UNIT-II

Network Analysis

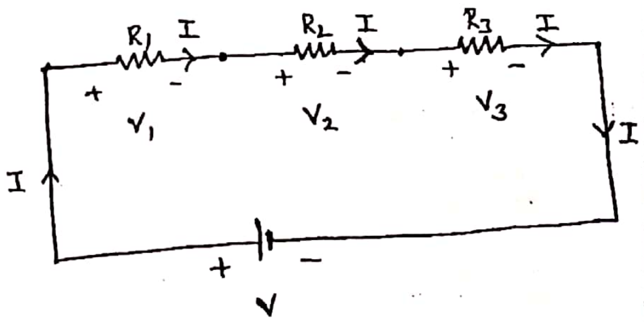
Network Reduction Techniques - Series and parallel connection of resistive networks, - star to Delta and Delta to star Transformations for Resistive Networks, - Mesh Analysis - Nodal Analysis.

Network Theorems: Thevenin's theorem - Norton's theorem - Maximum Power Transfer theorem, - superposition theorem Illustrative problems.

Network Reduction Techniques:

Resistors in series:

Consider resistors R_1, R_2 and R_3 are connected in series. The combination is connected across a source of voltage V volts.



The current flowing through all of them is same indicated as I amp.

Let V_1, V_2 and V_3 are the voltages across the terminals of resistors R_1 R_2 and R_3 respectively.

Then According to KVL $V = V_1 + V_2 + V_3$

According to ohm's law $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$

Current through all of them is same i.e. I

$$V = IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$

Applying ohm's law to overall circuit

$$V = I R_{eq}.$$

i.e. total of equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For 'n' resistors in series $R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$.

For resistors connected in series:

→ the same current flows through each resistor.

→ the supply voltage 'V' is the sum of the individual voltage drops across the resistances.

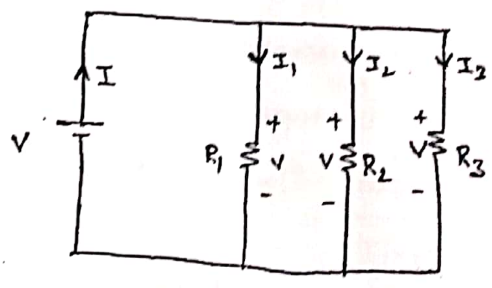
$$V = V_1 + V_2 + \cdots + V_n$$

→ the equivalent resistance is the largest of all the individual resistances.

→ the equivalent resistance is equal to the sum of the individual resistances.

Resistors in Parallel:

Consider resistors R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V'. In parallel circuit current passing through each resistor is different. Let total current drawn is I as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are I_1, I_2 and I_3. The voltage across each resistor R_1, R_2 and R_3 is same and equals the supply voltage V.



Apply ohm's law to each resistance.

$$V = I_1 R_1 \quad V = I_2 R_2 \quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2} \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

For overall circuit if ohm's law is applied,

$$V = I R_{eq} \quad I = \frac{V}{R_{eq}}$$

where R_{eq} = Total or equivalent resistance of the circuit

comparing the two equations $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

In general if 'n' resistances are connected in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

For two resistances R_1 and R_2 in Parallel $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

For resistors connected in Parallel:

→ the same potential difference gets across all the resistors.

→ the total current gets divided into the number of paths equal to the number of resistances in Parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \cdots + I_n.$$

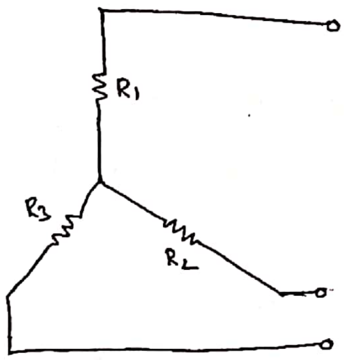
→ The reciprocal of the equivalent resistance of a Parallel circuit is equal to the sum of the reciprocal of the individual resistances.

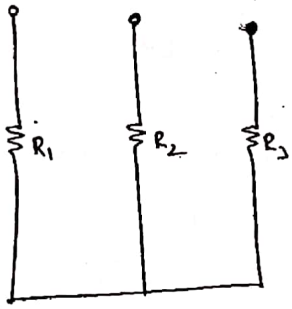
→ The equivalent resistance is the smallest of all the resistors.

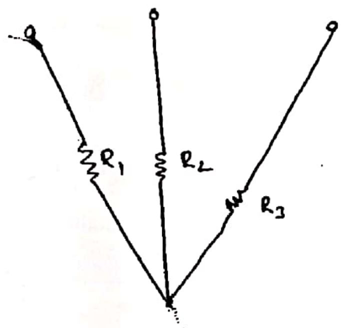
STAR-DELTA and DELTA-STAR Transformation:

The star - delta transformation is an technique useful in solving complex networks. Basically, any three circuit elements, i.e. resistive, inductive or capacitive, may be connected in two different ways. One way of connecting these elements is called the star connection or the Y connection. The other way of connecting these elements is called the delta (Δ) connection.

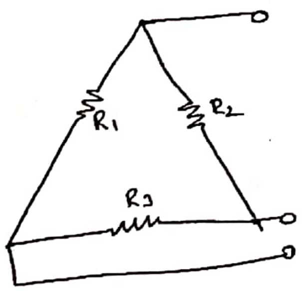
If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called star point, the resistances are said

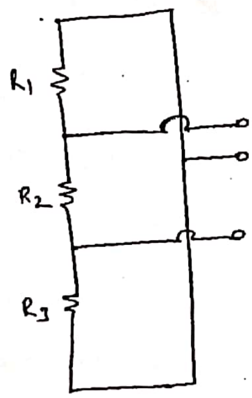


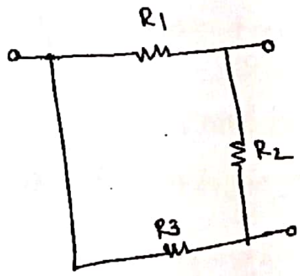




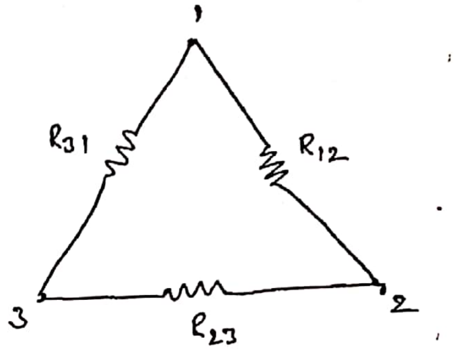
If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta.



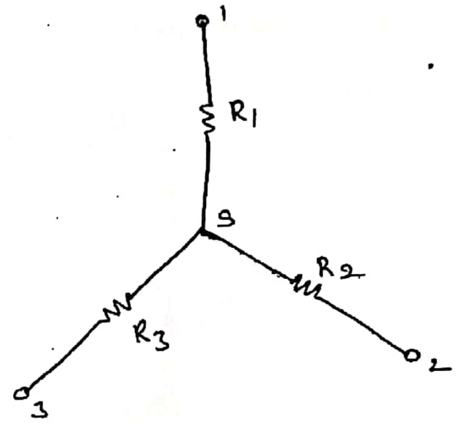




Delta - Star Transformation.



Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta. The terminals between which these are connected in Delta are named as 1, 2 and 3. It is always possible to replace these Delta connected resistances by three equivalent star connected resistances R_1, R_2, R_3 between the same terminals 1, 2 and 3.



Consider the terminals 1 and 2 in delta, find equivalent resistance between 1 and 2. we get the combination as R_{12} Parallel with $(R_{31} + R_{23})$.

Between 1 and 2 the resistance is $(\Delta) = \dfrac{R_{12}\ (R_{31} + R_{23})}{R_{12} + (R_{31} + R_{23})}$

Consider the terminals 1 and 2 in star, find equivalent resistance between 1 and 2. we get R_1 and R_2 in series.

Between 1 and 2 the resistance is $(Y) = R_1 + R_2$

To call this star as equivalent of given Delta it is necessary that the resistances calculated between terminals 1 and 2 in both the cases should be equal and hence equating above.

$$\frac{R_{12}\,(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} = R_1 + R_2 \longrightarrow (1)$$

Similarly if we find equivalent resistance as viewed through terminals 2 and 3 in both the cases and equating, we get

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \longrightarrow (2).$$

Similarly if we find the equivalent resistance as viewed through terminals 3 and 1 in both the cases and equating, we get

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \longrightarrow (3)$$

Now to find the values R_1, R_2, R_3

Subtract eq. (2) from (1)

$$\frac{R_{12}(R_{31}+R_{23}) - R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} = R_1 + R_2 - R_2 - R_3$$

$$\Rightarrow \quad R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \longrightarrow (4)$$

adding (4) and (3) we get

$$\frac{R_{12}R_{31} - R_{23}R_{31} + R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\frac{R_{12}R_{31} - R_{23}R_{31} + R_{31}R_{12} + R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} = 2R_1$$

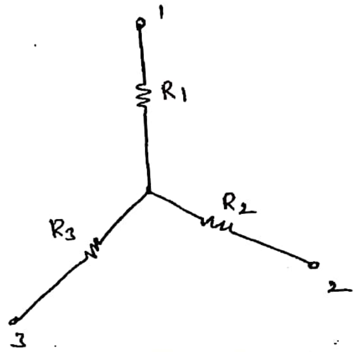
$$\Rightarrow \quad \boxed{R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (1) (2) & (3) we can get,

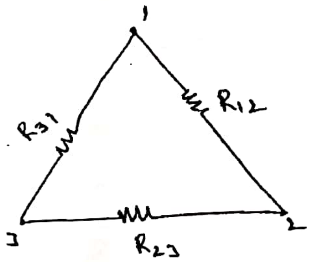
$$\boxed{R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}}}$$

$$\boxed{R_3 = \frac{R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}}}$$

Star - Delta Transformation:



Consider the three resistances R_1 R_2 and R_3 connected in star. It is always possible to replace these star connected resistances by three equivalent Delta connected resistances R_{12} R_{23} and R_{31} between the same terminals.



Now we need to find the values of R_{12} R_{23} R_{31} in terms of R_1 R_2 and R_3. For this we can use set of equations derived in previous topic. From the result of Delta - star transformation we know that

$$R_1 = \frac{R_{12}\,R_{31}}{R_{12}+R_{23}+R_{31}} \longrightarrow (1)$$

$$R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}} \longrightarrow (2)$$

$$R_3 = \frac{R_{23}\ R_{31}}{R_{12} + R_{23} + R_{31}} \longrightarrow (3)$$

Now find $(1)\times(2) + (2)\times(3) + (3)\times(1)$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$= \frac{R_{12}\ R_{31}\ R_{23}\ (R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}\ R_{31}\ R_{23}}{R_{12} + R_{23} + R_{31}} \longrightarrow (4)$$

Now find $(4)/(3)$, $(4)/(1)$ and $(4)/(2)$ we get

$$\boxed{\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_{12}}$$

$$\boxed{\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_{23}}$$

$$\boxed{\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_{31}}$$

Mesh Analysis:

Mesh and nodal analysis are two basic important techniques used in finding solution for a network. The suitability of either mesh or nodal analysis to a Particular Problem depends mainly on the number of voltage sources or current sources. If a network has a large number of voltage sources it is useful to use mesh analysis. The network has more current sources, nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non planar circuits, mesh analysis is not applicable. A circuit is said to be planar if it can be drawn on a plane surface without crossover. A non planar circuit cannot be drawn on a planee surface without a crossover.

'Mesh' is defined as a loop which does not contain any other loops within it.

Steps for Mesh Analysis:

step 1 : check whether the circuit is planar or not

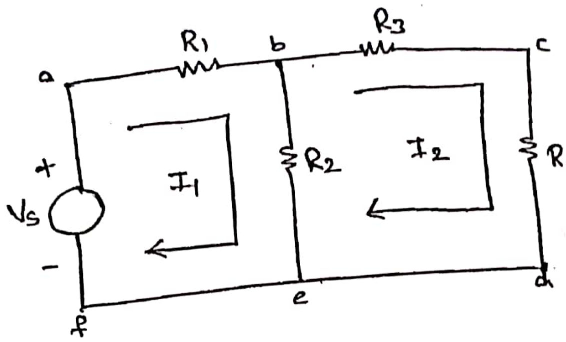
step 2 : choose the number of meshes in the circuit and Assign the mesh currents. ($I_1, I_2, I_3 \cdots$)

step 3 : Assign the directions for all mesh currents [clock or Anti clock wise]

step 4 : Apply kirchhoff's voltage law for each mesh and write the mesh equations in terms of unknown mesh currents.

step 5 : Solve the mesh equation for solution of the circuit.

Example:



The given circuit is a planar circuit. It consists of two meshes: abefa and bcdeb.

Let us assume mesh currents I_1 and I_2 with directions as indicated in the figure. Considering the loop abefa alone we observe that current I_1 is passing through R_1 and $(I_1 - I_2)$ is passing through R_2. By applying KVL, we can write

$$-V_s + I_1 R_1 + R_2 (I_1 - I_2) = 0$$

$$\Rightarrow \quad I_1 R_1 + R_2 (I_1 - I_2) = V_s \longrightarrow (1)$$

Similarly, if we consider the second mesh bcdeb, the current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. By applying KVL around the second mesh, we have

$$R_2 (I_2 - I_1) + R_3 I_2 + R_4 I_2 = 0 \longrightarrow (2)$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1 (R_1 + R_2) - I_2 R_2 = V_s$$

$$-I_1 R_2 + (R_2 + R_3 + R_4) I_2 = 0$$

By solving the above equations, we can find the currents I_1 and I_2.

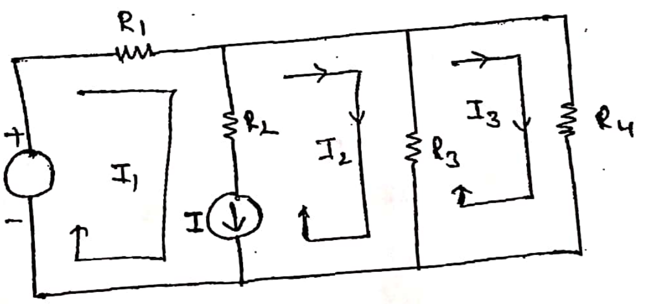
* Number of Mesh equations: In general, a circuit consists of B branches and N nodes including the reference node then the number of linearly independent mesh equations.

$$M = B - (N-1)$$

Super Mesh Analysis:

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis. One way to overcome this difficulty is by applying the super-mesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source.

Example:



Here, the current source I is the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$$R_1 I_1 + R_3 (I_1 - I_3) - V = 0$$

or $$R_1 I_1 + R_3 I_2 - R_3 I_3 = V$$

considering the mesh 3, we have

$$R_3 (I_3 - I_2) + R_4 I_3 = 0$$

Finally, the current I from the current source is equal to the difference between the two mesh currents i.e.

$$I_1 - I_2 = I$$

we have, thus, formed three mesh equations which we can solve for the three unknown currents in the network.

Nodal Analysis:

Nodal Analysis is one of the important technique used in finding solution of a network. If the network has more number of current sources, nodal analysis is more useful.

Steps for Nodal Analysis:

step 1: count the number of Principle nodes in the Network.

step 2: Assume any one node as reference node [voltage at reference node is always zero volts]

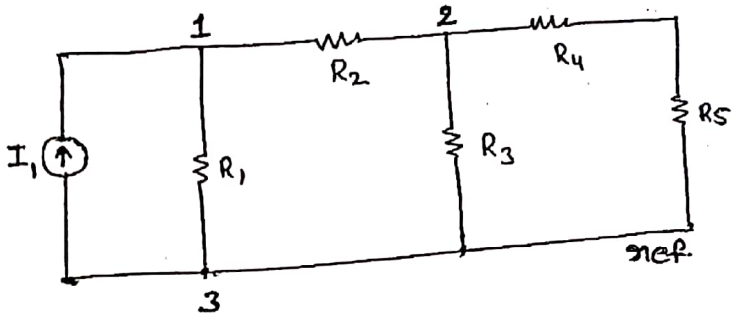
step 3: Assign the Node voltages ($v_1, v_2 \cdots\cdots$) at all principle nodes

step 4: By assuming node at highest Potential apply Kirchhoff's current law at each node, and write Node equations.

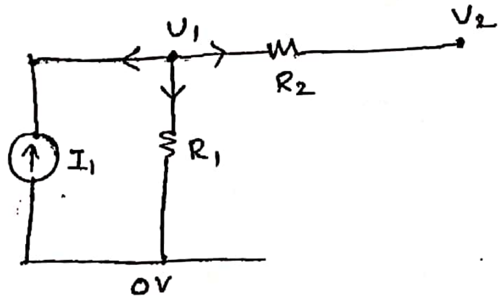
step 5: solve the node equations for solution of the network.

Example:

Consider the network shown in which there are '3' Principle nodes 1, 2, and 3. Assume node '3' as reference node and consider V_1 and V_2 are the voltages at 1 and 2 respectively.

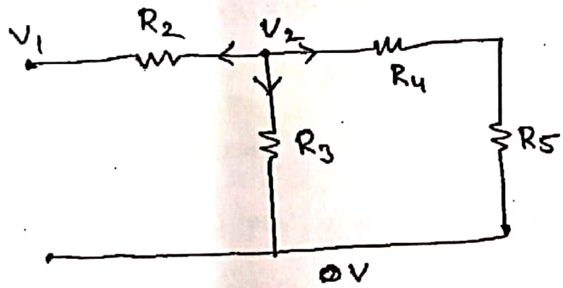


To write node equation at node 1 assume node 1 is at highest Potential and all currents are leaving from the node. Apply KCL and write node equation as



$$-I_1 + \frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

Similarly while writing the node equation at node 2 assume node '2' is at highest Potential and all currents are leaving from the node. Apply KCL and write node equation



$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_3} + \frac{V_2 - 0}{R_4 + R_5} = 0$$

Rearrang the above equations, we have.

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] + V_2\left[\frac{-1}{R_2}\right] = I_1$$

$$V_1\left[\frac{-1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0$$

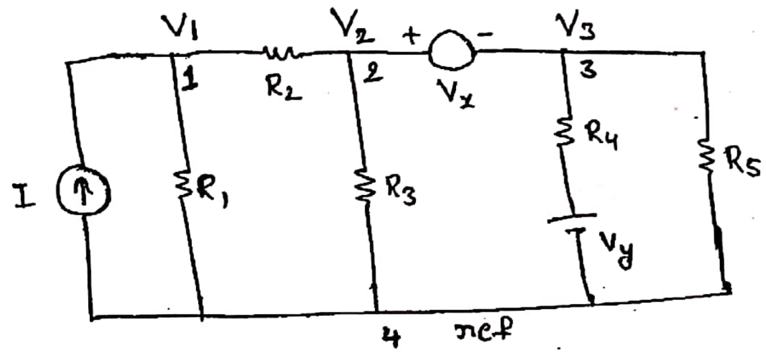
Solve the above equations for solution of the network.

Supernode Analysis:

Suppose any the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. one way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual.

Example:

It is clear from the figure that the node '4' is the reference node.



Applying KCL at the node 1 we get

$$-I + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

Due to the presence of voltage source V_x in between nodes 2 and 3, it is slightly difficult to find out the current The supernode technique can be applied in this case.

Accordingly, we can write the combined equation for nodes 2 and 3 as under

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_y}{R_4} + \frac{V_3}{R_5} = 0$$

The other equation is

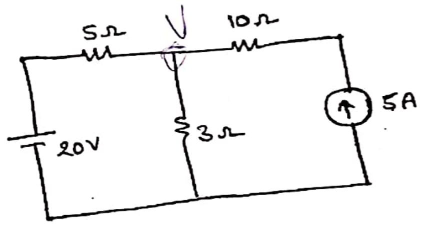
$$V_2 - V_3 = V_x$$

From the above three equations; we can find the three unknown voltages.

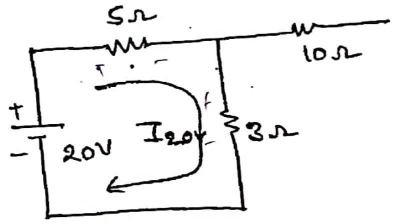
Superposition Theorem:

The superposition theorem states that "In any linear, bilateral network containing two or more sources, the response (current or voltage) in any element when all the sources acting is equal to the algebraic sum of the responses caused by individual sources acting alone, while considering individual sources all other sources are replaced with their internal resistances".

Example:

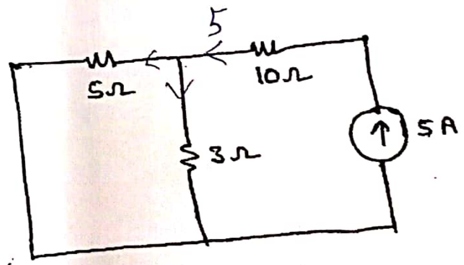


Consider the circuit which contains two sources. Now let us find the current passing through the 3Ω resistor in the circuit.



According to the superposition theorem, the current I_{20V} due to the 20V voltage source with 5A source open circuited

$$= \frac{20}{5+3} = 2.5\,A$$



The current I_S due to the 5A source with 20V source short circuited is

$$I_S = 5 \times \frac{5}{3+5} = 3.125\,A$$

~~Let us~~ Total current passing through 3Ω resistor is

$$2.5 + 3.125 = 5.625\,A.$$

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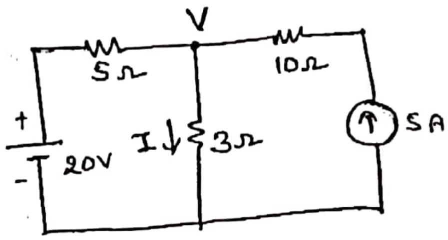
Let us verify the above result by applying nodal Analysis.

The current passing in the 3Ω due to both sources should be 5.625 A.



Applying nodal analysis, we have

$$\frac{V-20}{5} + \frac{V}{3} + (-5) = 0$$

$$V \cdot \left[\frac{1}{5} + \frac{1}{3}\right] = 5 + 4$$

$$V = 9 \times \frac{15}{8} = 16.875\,V$$

The current passing through the 3Ω resistor is equal to $V/3$

i.e $$I = \frac{16.875}{3} = 5.625\,A$$

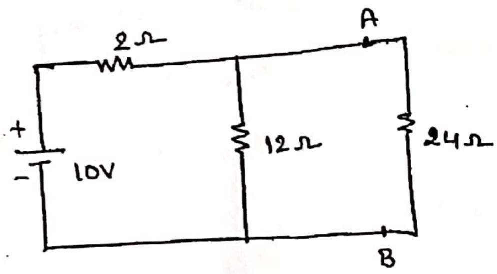
So the superposition theorem is verified.

Thevenin's Theorem:

Thevenin's theorem states that "any two terminal linear bilateral network having a number of voltage, current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where voltage is thevenins voltage is equal to the open-circuit voltage across the two terminals of the network and resistance is thevenins resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances."

Example:

In the circuit shown, if the 24Ω load resistance is connected to thevenin's equivalent circuit, it will have



the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, Let us find the current passing through the 24Ω resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

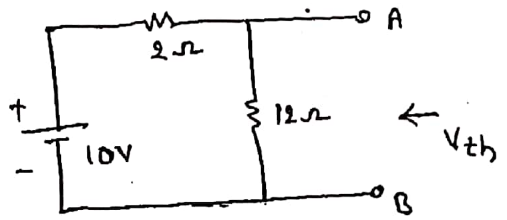
$$I_T = \frac{10}{2 + (12 \| 24)} = \frac{10}{10} = 1\,A$$

$$I_{24} = 1 \times \frac{12}{12+24} = 0.33\,A.$$

The voltage across the 24Ω resistor $= 0.33 \times 24 = 7.92\,V.$

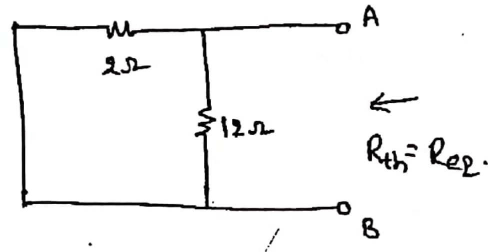
Now let us find Thevenin's equivalent circuit.

The Thevenin's voltage is equal to the open circuit voltage across the terminals 'AB'. i.e. the voltage across the 12Ω resistor. when the load resistance is disconnected from the circuit, the Thevenin voltage



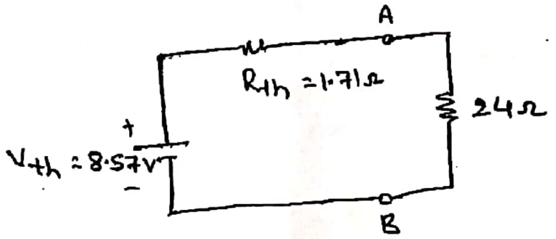
$$V_{th} = 10 \times \frac{12}{12+2} = 8.57\,V.$$

The resistance into the open circuit terminals is equal to the Thevenin resistance.



$$R_{th} = \frac{12 \times 2}{12+2} = 1.71\,\Omega$$

Thevenins equivalent circuit is shown in figure.



Now let us find the current passing through the 24Ω resistance and voltage across it due to Thevenin's equivalent circuit.

$$I_{24} = \frac{8.57}{24+1.71} = 0.33\,A.$$

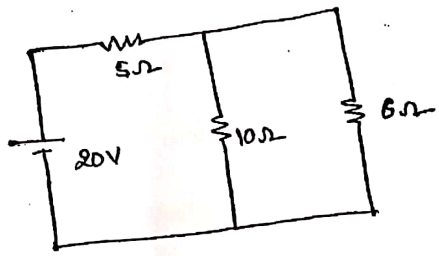
The voltage across the 24Ω resistance is equal to 7.92 V.

Thus it is proved that R_L (= 24Ω) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Norton's Theorem :

Norton's Theorem states that "any two terminal linear, bilateral network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with an resistance. The value of the current source is the short-circuit current between the two terminals of the network and the resistance ce is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.

Example:



In the circuit, if the load resistance of 6Ω is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6Ω resistor due to the original circuit.

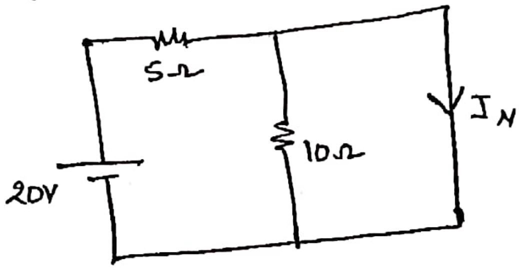
$$I_6 = I_T \times \frac{10}{10+6}$$

$$I_T = \frac{20}{5+(10\|6)} = 2.285A$$

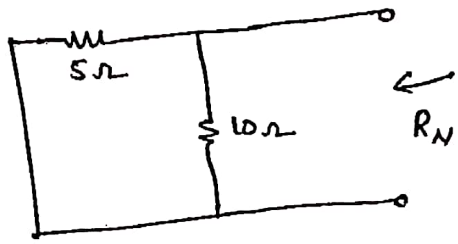
$$I_6 = 2.285 \times \frac{10}{16} = 1.43A.$$

i.e. the voltage across the 6Ω resistor is 8.58V $\boxed{6\times1.43}$. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short-circuited terminals

$$\therefore \quad I_N = \frac{20}{5} = 4A.$$

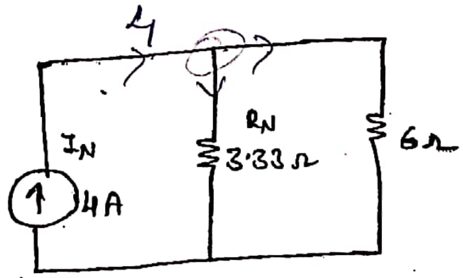


Norton's resistance is equal to the parallel combination of 5Ω and 10Ω resistors.



$$R_N = \frac{5 \times 10}{5+10} = 3.33\Omega$$

The Norton's equivalent is shown in figure. Now let us find the current passing through



the 6Ω resistor and the voltage across it due to Norton's equivalent circuit.

$$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43A$$

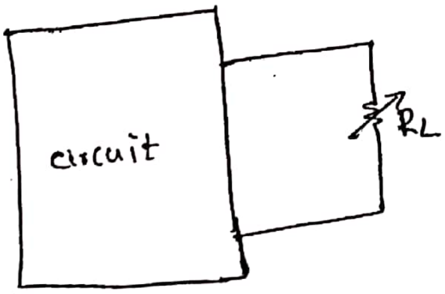
The voltage across the 6Ω resistor = 1.43 × 6

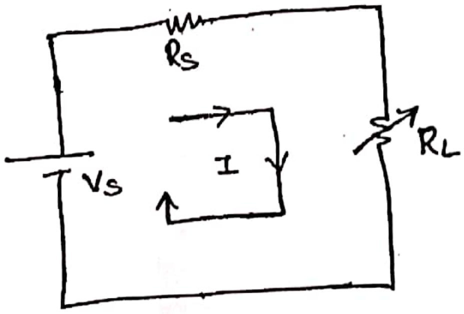
= 8.58V.

Thus it is proved that R_L (=6Ω) has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

Maximum Power Transfer Theorem:

This theorem deals with transfer of maximum power from a source to load and stated as " In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to internal resistance of the source as viewed from the load terminals, with load removed and all e.m.f. sources replaced by their internal resistances.





In many circuits basically consists of sources, it is necessary to transfer maximum power from the source to load. Now find the necessary conditions so that the power delivered by the source to the load is maximum.

Current in the circuit is $I = \dfrac{V_s}{R_s + R_L}$

Power delivered to the load R_L is $P = I^2 R_L$

$$= \frac{V_s^2 R_L}{(R_s + R_L)^2}$$

To determine the value of R_L for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to R_L i.e. when $\frac{dP}{dR_L}$ equal zero

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_s^2 R_L}{(R_s+R_L)^2}\right]$$

$$= \frac{V_s^2 (R_s+R_L)^2 - V_s^2 2R_L (R_s+R_L)}{(R_s+R_L)^4}$$

$$(R_s+R_L)^2 - 2R_L(R_s+R_L) = 0$$

$$\Rightarrow (R_s+R_L)^2 = 2R_L(R_s+R_L)$$

$$\Rightarrow R_s + R_L = 2R_L$$

$$\boxed{R_s = R_L}$$

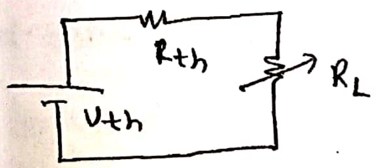
So maximum power will be transferred to the load resistance when load resistance is equal to the source resistance.

Max. Power: substituting condition $R_s = R_L$ in power equation $P = \frac{V_s^2 R_L}{(R_s+R_L)^2}$

$$\Rightarrow P_{max} = \frac{V_s^2 R_s}{(2R_s)^2} = \frac{V_s^2 \cancel{R_s}}{4R_s^{\cancel{2}}} = \frac{V_s^2}{4R_s}$$

Any circuit across load terminals can be reduced in to Thevenin's equivalent circuit.

Maximum power can also be find using



$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

Under the conditions of maximum power transfer, the efficiency is only 50%.