Q_{o,ij}, R_{o,\zeta\lambda}$	=	Observer design weighting matrices
η	=	Attitude-error weighting parameter

II. Introduction

Maintaining precise attitude during docking is a classic problem in spacecraft Guidance, Navigation, and Control (GNC). Modern small satellites achieve this with internal reaction wheels for torque authority and star trackers for high-accuracy attitude sensing. The challenge is to close the loop when measurements are noisy, actuators saturate, and disturbances—such as debris impacts—occur.

In this project, we design an estimation-and-control architecture that allows a spacecraft to hold a fixed orientation so a Space Cat can successfully dock. Four reaction wheels provide up to 2Nm of torque, while a star-tracker observes three cataloged guide stars.

*BS Student, Aerospace Engineering, Grainger College of Engineering, 104 S Wright St, Urbana, Illinois 61820, jyli3
†BS Student, Aerospace Engineering, Grainger College of Engineering, 104 S Wright St, Urbana, Illinois 61820, veranga2

We (i) derive and linearize the rigid-body dynamics and vision measurement model, (ii) verify controllability and observability, (iii) tune an LQR controller and Luenberger observer, and (iv) validate performance in Monte-Carlo simulation with sensor noise, wheel saturation, and random debris impulses. Results show greater than 80% docking success and sub-degree pointing accuracy, satisfying the desired safety criterion.

III. Theory

A. Wheel and Star Placement

Before we define the controller for the spacecraft, we must first define the placements of the reaction wheels and cataloged guide stars. Tables 1 and 2 show the chosen placements of the respective reaction wheels and stars within the spherical coordinates relative to the spacecraft body.

Table 1 Reaction wheel placements

Wheel (k)	α (rad)	β (rad)
1	$\frac{\pi}{2}$	0
2	$-\frac{\pi}{2}$	0
3	π	0
4	0	$-\frac{\pi}{2}$

Table 2 Star placements

Star (n)	α (rad)	δ (rad)
1	0.058	0
2	-0.0289	0.050
3	-0.0289	-0.05

Defining the rotation matrix R_{ij} each star placement can be expressed as a cartesian coordinate $s_i^{(n)}$. These coordinates are then projected onto the focal plane using a pinhole camera model, forming the measurement vector y_δ defined as:

$$y_k^{(n)} = \frac{s_{k+1}^{(n)}}{s_1^{(n)}}. \tag{1}$$

B. Spacecraft Dynamics

The spacecraft is modeled as a rigid body in free space where external disturbances are negligible and translational motion is independently regulated; therefore, we only consider rotational dynamics. The respective dynamics are governed by the Euler equations defined with the spacecraft's moment of inertia J_{ij}, the angular velocity vector in the body frame ω_i, and the applied torque τ_i together expressed as

$$J_{ij}\dot{\omega}_j = \tau_i - \varepsilon_{ijk}\omega_j(J_{kl}\omega_l). \tag{2}$$

C. Linearization

Defining the state vector x_i and control input vector u_ζ as the following

$$x_i = \begin{bmatrix}\psi & \theta & \phi & \omega_1 & \omega_2 & \omega_3\end{bmatrix}^T, \quad u_\zeta = \begin{bmatrix}\tau_1 & \tau_2 & \tau_3 & \tau_4\end{bmatrix}^T \tag{3}$$

the non-linear dynamics of the system $f_i(x_j, u_\zeta)$ is defined as

$$f_i = \begin{cases} T_{ij}x_{j+3} & i \in \{1,2,3\} \\ J_{ij}^{-1}\left(\tau_j - \varepsilon_{ijk}\omega_k(J_{lm}\omega_m)\right) & i \in \{4,5,6\} \end{cases}, \tag{4}$$

with the first three rows corresponding to the Euler angle derivatives with transformation matrix $T_{ij}(\psi,\theta,\phi)$, and the remaining rows correspond to the Euler torque dynamics. The system is then linearized around a desired equilibrium point x_e and u_e to produce controller matrices $A, B,$ and C, calculated as

$$A_{ij} = \frac{\partial f_i}{\partial x_j}\Big|_{x=x_e,u=u_e}, \quad B_{i\zeta} = \frac{\partial f_i}{\partial u_\zeta}\Big|_{x=x_e,u=u_e}, \text{ and } \quad C_{\beta j} = \frac{\partial y_\delta}{\partial x_j}\Big|_{x_e}, \tag{5}$$

and x_e and u_e are the chosen equilibrium vectors respectively.

D. Control

To verify that the system can be controlled by the available reaction wheels, we check the controllability of the linearized system. The controllability matrix $W_{i\sigma}$ is defined as:

$$W_{i\sigma} = [B_{i\zeta}, A_{ij}B_{j\zeta}, A_{ij}^2 B_{j\zeta}, \dots, A_{ij}^{n-1}B_{j\zeta}] \tag{6}$$

where n is the dimension of the state vector. The system is controllable if and only if the matrix $W_{i\sigma}$ has full rank, that is: $\text{rank}(W_{i\sigma}) = n$, confirming that any state in the n-dimensional space can be reached using some sequence of inputs u_ζ.

Once controllability is confirmed, the weight matrices Q_{ij} and $R_{\zeta\lambda}$ can be defined to solve the Riccati equation to obtain the LQR gain matrix $K_{\zeta j}$. Similarly, the observer gain $L_{i\delta}$ is obtained by solving the dual Riccati equation using measurements weight matrices $Q_{o,ij}$ and $R_{o,\zeta\lambda}$.

E. Obtaining Values

To fully devise a controller for the system, the $K_{\zeta j}$ and $L_{i\delta}$ matrices need to be obtained*. Defining the equilibrium state as

$$x_e = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T, \quad u_e = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T \tag{7}$$

the following

$$A_{ij} = \begin{bmatrix} 0 & \dots & 0 & 0 & 1 \\ 0 & \dots & 0 & 1 & 0 \\ 0 & \dots & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{i\zeta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0.038 & 0 \\ -0.042 & 0.042 & 0 & 0 \\ 0 & 0 & 0 & 0.038 \end{bmatrix}, \text{ and}$$
$$C_{\beta j} = \begin{bmatrix} -2.634 & 0 & \dots & 0 & 0 \\ 0 & 2.625 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -2.627 & 0.003 & \dots & 0 & 0 \\ -0.004 & 2.632 & \dots & 0 & 0 \end{bmatrix} \tag{8}$$

matrices can be obtained from Eq.(5). Subsequently, the A_{ij} and $B_{i\zeta}$ matrices are used to obtain the $W_{i\sigma}$ matrix which is proven to satisfy the controllability condition $\text{rank}(W_{i\sigma}) = n$. This allows for the development of the gain matrices $K_{\zeta j}$ and $L_{i\beta}$. Given the chosen weight matrices

$$\begin{aligned} &Q_{ij} = \text{diag}\,(10000, 10000, 10000, 1, 1, 1)\,, \quad R_{\zeta\lambda} = I_4, \\ &Q_{o,ij} = I_6, \text{ and } R_{o,\zeta\lambda} = I_{12} \end{aligned} \tag{9}$$

the resulting gains are:

$$K_{\zeta j} = \begin{bmatrix} 0 & -70 & \dots & -40.962 & 0 \\ 0 & 70.712 & \dots & 40.962 & 0 \\ 0 & 0 & \dots & 0 & 72.232 \\ 100 & 0 & \dots & 0 & 72.232 \end{bmatrix} \text{ and } L_{i\delta} = \begin{bmatrix} -0.694 & 0 & \dots & -0.692 & -0.001 \\ 0 & 0.691 & \dots & 0.001 & 0.693 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0.576 & \dots & 0.001 & 0.578 \\ -0.578 & 0 & \dots & -0.577 & -0.001 \end{bmatrix}. \tag{10}$$

*See [1] for integration of **Obtaining Values**

IV. Experimental Methods

We first implemented a baseline LQR controller with identity weight matrices $Q = I_6$, $R = I_4$, which produced a low success rate in docking simulations. Since attitude is critical to docking, we introduced a scalar weight η to increase the cost on Euler angle errors. The modified cost matrix was defined as:

$$Q = \mathrm{diag}(\eta, \eta, \eta, 1, 1, 1), \tag{11}$$

where to tune η, we performed a sweep over values:

$$\eta \in \left\{1, 10, 10^2, 10^3, 10^4, 10^5\right\}. \tag{12}$$

For each value of η, we ran 100 simulations of a 60-second docking attempt. Each trial included randomized initial conditions, sensor noise, actuator saturation, and randomly directed asteroid impacts. The randomized initial Euler angles (ψ, θ, ϕ) and angular velocities $(\omega_1, \omega_2, \omega_3)$ were drawn from a zero-mean Gaussian distribution with standard deviation of 0.05 (radians and radians/sec, respectively).

A trial was deemed successful if the spacecraft reached a docked configuration, defined by sub-degree orientation error and near-zero angular velocity. The value $\eta = 10^4$ yielded the highest success rate (approximately 85%), satisfying the AE 353 safety requirement that the system succeed in more than 80% of trials.

V. Results and Discussion

We evaluated the closed-loop performance of the control and estimation architecture through a series of Monte Carlo simulations under noise, actuator saturation, and random debris impacts.

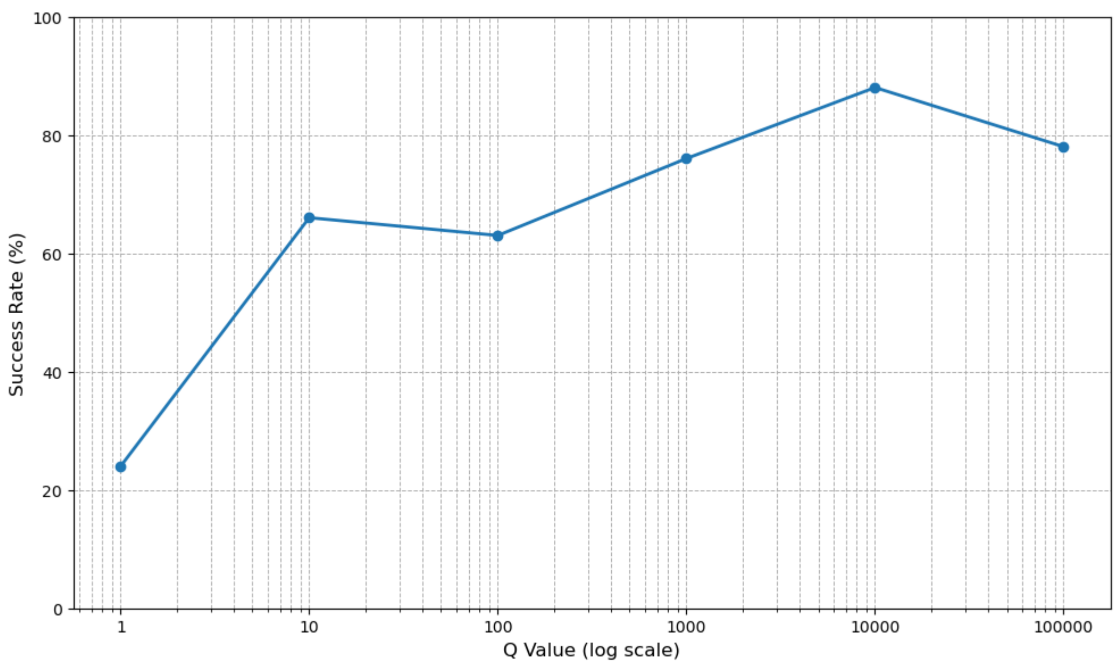

Fig. 1 Success rate is plotted as a function of attitude-weight parameter η in Q.

Figure 1 shows the effect of tuning the attitude-error weight η in the LQR cost function. The system achieved maximum performance at $\eta = 10^4$, with an $\sim$ 85% success rate over 100 trials, satisfying the desired safety requirement of greater than 80% success.

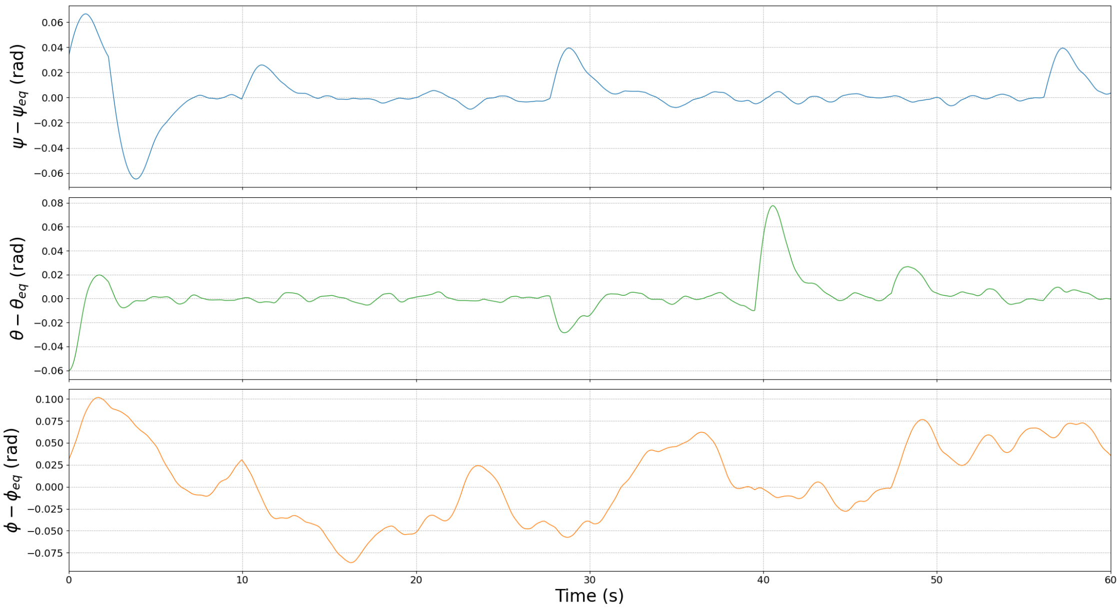

Fig. 2 Euler angle error $x - x_e$ during a representative docking trial.

Figure 2 plots the true state error of Euler angles (ψ, θ, ϕ) in $x - x_e$ for a representative docking trial. The Euler angles decay toward zero, and the system converges to the desired docked state within the simulation time even with the evident disturbances from debris impact.

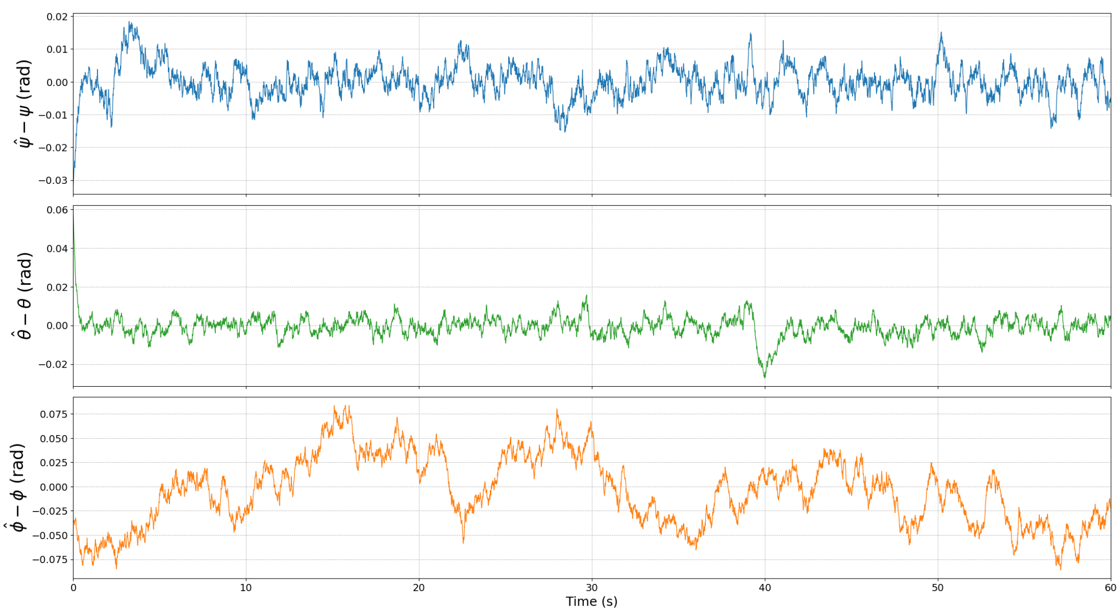

Fig. 3 Observer error $\hat{x} - x$ for the same trial.

Figure 3 shows the observer error of Euler angles (ψ, θ, ϕ) in $\hat{x} - x$ over the same trial. The Luenberger observer tracks the state with sub-degree and sub-rad/s accuracy, even in the presence of sensor noise and nonlinear measurement projection.

Taken together, these results validate that the spacecraft control system is both robust and effective: it consistently achieves precise pointing, uses reasonable control effort, and maintains accurate state estimation from noisy star-tracker measurements and disturbances.

VI. Conclusion

We developed and validated a spacecraft control system that enables safe, accurate, and robust docking using only star-tracker measurements and internal reaction wheels. The system combines a linear-quadratic regulator with a Luenberger observer, both designed from a linearized six-state model of rigid-body dynamics and projective measurements.

Through a sweep of the attitude-error weight η, we identified $\eta = 10^4$ as the optimal tuning parameter, yielding an 85% success rate across 100 randomized trials. Closed-loop simulations confirmed that the controller achieves sub-degree pointing accuracy and angular stabilization, while the observer tracks the full state with small steady-state error despite nonlinear measurements and sensor noise.

These results demonstrate that the design satisfies the safety criterion and is robust to realistic disturbances. Future improvements could include extending the estimator to handle partial star occlusion, or integrating nonlinear filtering for improved robustness at large angles.

Appendix

A. Space Cat Debriefs

The following are transcribed post-docking debriefs from our cat-pilots:

Pilot Whiskers: "That was chaotic and fun! When the first asteroid hit, I thought I was in trouble. But the spacecraft stabilized fast. The auto-pilot held course, tracked the stars, and got me back safe."

Commander Catnip: "Docking was smooth, confident, and way less terrifying than the first few flights. The engineers really stepped up their game and improved their model! Honestly, I'd fly this thing again just for fun! "

B. Video

Additionally, a link to video addressing the project can be found here.

Acknowledgments

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References

[1] Veranga, J., "SP25_AE353_Code/Design_Project_3," April 11, 2025. URL `https://github.com/muraSHUki/SP25_AE353_Code.git`.