Student Information

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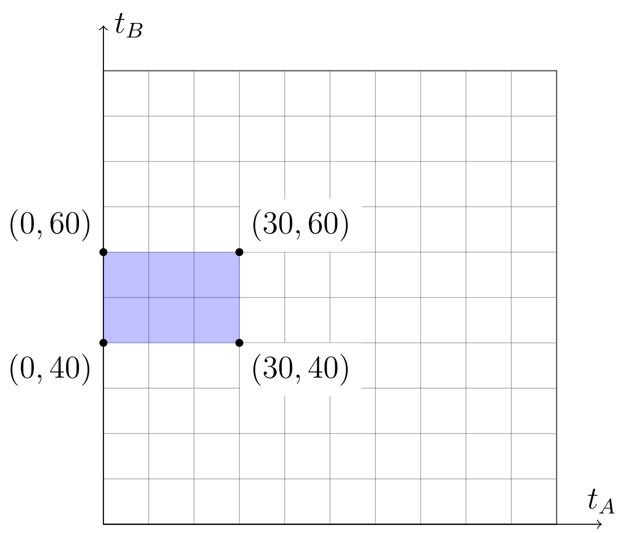
Answer 1

a)

- Since both T_A and T_B are uniformly distributed, their probability density functions are $f_A = f_B = \frac{1}{100}$ with $b = 100$ and $a = 0$. Since they are independent, the joint density function is $f(t_A, t_B) = f_A \cdot f_B = \frac{1}{10{,}000}$.
- The joint cumulative distribution function is $F(t_A, t_B) = \iint \frac{dx \cdot dy}{10{,}000} = \int \frac{x \cdot dy}{10{,}000} = \frac{x \cdot y}{10{,}000}$.

b)

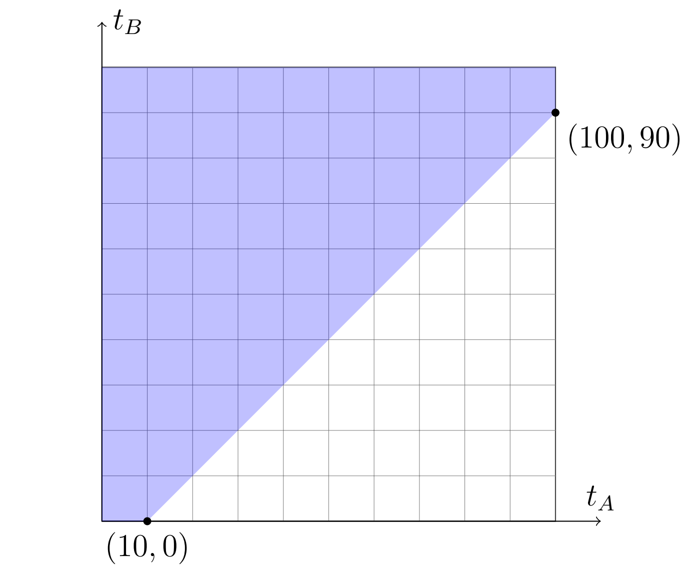
Let's draw a 100×100 square to illustrate the probabilites. Since the probability density function is a constant function, the simple area of a region over ten thousand would give us the probability of an event being inside the region.



The blue region depicts the probability $P\{T_A < 30 \cap 40 < T_B < 60\}$. The probability is equal to the volume of the space underneath it, where the space is bounded by the probability density function f from the above. Since f is constant, the volume simply equals to $30 \cdot 20 \cdot \frac{1}{10000} = \frac{600}{10000} = \frac{3}{50} = 0.06$.

c)

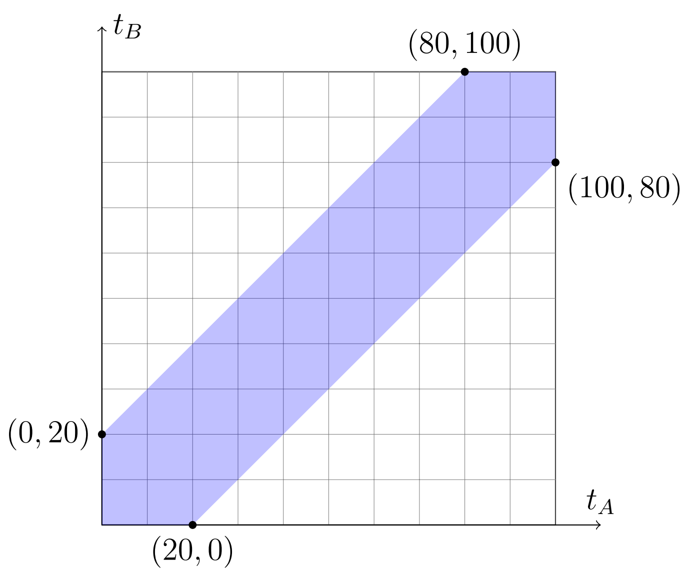
For this question, we must calculate the area of the surface of square with the constraint $t_A < t_B + 10$.



The area of the surface is $100 \cdot 100 - 90 \cdot 90 \cdot \frac{1}{2} = 5950$. The volume of the space is $5950 \cdot \frac{1}{10000} = \frac{5950}{10000} = \frac{119}{200} = 0.595$.

d)

For this question, the inequality $|t_A - t_B| < 20$ must be satisfied. This equals to $-20 < t_A - t_B < 20$ which means $t_B - 20 < t_A < t_B + 20$.



The area of the surface is $100 \cdot 100 - 2 \cdot 80 \cdot 80 \cdot \frac{1}{2} = 3600$. The volume of the space is $3600 \cdot \frac{1}{10000} = \frac{3600}{10000} = \frac{9}{25} = 0.36$.

Answer 2

a)

This probability is binomially distributed with parameters $n = 150$, because 150 people are selected from the population, and $p = 0.6$, because frequent shoppers are 60% of the population. Since n is large, normal approximation to binomial distribution with parameters $\mu = n \cdot p$ and $\sigma = \sqrt{n \cdot p \cdot (1 - p)}$ can be used. In that case, $\mu = 90$ and $\sigma = 6$. In order for the 65% of the customers to be frequent shoppers, at least 97.5 of them should be frequent shoppers.

$$\begin{aligned}
\boldsymbol{P}\{X > 97.5\} &= \boldsymbol{P}\left\{\frac{X - 90}{6} > \frac{97.5 - 90}{6}\right\} \\
&= \boldsymbol{P}\left\{\frac{X - 90}{6} > \frac{7.5}{6}\right\} \\
&= \boldsymbol{P}\left\{\frac{X - 90}{6} > 1.25\right\} \\
&= 1 - \boldsymbol{P}\left\{\frac{X - 90}{6} < 1.25\right\} \\
&= 1 - \Phi(1.25) \\
&= \Phi(-1.25) \\
&= 0.1056
\end{aligned}$$

b)

Similarly, this probability is binomially distributed with parameters $n = 150$, and $p = 0.1$. Normal approximation has $\mu = 15$ and $\sigma = \sqrt{13.5} = 3.6742$ as parameters. At most 22.5 of customers must be rare shoppers.

$$\begin{aligned}
\boldsymbol{P}\{X > 22.5\} &= \boldsymbol{P}\left\{\frac{X - 15}{\sqrt{13.5}} > \frac{22.5 - 15}{\sqrt{13.5}}\right\} \\
&= \boldsymbol{P}\left\{\frac{X - 15}{\sqrt{13.5}} > \frac{7.5}{\sqrt{13.5}}\right\} \\
&= \boldsymbol{P}\left\{\frac{X - 15}{\sqrt{13.5}} > 2.0412\right\} \\
&= 1 - \boldsymbol{P}\left\{\frac{X - 15}{\sqrt{13.5}} < 2.0412\right\} \\
&= 1 - \Phi(2.0412) \\
&= \Phi(-2.0412) \\
&= 0.0206
\end{aligned}$$

Answer 3

Answer 4

a)

b)

c)