Student Information

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Answer 1

a)

Since the sample is small and the standard deviation of the population is unknown, Student's t distribution can be used. The critical value of t distribution is $t_{\alpha/2} = t_{0.01} = 2.602$ with $n-1 = 15$ degrees of freedom.

$$\bar{X} = \frac{8.4 + 7.8 + 6.4 + 6.7 + 6.6 + 6.6 + 7.2 + 4.1 + 5.4 + 6.9 + 7.0 + 6.9 + 7.4 + 6.5 + 6.5 + 8.5}{16}$$
$$= 6.806$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$
$$= \sqrt{\frac{\sum_{i=1}^{16}(X_i - 6.806)^2}{15}}$$
$$= 1.055$$

The 98% confidence interval $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ becomes $6.806 \pm 2.602 \cdot \frac{1.055}{4} = [6.120, 7.493]$.

b)

Let μ_1 be the initial gasoline consumption per 100 kilometers and μ_2 be the improved gasoline consumption per 100 kilometers. Therefore, the null hypothesis H_0 is $\mu_1 = \mu_2$ and the alternative hypothesis H_A is $\mu_1 > \mu_2$. Since we don't know the population standard deviation, we can use T-statistic.

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$
$$= \frac{6.806 - 7.5}{1.055/4}$$
$$= -2.629$$

The rejection region is $R = (-\infty, -t_{\alpha}] = (-\infty, -t_{0.05}] = (-\infty, -1.753]$ with 15 degrees of freedom, since we are using a left-tail alternative. Since $t \in R$, we can reject the null hypothesis. There is sufficient evidence that the improvement was effective.

c)

Since $\bar{X} = 6.806 > 6.5$, we can immediately accept the null hypothesis because the T-statistic is positive and the rejection region includes zero.

Answer 2

a)

b)

c)

d)

Answer 3

Answer 4