

Student Information

Full Name : Murat Bolu
Id Number : 2521300

Answer 1

Part a)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Therefore, it is a contradiction.

Part b)

$$\begin{aligned} p \rightarrow ((q \vee \neg q) \rightarrow (p \wedge q)) &\equiv p \rightarrow (T \rightarrow (p \wedge q)) && \text{Negation Law, Table 6} \\ &\equiv p \rightarrow (\neg T \vee (p \wedge q)) && \text{Table 7, 1st Line} \\ &\equiv p \rightarrow (F \vee (p \wedge q)) && \text{Negation of Truth} \\ &\equiv p \rightarrow ((p \wedge q) \vee F) && \text{Commutative Law, Table 6} \\ &\equiv p \rightarrow (p \wedge q) && \text{Identity Law, Table 6} \\ &\equiv \neg p \vee (p \wedge q) && \text{Table 7, 1st Line} \\ &\equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{Distributive Law, Table 6} \\ &\equiv (p \vee \neg p) \wedge (\neg p \vee q) && \text{Commutative Law, Table 6} \\ &\equiv T \wedge (\neg p \vee q) && \text{Negation Law, Table 6} \\ &\equiv (\neg p \vee q) \wedge T && \text{Commutative Law, Table 6} \\ &\equiv \neg p \vee q && \text{Identity Law, Table 6} \end{aligned}$$

Answer 2

- a) $\forall x \exists y W(x, y)$
- b) $\exists y \forall x (\neg F(x, y))$
- c) $\forall x (W(x, P) \rightarrow A(a, x))$, where a is Ali.
- d) $\exists x (W(b, x) \wedge F(t, x))$, where b is Büşra and t is TÜBİTAK.
- e) $\exists x \exists y \exists z (S(x, y) \wedge S(x, z) \wedge \neg(y = z))$
- f) $\forall x \forall y \forall z ((W(x, z) \wedge W(y, z)) \rightarrow (x = y))$, where x and y are students and z is a project.

g) $\exists x \exists y \exists z (W(y, x) \wedge W(z, x) \wedge \neg(y = z) \wedge \forall t (W(t, x) \rightarrow ((y = t) \vee (z = t))))$, where x is a project and y, z and t are students.

Answer 3

1.	$p \rightarrow q$	Premise
2.	$(q \wedge \neg r) \rightarrow s$	Premise
3.	$\neg s$	Premise
4.	p	Assumption
5.	$\neg r$	Assumption
6.	q	\rightarrow e, 4, 1
7.	$q \wedge \neg r$	\wedge i, 6, 5
8.	s	\rightarrow e, 7, 2
9.	\perp	\neg e, 3, 8
10.	$\neg\neg r$	\neg i, 6 - 9
11.	r	$\neg\neg$ e, 10
12.	$p \rightarrow r$	\rightarrow i, 4 - 11

Answer 4

If we translate the sentences to logical formulas:

- Ayşe: p
- Barış: $s \rightarrow \neg q$
- Can: $p \rightarrow (q \wedge r)$
- Duygu: $r \rightarrow s$

Now, we need to show that $p, p \rightarrow (q \wedge r), r \rightarrow s \vdash \neg(s \rightarrow \neg q)$.

1.	p	Premise
2.	$p \rightarrow (q \wedge r)$	Premise
3.	$r \rightarrow s$	Premise
4.	$q \wedge r$	\rightarrow e, 1, 2
5.	q	\wedge e, 4
6.	r	\wedge e, 4
7.	s	\rightarrow e, 6, 3
8.	$s \rightarrow \neg q$	Assumption
9.	$\neg q$	\rightarrow e, 7, 8
10.	\perp	\neg e, 5, 9
11.	$\neg(s \rightarrow \neg q)$	\neg i, 8 - 10

Answer 5

1.	$\forall x (P(x) \rightarrow (Q(x) \rightarrow R(x)))$	Premise
2.	$\exists x (P(x))$	Premise
3.	$\forall x (\neg R(x))$	Premise
4.	$P(c)$	Assumption
5.	$P(c) \rightarrow (Q(c) \rightarrow R(c))$	\forall e, 1
6.	$Q(c) \rightarrow R(c)$	\rightarrow e, 4, 5
7.	$Q(c)$	Assumption
8.	$R(c)$	\rightarrow e, 7, 6
9.	$\neg R(c)$	\forall e, 3
10.	\perp	\neg e, 8, 9
11.	$\neg Q(c)$	\neg i, 7 - 10
12.	$\exists x (\neg Q(x))$	\exists i, 11
13.	$\exists x (\neg Q(x))$	\exists e, 2, 4 - 12