Derivation and Execution of Cat-Bot Controller

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The following report presents the development and implementation of a state-feedback controller for a wheeled robot, referred to as the "Cat-Bot." Using a differential-drive mechanism, the robot is stabilized and allowed to maneuver along an axis toward the predicted landing positions of cat pilots. To achieve this, a state-space model was developed along with subsequent simulations in PyBullet to validate the controller's effectiveness.

Nomenclature

r_w = 0.325
m_w = 2.4
J_w = 0.12675
r_b = 0.3
m_b = 12.0
J_b = 0.8
g = 9.81
ζ = Wheel Position
$\dot{\zeta}$ = Wheel Velocity
θ = Pitch Angle
$\dot{\theta}$ = Pitch Rate
τ = Wheel Torque

I. Introduction

DIFFERENTIAL drive transmission controls the movement of a vehicle by varying the torques applied to each wheel, allowing the vehicle to make sharp movements. This transmission is favored in the production of mobile robots of various sizes and specializations due to its lack of complexity, allowing machines to be quickly and cheaply produced. At the household level, autonomous vacuum cleaners rotate in place and maneuver various obstacles; they are especially

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helpful to people with disabilities hindering their mobility, as this transmission powers electric wheelchairs and other rehabilitation devices.

Beyond these established uses, advancements are being created as to what these vehicles can do and where they can go. Coupled with the use of AI and autonomous navigation, mobile robots are essential to path mapping, aiding in the development of self-driving vehicles. The biggest drawback to differential drive is the struggle with traversing rough and changing environments. The wheels can be prone to slipping, impeding precise movements. In Aerospace engineering, discovering ways to improve terrain adaptation is important for future space exploration.

This project will focus on the design and implementation of a controller that will allow the vehicle to move only forward and backward. The torque applied to both wheels must be varied throughout the simulation to a maximum of ±5 N·m to move ±2.5 m from the starting position. The robot's movements shall be controlled to keep the chassis upright when catching the pilots.

When testing the robot, the wheels will roll without slipping. The team will adjust the torques of the wheels and the governing equations will display the wheel position and velocity, the pitch angle, and the pitch rate. The modeling process will require addressing engineering challenges:

- Linearize the model in the space-state form about an equilibrium point concerning the desired wheel position.
- Design in theory and implement an asymptotically stable linear state feedback controller in simulation.
- Simulate the final launch system, making sure it has a high probability of catching the Cat- pilots.

Ultimately, the goal of the model is to guarantee the safety of the Cat- pilots to the highest degree. The process of creating the launch system utilizes control methods essential to keeping people safe.

II. Theory

Equations of Motion

The motion of the Cat-Bot is governed by the ordinary differential equation given as

$$M(q)\ddot{q} + N(q,\dot{q}) = F(q)r \tag{1}$$

Where given matrices for terms expand it into

$$\begin{bmatrix} \frac{J_w}{r_w^2} + m_b + m_w & m_b r_b \cos(\theta) \\ m_b r_b \cos(\theta) & J_b + m_b r_b^2 \end{bmatrix} \begin{bmatrix} \ddot{\zeta} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -m_b r_b \sin(\theta)\dot{\theta}^2 \\ -g m_b r_b \sin(\theta) \end{bmatrix} = \begin{bmatrix} \frac{\tau}{r_w} \\ -\tau \end{bmatrix} \tag{2}$$

Representing the full nonlinear dynamics of the system

Further, we can isolate the acceleration term within the equation yielding

$$\ddot{q} = M^{-1}(q)\left(F(q)r - N(q, \dot{q})\right) \tag{3}$$

Additionally, defining the state-space representation

$$f = \begin{bmatrix} \dot{\zeta} & \dot{\theta} & \ddot{\zeta} & \ddot{\theta} \end{bmatrix}^T \tag{4}$$

Linearize Around Equilibrium

To linearize the system, we define an equilibrium point around our desired conditions

$$\zeta_e \quad \theta_e \quad \dot{\zeta}_e \quad \dot{\theta}_e \quad \tau_e \tag{5}$$

along with expressions of perturbations around the equilibrium

$$x = m - m_e \quad u = n - n_e \tag{6}$$

where

$$m = \begin{bmatrix} \zeta & \theta & \dot{\zeta} & \dot{\theta} \end{bmatrix}^T \quad n = \begin{bmatrix} \tau \end{bmatrix} \tag{7}$$

The system is then linearized using Jacobians of the respective matrices

$$A = \left.\frac{\partial f}{\partial x}\right|_{m_e} \quad B = \left.\frac{\partial f}{\partial u}\right|_{n_e} \tag{8}$$

State Feedback Control

Commonly, state feedback controllers are designed using

$$u = -Kx \tag{9}$$

therefore obtaining the expression

$$F = A - BK \tag{10}$$

Where F is the closed-loop system matrix, where its respective eigenvalues determine the stability

$$s = \lambda(F) \tag{11}$$

Where gain matrix K is computed using Python's control.place function to achieve desired eigenvalues for stability

III. Experimental Methods

Obtaining values

Define desired equilibrium values

$$\zeta_e = 1 \quad \theta_e = 0 \quad \dot{\zeta}_e = 0 \quad \dot{\theta}_e = 0 \quad \tau_e = 0 \tag{12}$$

Meaning the Cat-Bot is at the desired position, standing upright, and not moving.

Yielding

$$m_e = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T \quad n_e = \begin{bmatrix} 0 \end{bmatrix} \tag{13}$$

Which can then be used to determine the A and B matrices

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7.67 & 0 & 0 \\ 0 & 33.659 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0.573 \\ -1.629 \end{bmatrix} \tag{14}$$

IV. Results and Discussion

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V. Conclusion

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References

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Appendix

[Link to git]

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