

Project #1 - 4-bit adder

Half Adder

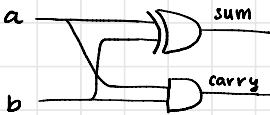
a	b	sum	carry
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

$\underbrace{\text{sum}}_{\text{XOR}} \quad \underbrace{\text{carry}}_{\text{AND}}$

← note full capacity of sum & carry is not met with 2 bits.

recall

$$\text{XOR} = A\bar{B} + \bar{A}B$$



full-adder

a	b	c	sum	carry
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

Carry

BC	00	10	11	01
A	0	0	0	1
0	0	0	1	0
1	0	1	1	1

$$BC + AB + AC = AB + C(A \oplus B)$$

		Sum			
BC		00	01	11	10
A		00	01	11	10
0	0	0	1	0	1
1	1	1	0	1	0

$$\begin{aligned}
 & A\bar{B}\bar{C} + \bar{A}B\bar{C} + ABC + \bar{A}\bar{B}C \\
 & A(\bar{B}\bar{C} + BC) + \bar{A}(\bar{B}C + B\bar{C}) \\
 & A(B \oplus C) + \bar{A}(B \oplus C) \\
 & = A(\bar{B} \oplus C) + \bar{A}(B \oplus C) \\
 & = A \oplus (B \oplus C) \leftarrow \text{XOR is symmetric} \\
 & = A \oplus B \oplus C
 \end{aligned}$$

