

# Student Information

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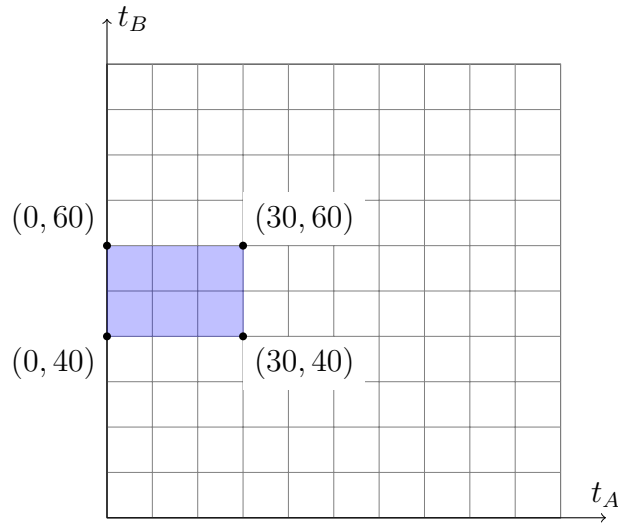
## Answer 1

a)

- Since both  $T_A$  and  $T_B$  are uniformly distributed, their probability density functions are  $f_A = f_B = \frac{1}{100}$  with  $b = 100$  and  $a = 0$ . Since they are independent, the joint density function is  $f(t_A, t_B) = f_A \cdot f_B = \frac{1}{10,000}$ .
- The joint cumulative distribution function is  $F(t_A, t_B) = \iint \frac{dx \cdot dy}{10,000} = \int \frac{x \cdot dy}{10,000} = \frac{x \cdot y}{10,000}$ .

b)

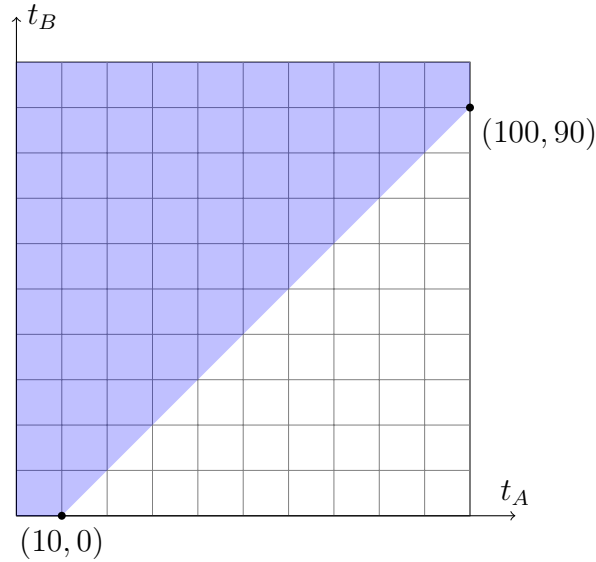
Let's draw a  $100 \times 100$  square to illustrate the probabilities. Since the probability density function is a constant function, the simple area of a region over ten thousand would give us the probability of an event being inside the region.



The blue region depicts the probability  $P\{T_A < 30 \cap 40 < T_B < 60\}$ . The probability is equal to the volume of the space underneath it, where the space is bounded by the probability density function  $f$  from the above. Since  $f$  is constant, the volume simply equals to  $30 \cdot 20 \cdot \frac{1}{10000} = \frac{600}{10000} = \frac{3}{50} = 0.06$ .

c)

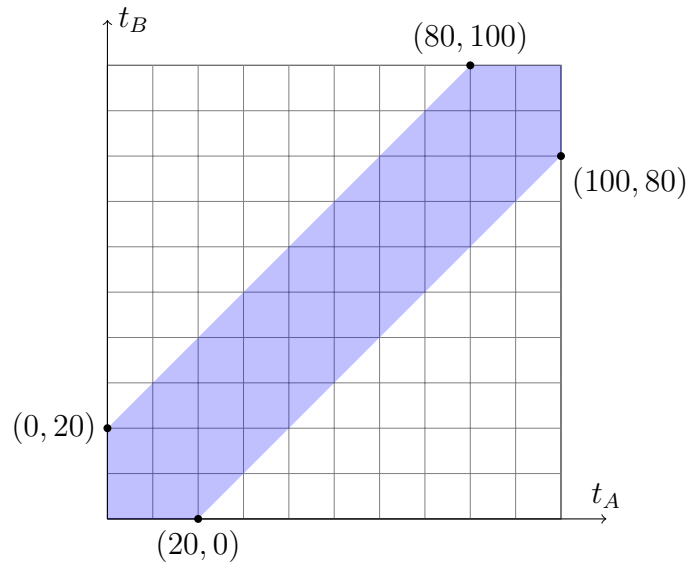
For this question, we must calculate the area of the surface of square with the constraint  $t_A < t_B + 10$ .



The area of the surface is  $100 \cdot 100 - 90 \cdot 90 \cdot \frac{1}{2} = 5950$ . The volume of the space is  $5950 \cdot \frac{1}{10000} = \frac{5950}{10000} = \frac{119}{200} = 0.595$ .

d)

For this question, the inequality  $|t_A - t_B| < 20$  must be satisfied. This equals to  $-20 < t_A - t_B < 20$  which means  $t_B - 20 < t_A < t_B + 20$ .



The area of the surface is  $100 \cdot 100 - 2 \cdot 80 \cdot 80 \cdot \frac{1}{2} = 3600$ . The volume of the space is  $3600 \cdot \frac{1}{10000} = \frac{3600}{10000} = \frac{9}{25} = 0.36$ .

## Answer 2

a)

This probability is binomially distributed with parameters  $n = 150$ , because 150 people are selected from the population, and  $p = 0.6$ , because frequent shoppers are 60% of the population. Since  $n$  is large, normal approximation to binomial distribution with parameters  $\mu = n \cdot p$  and  $\sigma = \sqrt{n \cdot p \cdot (1 - p)}$  can be used. In that case,  $\mu = 90$  and  $\sigma = 6$ . In order for the 65% of the customers to be frequent shoppers, at least 97.5 of them should be frequent shoppers.

$$\begin{aligned} P\{X > 97.5\} &= P\left\{\frac{X - 90}{6} > \frac{97.5 - 90}{6}\right\} \\ &= \end{aligned}$$

b)

## Answer 3

## Answer 4

a)

b)

c)