

Basic Electrical Engineering

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(V) UNIT-I D.C Circuits

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UNIT-I

D.C Circuits

Concept of Charge

The charge is a particle, which experiences a force when it is placed in a electric (or) magnetic fields.

It is denoted as "Q" or "q".

The unit of charge is "Coloumb (C)".

1 coulomb = charge on 6.24×10^{18} electrons

i.e $1\text{ coulomb} = 1.602 \times 10^{19}$ electrons.

Electric Current (I)

The electric current can be defined as "rate of flow of charge(i.e electrons) in an electric circuit (or) in any conductive or semi conductive medium".

$$\text{i.e } I = \frac{dq}{dt} \Rightarrow I = \frac{Q}{t}$$

where I = Average current in circuit

Q = Total charge transferred in circuit

t = time taken for charge.

Units for current is Amperes (Amp) (or) (Coloumbs/sec)

1-Ampere :- "When 1-coulomb of charge is transferred in a circuit in 1-second, then 1-Ampere current is flowing in that circuit."

Electric Voltage (or) Electric Potential (V)

Potential difference (or) Voltage

The voltage (v) is defined as "an electromotive force which causes the flow of charge (i.e electrons) in an electric circuit".

The ability of charge particle to do work in an electric circuit is called "electric potential".

$$\text{i.e } V = \frac{dW}{dq} \Rightarrow V = \frac{W}{Q}$$

where V = Total Voltage across two terminals

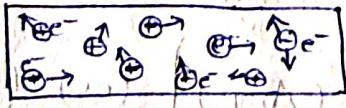
Q = Charge in electric circuit

W = Work done in electric circuit

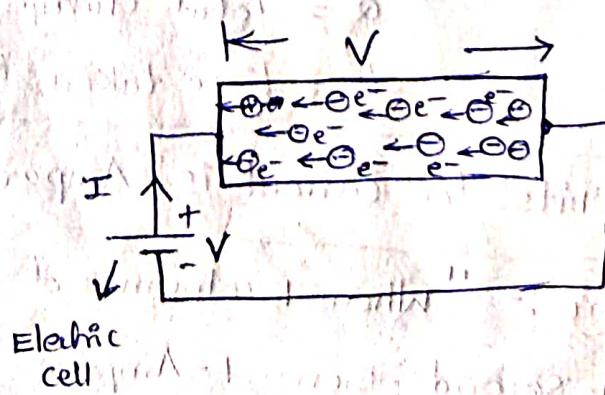
Units for voltage is Volts (V) (or) Jouls/Coloumb)

1 volt: When 1-coloumb of charge is transferred between two points with 1-joule of work, then 1-volt potential is exist between the two points.

Ex:-



Inside piece of conductor



Electric Power or Power (P)

The electric power can be defined as "the rate of electric work done in an electric circuit is called as "Power"

(or) "the rate of flow of electrical energy in electric circuit."

$$\text{i.e } P = \frac{dW}{dt} \Rightarrow P = \frac{dW}{dt} \times \frac{dq}{dt} \Rightarrow P = VI$$

i.e. Power is Product of Voltage & Current

Units for Power is Watts (W)

Electrical Energy (E)

An Electrical energy is the total amount of electrical work done in an electric circuit

$$\text{Units for Electrical Energy is, Watt-hr or kWh}$$

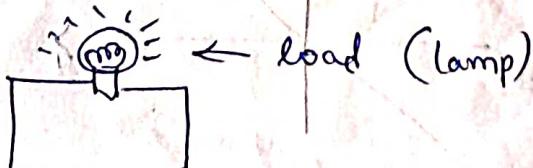
$$E = VIT$$

Electric circuit

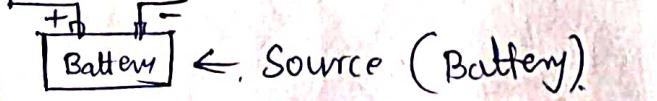
An electric circuit is a closed conducting path through which an electric current flows

(or)

A closed path which consists of Supply Source & load with conductors is called "Electric circuit"

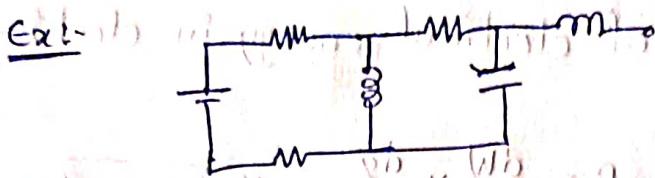


load (lamp)



Source (Battery)

Electrical Network:- Network is a combination of circuit & which consist of more no. of sources & loads, and it may be closed or open circuit (in any arrangement.)



Types of Circuit Elements:-

Active Elements:- An elements which are capable of supplying (or) delivering the power is called "Active elements"

Ex:- Voltage Source

Current Source

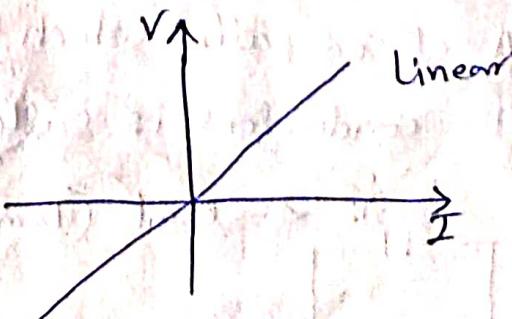
Passive Elements:- An elements which are capable of receiving (or) utilizing the power is called "Passive elements"

Ex:- Resistor, Inductor, Capacitor, etc

Linear Elements:-

The V-I characteristics of an elements is at all times a straight line & passing through the origin, thus elements are called "linear elements"

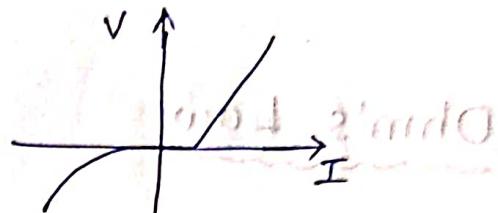
Example :- R, L, C



Non-Linear Elements :-

The V-I characteristics of an elements is at all times not a straight line and not passing through a origin, thus elements are called "Non-linear elements".

Ex:- Transistor, Diode



Bilateral Elements :-

Those elements characteristics & behaviour are independent of direction of current, thus elements are called "Bilateral Elements".

Ex:- Resistor, Inductor, Capacitor, Transformer

Unilateral Elements :-

Those elements characteristics & behaviour are dependent on direction of current, thus elements are called unilateral Elements.

Ex:- Diode, Triode.

Lumped Elements :-

The elements which are possible to separate from the network physically, these elements are called Lumped elements

Ex:- R, L, C & transformer, diode,

Distributed Elements :-

The elements which are not possible to separate from the network physically,

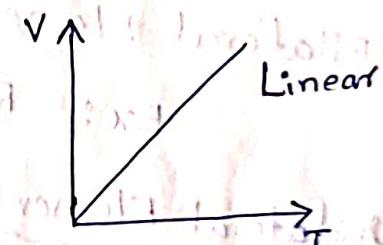
Ex:- Transmission line/ R-L-C parameters.

Ohm's Law

Statement :- Ohm's Law states that "the current passing through a conductor is directly proportional to the applied voltage across the conductor, at constant temperature 'T'."

$$\text{i.e } V \propto I$$

$$\text{Cor. } I \propto V$$



$$\therefore V = IR$$

$$\text{Also } I = \frac{V}{R}, \text{ & } R = \frac{V}{I}$$

Where 'R' is resistance of conductor. (Units Ω).

Limitations of Ohm's Law :-

- ① It is ~~not~~ applicable to temperature varying devices
- ② It is not applicable to non-linear devices
- ③ It is not applicable to semi-conductive materials.

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Electrical Circuit Elements

In Electrical ~~Field~~ field we have 3-types of electrical circuit elements, as

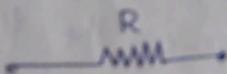
① Resistor (R)

② Inductor (L)

③ Capacitor (C)

Resistance (R) :- "The property of material which opposes or restrict the flow of electrons through material is called "Resistance". The element having this property is called "Resistor (R)"

→ It's symbol is



→ Units are "Ohm" is " Ω "

→ Power absorbed by resistor $P = VI \Rightarrow P = (IR)I = I^2 R$

$$\Rightarrow V(\frac{V}{R}) = \frac{V^2}{R}$$

→ Energy lost in the resistor in a time 't'

$$W = \int_0^t P dt = P.t = I^2 R t \propto \frac{V^2}{R} t.$$

→ Reciprocal of the resistance is called "Conductance" (G)

$$\text{i.e. } G = \frac{1}{R} \rightarrow \text{unit "mho (M)"}$$

→ 1 M is defined as "when 1V voltage is applied across a resistor, then 1A current flows, If having resistance is '1-M'."

Electrical Circuit Elements

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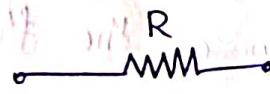
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→ Energy lost in the resistor in a time 't'

$$W_I = \int_0^t P dt = P.t = I^2 R t = \frac{V^2}{R} t.$$

→ Reciprocal of the resistance is called "Conductance" ("G")

$$\text{i.e } G = \frac{1}{R} \rightarrow \text{unit } \text{mho } (\text{S})$$

→ 1Ω is defined as "when 1-V voltage is applied across a resistor, then 1A current flows, it having resistance is ' 1Ω '"

Factors affecting resistance :-

① Length of material (l)

② Cross-sectional area (a)

③ Type of material (Nature of material (P))

④ Temperature (T).

$$\text{i.e. } R \propto \frac{l}{a} \Rightarrow R = \frac{P l}{a}$$

where l = length of material

a = area of cross-sectional

P = Resistivity (Specific Resistance)

Resistivity (Specific Resistance) :- (P)

It is a fundamental property (natural property) of material which opposes the flow of current is called resistivity (P).

$$P = \frac{R a}{l}$$

units are " $\Omega \cdot m$ "

Conductance (G) : The property of material which allows the flow of electrons in a conductor is called as "conductance".

$$\text{i.e. } G_1 = \frac{1}{R} \Rightarrow \frac{a}{P l}$$

-i.e. reciprocal of 'R' is called "G".

units are "Siemens"

→ The reciprocal of resistivity is called "conductivity (σ)".

$$\sigma = \frac{1}{P}$$

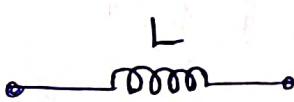
units $(\Omega \cdot m)^{-1}$

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Inductance (L) : Inductance is the property of a material coil or element which stores the electrical energy in the form of magnetic field.

(or)

"the property of a material which opposes ~~the~~ any change of magnitude (or) direction of electric current passing through the material (or) element."

→ It's symbol is 

→ Units are "Henry (H)"

→ The voltage across the inductor is directly proportional to the rate of change of current

$$V(t) \propto \frac{di(t)}{dt}$$

i.e.

$$V_L = L \frac{di}{dt}$$

Voltage across inductor

The current flowing through the inductor is

$$i_L = \frac{1}{L} \int_0^t V dt + i_0$$

Power absorbed by the inductor is

$$P = V \times i \Rightarrow L i \cdot \frac{di}{dt} \text{ watts.}$$

Energy stored in Inductor :-

The total energy in a inductor is

$$W_L = \int P_L(t) dt$$

$$= \int L \cdot i \cdot \frac{di}{dt} dt$$

$$= \int L \cdot \frac{i^2}{2} di \Rightarrow W_L = L \cdot \frac{i^2}{2}$$

$$W_L = \frac{1}{2} L i^2 \text{ Joules}$$

Capacitance (C)

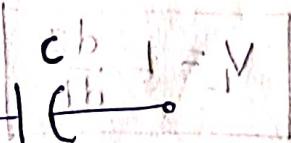
Capacitance is the "property of a element (or material

which stores electrical energy in the form of electric field.

(or)

the property of element which opposes sudden changes in magnitude of applied voltage across the element.

→ It's symbol is



→ Units are "Farads (F)"

→ The amount of charge q that can be stored in a capacitor of capacitance 'C' depends on applied voltage (V)

$$\text{i.e. } C = \frac{Q}{V}, i_C = C \frac{dV_C}{dt}, q = \frac{dq}{dt}$$

$$\text{i.e. } i_C = C \frac{dV}{dt}$$

current flowing capacitor

→ Voltage across capacitor

$$V_C = \frac{1}{C} \int i dt + V_0$$

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→ Power absorbed by capacitor

$$P = V^2 = V C \frac{dV}{dt}$$

→ Energy stored in capacitor :-

The total energy in a capacitor is

$$\begin{aligned} W_C &= \int (P_C(t)) dt \\ &= \int V C \frac{dV}{dt} dt \end{aligned}$$

$$\Rightarrow W_C = C \cdot \frac{V^2}{2} \quad \boxed{W_C = \frac{1}{2} C V^2 \text{ Joules}}$$

Problems :-

Q:- Find the energy dissipation and stored by the

passive elements R & L, C.

(i) A resistance 5Ω carries current of $2A$ for ~~3 hours~~

(ii) A $10H$ inductor carries a current $5A$ at

(iii) $300\mu F$ capacitor having applied voltage $20V$

$$\text{Ans: } E_R = 60 \text{ Wh}$$

$$E_L = 125 \text{ J}$$

$$E_C = 6000 \text{ J}$$

Q Find the values of R, L & C

(a) A resistance carries $2A$ at $500V$

(b) A inductor carries $2A$ and $20J$ of energy is stored

(c) A capacitor with $500V$ across it and $20J$ is stored energy

$$\text{Ans: } R = 250\Omega$$

$$L = 10H$$

$$C = 1600\mu F$$

Voltage and Current Sources

These are classified as two types

① Independent Sources

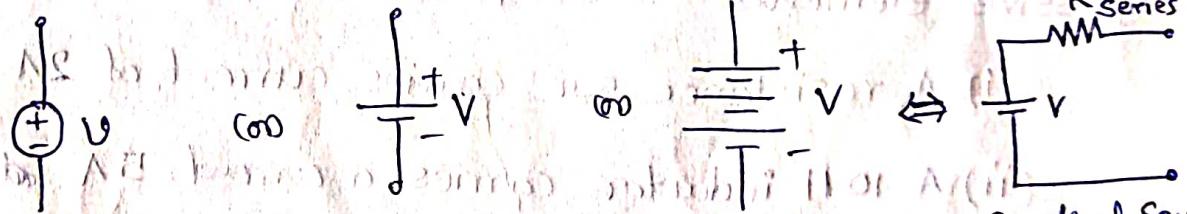
② Dependent Sources.

① Independent Sources : The values of energy sources does not depend on other voltages (or) currents in the network, thus sources are called "Independent Sources".

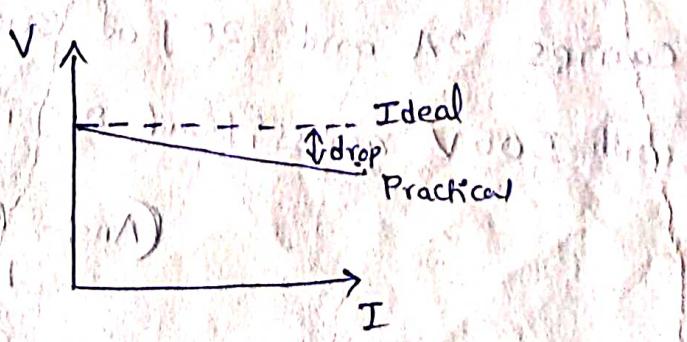
(i) Independent Voltage Source :-

The energy source which maintains constant voltage at its terminals irrespective of the current delivered to the external circuit, is called "Independent Ideal voltage Source".

→ Symbol is



→ Practically it has a small internal resistance in series, due to this some amount of voltage drop is exist as shown below



* Internal resistance of an ideal voltage source

is $R=0$ i.e $V_p = V_{op}$

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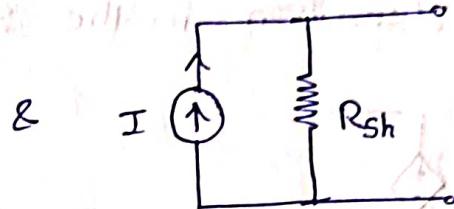
(ii) Independent Current Source

The source which delivers a current of constant magnitude at its terminals irrespective of applied voltage across its terminals is called "Independent Ideal Current Source".

→ Symbol is

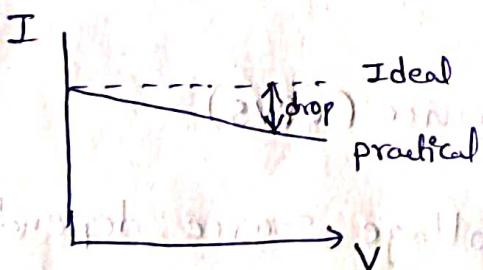


ideal source



Practical Source

→ But practically it has high internal resistance connected parallel with it, due to this some amount of drop will exist as shown below.



* Internal resistance of an ideal current source i.e. $R = \infty$ i.e. infinite

(2) Dependent Sources :-

The values of energy sources depends on other voltages (or) currents in the network, thus sources are called "Dependent Sources".

These are classified as 4-types

- ① voltage Dependent dependent Voltage Source (VDVS)
- ② Voltage Dependent Current source (VDCS)
- ③ Current Dependent Voltage source (CDVS)
- ④ current Dependent current source (CDCS)

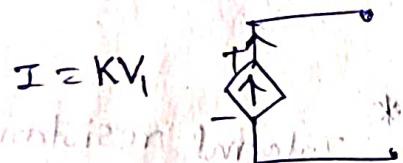
① Voltage Dependent Voltage Source (VDVS)

The value of this voltage source depends on function of voltages in elsewhere in the same circuit.



② Voltage Dependent Current source (VDCS)

The value of this current source depend on function of voltages ~~currents~~ in elsewhere in the same circuit.



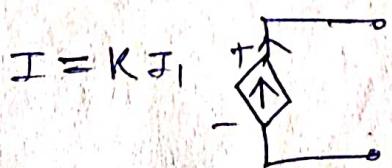
③ Current Dependent Voltage source (CDVS)

The value of this voltage source depend on function of currents in somewhere in the same circuit



④ Current dependent Current source (CDCS)

The value of this current source depend on the function of current in somewhere in the same circuit



Series & Parallel connections of

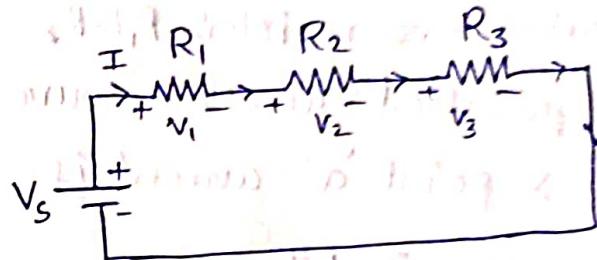
R, L & C electrical elements

① Resistors connected in Series

In a series circuit, all the components are connected such that there is only one closed path & same current flows through all the components. As shown in below figure

consider 3-Resistors

R_1 , R_2 & R_3 are connected in series with a voltage source V_s & causes the current I .



The total (or) equivalent resistance of circuit

is

$$R_{eq} = R_1 + R_2 + R_3$$

Note: ① In series circuit Current is Same

$$I_s = I$$

② In series circuit Voltage is divided

i.e.

$$V_s = V_1 + V_2 + V_3$$

→ Application of series connection is to increase the resistance & to limit the current flow.

→ Resistances, Voltages & powers are additive in series connection.

→ Different current rating devices unable to connect in series.

② Parallel connection of Resistors

In parallel circuit "all the components" in that circuit are connected in a manner that there are more than one path ~~as~~ branches current can flows.

As shown below

consider a circuit

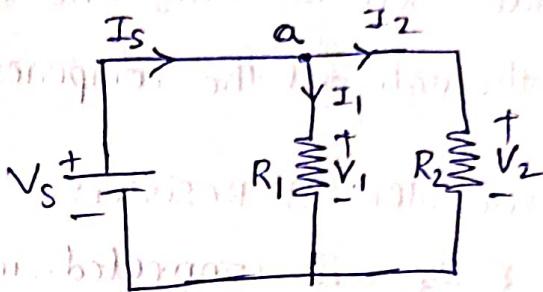
having two resistors R_1 & R_2 in parallel across a source

V_s & point 'a' current is

$$I = I_1 + I_2$$

The equivalent Resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Note :-

① Parallel circuit Voltage is Same

$$V_s = V_1 = V_2$$

② Current is divided i.e

$$I_s = I_1 + I_2$$

Advantages :-

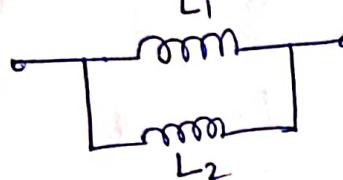
- ① If there is a discontinuity or break in any one branch, the current will still flow in the other branch.
- ② The power & current rating can be distributed to the electrical appliances is same.
- ③ These are used in all house hold & industrial wiring.

③ Inductors in Series



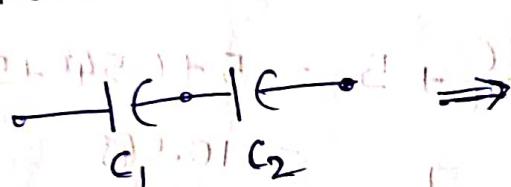
$$L_{eq} = L_1 + L_2$$

④ Inductors in parallel



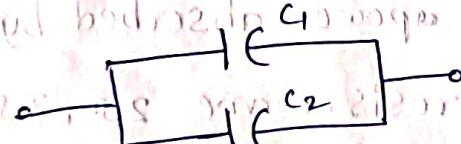
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

⑤ Capacitors in series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

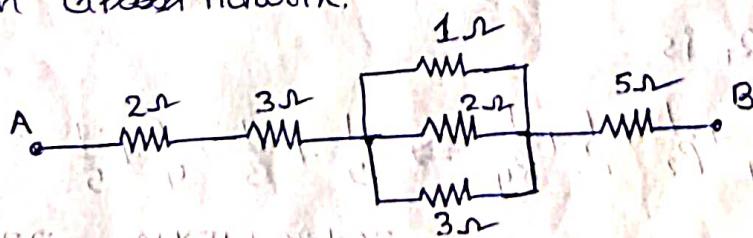
⑥ Capacitors in parallel



$$C_{eq} = C_1 + C_2$$

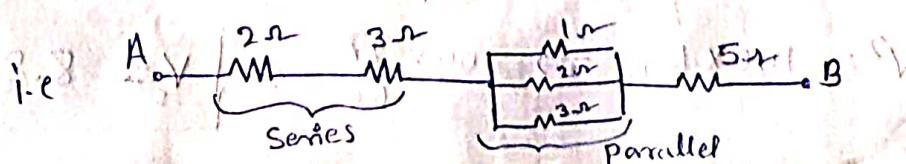
Problem :-

- ① find the total Resistance between the terminal A & B for given circuit network.

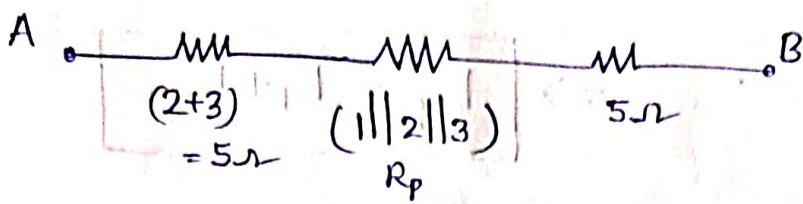


Solution:- In given circuit 2Ω & 3Ω resistors in series

i.e. $2\Omega, 1\Omega, 3\Omega$ are in parallel.



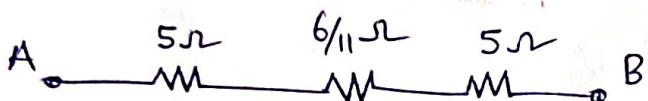
The circuit modified as



$$R_p = \frac{1 \times 2 \times 3}{2 + 6 + 3} = \frac{6}{11} \Omega$$

$$\therefore R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3}$$

then



Now

$$\text{Total Resistance } R_{AB} = 5 + \frac{6}{11} + 5 = 5 + 0.545 + 5 \\ \Rightarrow 10.545$$

$$R_{AB} = 10.545 \Omega$$

- ② Determine the voltage if the total power absorbed by the resistors is 100 watts, thus resistor are $2\Omega, 3\Omega, 4\Omega$ & 5Ω respectively are connected in parallel.

Solution:

Given data: $R_1 = 2\Omega, R_2 = 3\Omega, R_3 = 4\Omega, R_4 = 5\Omega$ & $P = 100\text{Watt}$

Then in parallel R_T is

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \Rightarrow \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \\ = \frac{30 + 20 + 15 * 12}{60} = \frac{77}{60} \Rightarrow R_T$$

$$\text{The total power absorbed } P = \frac{V^2}{R_T} = 100\text{W}$$

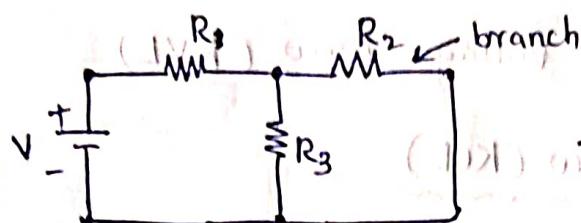
$$V^2 = 100 \times R_T \Rightarrow 100 \times \frac{77}{60}$$

$$V_s = 8.827 \text{ Volts}$$

Basic Definitions

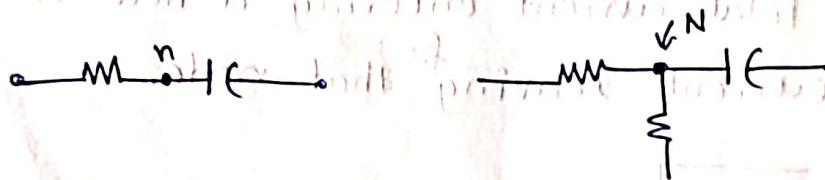
Branch :- "A part of a network which is connected between two nodes, usually called as 'branch'".

Ex :-

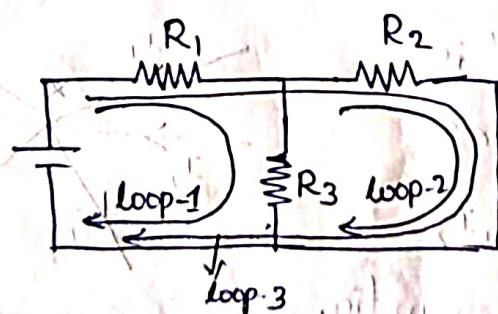


\Rightarrow We have 4-branches

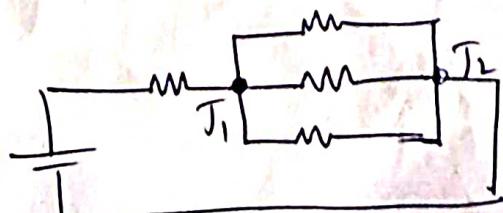
Node :- "A point at which two or more circuit elements are joined together is called 'node'.



Mesh (or) Loop :- "A closed path which starts and ending at same node and which is not passing any node (or) branch twice is called mesh (or) loop.



Junction Point :- A point of a network at which three or more no. of circuit elements are joined together



(01)

Kirchhoff's Laws

Kirchhoff's Laws are classified into two types

① Kirchhoff Current Law (KCL)

② Kirchhoff Voltage Law (KVL)

Kirchhoff Current Law (KCL)

Statement :- KCL states that algebraic sum of currents at any node is equal to "zero". i.e

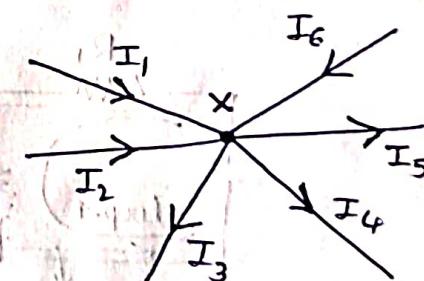
the total current entering a node is equal to the total current leaving that node.

i.e
$$\sum_{\text{at node}} I = 0$$

$$\sum I_{\text{ent}} = \sum I_{\text{leav}}$$

Explanation :-

consider a node 'x' is connected by a 6-brances carrying a currents I_1, I_2, I_3, I_4, I_5 & I_6 as shown in fig.



Let the currents entering into the node are positive currents & leaving currents are negative currents as

I_1, I_2, I_6 are positive

& I_3, I_4, I_5 are negative

Then, according to KCL

$$\sum I = 0 \text{ i.e. } I_1 + I_2 - I_3 - I_4 - I_5 + I_6 = 0$$

(or)

$$I_1 + I_2 + I_6 = I_3 + I_4 + I_5$$

i.e from KCL

Total current entering into node = Total current leaving from that node.

→ It is useful for Nodal analysis

Kirchhoff Voltage Law (KVL)

Statement :- KVL states that algebraic sum of the voltages around the closed loop (or) circuit is equal to zero

i.e

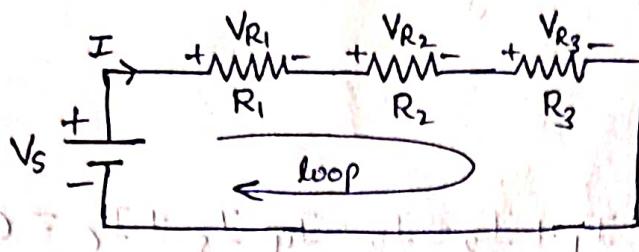
$$\sum_{\text{In Loop}} V = 0$$

i.e the algebraic sum of the emf's (or) voltages of sources in loop is equal to sum of the voltage drop across the circuit elements of in that loop

i.e

$$\sum_{\text{Loop}} V_s = \sum_{\text{Loop}} (I R)_{\text{drop}}$$

Explanation:- consider a circuit which is having resistors $R_1, R_2 \& R_3$ & supply source V_s as shown below



According to KVL

$$V_s - V_{R_1} - V_{R_2} - V_{R_3} = 0$$

$$V_s = V_{R_1} + V_{R_2} + V_{R_3}$$

$$= IR_1 + IR_2 + IR_3$$

$$V_s = I(R_1 + R_2 + R_3) \Rightarrow I = \frac{V_s}{R_1 + R_2 + R_3}$$

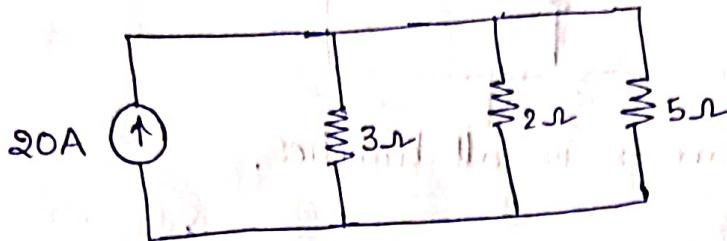
- Indicate the sign conventions for elements by the direction of current
- It is useful for mesh analysis.

Steps for applying Kirchhoff Laws:-

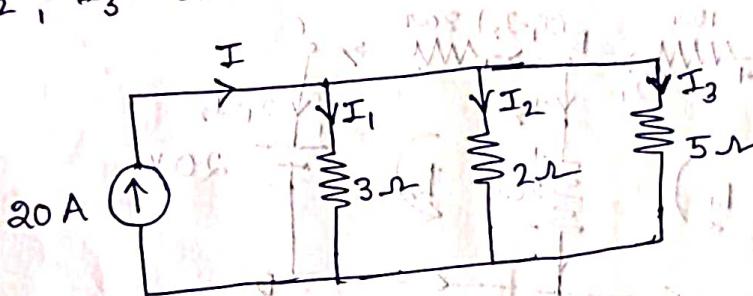
- Step ① Draw the circuit diagram from given information
- ② Insert the all values of sources with appropriated polarities and all resistance values
- ③ By using KCL mark the all branch currents with some assumed directions at the nodes.
- ④ Mark the all the polarities of voltage drops of rises & drops
- ⑤ Apply KVL for different closed loops
- ⑥ Solve the equations by based on solution

Problems:-

PQ1 Determine the current in all resistors in the given circuit below by using KCL, for a given current of 20A.

Solution:-

Let the total current I , & respective branch currents I_1, I_2, I_3 as shown in the circuit & let the voltage 'V'



According to KCL

$$I = I_1 + I_2 + I_3$$

$$\therefore I_1 = \frac{V}{3}$$

$$\because I = \frac{V}{R}$$

$$20 = \frac{V}{3} + \frac{V}{2} + \frac{V}{5} \quad \therefore I_2 = \frac{V}{2}$$

$$20 = V \left[\frac{1}{3} + \frac{1}{2} + \frac{1}{5} \right] \quad \therefore I_3 = \frac{V}{5}$$

$$20 = V \left[\frac{10 + 15 + 6}{30} \right] \Rightarrow V = \frac{20 \times 30}{31} \Rightarrow V = 19.35 V$$

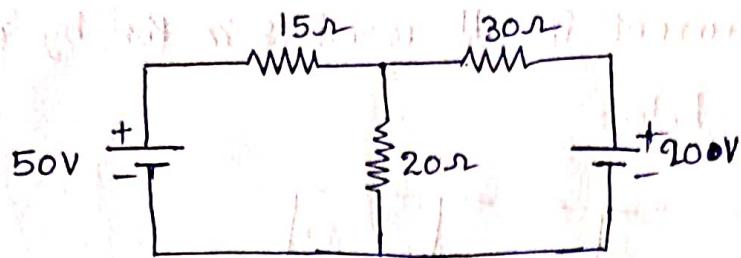
Now branch currents

$$\text{Current in } 3\Omega \text{ Resistor } I_{3\Omega} = I_1 = \frac{V}{3} = \frac{19.35}{3} \Rightarrow I_1 = 6.45 A$$

$$\text{Current in } 2\Omega \text{ Resistor } I_{2\Omega} = I_2 = \frac{V}{2} = \frac{19.35}{2} = I_2 = 9.67 \text{ Amp}$$

$$\text{Current in } 5\Omega \text{ Resistor } I_{5\Omega} = I_3 = \frac{V}{5} = \frac{19.35}{5} = I_3 = 3.87 \text{ Amp}$$

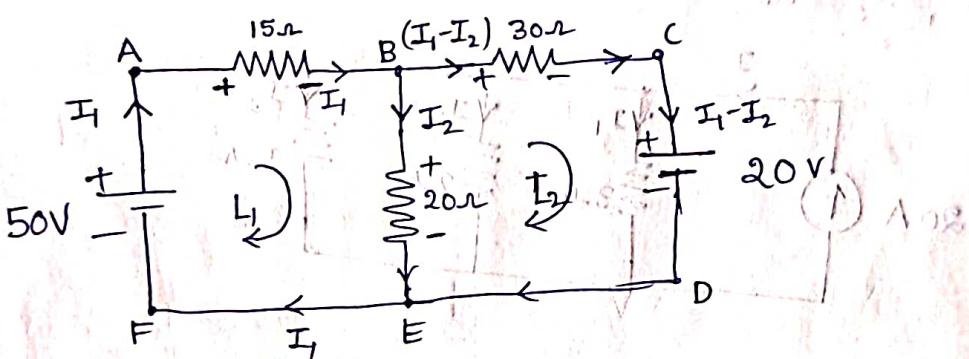
P② Apply Kirchhoff's laws to the given circuit



Determine the currents in all branches.

Solutions:-

Steps ① & ② Draw the circuit diagram with the values & polarities & branch currents by 50V source as reference, by KCL



→ Apply KVL for different loops L_1 & L_2

Loop 1 : ABCFA

$$50 - (I_1 \times 15) - (I_2 \times 20) = 0$$

$$15I_1 + 20I_2 = 50 \quad \text{--- (1)}$$

Loop 2 :- BCDEB

$$- 30 \times (I_1 - I_2) - 20 + (20 \times I_2) = 0$$

$$- 30I_1 + 30I_2 + 20I_2 = 20$$

$$- 30I_1 + 50I_2 = 20 \quad \text{--- (2)}$$

By solving the equations ① & ②

(or) use the
casio

Multiply eq ① x 2 & ~~eq ② x 3~~

then

$$\begin{array}{r} 30I_1 + 40I_2 = 100 \\ -30I_1 + 50I_2 = 20 \\ \hline 90I_2 = 120 \Rightarrow I_2 = \frac{120}{90} \Rightarrow I_2 = 1.33 \text{ Amp} \end{array}$$

Substitute I_2 in eq ① or ②

$$15I_1 + (20 \times 1.33) = 50$$

$$I_1 = \frac{(50 - 26.6)}{15} \Rightarrow I_1 = 1.56 \text{ Amp}$$

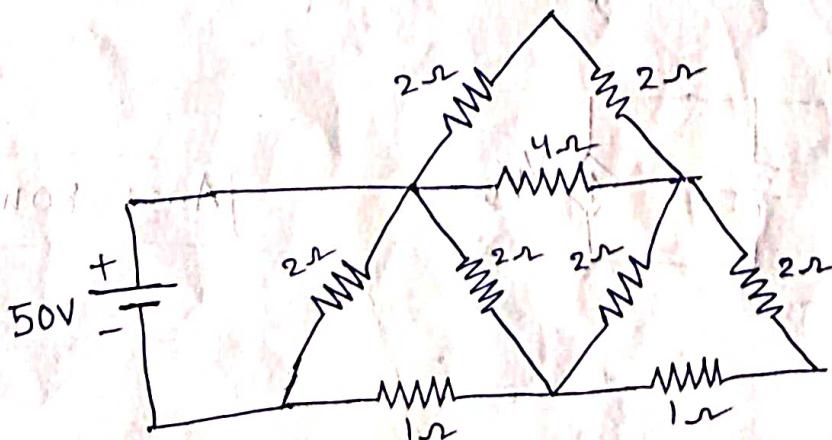
Now branch current

Current in 15Ω Resistor $I_{15\Omega} = I_1 = 1.56 \text{ Amp}$

Current in 20Ω Resistor $I_{20\Omega} = I_2 = 1.33 \text{ Amp}$

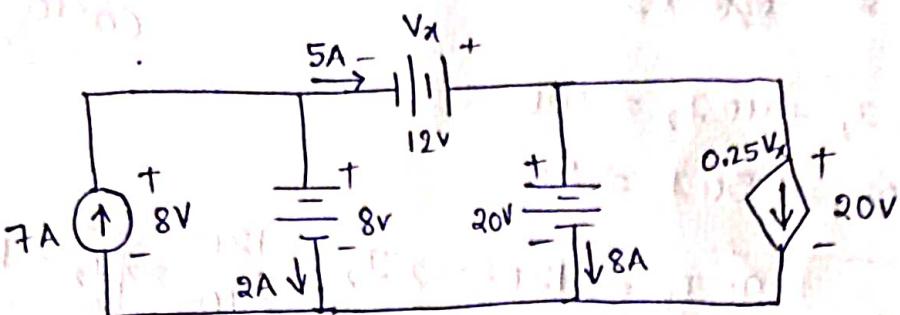
Current in 30Ω Resistor $I_{30\Omega} = (I_1 - I_2) = 0.23 \text{ Amp}$

P(3) :- Determine the current delivered by the source in the circuit shown in below.



Ans: $I = 47.41 \text{ Am}$

Q: Calculate the power absorbed by each component in the circuit shown in figure.



Solution:- In a given circuit we have 5-components a)

3-voltage Sources

1- Current source & 1-voltage dependent current source.

Now Power absorbed by

$$7A \text{ current source is } P_{7A} = V \times I = 7 \times 8 \Rightarrow P_{7A} = 56W$$

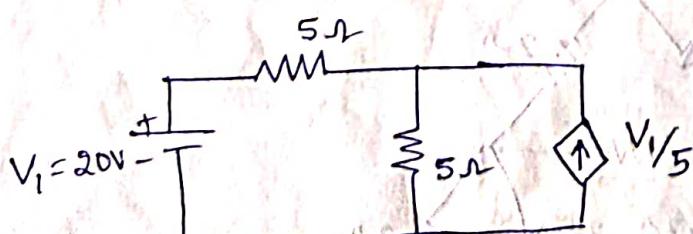
$$8V \text{ voltage source is } P_{8V} = 8 \times (-2) = -16 \Rightarrow P_{8V} = -16W$$

$$12V \text{ voltage source is } P_{12V} = 5 \times 12 = 60 \Rightarrow P_{12V} = 60W$$

$$20V \text{ voltage source is } P_{20V} = 20 \times (-8) = -160 \Rightarrow P_{20V} = -160W$$

$$0.25V_x \text{ VDCS is } P = (0.25 \times 12) \times (20) = 60 \Rightarrow P_{VDCS} = 60W$$

Q:- Find the power supplied by dependent source.



Ans: 80W

Analysis of Simple DC-circuits

Analysis of DC-circuits can be done by Network Reduction techniques as

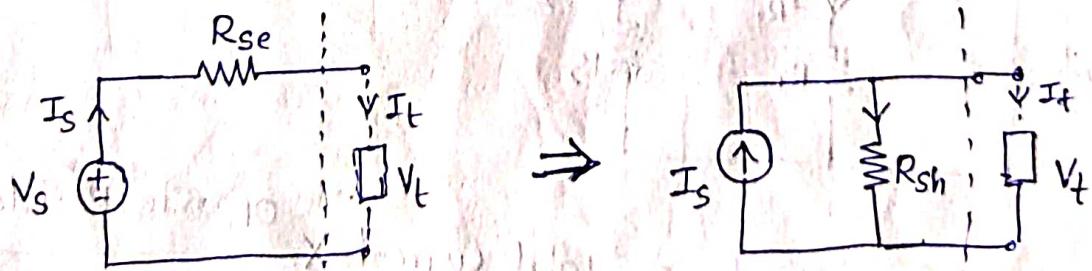
- Source transformation
- Current division rule
- Voltage division rule
- Nodal Analysis
- Mesh Analysis
- Star-Delta or Delta-star transformation:-

Source Transformation:-

Source transformation is technique for "reducing" complexity of network by replacing voltage source by current source (or) current source by voltage source.

Practically we know that voltage source has a series resistance and current source has a parallel shunt resistance.

Let the Voltage & current sources as



When $R_{se} = R_{sh}$, then Source Transformation is possible

$$V_t = V_s - I_s R_{se} \quad (\because I_s = I_t)$$

$$I_t = \frac{V_s - V_t}{R_{se}}$$

$$I_t = \frac{V_s}{R_{se}} - \frac{V_t}{R_{se}} \Rightarrow I_t = I_s + \frac{V_t}{R_{se}} \quad \text{--- (1)}$$

from above

$$I_s = \frac{V_t}{R_{sh}} + I_t$$

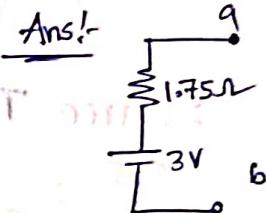
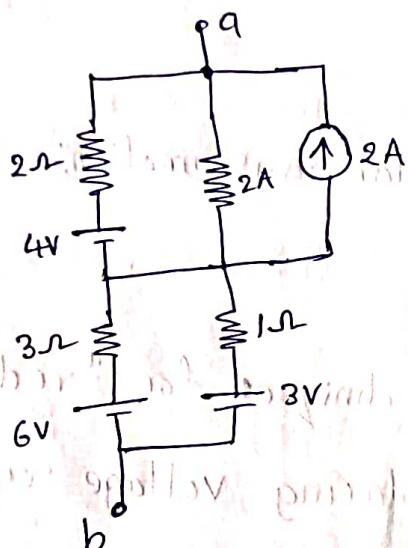
$$I_t = I_s - \frac{V_t}{R_{sh}} \quad \text{--- (2)}$$

From above eq① & eq② when $R_{se} = R_{sh}$ then S.T is possible

As... ~~voltage~~ voltage source V_s and internal resistance R_{se} can be replaced by equivalent current source I_s in parallel with shunt resistor i.e. $I_s = \frac{V_s}{R_s}$ & $R_{sh} = R_s$.

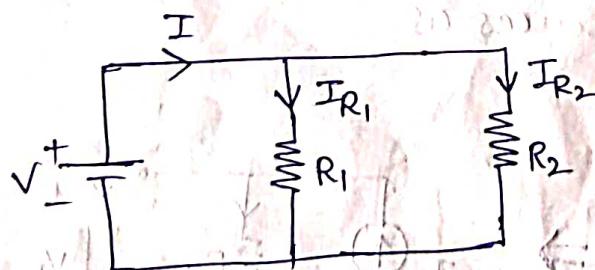
Prob!:-

Convert given circuit into single voltage source.



Current Division in parallel circuit

For a given circuit current division applied as



Opposite Resistance
Total Current X
Total Resistance

i.e. Current in branch = Resistance \times Total Current \times Opposite Resistance / Total Resistance

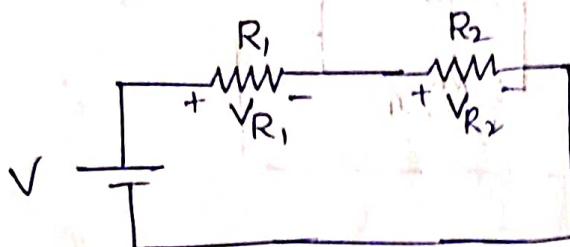
$$\text{i.e. Current in } I_{R_1} = I \times \frac{R_2}{R_1 + R_2}$$

$$I_{R_2} = I \times \frac{R_1}{R_1 + R_2}$$

$$\text{i.e. } I_{R_n} = I \cdot \frac{R_1 \cdot R_2 \dots R_{n-1}}{\sum R_1 R_2 \dots R_n}$$

Voltage division in series circuits

VDR - is apply for a given circuit



$$\text{i.e. Voltage across any branch} = \frac{\text{Total Voltage} \times \frac{\text{Some Resistance}}{\text{Total Resistance}}}{\text{Total Current} \times \text{Some Resistor}}$$

i.e. Voltage across R_1 is

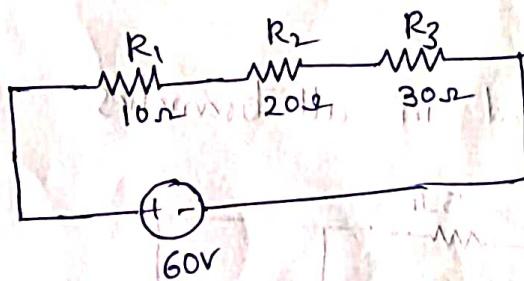
$$V_{R_1} = V \times \frac{R_1}{R_1 + R_2}$$

$$\text{i.e. } V_{R_n} = V \cdot \frac{\sum R_1, R_2, \dots, R_n}{\sum R_n}$$

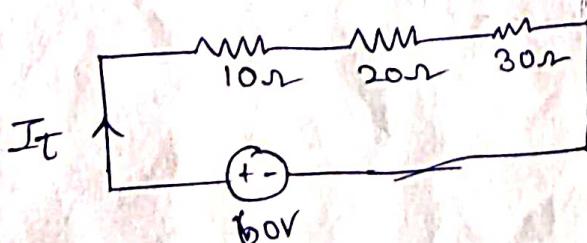
$$V_{R_2} = V \times \frac{R_2}{R_1 + R_2}$$

Problems:-

①. Find the voltage across 3-resistors by using VDR.



Solution:- Let total current I_T & total voltage $V_t = 60V$



$$R_T = R_1 + R_2 + R_3$$

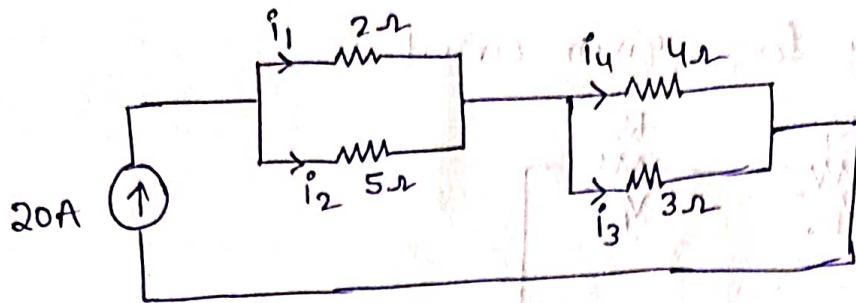
$$= 10 + 20 + 30 = 60\Omega$$

$$I_T = \frac{V_t}{R_T} = \frac{60}{60} = 1A$$

$$V_{10\Omega} = I_T R_{10} = 1 \times 10 = 10V$$

$$V_{20\Omega} = 20 \times 1 = 20V \quad \& \quad V_{30} = 30V$$

PQ Find currents i_1 , i_2 , i_3 , & i_4 in given circuit.



Solution

In a given circuit total current $I = 20\text{A}$, then

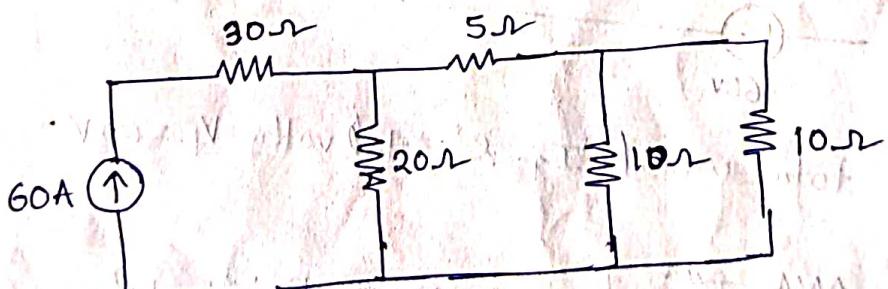
$$i_1 = I \times \frac{5}{2+5} = 20 \times \frac{5}{7} = \frac{100}{7} \Rightarrow i_1 = 14.28 \text{ Amp}$$

$$i_2 = 20 \times \frac{2}{2+5} = \frac{40}{7} \Rightarrow i_2 = 5.71 \text{ Amp}$$

$$i_3 = 20 \times \frac{4}{4+3} = \frac{80}{7} = 11.42 \text{ Amp}$$

$$i_4 = 20 \times \frac{3}{4+3} = \frac{60}{7} \Rightarrow i_4 = 8.57 \text{ Amp}$$

Prob! :- Find the current in all branches.



Nodal analysis :-

- This method is mainly based on "Kirchhoff Current Law"
- If a network has ' n ' nodes & ' j ' junction points, then we will get $(n-1)$ of nodal equations.
 $\text{cos } (j-1)$

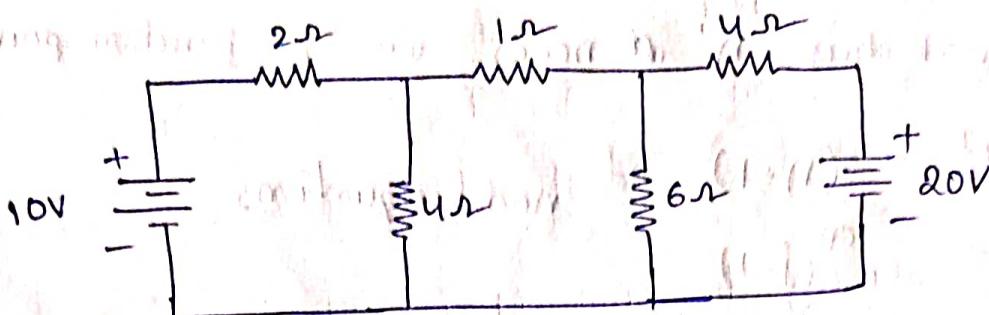
Steps for Nodal Analysis:-

- Step ① choose the nodes & node voltages
- Step ②. Assume the ~~one~~ one of the node as "base node" or reference node & Assume remaining all ~~other~~ nodes are high potential nodes leaving ~~leaving~~ high potential node
- Step ③ Assume the branch currents from high potential node by using KCL at node
- Step ④ obtain the current ~~as~~ nodal equations in terms of node voltages
- Step ⑤ Solving the nodal equations we get node voltages w.r.t them find branch currents.

Note :- If a network has a more no. of voltage sources then better to use mesh analysis otherwise nodal analysis if network ~~also~~ has $m < n$ then nodal method is preferred.

problem:-

Find the current through each resistor of circuit shown in figure, by using nodal analysis.

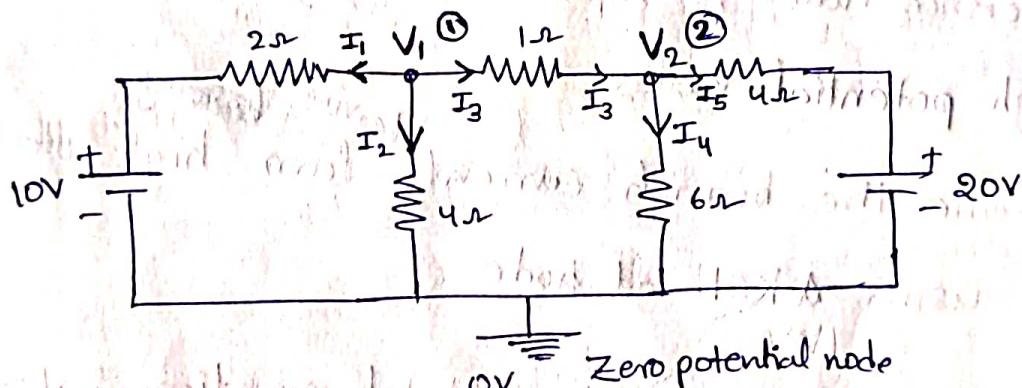


Solution:-

→ choose nodes & node voltages, Let a one node as reference

→ Assume branch currents as I_1, I_2, I_3 at node ①.

and I_4, I_5 at n② as



→ At node 1, by applying KCL

$$-I_1 - I_2 - I_3 = 0$$

$$\text{We know } I_1 = \frac{V_1 - 10}{2}$$

Now

$$-\left[\frac{V_1 - 10}{2}\right] - \left[\frac{V_1}{4}\right] - \left[\frac{V_1 - V_2}{1}\right] = 0$$

$$I_2 = \frac{V_1 - 0V}{4} = \frac{V_1}{4}$$

$$-2(V_1 - 10) - V_1 - 4[V_1 - V_2] = 0$$

$$I_3 = \frac{V_1 - V_2}{1}$$

$$-2V_1 + 20 - V_1 - 4V_1 + 4V_2 = 0$$

$$-7V_1 + 4V_2 = -20 \quad \text{--- (1)}$$

→ At node-2 :

$$I_3 - I_4 - I_5 = 0$$

$$\frac{V_1 - V_2}{1} - \frac{V_2}{6} - \frac{V_2 - 20}{4} = 0$$

$$12(V_1 - V_2) - 2V_2 - 3(V_2 - 20) = 0$$

$$12V_1 - 17V_2 = -60 \quad \text{--- (2)}$$

By solving eq(1) & eq(2).

$$V_1 = 8.169V \quad \text{and} \quad V_2 = 9.295V$$

Now branch currents

$$I_1 = \frac{V_1 - 10}{2} = \frac{8.169 - 10}{2} = -0.9155$$

i.e. $I_1 = 0.9155 \text{ Amp}$ \rightarrow

\leftarrow -ve means opposite direction

$$I_2 = \frac{V_1}{4} = \frac{8.169}{4} = 2.042 \Rightarrow I_2 = 2.042 \text{ A}$$

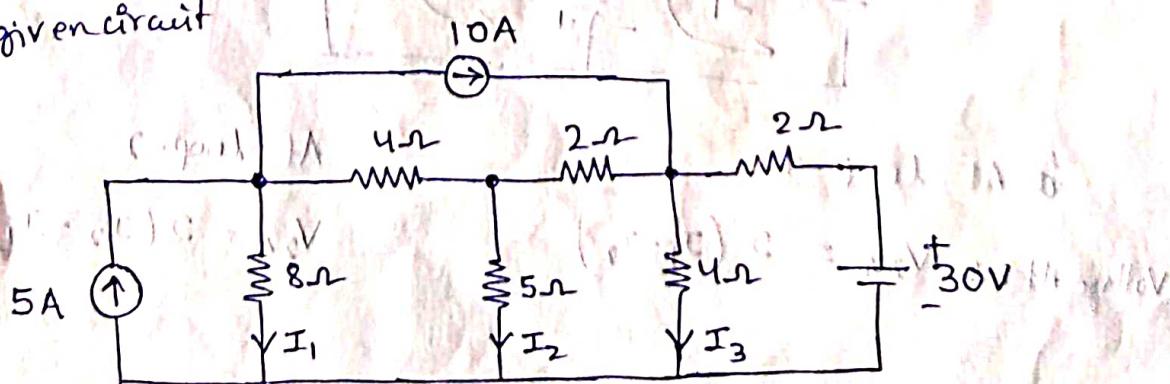
$$I_3 = \frac{V_1 - V_2}{1} = 1.126 \Rightarrow I_3 = 1.126 \text{ A}$$

$$I_4 = \frac{|V_2|}{6} = 1.361 \Rightarrow I_4 = 1.361 \text{ A}$$

$$I_5 = \frac{V_2 - 20}{4} = -2.676 \Rightarrow I_5 = 2.676 \text{ A}$$

Problem:- Find the different node voltages & currents $I_1, I_2, \text{ & } I_3$

for given circuit



Mesh analysis (or) Loop Analysis

→ This method is mainly based on KVL.

→ If a network have 'b' of branches, 'J' of junctions, then we will get the independent mesh equations.

$$\text{i.e } m = b - (J - 1) \text{ or } b - (n - 1)$$

Steps for mesh analysis :-

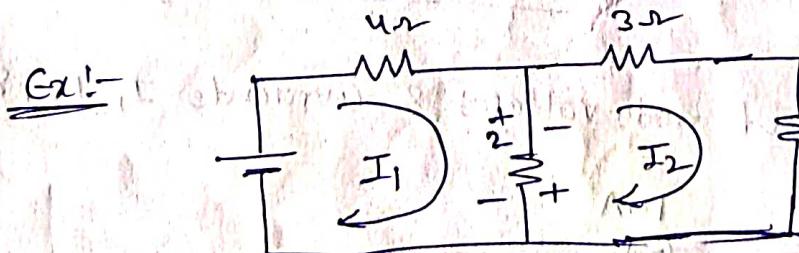
Step ①. find the various loops in a network.

②. Indicate the assumed loop currents directions and w.r.t to that mark the polarities.

Note: Assumed loop current is higher value compare with other loop currents.

Step ③ choose minimum no. of loops and write the loop equations by applying KVL.

Step ④ By solving the loop equations we get loop currents.



At loop-1

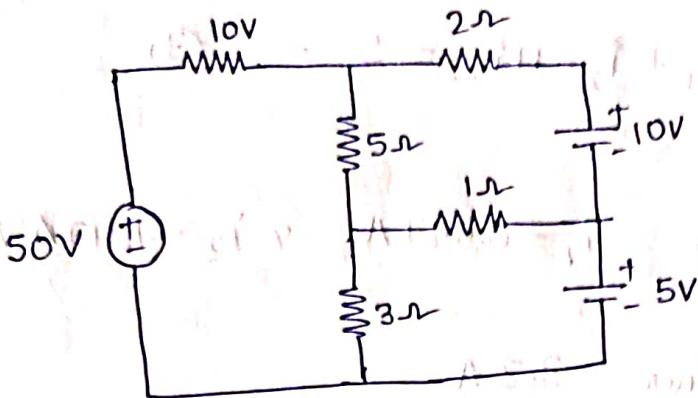
$$\text{Voltage At } 2\Omega \text{ } V_{2\Omega} = 2(I_1 - I_2)$$

At loop-2

$$V_{2\Omega} = 2(I_2 - I_1)$$

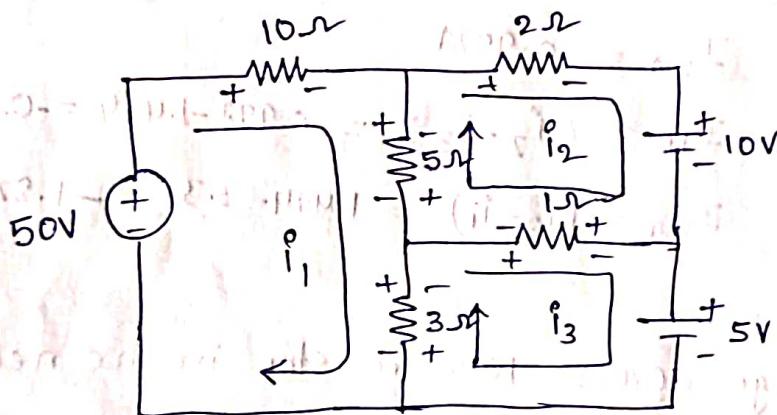
Problem!:-

① Determine mesh current by using mesh analysis



Solution!:-

Assuming loop currents i_1, i_2 & i_3



Step ② At loop-1, Apply KVL

$$50 - 10i_1 - 5(i_1 - i_2) - 3(i_1 - i_3) = 0$$

$$-18i_1 + 5i_2 + 3i_3 + 50 = 0$$

$$-18i_1 + 5i_2 + 3i_3 = -50 \quad \text{--- (1)}$$

At loop-2,

$$-2i_2 - 10 - 1(i_2 - i_3) - 5(i_2 - i_1) = 0$$

$$5i_1 - 8i_2 + i_3 = 10 \quad \text{--- (2)}$$

At loop ③

$$-(i_3 - i_2) - 5 - 3(i_3 - i_1) = 0$$

$$3i_1 + i_2 - 4i_3 = 5 \quad \text{--- } ③$$

by solving eqn

$$i_1 = 3.300 \text{ Amp} \quad i_2 = 0.997 \text{ A} \quad 8i_3 = 1.474 \text{ A}$$

Current in 10Ω $I_{10\Omega} = 3.3 \text{ A}$

$$I_{5\Omega} = \frac{3.3 + 0.997}{8} (i_1 - i_2)$$

$$= 3.3 - 0.997 = 2.303 \text{ A}$$

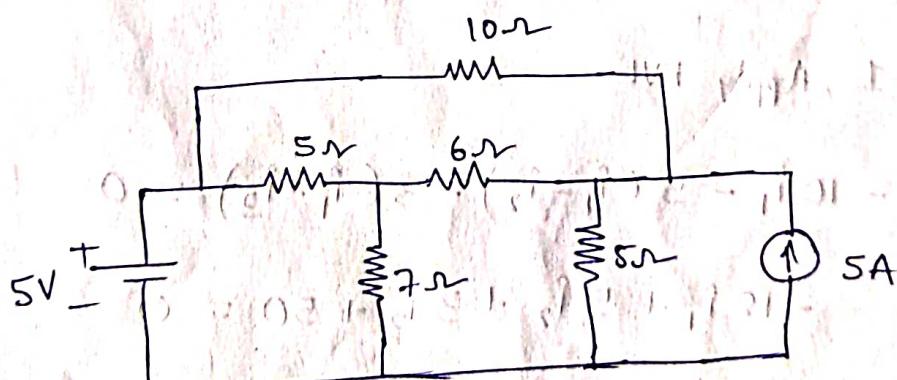
$$I_{2\Omega} = 0.997 \text{ A}$$

$$I_{1\Omega} = (i_2 - i_3) = 0.997 - 1.474 = -0.477 \text{ A}$$

$$I_{3\Omega} = (i_3 - i_1) = 1.474 - 3.3 = -1.825 \text{ A}$$

Prob ②

Find the voltage across 10Ω resistor in the network



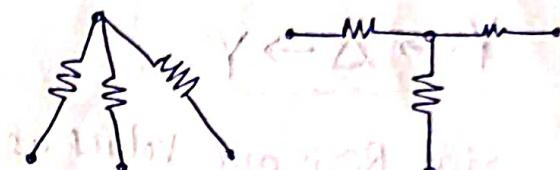
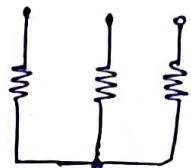
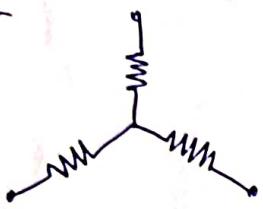
Ans: 9.138 V

Star to Delta & Delta to Star Transformation

Star connection (Δ)

If 3-resistors are connected in such a manner that one end of each resistors are connected at a junction point (star point), then thus 3-resistors are said to be in "star connection".

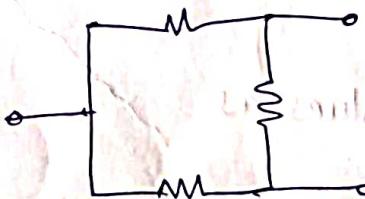
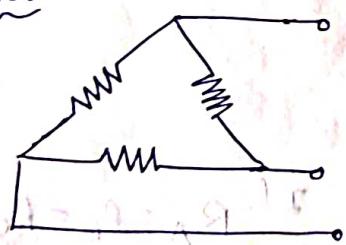
Ex:-



Delta Connection (Δ)

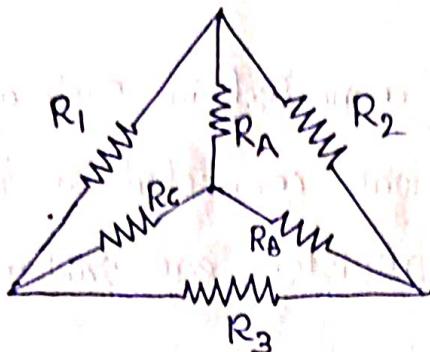
In 3-resistors, one end of the first resistor is connected to the first end of second, the second end of second to first end of third and so on to complete a loop then thus resistors are said to be connected in "Delta".

Ex:-



- These Δ & Δ connections are used in 3-phase systems & machines.
- These connection can increases the voltages (or current) in 3- ϕ systems.

Star -to-Delta ($\gamma \rightarrow \Delta$) Transformation



$\gamma \leftrightarrow \Delta \rightarrow \gamma$

Star Resistance values as

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} \quad \text{if } R_1 = R_2 = R_3 = R \text{ are same}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_Y = \frac{R_\Delta}{3}$$

$\Delta \rightarrow \gamma$

Delta Resistance values as

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

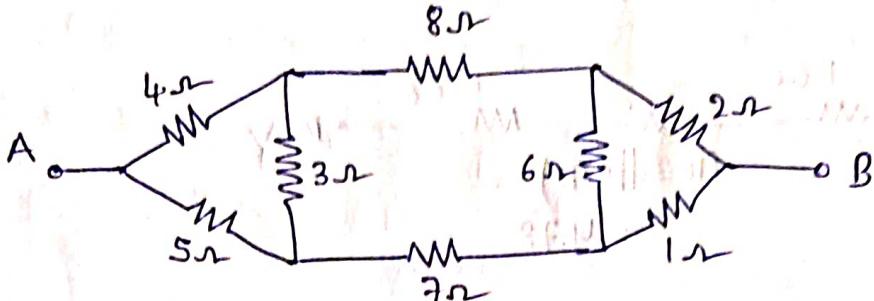
$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$\text{If } R_A = R_B = R_C = R \text{ then } R_\Delta = 3 R_Y$$

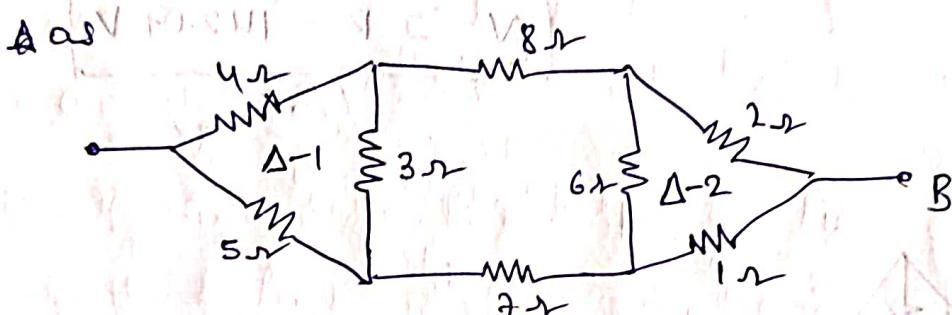
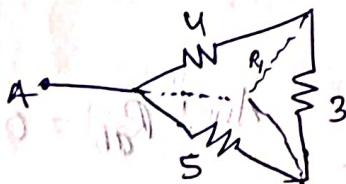
(20)

problem! -

PQ. Find the voltage to be applied across A-B, if drive current of 20A

Solution! -

In a given circuit $\Delta\text{-}\Delta\text{-}\Delta$ connections are present

 $\Delta-1$ 

$$R_1 = \frac{4 \times 3}{4+3+5} = \frac{12}{12} = 1 \Omega$$



$$R_1 = \frac{6 \times 2}{6+2+1} = \frac{12}{9} = 1.33 \Omega$$

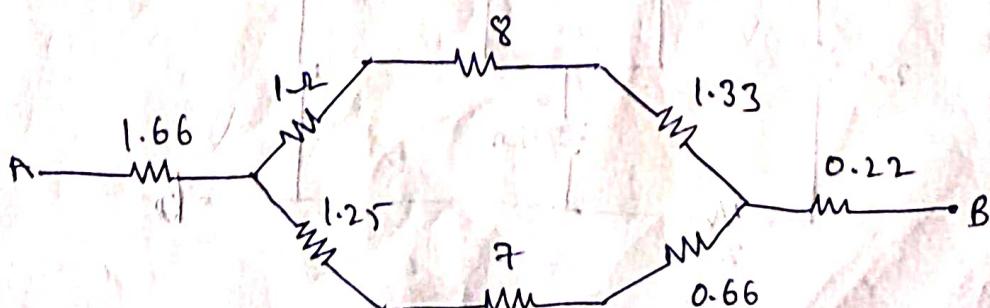
$$R_2 = \frac{3 \times 5}{3+5+4} = \frac{15}{12} = 1.25 \Omega$$

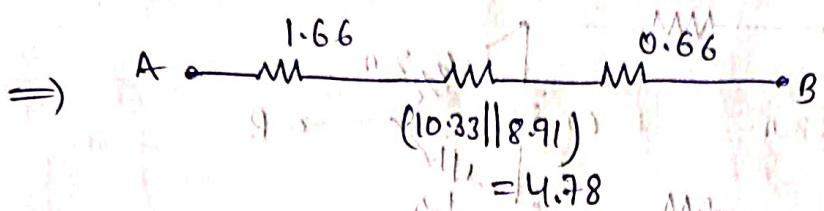
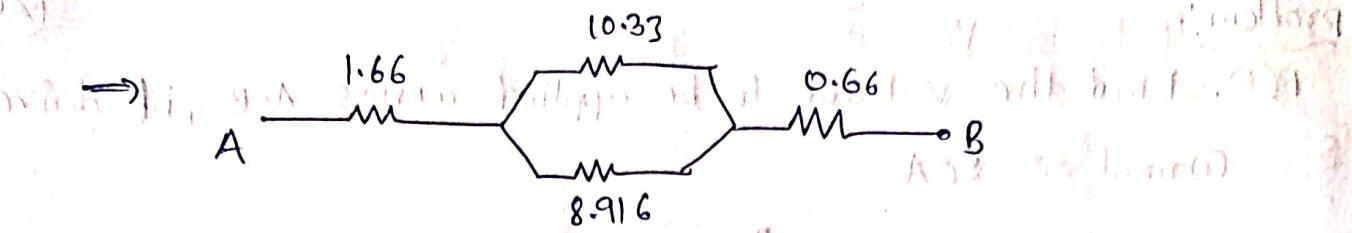
$$R_2 = \frac{2}{9} = 0.22 \Omega$$

$$R_3 = \frac{5 \times 4}{12} = 1.66 \Omega$$

$$R_3 = \frac{6}{9} = 0.66 \Omega$$

Circuit Redrawn as



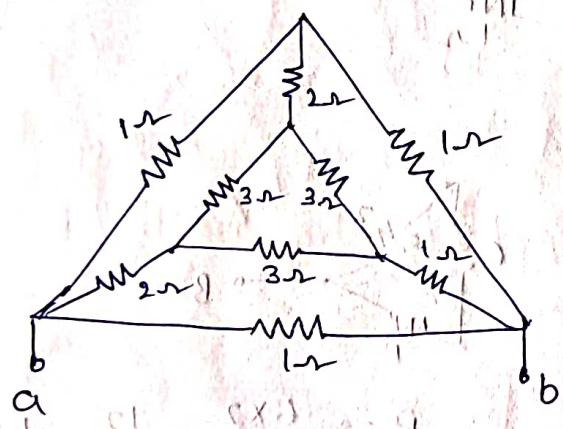


$$\Rightarrow \text{A} \xrightarrow{7.10\Omega} \text{B} \quad \Rightarrow \text{we have } I = 20\text{A}$$

$$R_{AB} = 7.10\Omega$$

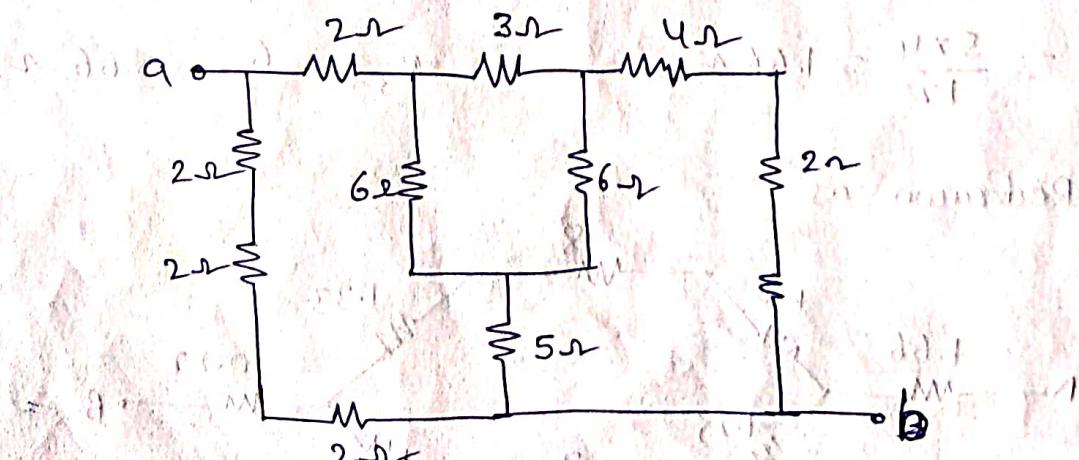
$$V = IR = 142.07\text{V}$$

P② Find R_{ab}



$$\text{Ans! } R_{ab} = 0.6\Omega$$

P③ Find R_{ab}



Network Theorems

The necessity of network theorems is to calculate single branch response in complicated network.

① Superposition theorem

② Thevenin's theorem

③ Norton's theorem

④ Maximum Power Transfer theorem

⑤ Reciprocity theorem

①. Superposition Theorem

Statement : In any linear, bilateral network consist of two (or) more sources & Resistors , the response (i.e V or I) at any element in network is equal to the algebraic sum of the responses caused by individual sources acting separately while other sources are replaced by their internal resistances at ideal conditions

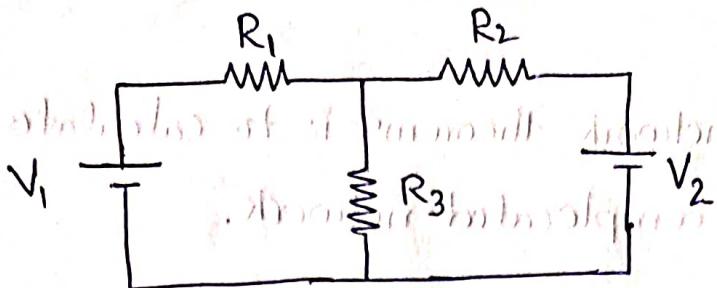
i.e Voltage source as \rightarrow short circuit

Current Source as \rightarrow Open circuit

Explanation :- consider a dc-circuit which is having two voltage sources V_1 & V_2 , and having 3-resistors

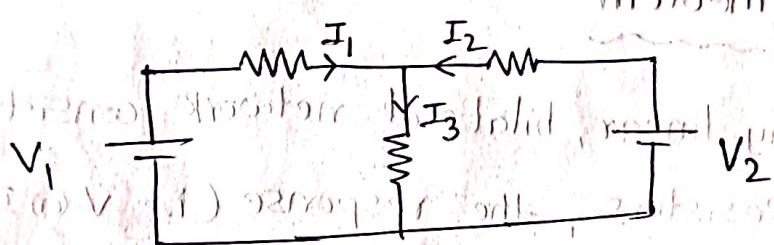
R_1 , R_2 & R_3 as shown below circuit .





for above circuit verify superposition theorem, take response at "R₃" - elements.

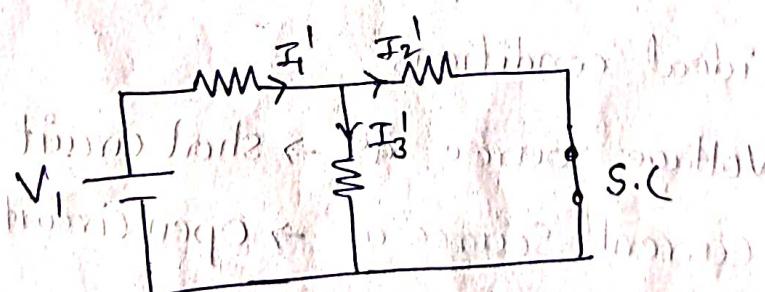
Step ① Let all sources as two sources are in Active Condition
i.e. V₁ & V₂ one in ON, then let respective currents as I₁, I₂ & I₃



$$I_3 = I_1 + I_2$$

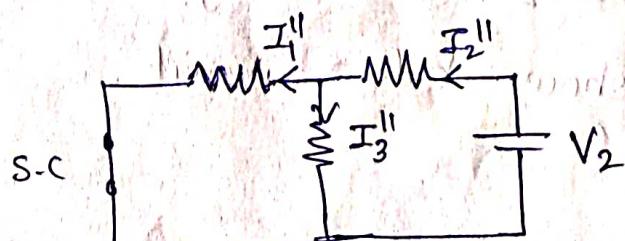
where I₃' is response at R₃ Resistor

Step ② Let Assume the V₁ = ON & V₂ = off state then



$$I_3' = I_1' - I_2'$$

Step ③ Assume the V₁ = off & V₂ = ON State



$$I_3'' = I_1'' + I_2''$$

According to Superposition theorem

$$I_3 = I_3' + I_3''$$

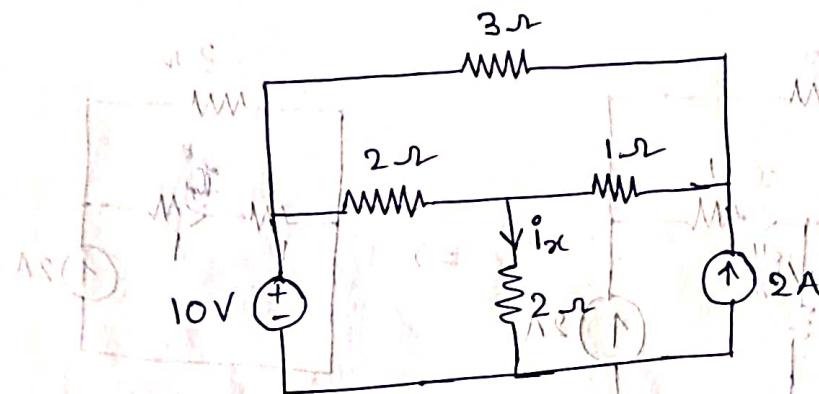
Similarly

$$I_1 = I_1' + I_1''$$

$$I_2 = I_2' + I_2''$$

Problem :-

Find ' i_x' for a given circuit using Superposition Theorem.



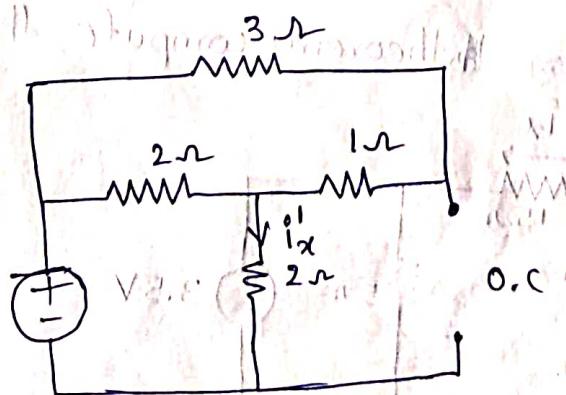
Solution:- In a given circuit we have 2-sources

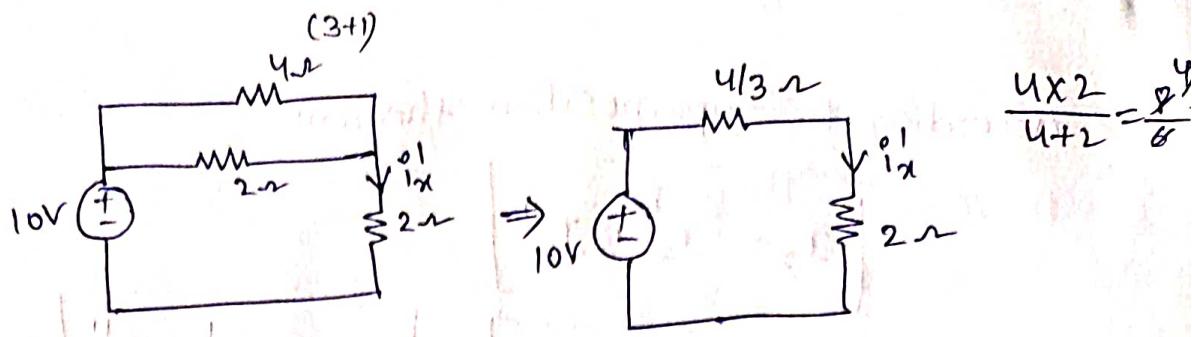
as 10V \rightarrow voltage source

& 2A \rightarrow current source

Let individual sources acting separately, then

Step ① 10V \rightarrow ON & 2A \rightarrow off , circuit drawn as

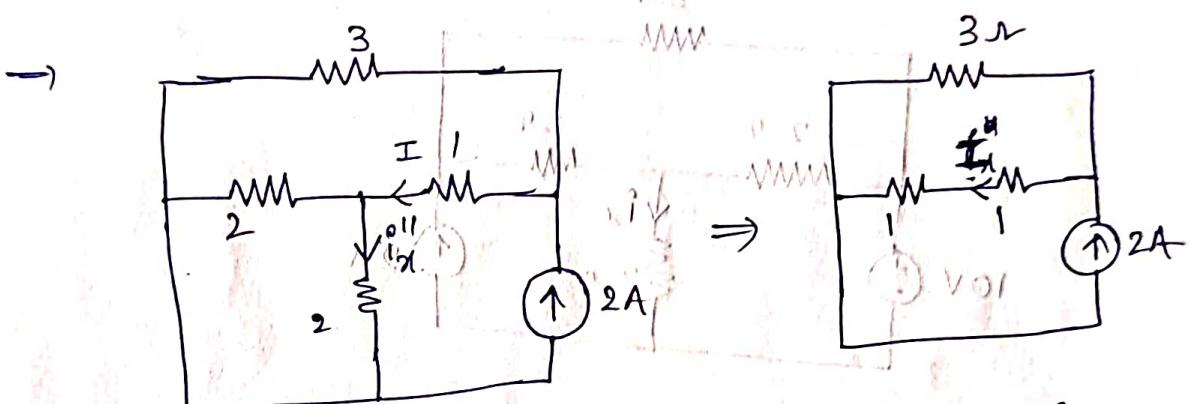




$$\frac{4 \times 2}{4+2} = \frac{8}{6}$$

Now $i_x^1 = \frac{10}{\frac{4}{3} + 2} = \frac{10}{\frac{10}{3}} = \frac{30}{10} \Rightarrow i_x^1 = 3A$

Step 2 Let 10V → OFF & 2A → ON, then



$$\frac{2 \times 2}{2+2} = \frac{4}{4}$$

$$I = \frac{2 \times 3}{2+3} = \frac{6}{5} = 1.2A$$

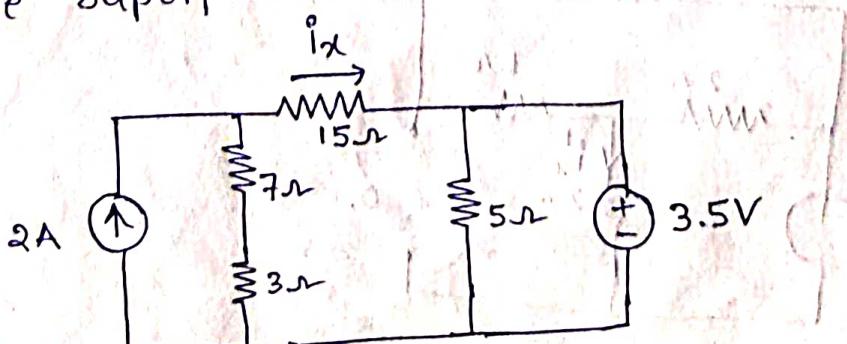
$$i_x^{11} = I \times \frac{2}{2+2} = 1.2 \times \frac{2}{2+2} = 0.6A$$

According to superposition theorem

$$i_x = i_x^1 + i_x^{11} \Rightarrow i_x = 3 + 0.6 \Rightarrow i_x = 3.6A$$

Problem:-

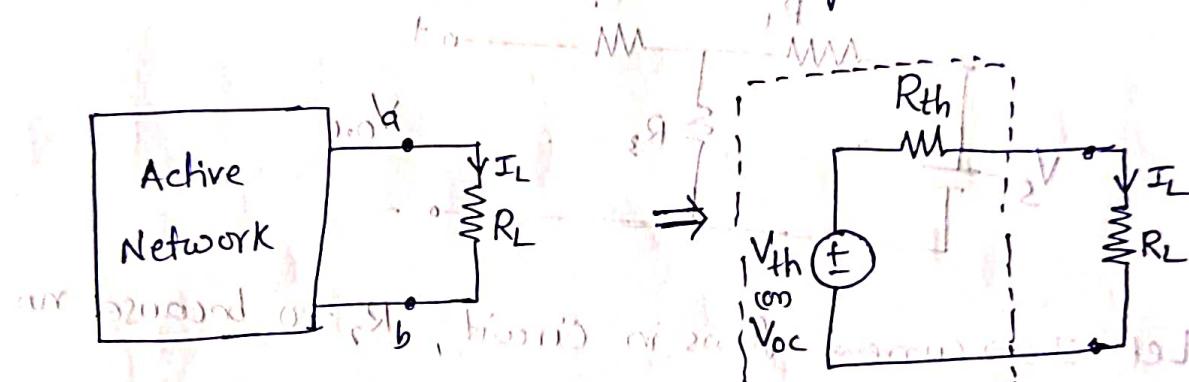
Using the superposition theorem compute the current "i_x"



Ans: 660 mA.

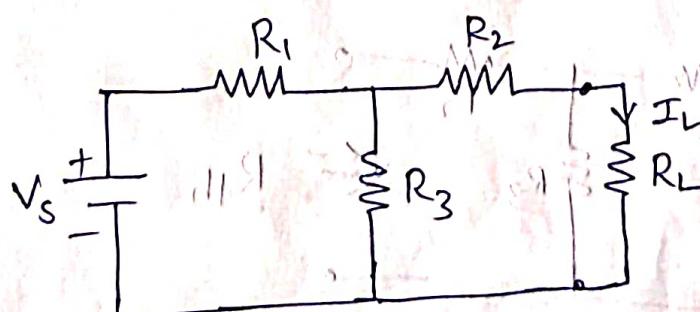
Thevenin's Theorem:

In any linear, bilateral network consist of more no. of sources and resistors can be replaced by an equivalent circuit of single voltage source in series with resistor, while all other sources are replaced by their internal resistances at ideal conditions as voltage source as short circuit (S.C) current source as open circuit



$$\therefore I_L = \frac{V_{th}}{R_{th} + R_L}$$

Explanation :- Consider a dc-circuit which is having Voltage Source (V_s), & resistors R_1, R_2 & R_3 are connected with a load Resistor ' R_L ' as shown below



Step find current in load ' R_L ' using thevenin's theorem

Step① To find ' I_L ' value, we have to find thevenin's equivalent circuit. To find equivalent circuit we have to find " V_{th} & R_{th} "

Step① To find ' V_{th} ', remove the load resistor ' R_L ' from the circuit, across open terminal find the voltage ($V_{o.c}$)

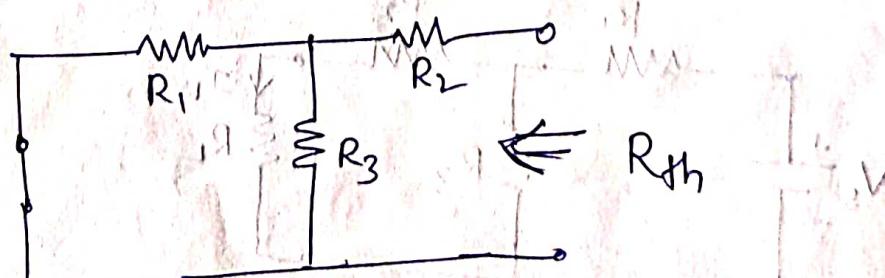
Then circuit modified as



Let $I \rightarrow$ current flows in circuit, $R_2 = 0$ because no current flows, then $V_{o.c} =$ voltage across R_3 resistor

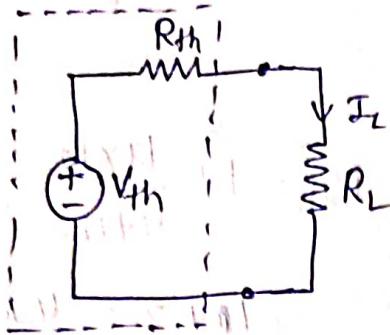
$$V_{o.c} = I R_3$$

Step② To find ' R_{th} ', voltage source is replaced by its internal resistance, as



$$\text{then } R_{th} = R_2 + (R_1 \parallel R_3)$$

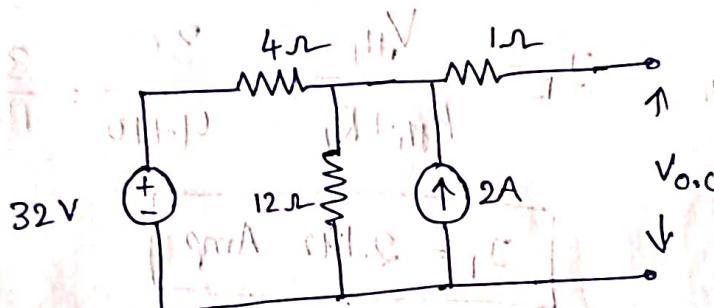
Step ③ The thevenins equivalent circuit drawn as



$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

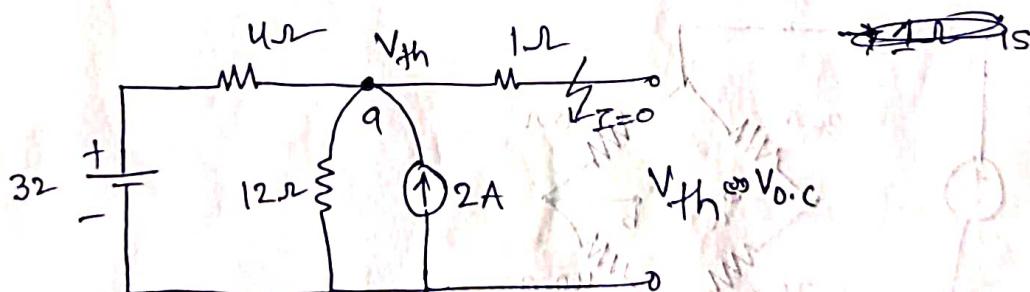
$$P_L = I_L^2 R_L$$

Problem :- ① Find current flowing in load resistor ' $R_L = 10\Omega$ ' using thevenin's theorem for a given circuit



Solution :-

Step 1: To find " $V_{oc} = V_{th}$ ", circuit redrawn as



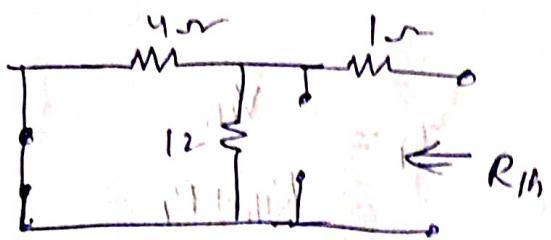
→ In 1Ω resistor $I_b = 0$, at node 'a' V_{th} will appear, then by using nodal analysis

$$\frac{V_{th} - 32}{4} + \frac{V_{th}}{12} - 2 = 0$$

$$\Rightarrow 3V_{th} - 96 + V_{th} - 24 = 0$$

$$V_{th} = \frac{120}{4} = \boxed{V_{th} = 30V}$$

Step ② :- To find ' R_{Th} ' all sources are replaced by their resistances

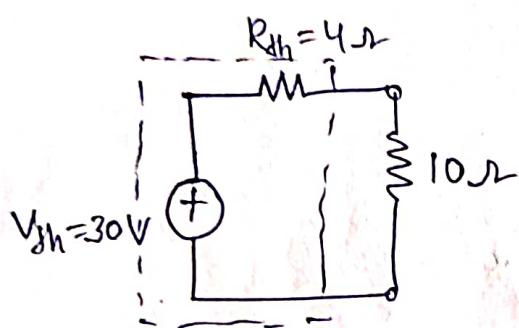


$$R_{Th} = 1 + \frac{4 \times 1}{4 + 12}$$

$$= 1 + 3 = 4\Omega$$

current flowing through load resistor can be found by

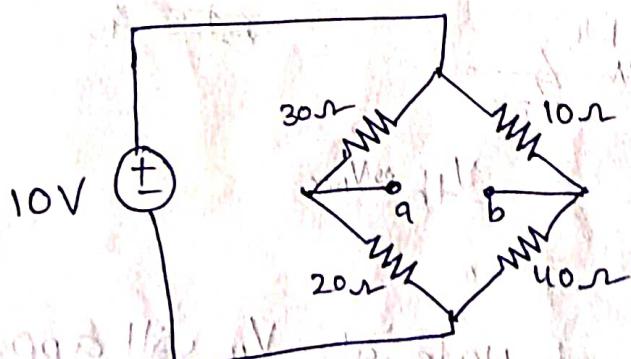
thevenin's equivalent circuit has



$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + 10} = \frac{30}{14} = 2.142 \text{ Amp}$$

$$I_L = 2.142 \text{ Amp}$$

Ques ② :- find thevenin's equivalent circuit for a given circuit



Ans! -

$$V_{Th} = -4V$$

$$R_{Th} = 20\Omega$$

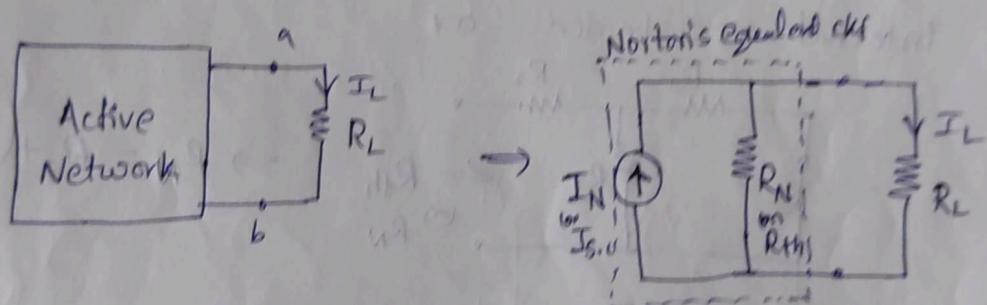
Norton's Theorem

(213)

In any linear, bilateral network consisted more no. of sources and resistors can be replaced by an equivalent circuit of single current source in parallel with resistor, while all other sources are replaced by their internal resistances at ideal conditions as

Voltage Source as short circuit (S.C.)

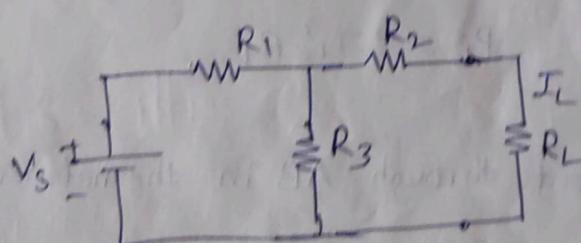
Voltage Source as open circuit (O.C.)



$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

Explanation :- consider a dc-circuit

which is having Voltage Source (V_S), resistors R_1 , R_2 & R_3 are connected with load resistor ' R_L ' as shown below



To find Norton's equivalent circuit, we have to find

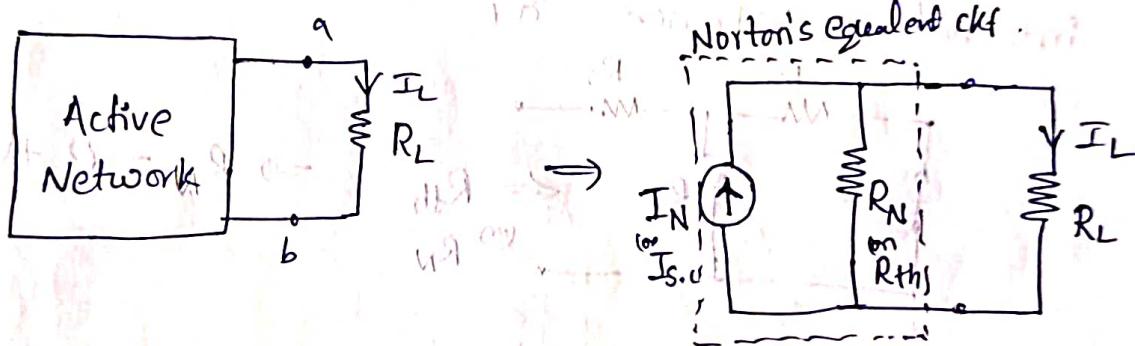
$$I_N \text{ & } R_N$$

Norton's Theorem

In any linear, bilateral network consist of more no. of sources and resistors can be replaced by an equivalent circuit of single current source in parallel with resistor, while all other sources are replaced by their internal resistances at ideal conditions as

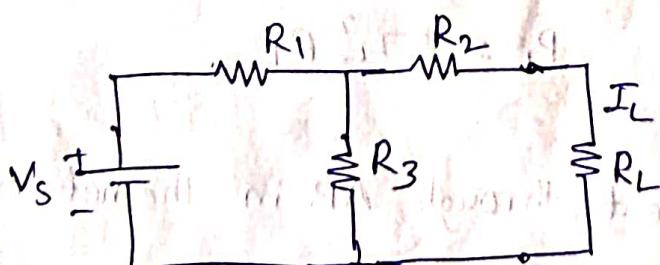
Voltage Source as short circuit (s.c)

Voltage Source as open circuit (o.c.)



Explanation :- Consider a dc-circuit

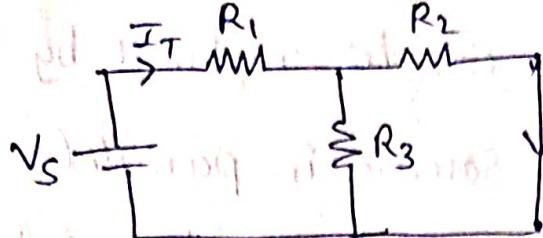
which is having Voltage Source (V_s), resistors R_1 , R_2 & R_3 are connected with load resistor ' R_L ' as shown below



To find Norton's equivalent circuit, we have to find

$$I_N \text{ & } R_N$$

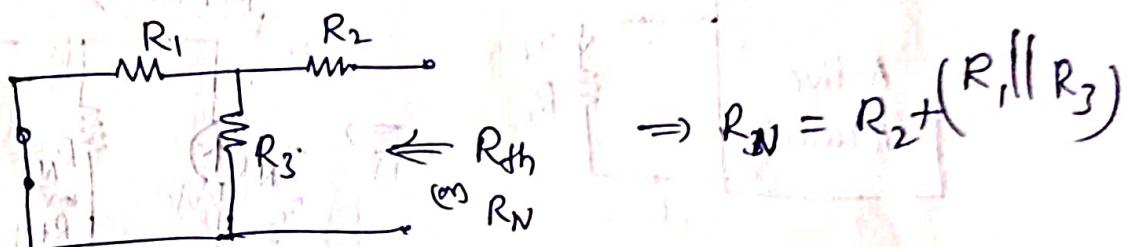
Step ① To find "IN" remove the ' R_L ' and short circuit the terminals, as norton's current I_N con $I_{S.C}$, then



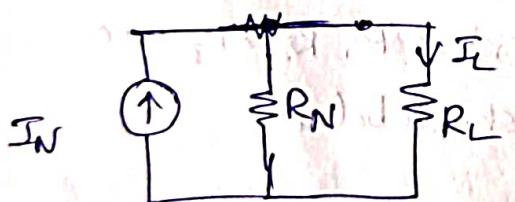
Here $I_{S.C}$ is current flowing R_2 i.e. $I_{S.C} = I_T \times \frac{R_3}{R_2 + R_3}$

I_T = Total current.

Step ② To find R_N or R_{th} , Voltage source is replaced by its internal resistance as



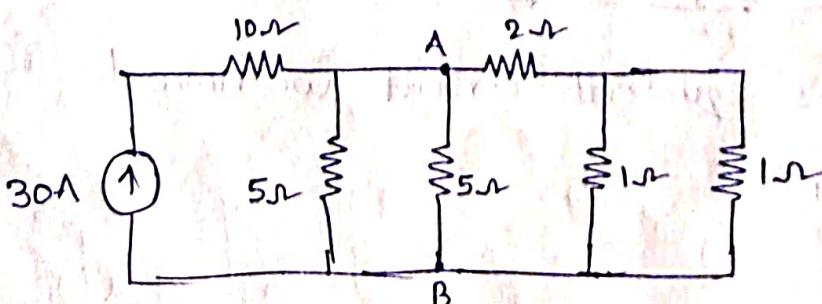
Step ③ The Norton's equivalent circuit as



$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

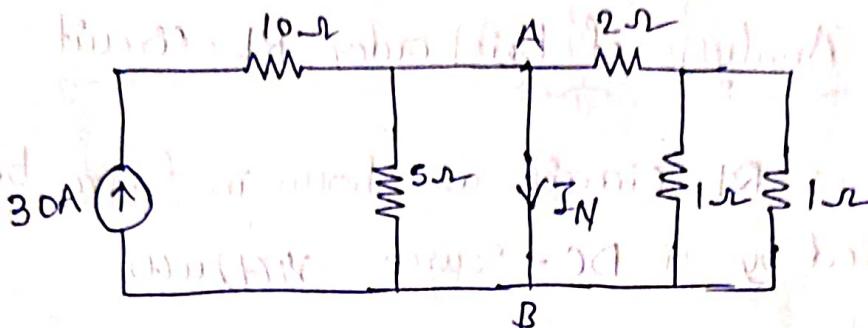
$$P_L = I_L^2 R_L$$

Prob:-
① Determine the current through AB in the network.



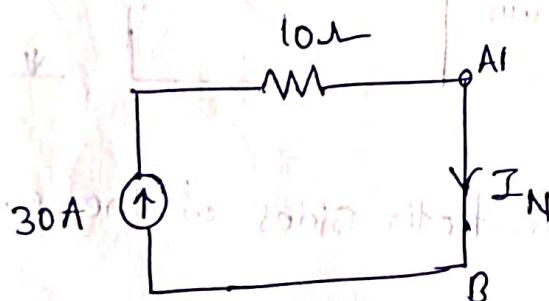
Solution:- In given circuit AB is 5Ω resistor is load Resistor $R_L = 5\Omega$

Step 1 To find I_N ; the AB branch is short circuited, then
Circuit redrawn as



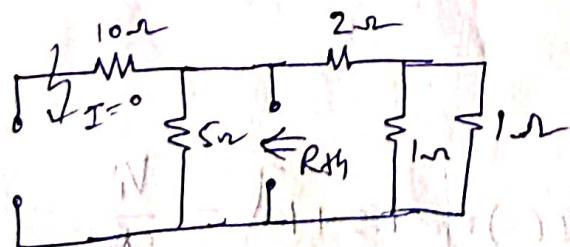
When AB-branch circuited, then all currents passes through AB branch, then no-current in 2Ω , 1Ω 's & 5Ω resistor

As



$$\text{since } I_N = 30 \text{ A}$$

Step 2 To find R_N , AB branch is removed then

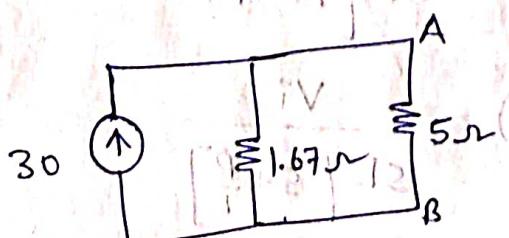


so

$$I_N = \left\{ (1 \parallel 1) + 2 \right\} \parallel 5$$

$$= \frac{2.5 \times 5}{2.5 + 5} = 1.67 \Omega$$

Step 3 Norton equivalent circuit



$$I_L = 30 \times \frac{1.67}{1.67 + 5}$$

$$I_L = 7.5 \text{ A}$$

Time-domain analysis of first order RL & RC circuits

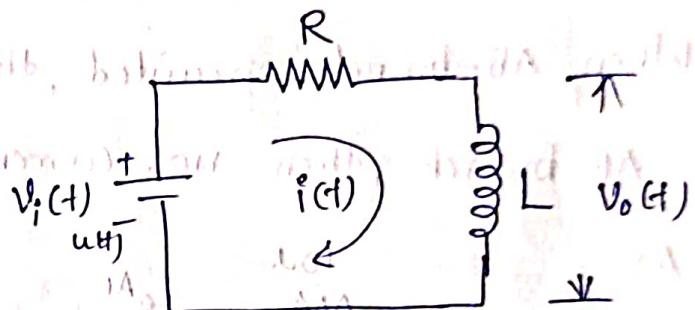
Time domain analysis (or) Time response analysis is nothing but the responses of circuits analysed in time domain.

Time Response Analysis of First order RL-circuit

Consider a series RL circuit as shown in figure below, which is excited by a DC - Source $v_i(t) u(t)$

Applying KVL to the

for given circuit



$$L \frac{di(t)}{dt} + R i(t) = v_i(t) u(t)$$

Taking Laplace transform on both sides of the equation

$$L \{s I(s) - i(0)\} + R I(s) = \frac{V_i}{s}$$

Initial $i(0)=0$, then

$$L s I(s) + R I(s) = \frac{V_i}{s}$$

$$I(s) = \frac{V_i}{s L [s + R]} \Rightarrow I(s) [Ls + R] = \frac{V_i}{s}$$

$$I(s) = \frac{V_i}{s [Ls + R]}$$

$$I(s) = \frac{V_i}{s L [s + \frac{R}{L}]}$$

using partial fraction for above equation

$$I(s) = \frac{A_1}{s} + \frac{A_2}{s + \frac{R}{L}}$$

where $A_1 = \left. s I(s) \right|_{s=0} = \frac{V_i}{L} \times \left. \frac{1}{s + \frac{R}{L}} \right|_{s=0} \Rightarrow A_1 = \frac{V_i}{R}$

$$A_2 = \left. \left(s + \frac{R}{L} \right) I(s) \right|_{s=-\frac{R}{L}} = \left. \frac{V_i}{L} \times \frac{1}{s} \right|_{s=-\frac{R}{L}} = A_2 = -\frac{V_i}{R}$$

Therefore $I(s) = \frac{V_i/R}{s} + \frac{-V_i/R}{s + R/L}$

$$I(s) = \frac{V_i}{R} \left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right]$$

Taking inverse Laplace transform, we get

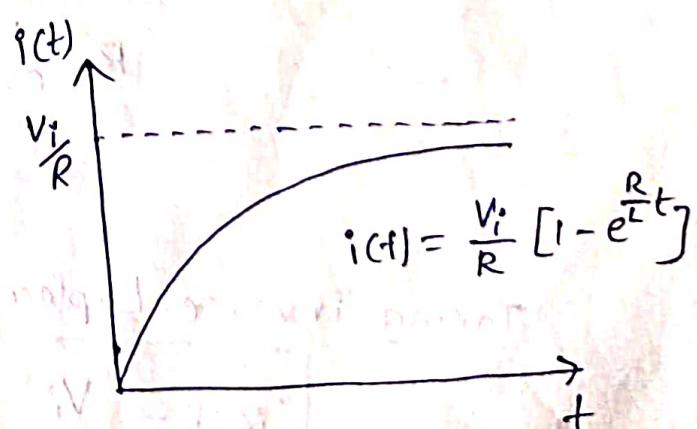
$$i(t) = \frac{V_i}{R} \left[1 - e^{-\frac{R}{L}t} \right]$$

By this equation time ~~equation~~ response of system is

for RL-circuit

Time constant value is

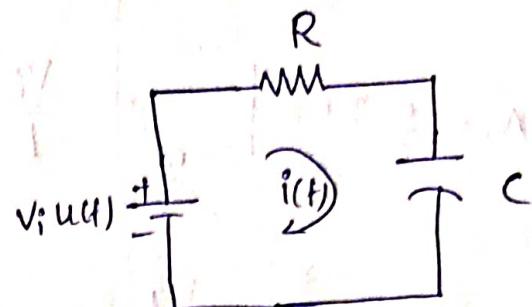
$$\boxed{\tau = \frac{R}{L}} \quad \text{Secans}$$



Time Response Analysis of First Order RC-Circuit

consider a series RC-circuit as shown figure, excited by DC-source $V_i(t)$.

using KVL to the circuit



$$\frac{1}{C} \int_{-\infty}^t i(t) dt + R i(t) = V_i(t)$$

$$\text{i.e. } R i(t) + \frac{1}{C} \int_{-\infty}^0 i(t) dt + \frac{1}{C} \int_0^t i(t) dt = V_i(t)$$

Taking Laplace transform on both sides.

$$R I(s) + \frac{1}{C} \left[\frac{I(s)}{s} + \frac{q(0^+)}{s} \right] = \frac{V_i}{s}$$

$$\because q(0^+) = 0$$

C is uncharged

Therefore

$$R I(s) + \frac{1}{C s} I(s) = \frac{V_i}{s}$$

$$I(s) \cdot \left[R + \frac{1}{C s} \right] = \frac{V_i}{s}$$

$$I(s) = \frac{\frac{V_i}{s}}{R + \frac{1}{C s}} \Rightarrow I(s) = \frac{\frac{V_i}{s}}{R \left[s + \frac{1}{R C} \right]}$$

$$I(s) = \frac{V_i}{R \left[s + \frac{1}{R C} \right]}$$

Taking inverse Laplace Transform

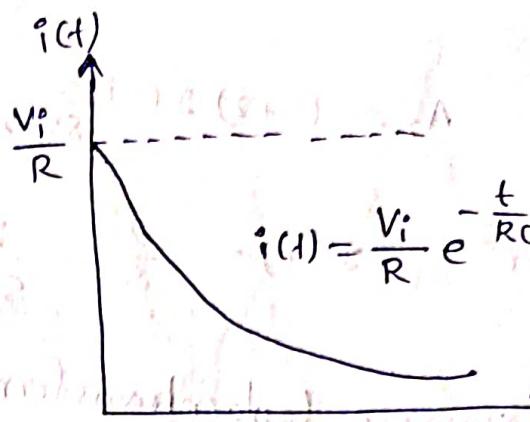
$$i(t) = \frac{V_i}{R} e^{-\frac{t}{R C}}$$

The time response of the first order series RC circuit is

Time constant

$$\tau = \frac{1}{RC}$$

sec.



problem!-

Derive the expression for the current in the circuit as shown in figure

Solution!-

Applying KVL to the given

$$15 \frac{di(t)}{dt} + 30i(t) = 60$$

$$\frac{d i(t)}{dt} + 2i(t) = 4$$

Taking Laplace transform on both sides, we get

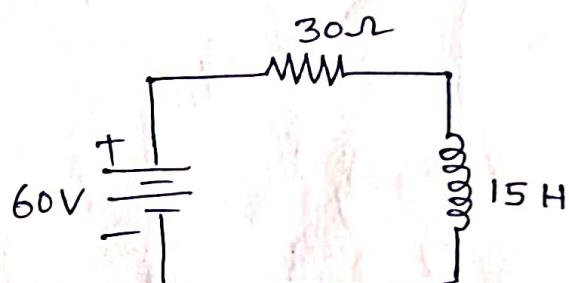
$$\{s I(s) + -i(0^-)\} + 2 I(s) = \frac{4}{s}$$

since $i(0^-) = 0$, we get

$$I(s) = \frac{4}{s(s+2)}$$

using partial fraction

$$I(s) = \frac{4}{s(s+2)} = \frac{A_1}{s} + \frac{A_2}{s+2}$$



where $A_1 = s I(s) \Big|_{s=0} = \frac{4}{s+2} \Big|_{s=0} = 2$

$$A_2 = (s+2) I(s) \Big|_{s=-2} = \frac{4}{s} \Big|_{s=-2} = -2$$

Therefore $I(s) = \frac{2}{s} - \frac{2}{s+2}$

Taking Inverse Laplace transform, we get

$$i(t) = 2(1 - e^{-2t}) A$$