CENG280 Homework 1

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1 Question 1

a.

$$((a \cup b)^* \ aa \ (a \cup b)^* \ bb \ (a \cup b)^*) \cup ((a \cup b)^* \ bb \ (a \cup b)^* \ aa \ (a \cup b)^*)$$

b.

Formal definition of the nondeterministic finite automaton M:

$$\begin{aligned} M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{&(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_1, a, q_2), \\ &(q_2, a, q_2), (q_2, b, q_2), (q_2, b, q_3), (q_3, b, q_7), \\ &(q_0, b, q_4), (q_4, b, q_5), (q_5, a, q_5), (q_5, b, q_5), \\ &(q_5, a, q_6), (q_6, a, q_7), (q_7, a, q_7), (q_7, b, q_7)\}, q_0, \{q_7\}) \end{aligned}$$

Drawing of the nondeterministic finite automaton M:

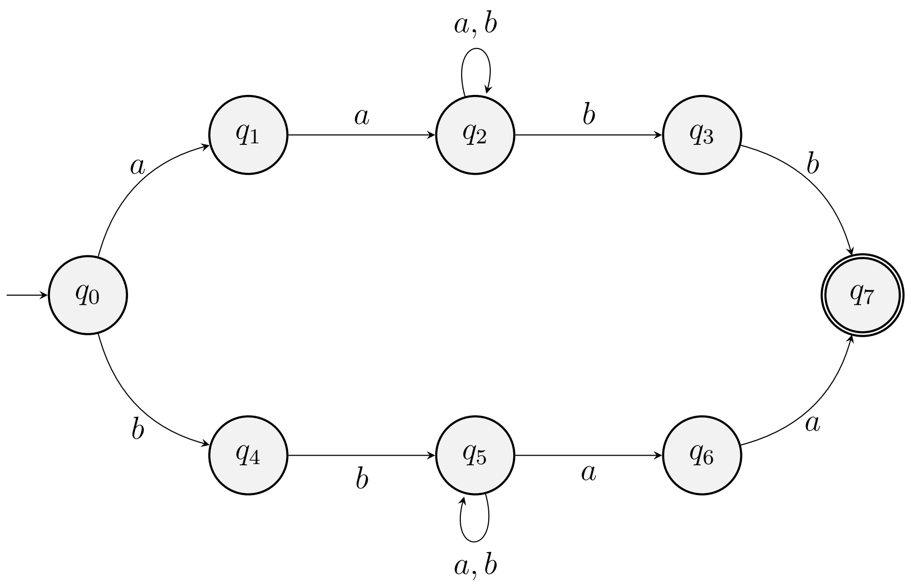

Figure 1: Nondeterministic Finite Automaton M

c.

Let's formally define the deterministic finite automaton M', which will be equivalent to M. We don't need to use $E(q_i)$ notation since there are no empty transitions in our NFA.

$$M' = (K', \Sigma, \delta', s', F')$$

Where $\Sigma = \{a, b\}$, same with the NFA.
Where $s' = \{q_0\}$, the set of all states that are reachable from the start state without reading any input.

We need to construct δ' by taking the set of start states, defining the transitions of it accordingly and defining transitions of the new states until all states are defined.

$$
\begin{aligned}
\text{Where}\quad \delta'(\{q_0\}, a) &= \{q_0, q_1\} \\
\delta'(\{q_0\}, b) &= \{q_0, q_4\} \\
\delta'(\{q_0, q_1\}, a) &= \{q_0, q_1, q_2\} \\
\delta'(\{q_0, q_1\}, b) &= \{q_0, q_4\} \\
\delta'(\{q_0, q_4\}, a) &= \{q_0, q_1\} \\
\delta'(\{q_0, q_4\}, b) &= \{q_0, q_4, q_5\} \\
\delta'(\{q_0, q_1, q_2\}, a) &= \{q_0, q_1, q_2\} \\
\delta'(\{q_0, q_1, q_2\}, b) &= \{q_0, q_2, q_3, q_4\} \\
\delta'(\{q_0, q_4, q_5\}, a) &= \{q_0, q_1, q_5, q_6\} \\
\delta'(\{q_0, q_4, q_5\}, b) &= \{q_0, q_4, q_5\} \\
\delta'(\{q_0, q_2, q_3, q_4\}, a) &= \{q_0, q_1, q_2\} \\
\delta'(\{q_0, q_2, q_3, q_4\}, b) &= \{q_0, q_2, q_3, q_4, q_5, q_7\} \\
\delta'(\{q_0, q_1, q_5, q_6\}, a) &= \{q_0, q_1, q_2, q_5, q_6, q_7\} \\
\delta'(\{q_0, q_1, q_5, q_6\}, b) &= \{q_0, q_4, q_5\} \\
\delta'(\{q_0, q_2, q_3, q_4, q_5, q_7\}, a) &= \{q_0, q_1, q_2, q_5, q_6, q_7\} \\
\delta'(\{q_0, q_2, q_3, q_4, q_5, q_7\}, b) &= \{q_0, q_2, q_3, q_4, q_5, q_7\} \\
\delta'(\{q_0, q_1, q_2, q_5, q_6, q_7\}, a) &= \{q_0, q_1, q_2, q_5, q_6, q_7\} \\
\delta'(\{q_0, q_1, q_2, q_5, q_6, q_7\}, b) &= \{q_0, q_2, q_3, q_4, q_5, q_7\}
\end{aligned}
$$

$$
\begin{aligned}
\text{Where } K' = \{&\{q_0\}, \{q_0, q_1\}, \{q_0, q_4\}, \{q_0, q_1, q_2\}, \\
&\{q_0, q_4, q_5\}, \{q_0, q_2, q_3, q_4\}, \{q_0, q_1, q_5, q_6\}, \\
&\{q_0, q_2, q_3, q_4, q_5, q_7\}, \{q_0, q_1, q_2, q_5, q_6, q_7\}\},
\end{aligned}
$$

the set of all possible sets of states.

Where $F' = \{\{q_0, q_2, q_3, q_4, q_5, q_7\}, \{q_0, q_1, q_2, q_5, q_6, q_7\}\}$,
the set of all sets in K' containing a common element with s'.

Drawing of the deterministic finite automaton M':

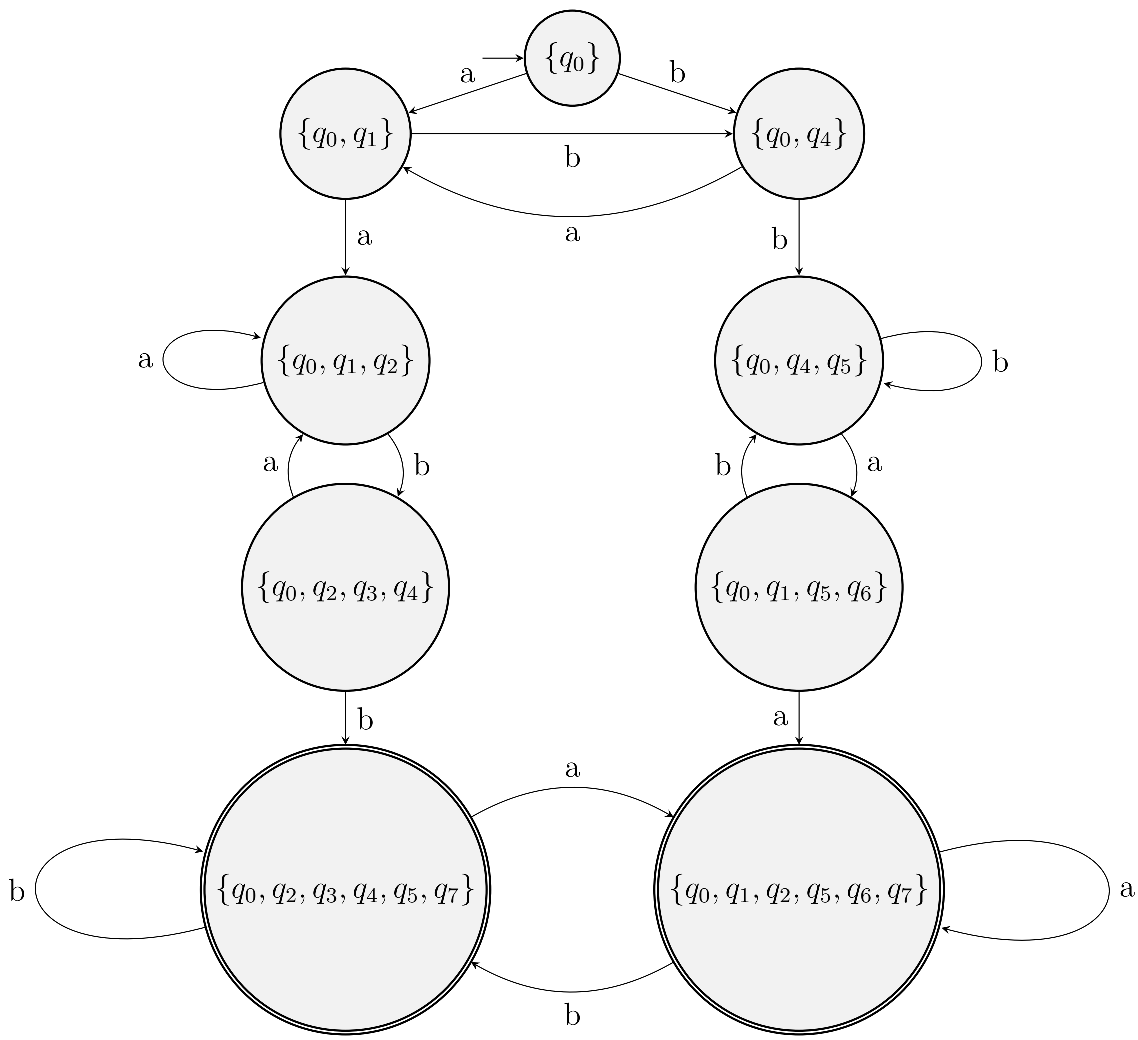

Figure 2: Deterministic Finite Automaton M'

d.

$$\begin{aligned}(\{q_0\}, bbabb) &\vdash_{M'} (\{q_0, q_4\}, babb)\\ &\vdash_{M'} (\{q_0, q_4, q_5\}, abb)\\ &\vdash_{M'} (\{q_0, q_1, q_5, q_5\}, bb)\\ &\vdash_{M'} (\{q_0, q_4, q_5\}, b)\\ &\vdash_{M'} (\{q_0, q_4, q_5\}, e)\end{aligned}$$

The string "*bbabb*" is not accepted by the DFA M' since $\{q_0, q_4, q_5\}$ is not an element of F'.

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_4, babb)\\ &\vdash_M (q_5, abb)\\ &\vdash_M (q_6, bb)\\ &\text{Machine halts.}\end{aligned}$$

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_0, babb)\\ &\vdash_M (q_0, abb)\\ &\vdash_M (q_1, bb)\\ &\text{Machine halts.}\end{aligned}$$

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_0, babb)\\ &\vdash_M (q_0, abb)\\ &\vdash_M (q_0, bb)\\ &\vdash_M (q_0, b)\\ &\vdash_M (q_0, e)\\ &\text{Not accepted.}\end{aligned}$$

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_4, babb)\\ &\vdash_M (q_5, abb)\\ &\vdash_M (q_5, bb)\\ &\vdash_M (q_5, b)\\ &\vdash_M (q_5, e)\\ &\text{Not accepted.}\end{aligned}$$

$$\begin{aligned}(q_0, bbabb) &\vdash_M (q_0, babb)\\ &\vdash_M (q_0, abb)\\ &\vdash_M (q_0, bb)\\ &\vdash_M (q_4, b)\\ &\vdash_M (q_5, e)\\ &\text{Not accepted.}\end{aligned}$$

This is not an exhaustive list but one can try every possible combination and see that "*bbabb*" is not accepted by the NFA M.

2 Question 2

a.

Assume L_2 is regular. That means $\overline{L_1}$ is also regular. By closure property of complementation of regular languages, $\overline{L_2}$ is also regular. Since $L_2 = \overline{L_1} \Rightarrow \overline{L_2} = \overline{\overline{L_1}} \Rightarrow \overline{L_2} = L_1,\ L_1$ is also regular. This means that by pumping lemma, there is an integer $p \geq 1$ such that any string $w \in L_1$ with $|w| \geq p$ can be partitioned into $w = xyz$ where $y \neq e$, $|xy| \leq p$, and $xy^iz \in L_1$ for all $i \geq 0$. Let $w = a^{p+1}b^p \in L_1$. Then w can be written as xyz, where $y = a^j$ because of the $|xy| \leq p$ constraint and $1 \leq j \leq p$. If we pump down the string by taking $i = 0$ in xy^iz, the new string w' becomes $w' = xz$. Considering the fact that the number of a's in the string was only one more than the number of b's in the string, pumping down makes the number of a's less than or equal to the number of b's, which makes $w' \notin L1$. This leads to a contradiction, therefore L_2 is not regular.

b.

The subset of L_5 where $m = n$ and $m \neq 0$ and $n \neq 0$ is equivalent to L_4. Therefore $L_4 \cup L_5 = L_5$ by set properties. L_5 is regular since it is equivalent to the language of the regular expression a^*b^*. L_6 is regular since it is a regular expression. Therefore $L_4 \cup L_5 \cup L_6 = L_5 \cup L_6$ which is regular by closure property of union of regular languages.