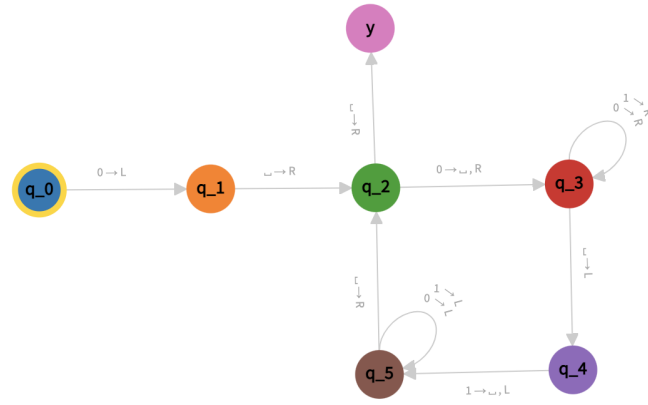


CENG280 Homework 3

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March 14, 2023

Answer 1



In the first Turing machine the start state is q_0 . The states q_0 and q_1 are placed only to make sure that the machine doesn't accept the empty string, as $N \geq 1$. The state q_2 deletes the symbol 0 in the initial position, moves the head to the right and the machine goes to the state q_3 . The state q_3 makes the head go left until the first \sqcup read, moves the head to the left and the machine goes to the state q_4 . The state q_4 deletes the symbol 1 in the final position, moves the head to the left and the machine goes to the state q_5 . The state q_5 makes the head go left until the first \sqcup read, moves the head to the left and the machine goes to the state q_2 . When the string is empty and the machine is in the state q_2 , the machine accepts the input string and halts in the state y .

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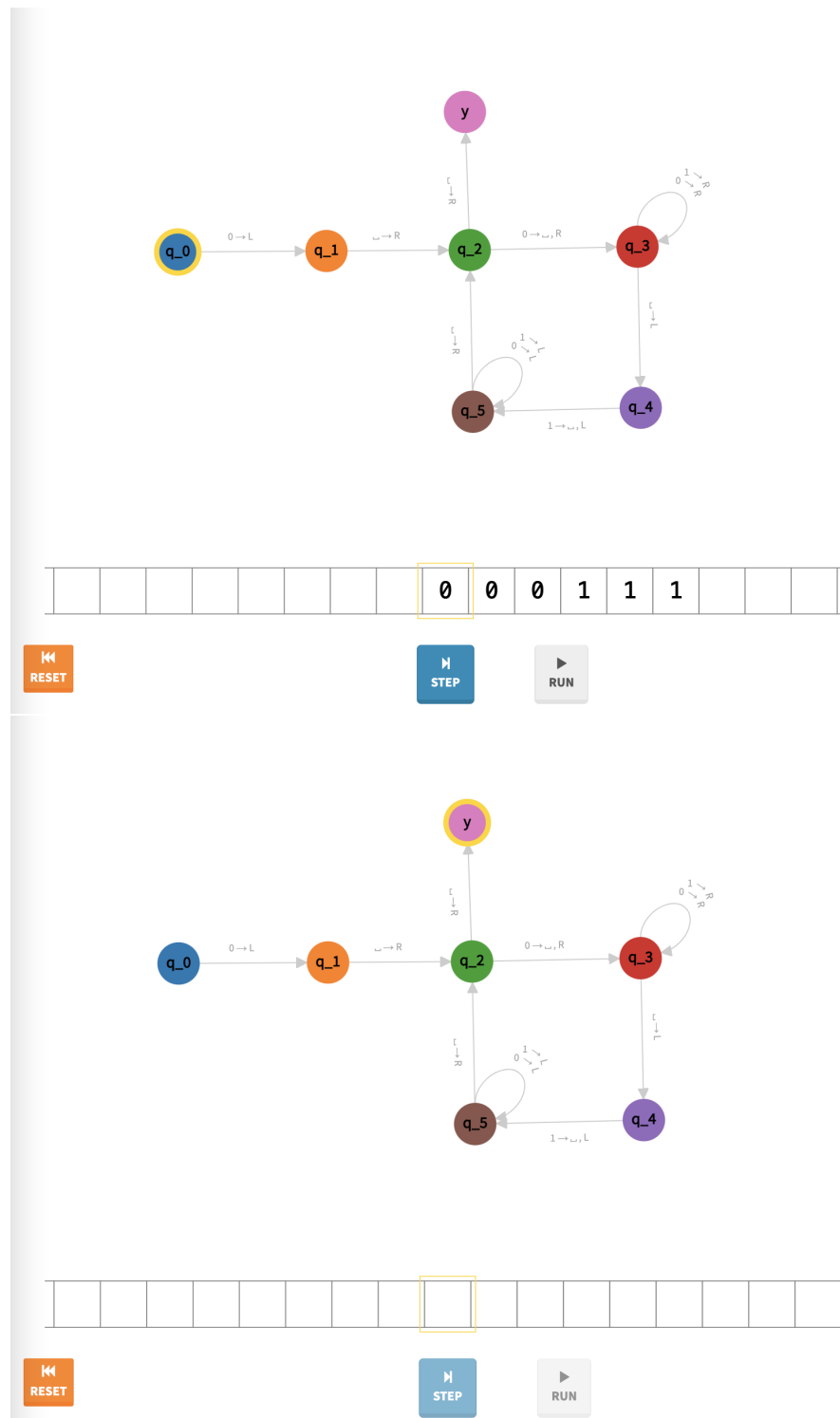
1 input: '000111'
2 blank: ' '
3 start state: q_0
4 table:
5   q_0:
6     0: {L: q_1}
7   q_1:
8     ' ': {R: q_2}
9   q_2:
10    0: {write: ' ', R: q_3}
11    ' ': {R: y}
12   q_3:
13     0: R
14     1: R
15     ' ': {L: q_4}
16   q_4:
17     1: {write: ' ', L: q_5}
18   q_5:
19     0: L
20     1: L
21     ' ': {R: q_2}
22   y:

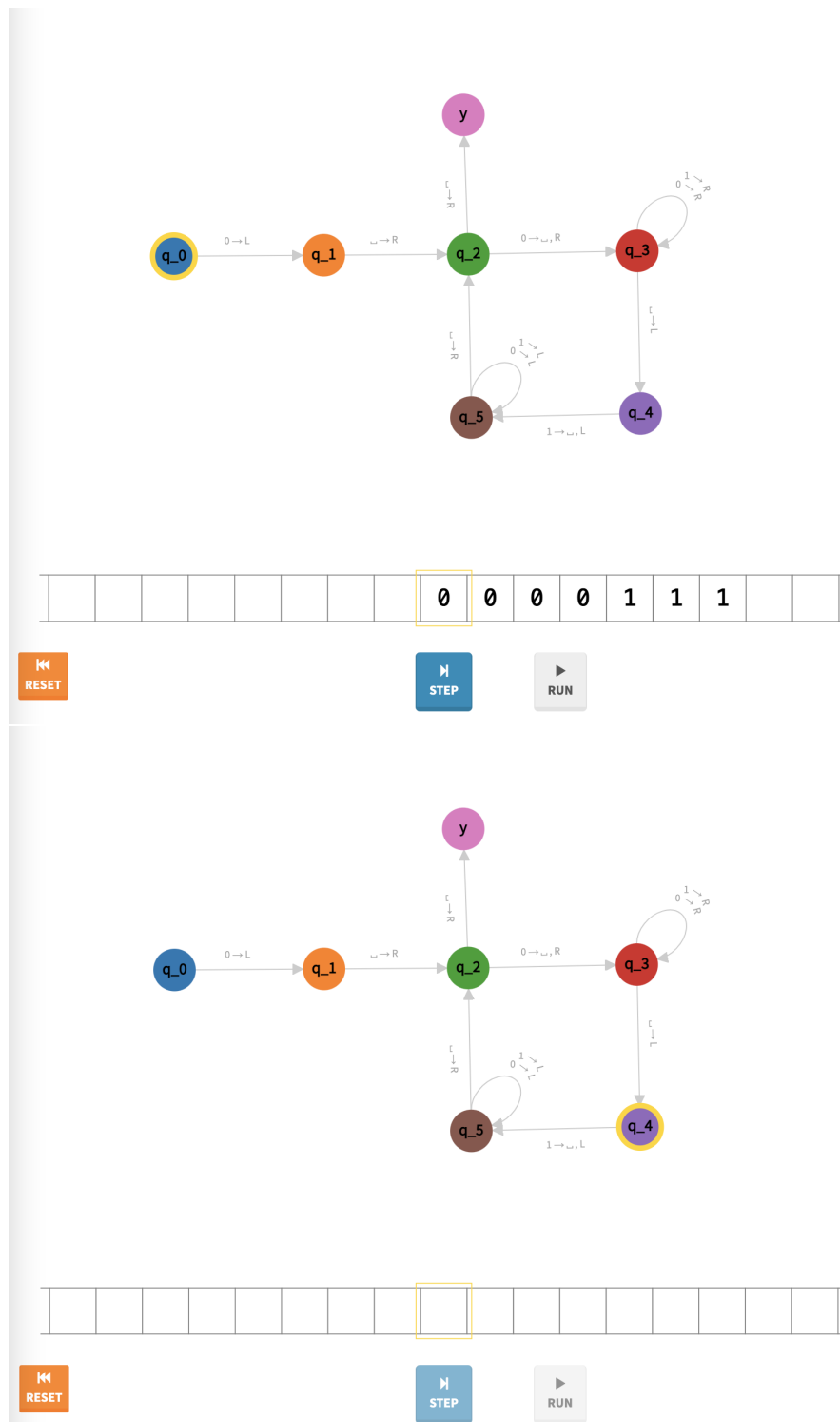
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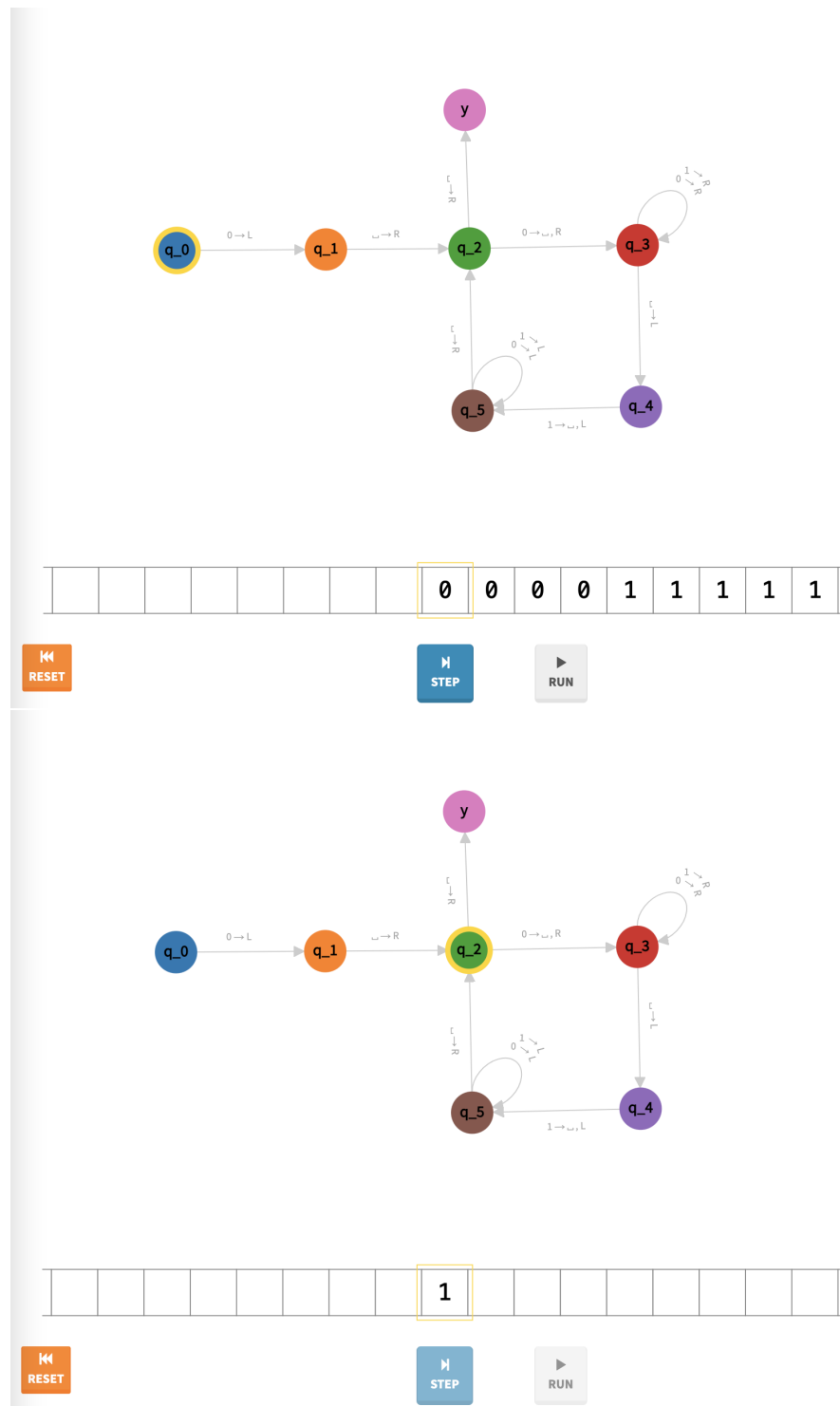
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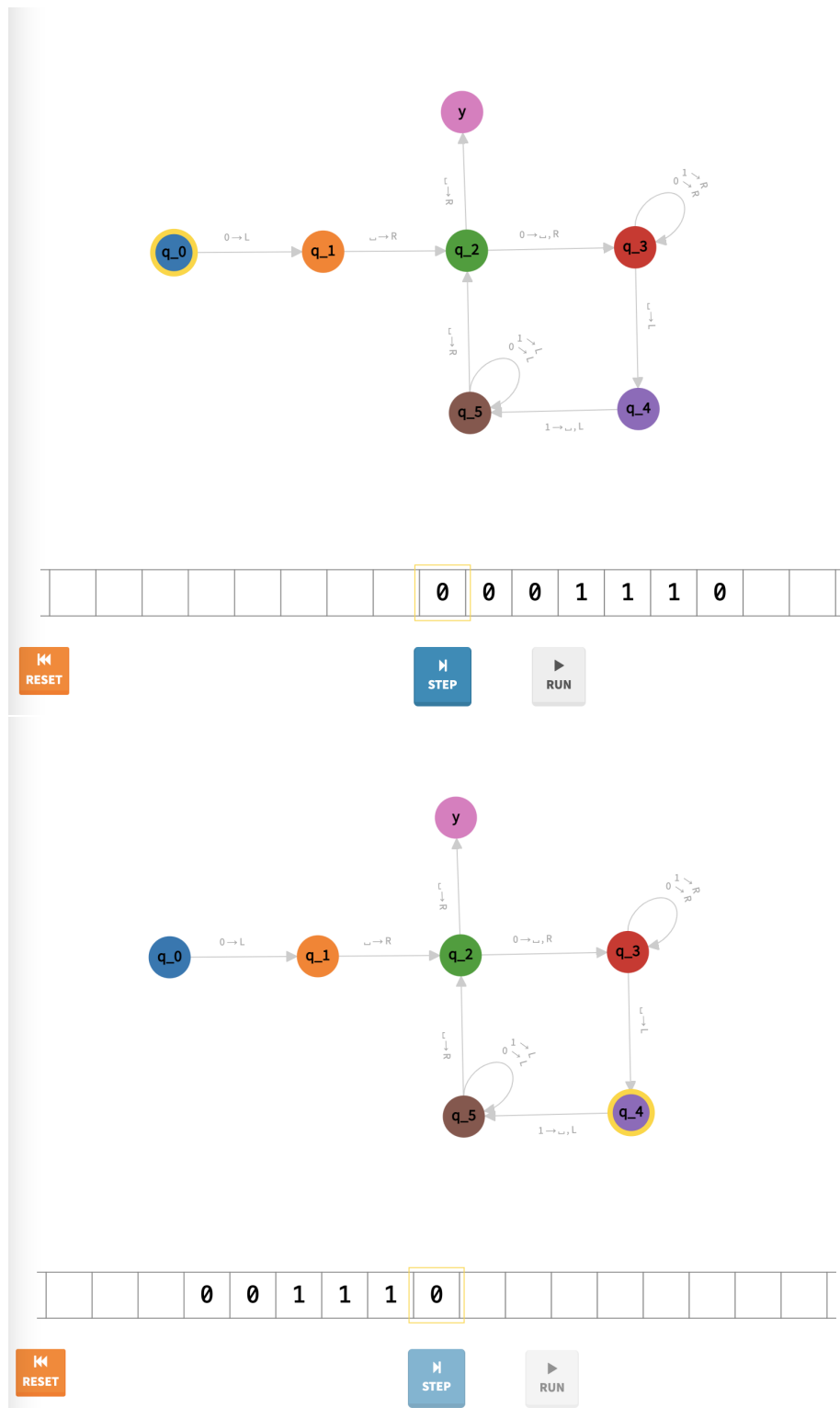
input: '0001110'
blank: ' '
start state: q_0
table:
q_0:
0: {L: q_1}
q_1:
' ': {R: q_2}
q_2:
0: {write: ' ', R: q_3}
' ': {R: y}
q_3:
0: R
1: R
' ': {L: q_4}
q_4:
1: {write: ' ', L: q_5}
q_5:
0: L
1: L
' ': {R: q_2}
y:

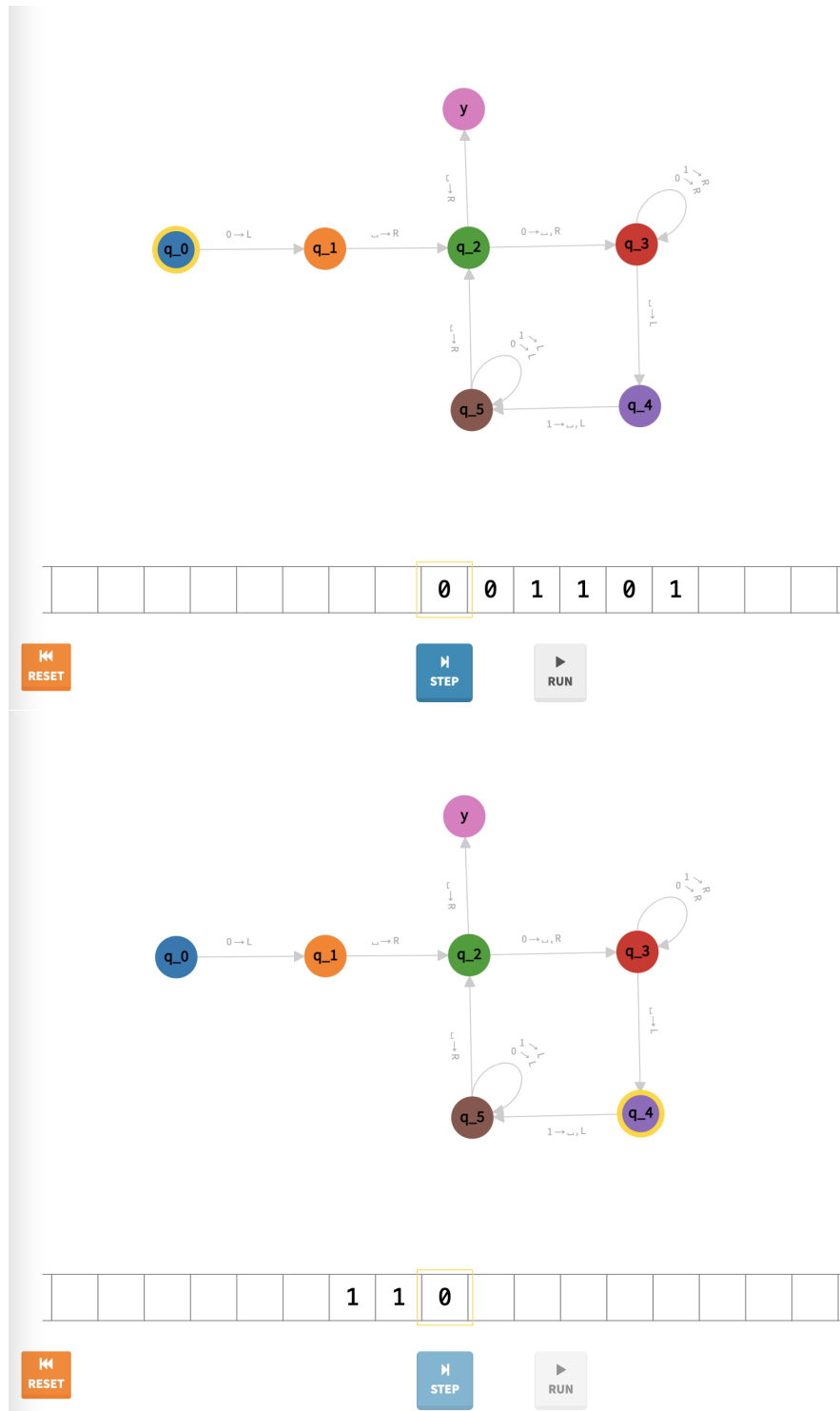
```

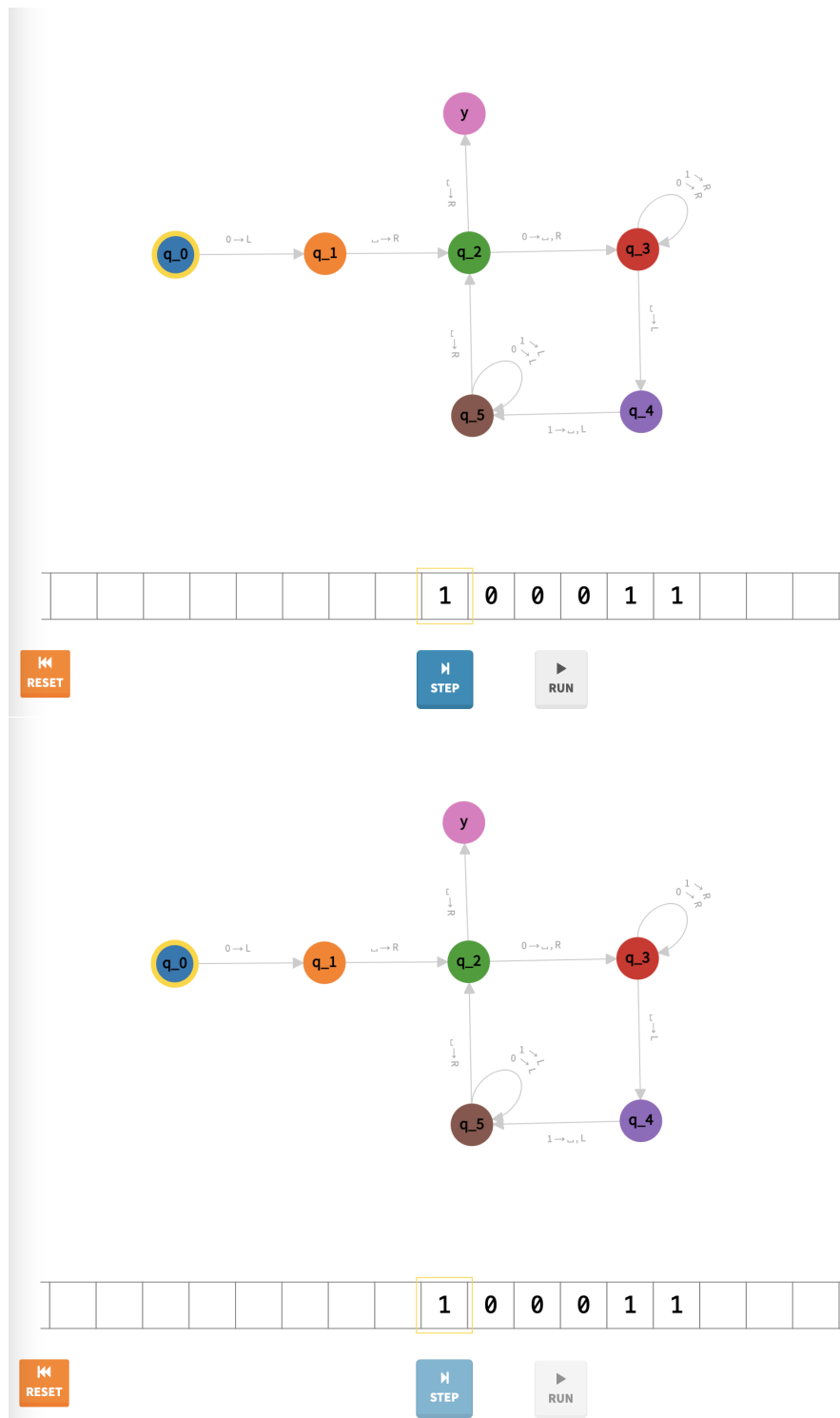




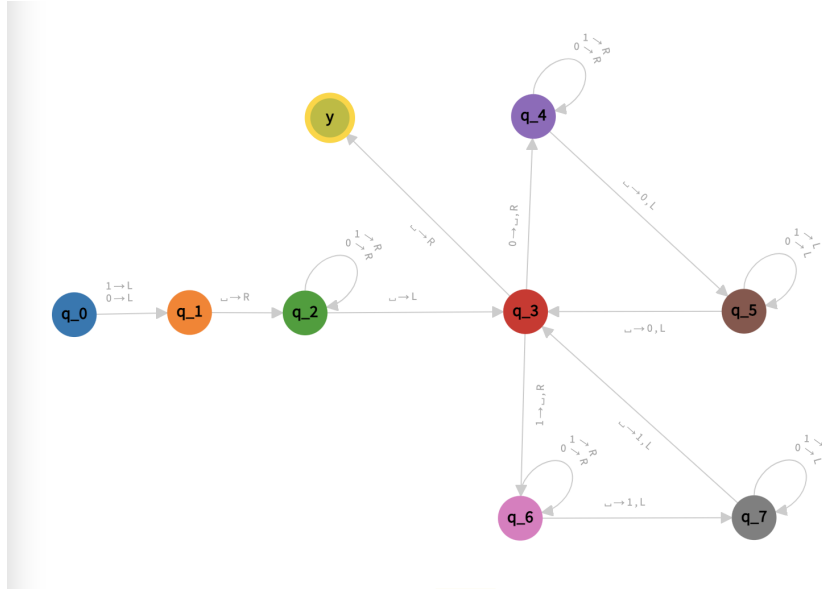








Answer 2



In the second Turing machine the start state is q_0 . The states q_0 and q_1 are placed only to make sure that the machine doesn't accept the empty string, as $e \notin \{0,1\}^+$. The state q_2 makes the head go right until the first \sqcup read, moves the head to the left and the machine goes to the state q_3 . The state q_3 deletes the symbol 0 (or 1), moves the head to the right and the machine goes to the state q_4 (or q_6). The state q_4 (or q_6) moves the head to the right until the first \sqcup read, writes the symbol 0 (or 1), moves the head to the left and the machine goes to the state q_5 (or q_7). The state q_5 (or q_7) moves the head to the left until the first \sqcup seen, writes the symbol 0 (or 1), and the machine goes the state q_3 . When the head reads \sqcup and the machine is in the state q_3 , the machine halts in the state y .

```

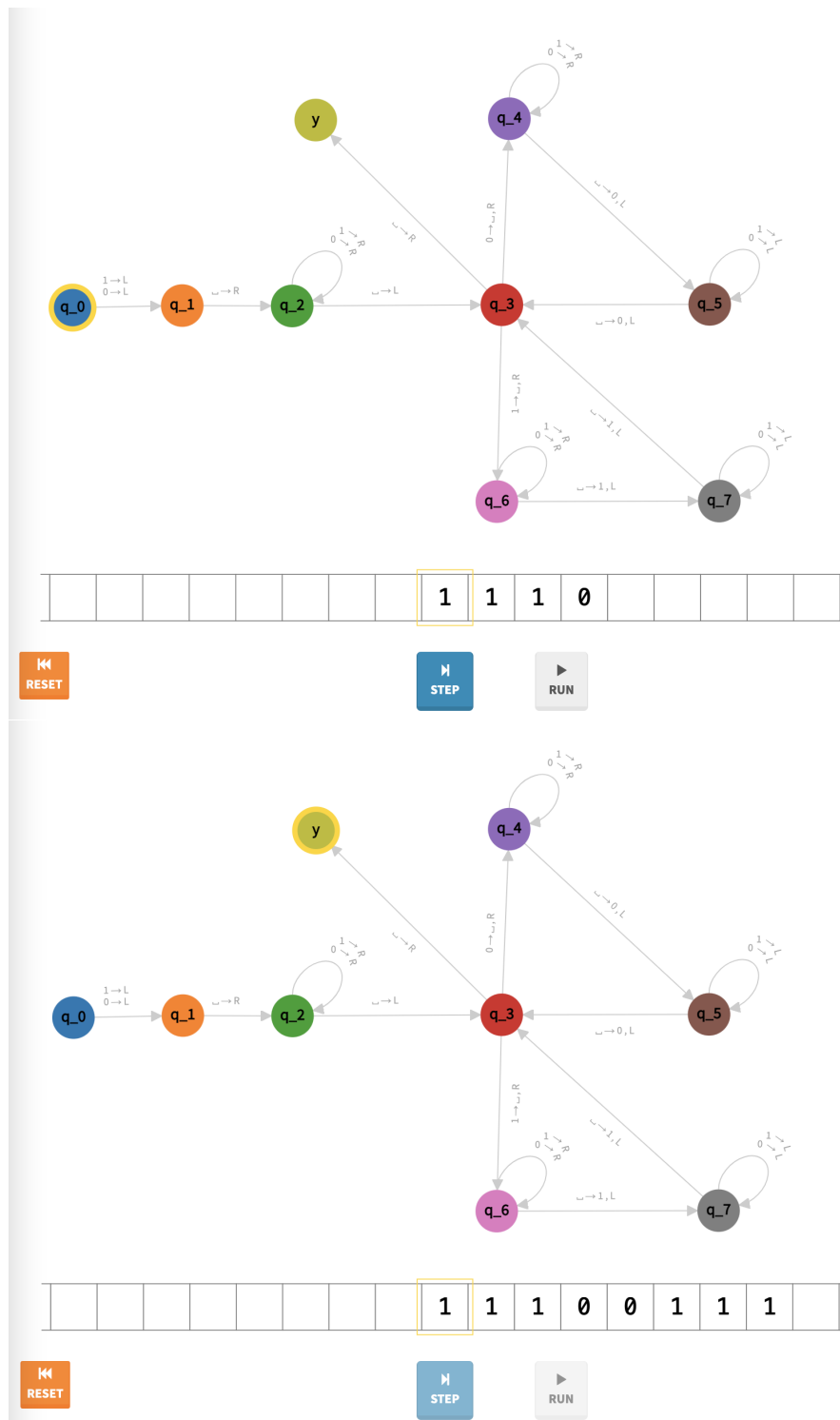
1 input: '1010001'
2 blank: ' '
3 start state: q_0
4 table:
5   q_0:
6     0: {L: q_1}
7     1: {L: q_1}
8   q_1:
9     ' ': {R: q_2}
10  q_2:
11    0: R
12    1: R
13    ' ': {L: q_3}
14  q_3:
15    0: {write: ' ', R: q_4}
16    1: {write: ' ', R: q_6}
17    ' ': {R: y}
18  q_4:
19    0: R
20    1: R
21    ' ': {write: 0, L: q_5}
22  q_5:
23    0: L
24    1: L
25    ' ': {write: 0, L: q_3}
26  q_6:
27    0: R
28    1: R
29    ' ': {write: 1, L: q_7}
30  q_7:
31    0: L
32    1: L
33    ' ': {write: 1, L: q_3}
34  y:

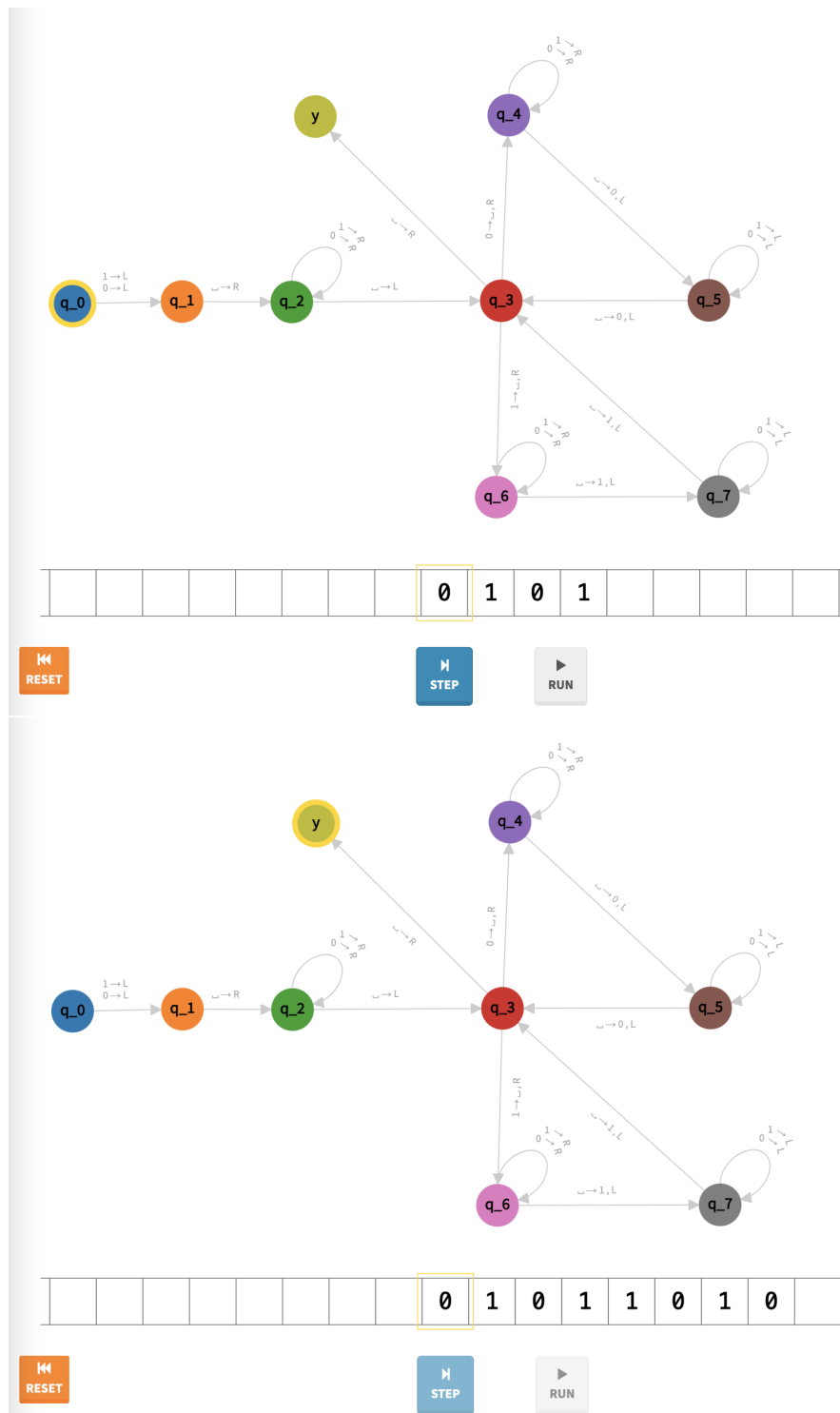
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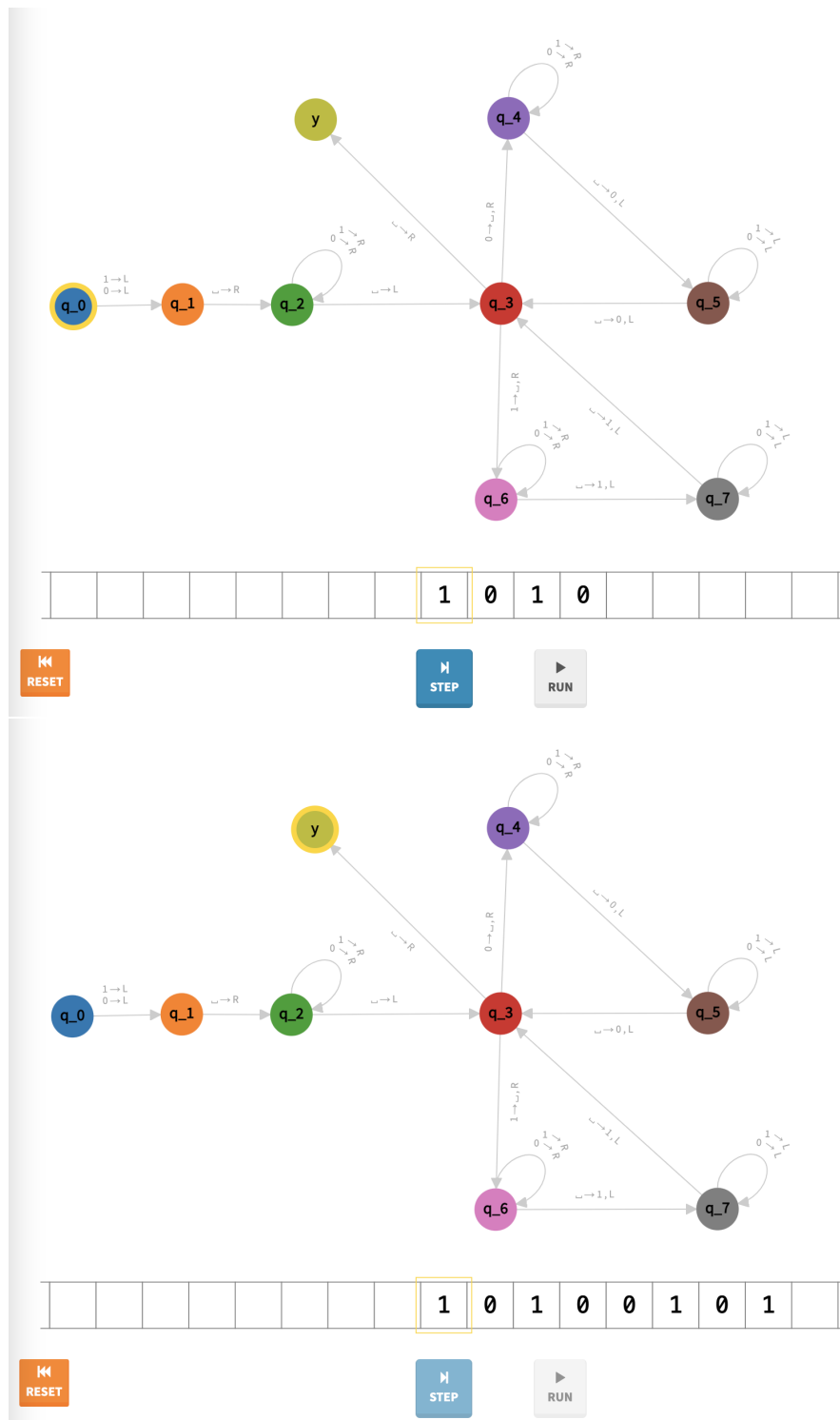
```

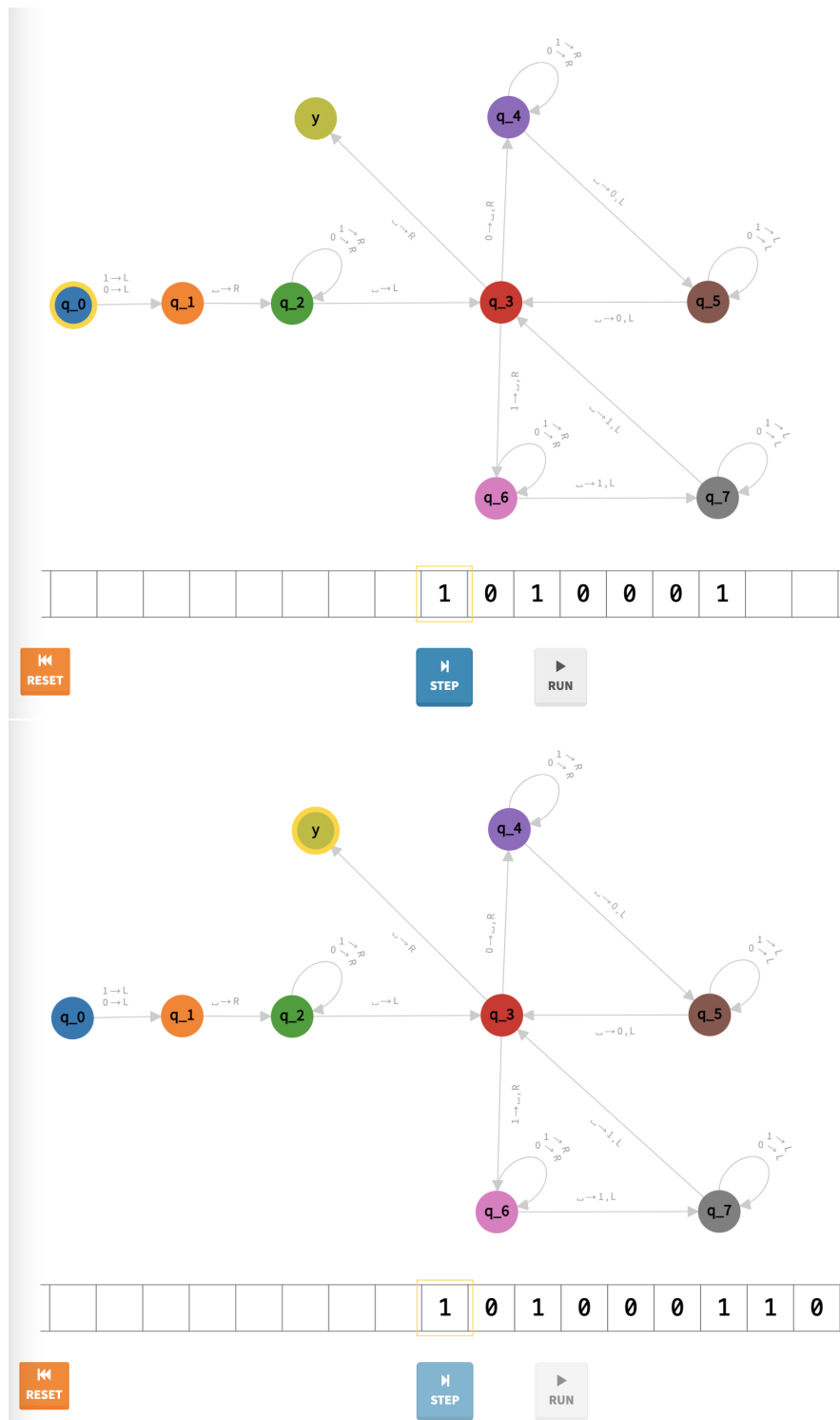
input:  '1011'
blank:  ' '
start state:  q_0
table:
q_0:
0:  L: q_1
1:  L: q_1
q_1:
' ': R: q_2
q_2:
0:  R
1:  R
' ': L: q_3
q_3:
0:  write: ' ', R: q_4
1:  write: ' ', R: q_6
' ': R: y
q_4:
0:  R
1:  R
' ': write: 0, L: q_5
q_5:
0:  L
1:  L
' ': write: 0, L: q_3
q_6:
0:  R
1:  R
' ': write: 1, L: q_7
q_7:
0:  L
1:  L
' ': write: 1, L: q_3
y:

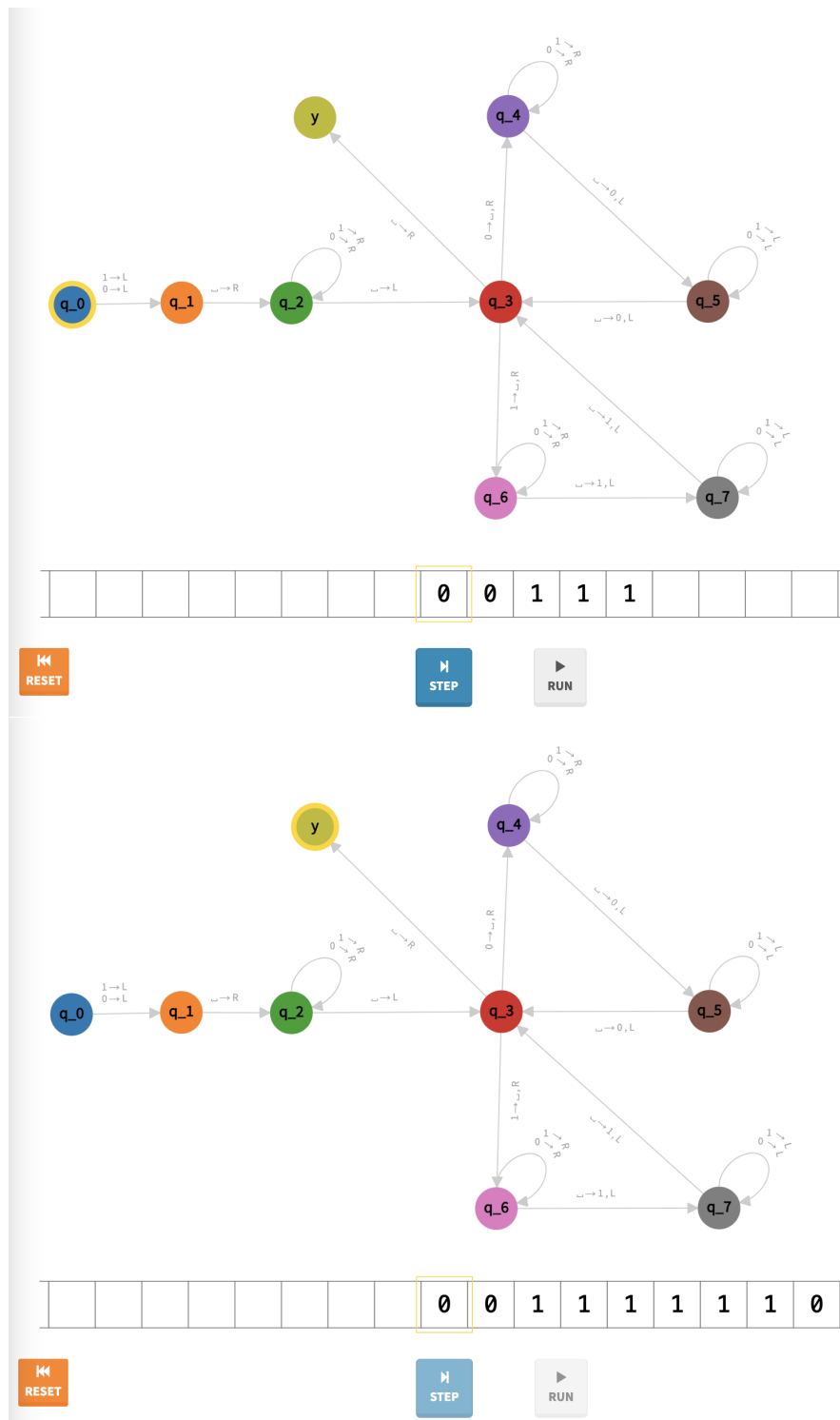
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Answer 3

Let M be a Turing machine with a two-dimensional tape formally defined by the quintuple $(K, \Sigma, \delta, s, H)$ where K is a finite set of states, Σ is an alphabet, δ is the transition function, s is the initial state, and H is the set of halting states. Let there be two more arrows, \uparrow and \downarrow , which will denote the head moving up or down on the tape. The tape goes to infinity for four sides, its center is marked with \triangleright and it possesses similar properties with the usual \triangleright .

$$\begin{aligned} \sqcup &\in \Sigma, \triangleright \in \Sigma, \uparrow \notin \Sigma, \rightarrow \notin \Sigma, \downarrow \notin \Sigma, \leftarrow \notin \Sigma \\ \delta &: (K \setminus H) \times \Sigma \rightarrow K \times (\Sigma \cup \{\uparrow, \rightarrow, \downarrow, \leftarrow\}) \\ \delta(q_0, \triangleright) &= (q_1, p) \Rightarrow p = \rightarrow, \forall q_0 \in K \setminus H \\ \delta(q_0, p_0) &= (q_1, p_1) \Rightarrow p_1 \neq \triangleright, \forall q_0 \in K \setminus H \wedge p_0 \in \Sigma \\ s &\in K \\ H &\subseteq K \end{aligned}$$

Each cell can be represented with a pair of integers. Assume it is similar to a Cartesian coordinate system with pairs (x, y) where x increases in rightward direction and y increases in upward direction. Therefore a configuration for M is an element of $K \times \mathbb{Z} \times \mathbb{Z} \times F$ where K is the set of states, \mathbb{Z} is the set of integers, and F is the set of all functions with the properties $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \Sigma$ and $f(0, 0) = \triangleright \forall f \in F$. F is defined in a way such that all elements of F are functions with finitely many symbols different from \sqcup in their ranges, more formally:

$$\begin{aligned} \exists X \in \mathbb{Z}, \forall y \in \mathbb{Z}, \forall f \in F, (|a| > X \Rightarrow f(a, y) = \sqcup \wedge f(-a, y) = \sqcup) \\ \exists Y \in \mathbb{Z}, \forall x \in \mathbb{Z}, \forall f \in F, (|a| > Y \Rightarrow f(x, a) = \sqcup \wedge f(x, -a) = \sqcup) \end{aligned}$$

K represents the current state, $\mathbb{Z} \times \mathbb{Z}$ represent the position of the head, and F represent the contents of the tape.

A step of computation, denoted with $(q_0, x_0, y_0, f_0) \vdash_M (q_1, x_1, y_1, f_1)$ is valid if and only if $\delta(q_0, f_0(x_0, y_0)) = (q_1, p)$ and

$$\begin{aligned} p = \uparrow &\Leftrightarrow y_1 = y_0 + 1 \wedge x_1 = x_0 \wedge f_1 = f_0 \\ p = \rightarrow &\Leftrightarrow x_1 = x_0 + 1 \wedge y_1 = y_0 \wedge f_1 = f_0 \\ p = \downarrow &\Leftrightarrow y_1 = y_0 - 1 \wedge x_1 = x_0 \wedge f_1 = f_0 \\ p = \leftarrow &\Leftrightarrow x_1 = x_0 - 1 \wedge y_1 = y_0 \wedge f_1 = f_0 \\ p \notin \{\uparrow, \rightarrow, \downarrow, \leftarrow\} &\Leftrightarrow x_1 = x_0 \wedge y_1 = y_0 \wedge f_1(x_1, y_1) = p \end{aligned}$$

Let \vdash_M^* be the reflexive transitive closure of \vdash_M . The language accepted by this machine is as follows. Let the machine M have exactly two halting states, y and n , and any other number of non-halting states. Let s be the starting state. Let the string w start from the position $(2, 0)$ and go leftwards. That is, $\forall i \in \mathbb{N}, 2 \leq i \leq |w| + 2 \Rightarrow w_{i-2} = f(i, 0)$ where $f \in F$ is the function defining the contents of the tape. Let the head start in the position $(1, 0)$. Let the machine M halt for all inputs, that is, it is a decider, an algorithm. The

machine M accepts this string if and only if it halts in the state y and rejects otherwise. More specifically:

$$w \in L(M) \Leftrightarrow (s, 1, 0, F) \vdash_M^* (y, x_n, y_n, F_n)$$

Let $g : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{N}$ be a bijection mapping pairs of integers to natural numbers. Let $g(0, 0) = 0$. We can map the two-dimensional tape to a semi-infinite one-dimensional tape by using g . Let M_s be the standard Turing machine with semi-infinite tape that will simulate M which has a two-dimensional tape. Let M_s be formally defined by the quintuple $(K_s, \Sigma_s, \delta_s, s_s, H_s)$, where

$$\begin{aligned} K_s &= K \\ \Sigma_s &= \Sigma \\ \delta_s(q_0, p_0) &= \delta(q_0, p_0) = (q_1, p_1) \Leftrightarrow p_1 \notin \{\uparrow, \rightarrow, \downarrow, \leftarrow\} \\ s_s &= s \\ H_s &= H \end{aligned}$$

If $p_1 \in \{\uparrow, \rightarrow, \downarrow, \leftarrow\}$, then the machine M_s moves the head to the appropriate position given by the function g . Therefore, the machine M can be simulated by the standard Turing machine M_s .