Student Information

Full Name : Murat Bolu
Id Number : 2521300

Answer 1

a)

The expected value $E(X)$ for random variables with finitely many outcomes is equal to $\sum_x xP(x)$. Therefore, assuming all dice are fair, i.e. all sides have the same probability of occurring,

$$E(\text{Blue die}) = 1 \cdot \tfrac{1}{6} + 2 \cdot \tfrac{1}{6} + 3 \cdot \tfrac{1}{6} + 4 \cdot \tfrac{1}{6} + 5 \cdot \tfrac{1}{6} + 6 \cdot \tfrac{1}{6} = 3.5$$

$$E(\text{Yellow die}) = 1 \cdot \tfrac{1}{8} + 1 \cdot \tfrac{1}{8} + 1 \cdot \tfrac{1}{8} + 3 \cdot \tfrac{1}{8} + 3 \cdot \tfrac{1}{8} + 3 \cdot \tfrac{1}{8} + 4 \cdot \tfrac{1}{8} + 8 \cdot \tfrac{1}{8} = 3$$

$$E(\text{Red die}) = 2 \cdot \tfrac{1}{10} + 2 \cdot \tfrac{1}{10} + 2 \cdot \tfrac{1}{10} + 2 \cdot \tfrac{1}{10} + 2 \cdot \tfrac{1}{10} + 3 \cdot \tfrac{1}{10} + 3 \cdot \tfrac{1}{10} + 4 \cdot \tfrac{1}{10} + 4 \cdot \tfrac{1}{10} + 6 \cdot \tfrac{1}{10} = 3$$

b)

I would roll the blue die thrice. By using the linearity of expectation,

$$E(\text{3 Blue dice})[0.5ex] = 3 \cdot E(\text{Blue die}) = 10.5$$

$$\begin{aligned} E(\text{One of each dice}) &= E(\text{Blue die}) + E(\text{Yellow die}) + E(\text{Red die}) \\ &= 3.5 + 3 + 3 \\ &= 9.5 \end{aligned}$$

$$10.5 > 9.5$$

c)

If the value of the yellow die is 8, then it is no longer a random variable. Its expected value is trivially 8. I would choose rolling one of each die this time, since,

$$\begin{aligned} E(\text{One of each dice}) &= E(\text{Blue die}) + E(\text{Yellow die}) + E(\text{Red die}) \\ &= 3.5 + 8 + 3 \\ &= 14.5 \end{aligned}$$

$$14.5 > 9.5$$

d)

By using Bayes theorem and the law of total probability,

$$\begin{aligned}
P(\text{Die is red} \mid \text{Value is 3}) &= \frac{P(\text{Value is 3} \mid \text{Die is red}) \cdot P(\text{Die is red})}{P(\text{Value is 3})} \\
&= \frac{P(\mathrm{V_3} \mid \mathrm{D_r}) \cdot P(\mathrm{D_r})}{P(\mathrm{V_3} \mid \mathrm{D_b}) \cdot P(\mathrm{D_b}) + P(\mathrm{V_3} \mid \mathrm{D_y}) \cdot P(\mathrm{D_y}) + P(\mathrm{V_3} \mid \mathrm{D_r}) \cdot P(\mathrm{D_r})} \\
&= \frac{\frac{2}{10} \cdot \frac{1}{3}}{\frac{1}{6} \cdot \frac{1}{3} + \frac{3}{8} \cdot \frac{1}{3} + \frac{2}{10} \cdot \frac{1}{3}} \\
&= \frac{\frac{2}{10}}{\frac{1}{6} + \frac{3}{8} + \frac{2}{10}} \\
&= \frac{\frac{1}{5}}{\frac{1}{6} + \frac{3}{8} + \frac{1}{5}} \\
&= \frac{24}{20 + 45 + 24} \\
&= \frac{24}{89}
\end{aligned}$$

e)

The total value can be 5 in four different ways, namely $(1,4),(2,3),(3,2),(4,1)$, where the first element represent the value of the blue die and the second element represent the value of the yellow die. Since these are independent events, simple multiplication is sufficient.

$$\begin{gathered}
P(\text{Blue die is 1}) \cdot P(\text{Yellow die is 4}) = \tfrac{1}{6} \cdot \tfrac{1}{8} = \tfrac{1}{48} \\
P(\text{Blue die is 2}) \cdot P(\text{Yellow die is 3}) = \tfrac{1}{6} \cdot \tfrac{3}{8} = \tfrac{3}{48} \\
P(\text{Blue die is 3}) \cdot P(\text{Yellow die is 2}) = \tfrac{1}{6} \cdot \tfrac{0}{8} = 0 \\
P(\text{Blue die is 4}) \cdot P(\text{Yellow die is 1}) = \tfrac{1}{6} \cdot \tfrac{3}{8} = \tfrac{3}{48} \\
\frac{1}{48} + \frac{3}{48} + 0 + \frac{3}{48} = \frac{7}{48}
\end{gathered}$$

Answer 2

a)

The event of a specific distributor offering a discount that day can be thought of being a Bernouilli trial, and the probability of a specific amount of distributors offering a discount that day can be calculated using Binomial distribution. Since Company A has a large amount of distributors and a small chance of offering a discount, we can use Poisson approximation of Binomial distribution to calculate the probability of at least four distributors offering a discount tomorrow.
The number of distributors are $n = 80$, chance of offering a discount is $p = 0.025$, and the expected value of discounts per day are $n \cdot p = 2$. The probability of at least four distributors offering a discount tomorrow is $F_Y(4) = 0.9473$, where F_Y is the cumulative distribution function of Poisson distribution with $\lambda = 2$.

b)

Assuming the customer is only going to buy a phone when there is a discount in A or B, one can calculate the probability of buying a phone by subtracting the probability of no discounts happening in two days from 1.
There will be no discounts if both A and B have no discounts.

- A will have no discounts with the probability $P(0) = 0.0183$, where P is the probability density function of Poisson distribution with $\lambda = 4$, since we are looking at a two day timespan.
- B will have no discounts with the probability $0.9 \cdot 0.9 = 0.81$.
- They will both have no discounts with the probability $0.0183 \cdot 0.81 = 0.0148$.

Therefore, the customer will be able to buy the phone in two days with the probability $1 - 0.0148 = 0.9852$.

Answer 3

The Octave code is as follows,

```
N = 1000;

% F is the first iteration where we roll one die of each color
F = rand(3, N);
F = F.*[6; 8; 10];
F = ceil(F);
for i = 1:N
  F(:,i) = getValues(F(:,i));
end
F = sum(F);

% S is the second iteration where we roll blue die thrice
S = rand(3, N);
S = S.*[6; 6; 6];
S = ceil(S);
S = sum(S);

% isBigger is the 1 x N matrix with binary outcomes
isBigger = (S > F);

% Let F and S be the total average and isBigger the percentage
% of times S was bigger than F
F = sum(F)/N;
S = sum(S)/N;
isBigger = sum(isBigger)/length(isBigger)*100;

printf("First option: %.2f\nSecond option: %.2f\
\nPercentage of cases: %2.2f%%\n", F, S, isBigger)

function out = getValues(x)
  b = x(1); y = x(2); r = x(3);
  b = [1 2 3 4 5 6](1, b);
  y = [1 1 1 3 3 3 4 8](1, y);
  r = [2 2 2 2 2 3 3 4 4 6](1, r);
  out = [b; y; r];
endfunction
```

with a screenshot of some outputs,

```
>> the1_code
First option: 9.40
Second option: 10.40
Percentage of cases: 57.00%
>> the1_code
First option: 9.44
Second option: 10.53
Percentage of cases: 57.70%
>> the1_code
First option: 9.52
Second option: 10.53
Percentage of cases: 56.90%
```

The findings agree with our calculations. First option is close to 9.5 and the second option is close to 10.5. The second option has a slight advantage compared to the first one, as we can see in the percentage of cases it triumphed over the first one. However, it is not a huge difference.