Student Information

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Answer 1

a)

- Since both T_A and T_B are uniformly distributed, their probability density functions are $f_A = f_B = \frac{1}{100}$ with $b = 100$ and $a = 0$. Since they are independent, the joint density function is $f(t_A, t_B) = f_A \cdot f_B = \frac{1}{10,000}$.

- The joint cumulative distribution function is $F(t_A, t_B) = \iint \frac{dx \cdot dy}{10,000} = \int \frac{x \cdot dy}{10,000} = \frac{x \cdot y}{10,000}$.

b)

Let's draw a 100×100 square to illustrate the probabilites. Since the probability density function is a constant function, the simple area of a region over ten thousand would give us the probability of an event being inside the region.
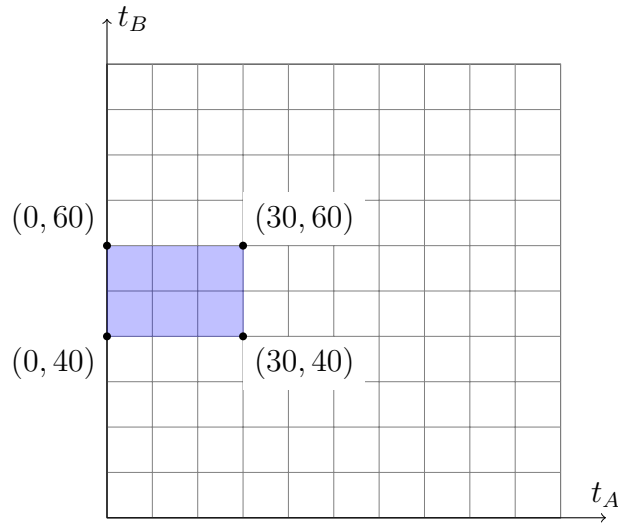
The blue region depicts the probability $P\{T_A < 30 \cap 40 < T_B < 60\}$. The probability is equal to the volume of the space underneath it, where the space is bounded by the probability density function f from the above. Since f is constant, the volume simply equals to $30 \cdot 20 \cdot \frac{1}{10000} = \frac{600}{10000} = \frac{3}{50} = 0.06$.
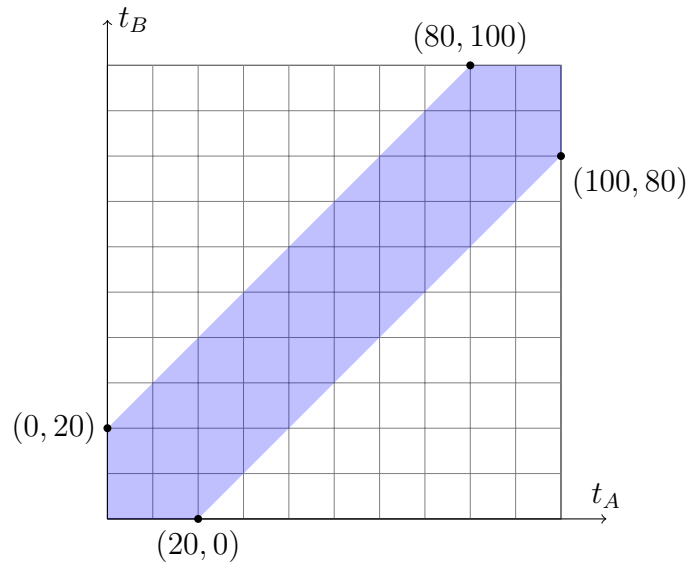
c)

For this question, we must calculate the area of the surface of square with the constraint $t_A < t_B + 10$.
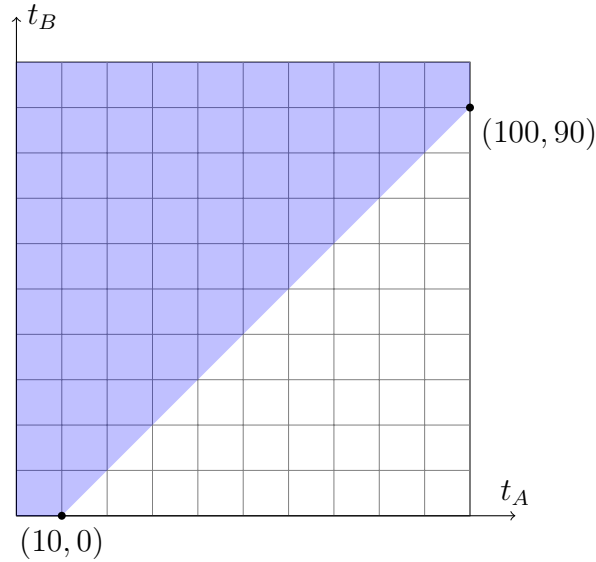
The area of the surface is $100 \cdot 100 - 90 \cdot 90 \cdot \frac{1}{2} = 5950$. The volume of the space is $5950 \cdot \frac{1}{10000} = \frac{5950}{10000} = \frac{119}{200} = 0.595$.

d)

For this question, the inequality $|t_A - t_B| < 20$ must be satisfied. This equals to $-20 < t_A - t_B < 20$ which means $t_B - 20 < t_A < t_B + 20$.

The area of the surface is $100 \cdot 100 - 2 \cdot 80 \cdot 80 \cdot \frac{1}{2} = 3600$. The volume of the space is $3600 \cdot \frac{1}{10000} = \frac{3600}{10000} = \frac{9}{25} = 0.36$.

Answer 2

a)

b)

Answer 3

Answer 4

a)

b)

c)