Student Information

Full Name : Murat Bolu
Id Number : 2521300

Answer 1

Part **a)**

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Therefore, it is a contradiction.

Part **b)**

$$
\begin{aligned}
p \to ((q \vee \neg q) \to (p \wedge q)) &\equiv p \to (T \to (p \wedge q)) && \text{Negation Law, Table 6} \\
&\equiv p \to (\neg T \vee (p \wedge q)) && \text{Table 7, 1}^{\text{st}} \text{ Line} \\
&\equiv p \to (F \vee (p \wedge q)) && \text{Negation of Truth} \\
&\equiv p \to ((p \wedge q) \vee F) && \text{Commutative Law, Table 6} \\
&\equiv p \to (p \wedge q) && \text{Identity Law, Table 6} \\
&\equiv \neg p \vee (p \wedge q) && \text{Table 7, 1}^{\text{st}} \text{ Line} \\
&\equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{Distributive Law, Table 6} \\
&\equiv (p \vee \neg p) \wedge (\neg p \vee q) && \text{Commutative Law, Table 6} \\
&\equiv T \wedge (\neg p \vee q) && \text{Negation Law, Table 6} \\
&\equiv (\neg p \vee q) \wedge T && \text{Commutative Law, Table 6} \\
&\equiv \neg p \vee q && \text{Identity Law, Table 6}
\end{aligned}
$$

Answer 2

a) $\forall x\, \exists y\, W(x, y)$

b) $\exists y\, \forall x\, (\neg F(x, y))$

c) $\forall x\, (W(x, P) \to A(a, x))$, where a is Ali.

d) $\exists x\, (W(b, x) \wedge F(t, x))$, where b is Büşra and t is TÜBİTAK.

e) $\exists x\, \exists y\, \exists z\, (S(x, y) \wedge S(x, z) \wedge \neg(y = z))$

f) $\forall x\, \forall y\, \forall z\, ((W(x, z) \wedge W(y, z)) \to (x = y))$, where x and y are students and z is a project.

g) $\exists x\, \exists y\, \exists z\, (W(y,x) \wedge W(z,x) \wedge \neg(y = z) \wedge \forall t\, (W(t,x) \rightarrow ((y = t) \vee (z = t))))$, where x is a project and y, z and t are students.

Answer 3

1.	$p \rightarrow q$	Premise
2.	$(q \wedge \neg r) \rightarrow s$	Premise
3.	$\neg s$	Premise
4.	p	Assumption
5.	$\neg r$	Assumption
6.	q	$\rightarrow$ e, 4, 1
7.	$q \wedge \neg r$	$\wedge$ i, 6, 5
8.	s	$\rightarrow$ e, 7, 2
9.	$\perp$	$\neg$ e, 3, 8
10.	$\neg\neg r$	$\neg$ i, 6 - 9
11.	r	$\neg\,\neg$ e, 10
12.	$p \rightarrow r$	$\rightarrow$ i, 4 - 11

Answer 4

If we translate the sentences to logical formulas:

- Ayşe: p
- Barış: $s \rightarrow \neg q$
- Can: $p \rightarrow (q \wedge r)$
- Duygu: $r \rightarrow s$

Now, we need to show that $p,\ p \rightarrow (q \wedge r),\ r \rightarrow s \vdash \neg(s \rightarrow \neg q)$.

1.	p	Premise
2.	$p \to (q \land r)$	Premise
3.	$r \to s$	Premise
4.	$q \land r$	$\to$ e, 1, 2
5.	q	$\land$ e, 4
6.	r	$\land$ e, 4
7.	s	$\to$ e, 6, 3
8.	$s \to \neg q$	Assumption
9.	$\neg q$	$\to$ e, 7, 8
10.	$\bot$	$\neg$ e, 5, 9
11.	$\neg(s \to \neg q)$	$\neg$ i, 8 - 10

Answer 5

1.	$\forall x\ (P(x) \to (Q(x) \to R(x)))$	Premise
2.	$\exists x\ (P(x))$	Premise
3.	$\forall x\ (\neg R(x))$	Premise
4.	$P(c)$	Assumption
5.	$P(c) \to (Q(c) \to R(c))$	$\forall$e, 1
6.	$Q(c) \to R(c)$	$\to$ e, 4, 5
7.	$Q(c)$	Assumption
8.	$R(c)$	$\to$ e, 7, 6
9.	$\neg R(c)$	$\forall$e, 3
10.	$\bot$	$\neg$ e, 8, 9
11.	$\neg Q(c)$	$\neg$ i, 7 - 10
12.	$\exists x\ (\neg Q(x))$	$\exists$i, 11
13.	$\exists x\ (\neg Q(x))$	$\exists$e, 2, 4 - 12