Student Information

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Answer

a)

The Octave code is as follows,

```
% Load statistics module for inverse standard normal distribution
pkg load statistics;

% Define the constants
poissLambdaBulk = 50;
poissLambdaContainer = 40;
poissLambdaOil = 25;

alphaBulk = 60;
alphaContainer = 100;
alphaOil = 120;

gamLambdaBulk = 0.1;
gamLambdaContainer = 0.05;
gamLambdaOil = 0.02;

weightThreshold = 300000;

alphaProbability = 1 - 0.98;
epsilonDifference = 0.03;
% Calculate Monte Carlo study size
z_alpha = norminv(1 - alphaProbability/2);
sizeMonteCarlo = ceil(0.25 * (z_alpha/epsilonDifference)^2);

% Function to generate poisson and gamma random variables and calculate total
   cargo
function result = generateTotalSize(poissLambda, alpha, gamLambda)
  U = rand;
  i = 0;
  F = exp(-poissLambda);
  while (F < U);
    i += 1;
    F += exp(-poissLambda) * poissLambda^i / gamma(i+1);
  endwhile
  result = 0;
  for j = 1:i;
    result += sum(-1/gamLambda * log(rand(alpha, 1)));
  endfor
endfunction
```

```
% Number of times total cargo exceeded threshold
cargoExceeds = 0;
% Vector of total cargo sizes
sums = [];

for i = 1:sizeMonteCarlo;
  sumBulk = generateTotalSize(poissLambdaBulk, alphaBulk, gamLambdaBulk);
  sumContainer = generateTotalSize(poissLambdaContainer, alphaContainer,
   gamLambdaContainer);
  sumOil = generateTotalSize(poissLambdaOil, alphaOil, gamLambdaOil);
  sumAll = sumBulk + sumContainer + sumOil;
  sums = [sums, sumAll]; % Append to sums vector
  cargoExceeds += (sumAll > weightThreshold);
endfor

% Probability that the total cargo exceeded the threshold
probabilityExceeds = cargoExceeds/sizeMonteCarlo;
% X is average daily total cargo
X = mean(sums);
% Standard deviation of X
stdDevX = std(sums);

fprintf("Estimated probability: %.3f\nExpected weight: %.2f\nStandard deviation
   : %.2f\n", probabilityExceeds, X, stdDevX);
```

with a screenshot of some outputs,

```
Estimated probability: 0.118
Expected weight: 260019.30
Standard deviation: 32740.17
```

In this part, we calculated the size of the Monte Carlo study by using the $N \geq 0.25(\frac{z_{\alpha/2}}{\varepsilon})^2$ formula. We did not have an estimate for the probability p, so we used the highest possible value of $p \cdot (1-p)$ which is 0.25. We have the size of the Monte Carlo study as 1504. Then, for each type of ship, we generate the Poisson random variable to determine the number of ships arriving. For each ship, we generate the gamma-distributed random variable to determine its cargo weight. We calculate the sum for each type of ship, then we calculate the total amount of cargo weight in a day by summing three different type of ships. Then, we determine if the total cargo weight passes the threshold. We repeat this process for 1504 times, the size of our Monte Carlo study. Our estimated probability of a day exceeding the cargo weight threshold is around 11%.

b)

Our estimate for the average amount of total cargo weight that arrives to the port every day is around 260 thousand tons. We calculated this value by taking the average of total cargo weight of every iteration.

c)

Our estimate for the standard deviation of the average total cargo weight is around 32 thousand tons. We calculated this value by taking the standard deviation of the sample generated by Monte Carlo simulation.