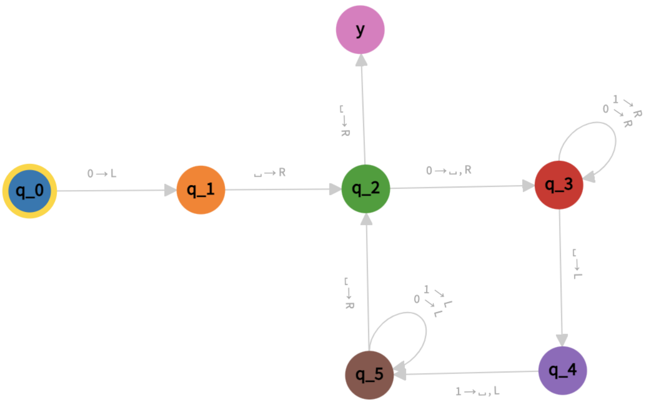
CENG280 Homework 3

Murat Bolu

March 14, 2023

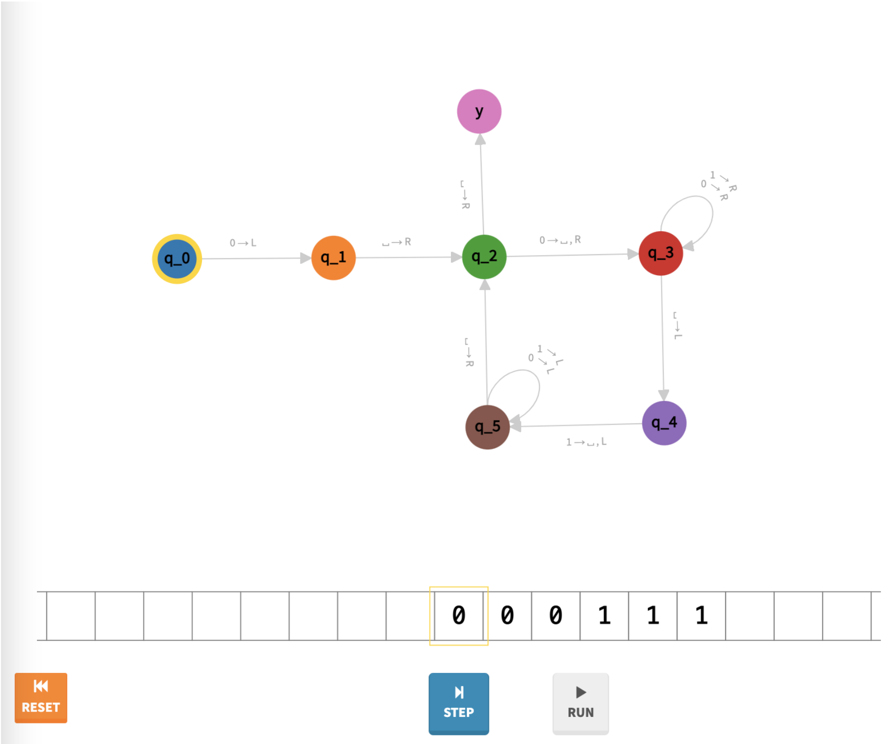
Answer 1

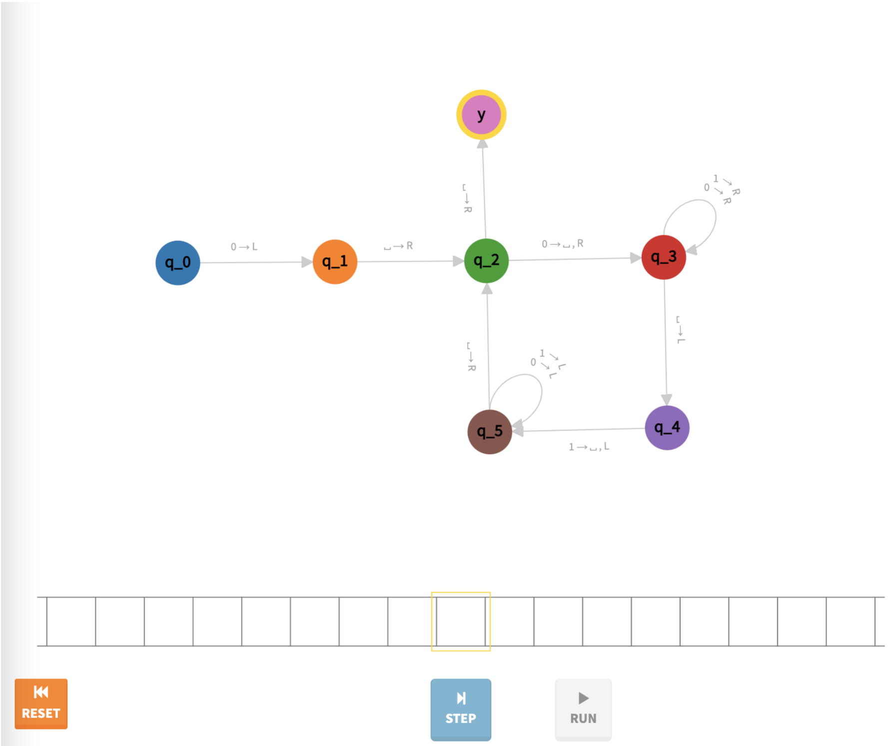


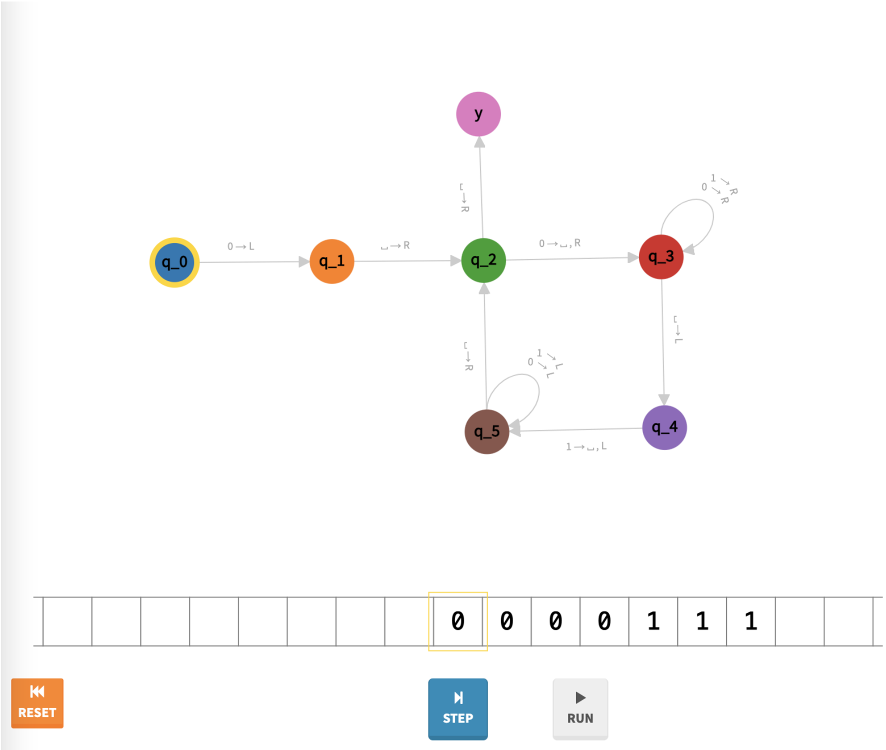
In the first Turing machine the start state is q_0. The states q_0 and q_1 are placed only to make sure that the machine doesn't accept the empty string, as $N \geq 1$. The state q_2 deletes the symbol 0 in the initial position, moves the head to the right and the machine goes to the state q_3. The state q_3 makes the head go left until the first $\sqcup$ read, moves the head to the left and the machine goes to the state q_4. The state q_4 deletes the symbol 1 in the final position, moves the head to the left and the machine goes to the state q_5. The state q_5 makes the head go left until the first $\sqcup$ read, moves the head to the left and the machine goes to the state q_2. When the string is empty and the machine is in the state q_2, the machine accepts the input string and halts in the state y.

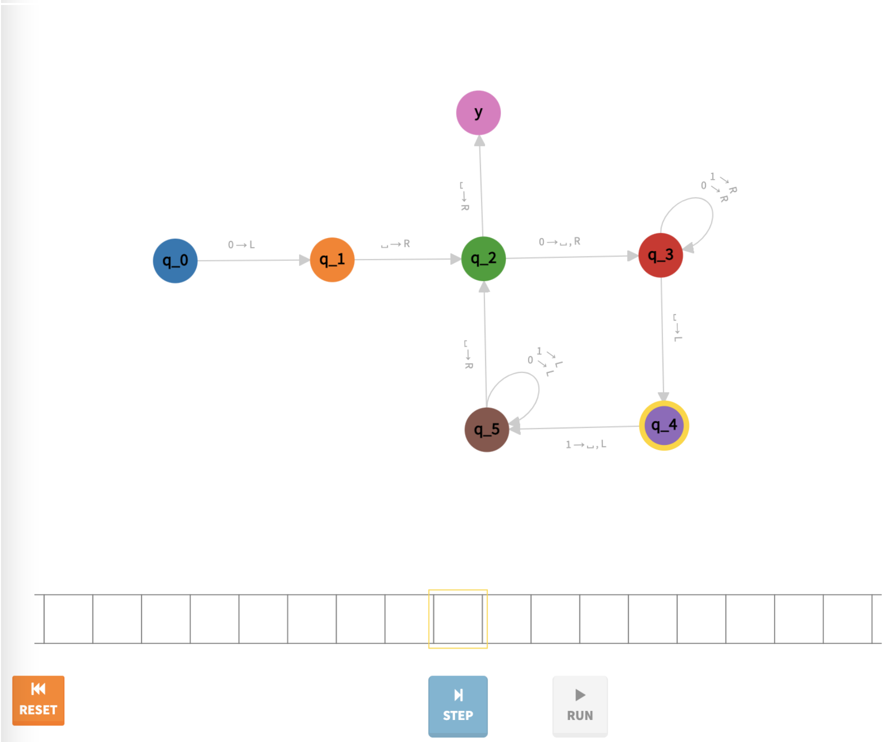
```
input: '000111'
blank: ' '
start state: q_0
table:
  q_0:
    0: {L: q_1}
  q_1:
    ' ': {R: q_2}
  q_2:
    0: {write: ' ', R: q_3}
    ' ': {R: y}
  q_3:
    0: R
    1: R
    ' ': {L: q_4}
  q_4:
    1: {write: ' ', L: q_5}
  q_5:
    0: L
    1: L
    ' ': {R: q_2}
  y:
```

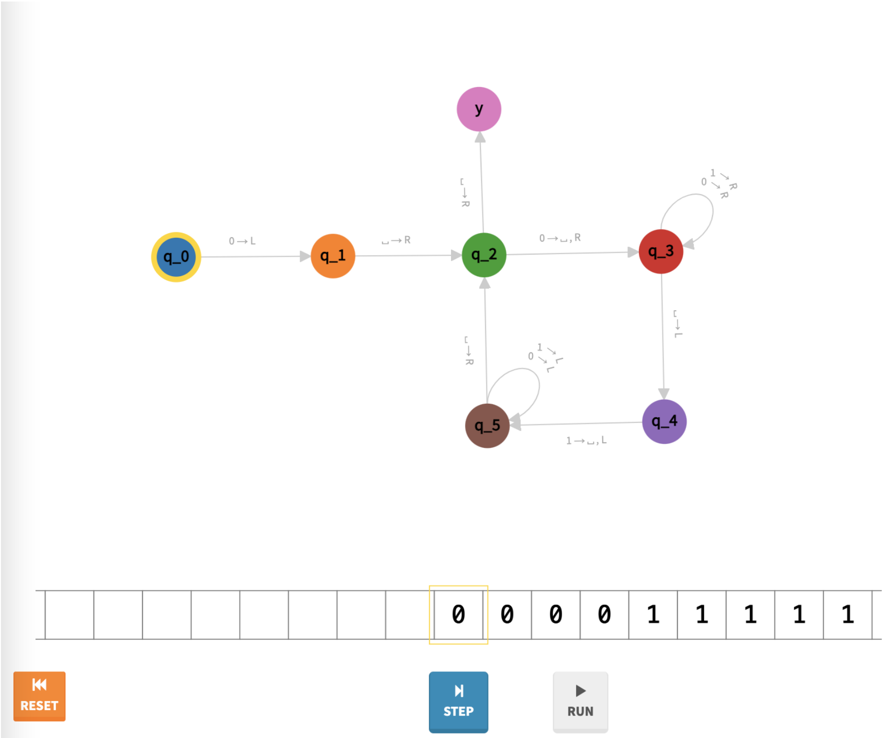
```
input: ’0001110’
blank: ’ ’
start state: q_0
table:
q_0:
0: {L: q_1}
q_1:
’ ’: {R: q_2}
q_2:
0: {write: ’ ’, R: q_3}
’ ’: {R: y}
q_3:
0: R
1: R
’ ’: {L: q_4}
q_4:
1: {write: ’ ’, L: q_5}
q_5:
0: L
1: L
’ ’: {R: q_2}
y:
```

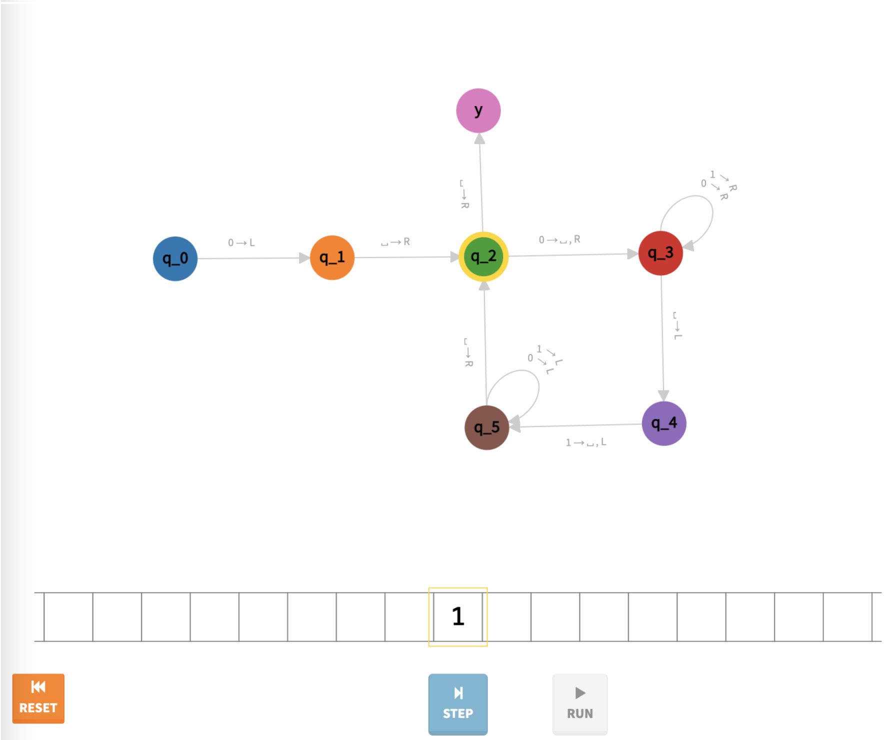

y
q_0
0 → L
q_1
␣ → R
q_2
0 → ␣, R
q_3
q_4
1 → ␣, L
q_5
0 0 0 1 1 1
RESET
STEP
RUN

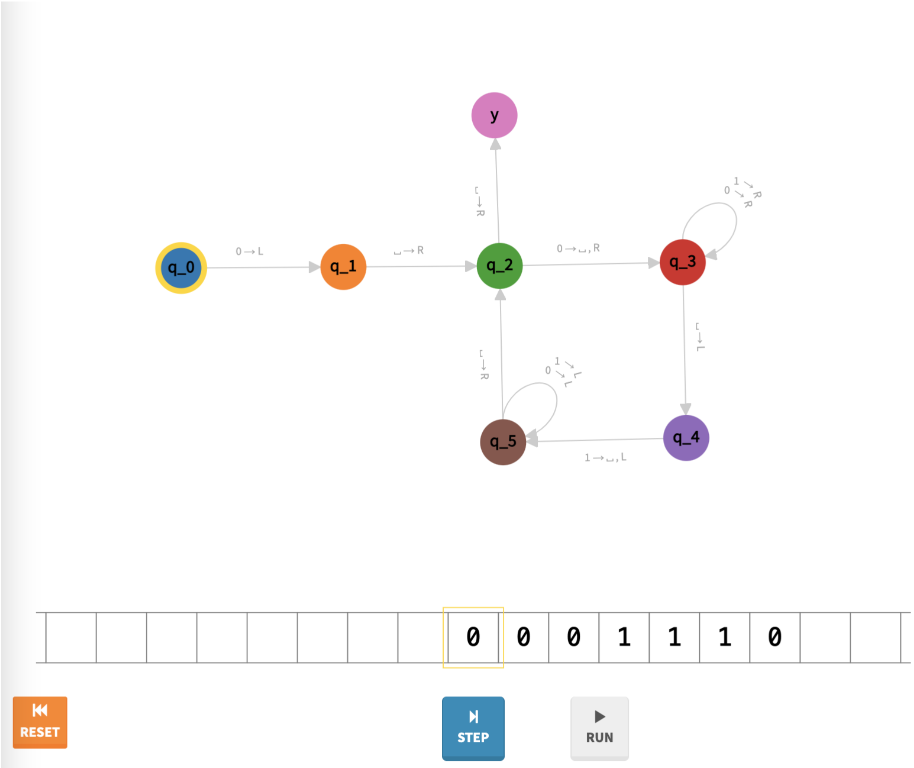

y
q_0
0 → L
q_1
␣ → R
q_2
0 → ␣, R
q_3
q_4
1 → ␣, L
q_5
RESET
STEP
RUN

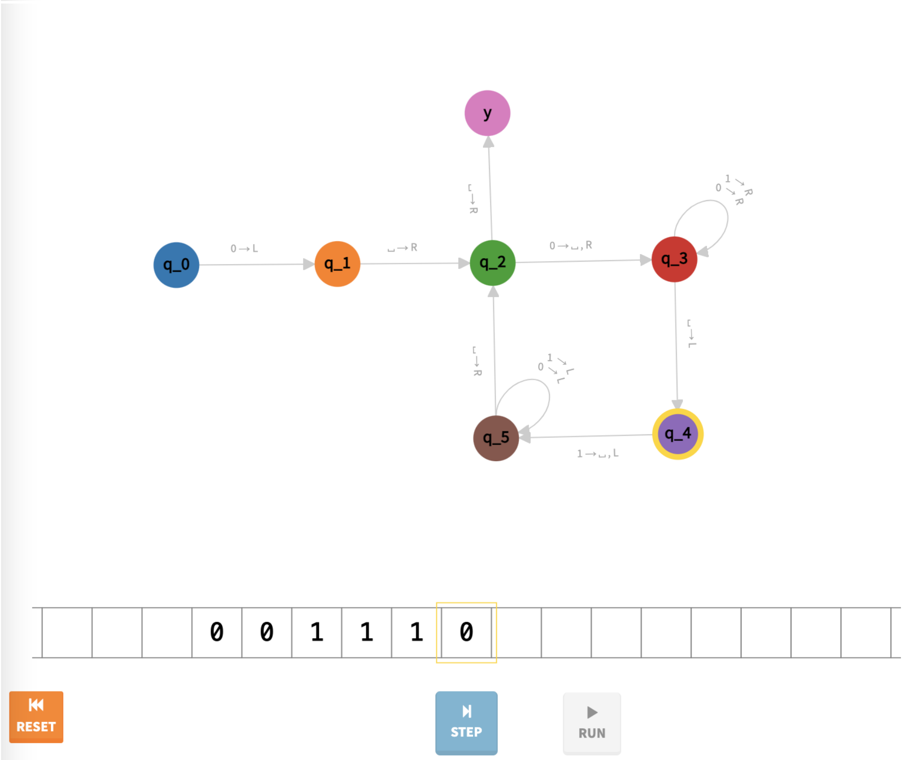

y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
1 → ␣, L
0 0 0 0 1 1 1
RESET
STEP
RUN

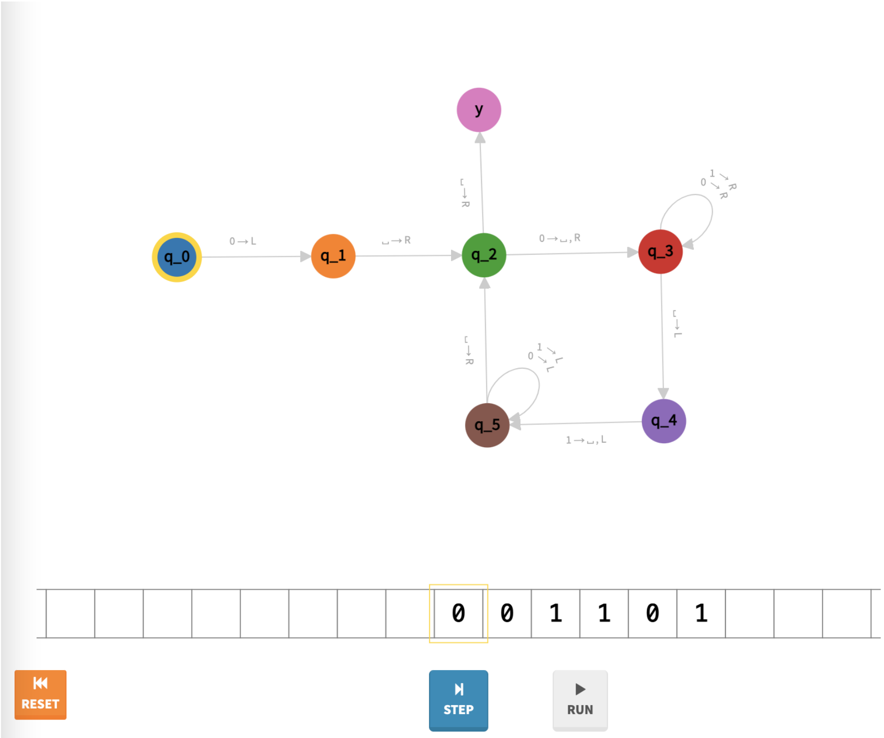

y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
1 → ␣, L
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
0 → ␣, R
1 → ␣, L
0 0 0 0 1 1 1 1 1
RESET
STEP
RUN

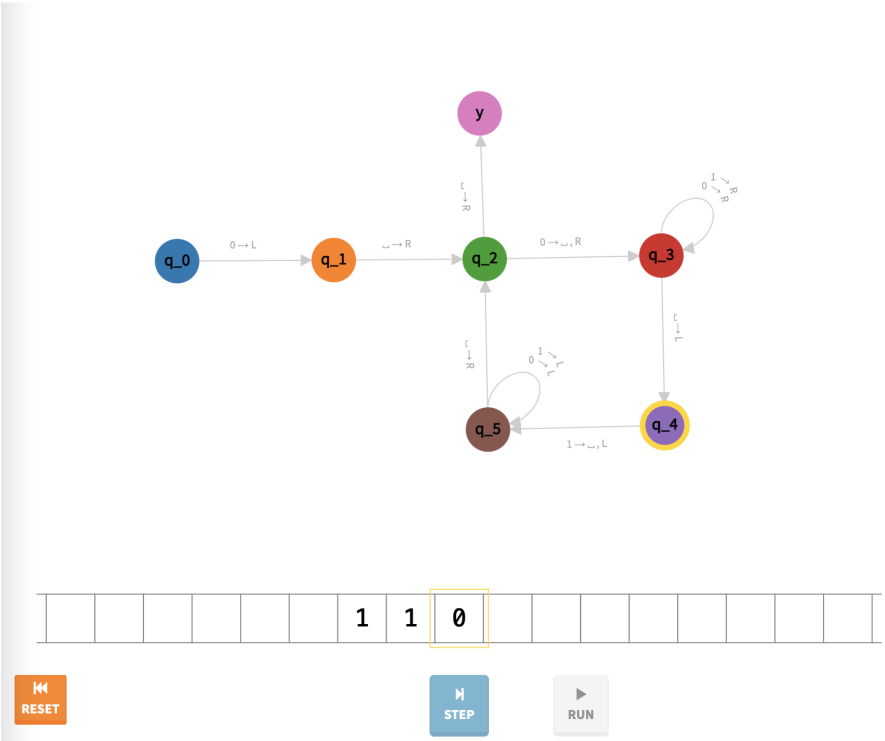

y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
0 → ␣, R
1 → ␣, L
1
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
␣ → R
0 → ␣, R
1 → R
0 → R
␣ → L
1 → ␣, L
1 → L
0 → L
␣ → R
0 0 0 1 1 1 0
RESET
STEP
RUN

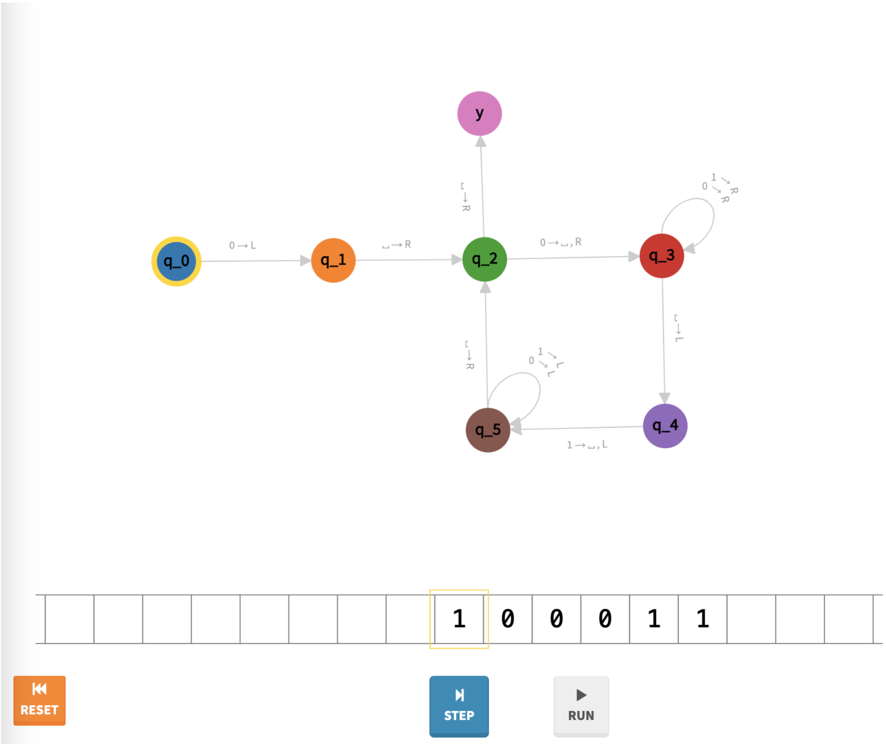

y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
␣ → R
0 → ␣, R
1 → R
0 → R
␣ → L
1 → ␣, L
1 → L
0 → L
␣ → R
0 0 1 1 1 0
RESET
STEP
RUN

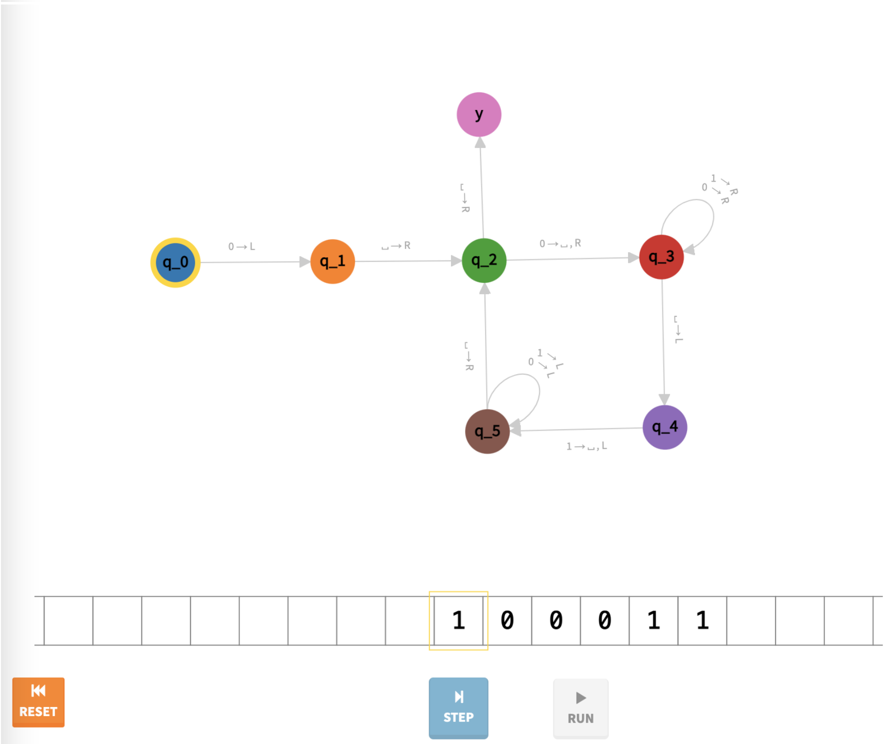


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
␣ → R
0 → ␣, R
1 → ␣, L
0 0 1 1 0 1
RESET
STEP
RUN

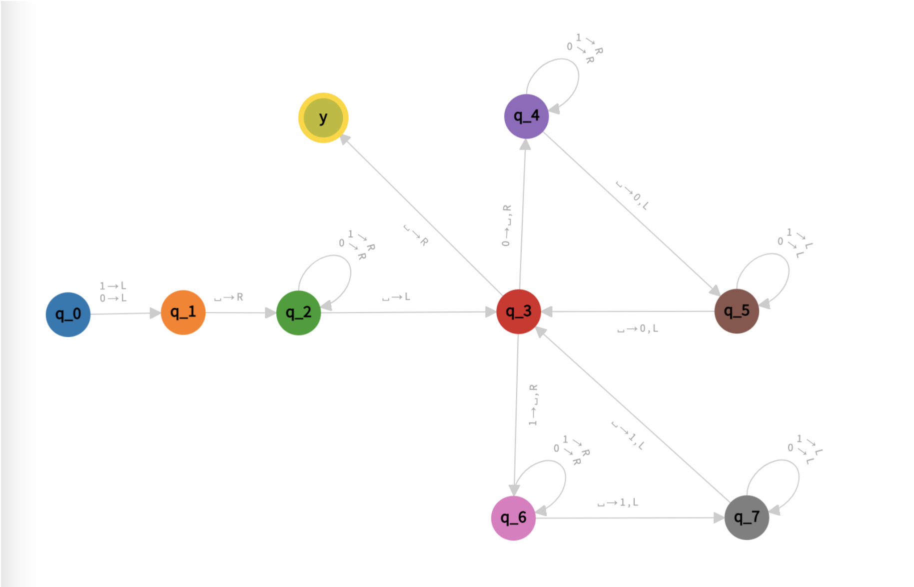


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
␣ → R
0 → ␣, R
1 → ␣, L
1 1 0
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
⊔ → R
0 → ⊔, R
1 → ⊔, L
1 0 0 0 1 1
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
0 → L
⊔ → R
0 → ⊔, R
1 → ⊔, L
1 0 0 0 1 1
RESET
STEP
RUN

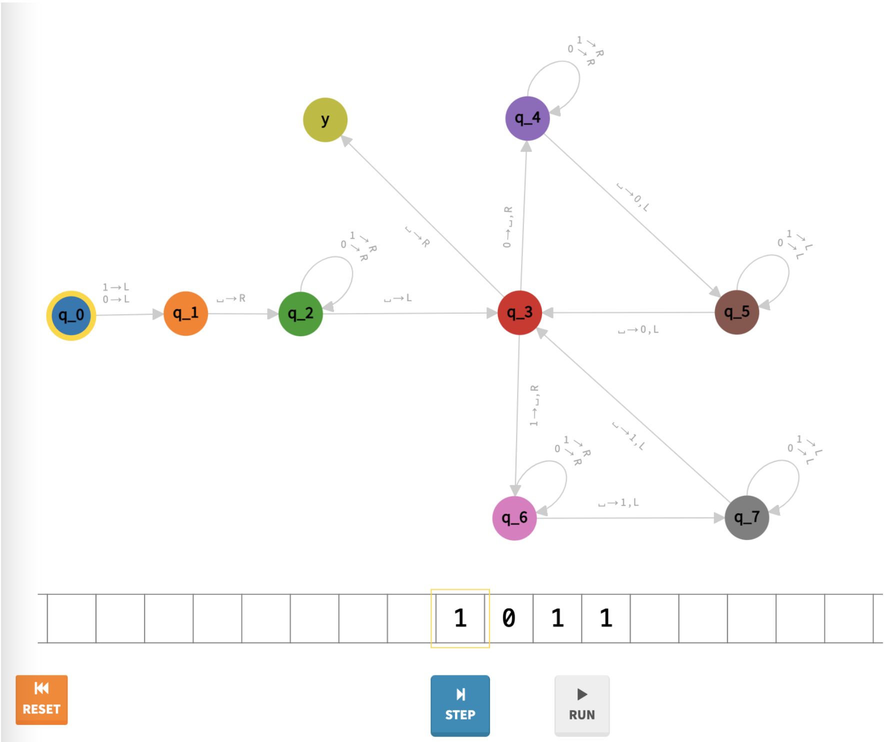
Answer 2

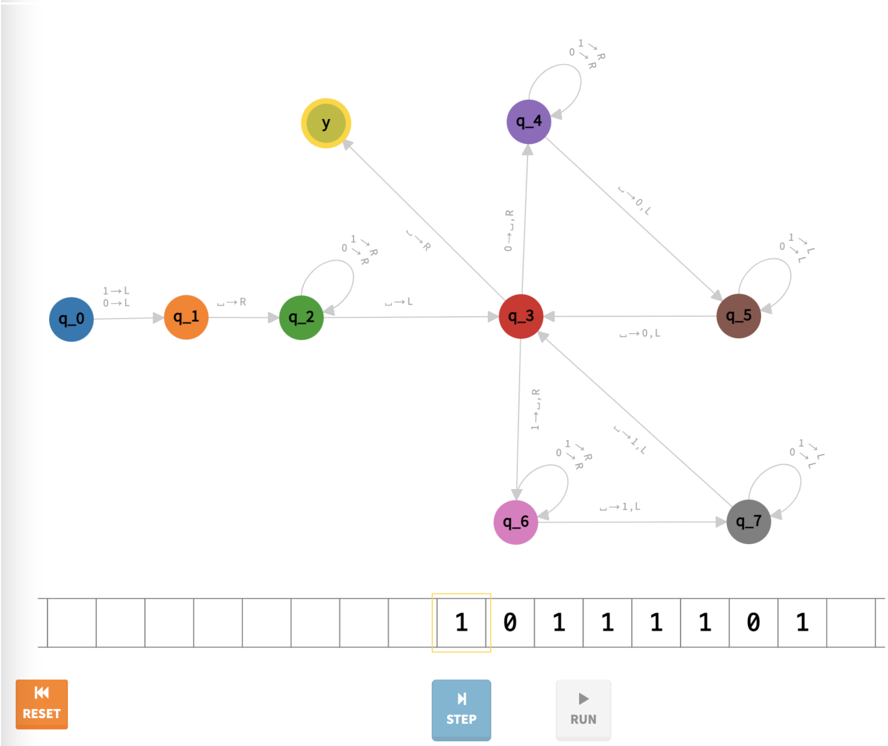


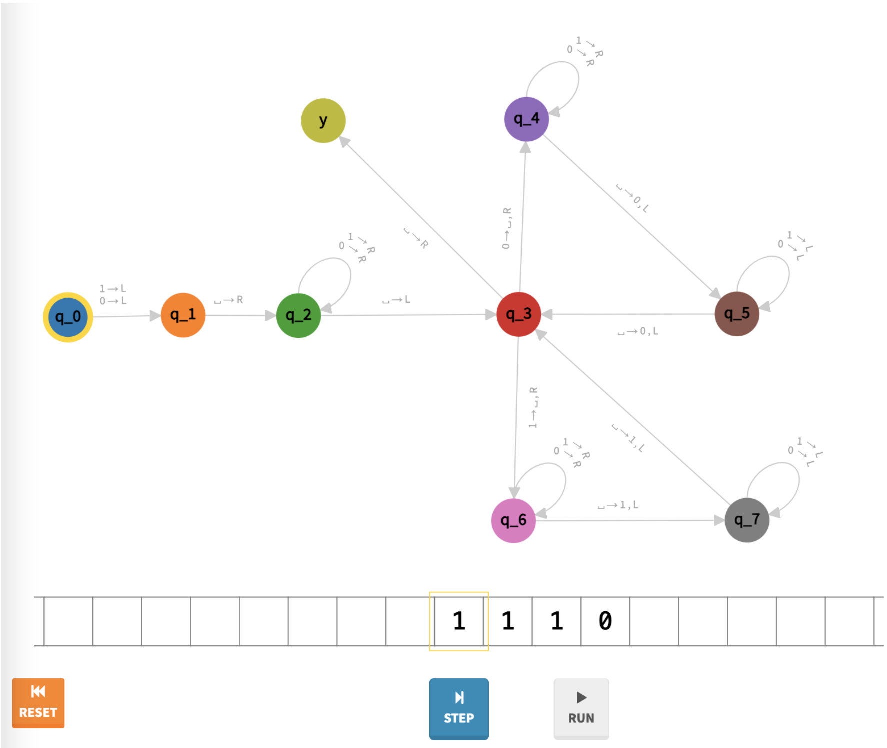
In the second Turing machine the start state is q_0. The states q_0 and q_1 are placed only to make sure that the machine doesn't accept the empty string, as $e \notin \{0,1\}^+$. The state q_2 makes the head go right until the first $\sqcup$ read, moves the head to the left and the machine goes to the state q_3. The state q_3 deletes the symbol 0 (or 1), moves the head to the right and the machine goes to the state q_4 (or q_6). The state q_4 (or q_6) moves the head to the right until the first $\sqcup$ read, writes the symbol 0 (or 1), moves the head to the left and the machine goes to the state q_5 (or q_7). The state q_5 (or q_7) moves the head to the left until the first $\sqcup$ seen, writes the symbol 0 (or 1), and the machine goes the state q_3. When the head reads $\sqcup$ and the machine is in the state q_3, the machine halts in the state y.

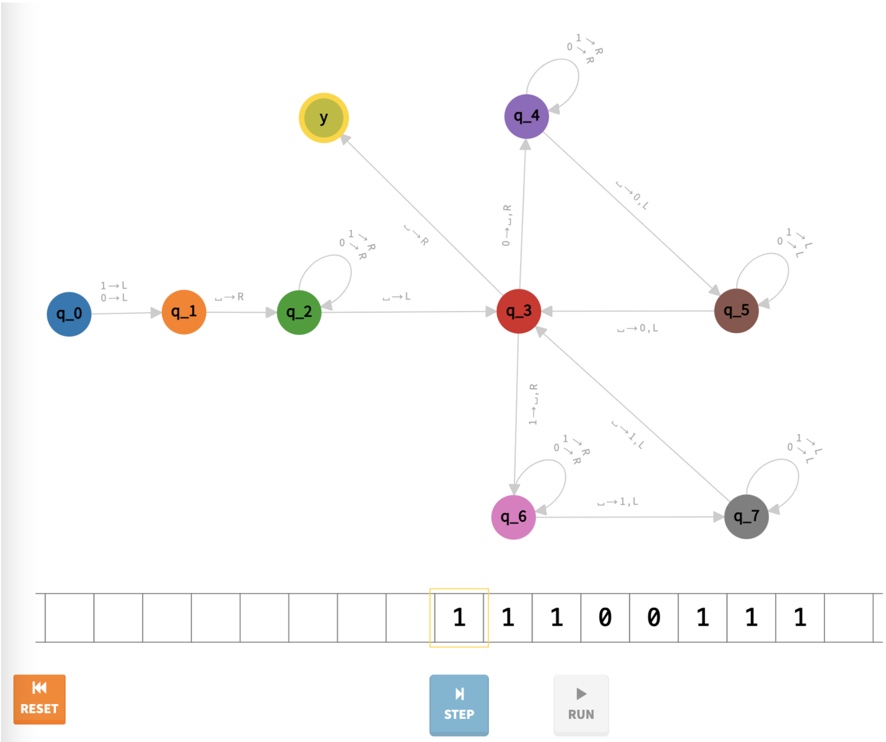
```
input: '1010001'
blank: ' '
start state: q_0
table:
  q_0:
    0: {L: q_1}
    1: {L: q_1}
  q_1:
    ' ': {R: q_2}
  q_2:
    0: R
    1: R
    ' ': {L: q_3}
  q_3:
    0: {write: ' ', R: q_4}
    1: {write: ' ', R: q_6}
    ' ': {R: y}
  q_4:
    0: R
    1: R
    ' ': {write: 0, L: q_5}
  q_5:
    0: L
    1: L
    ' ': {write: 0, L: q_3}
  q_6:
    0: R
    1: R
    ' ': {write: 1, L: q_7}
  q_7:
    0: L
    1: L
    ' ': {write: 1, L: q_3}
  y:
```

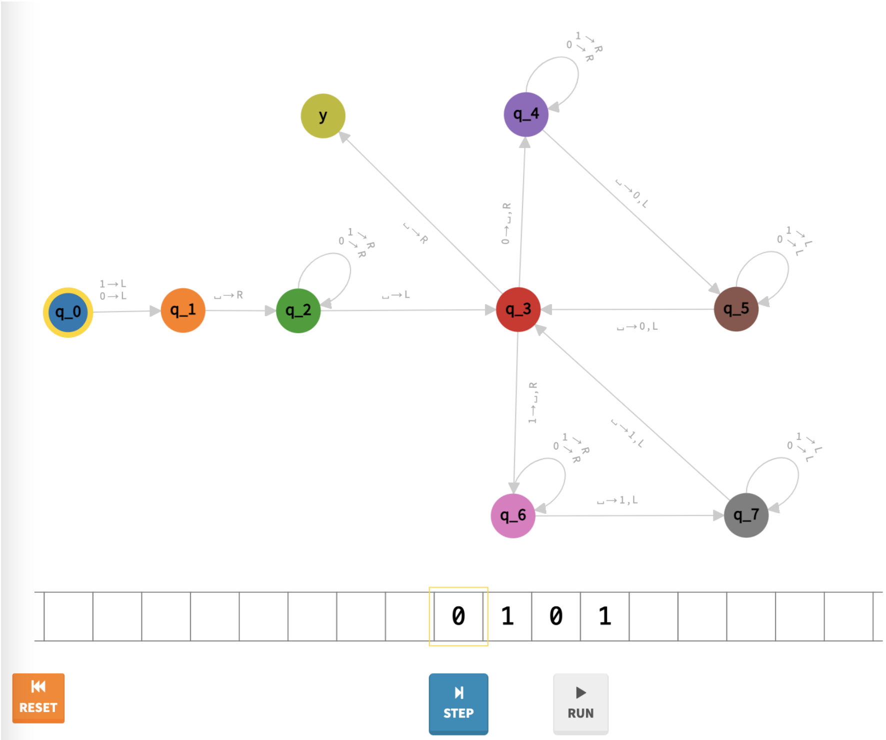
```
input: '1011'
blank: ' '
start state: q_0
table:
q_0:
0: L: q_1
1: L: q_1
q_1:
' ': R: q_2
q_2:
0: R
1: R
' ': L: q_3
q_3:
0: write: ' ', R: q_4
1: write: ' ', R: q_6
' ': R: y
q_4:
0: R
1: R
' ': write: 0, L: q_5
q_5:
0: L
1: L
' ': write: 0, L: q_3
q_6:
0: R
1: R
' ': write: 1, L: q_7
q_7:
0: L
1: L
' ': write: 1, L: q_3
y:
```

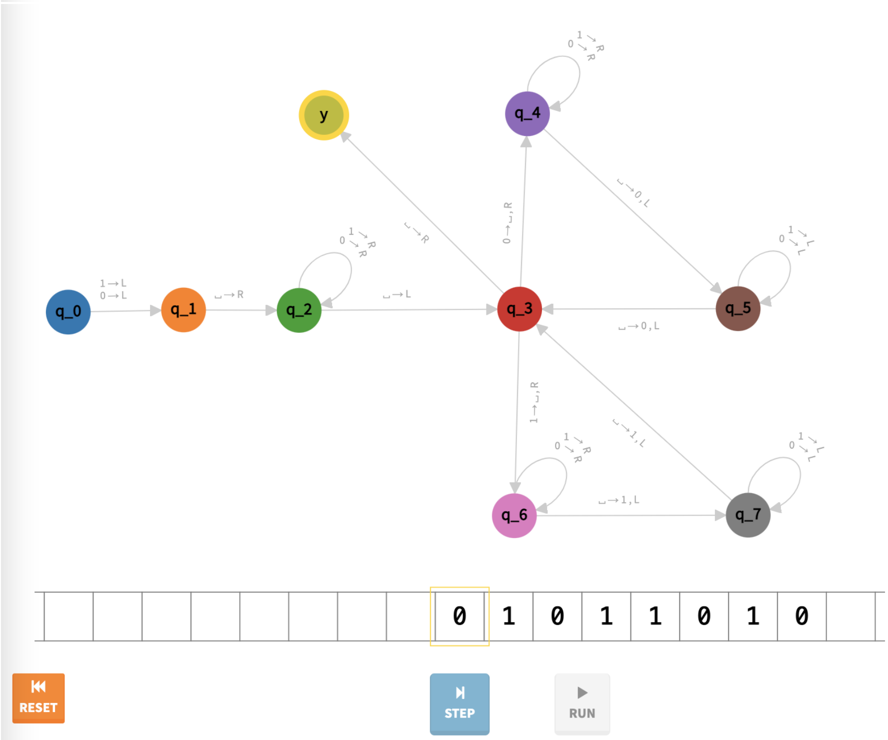

q_0
q_1
q_2
q_3
q_4
q_5
q_6
q_7
y
1 0 1 1
RESET
STEP
RUN

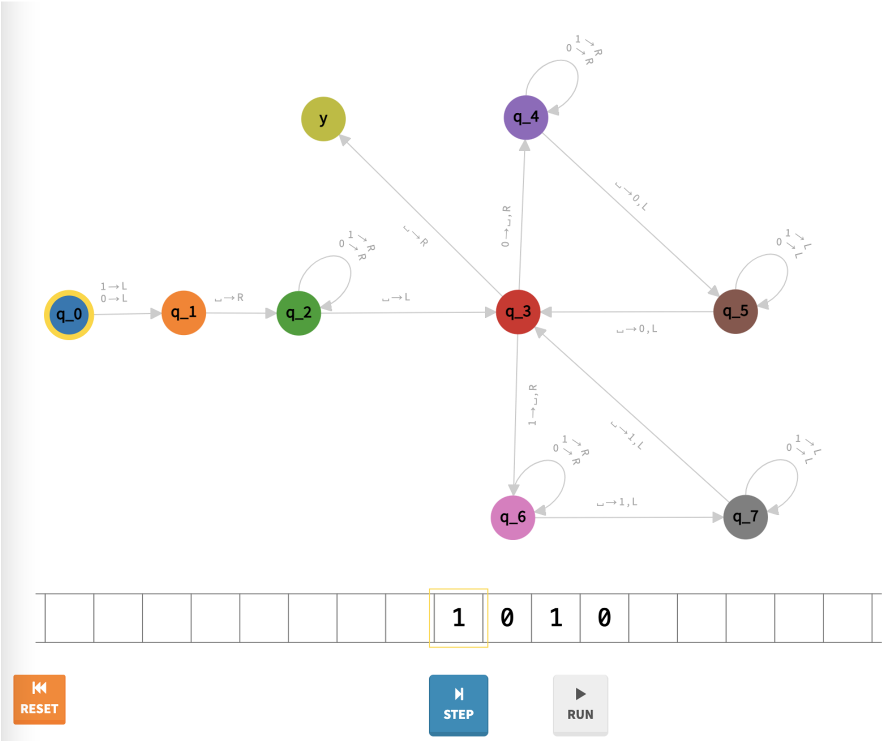

q_0
q_1
q_2
q_3
q_4
q_5
q_6
q_7
y
1 0 1 1 1 1 0 1
RESET
STEP
RUN

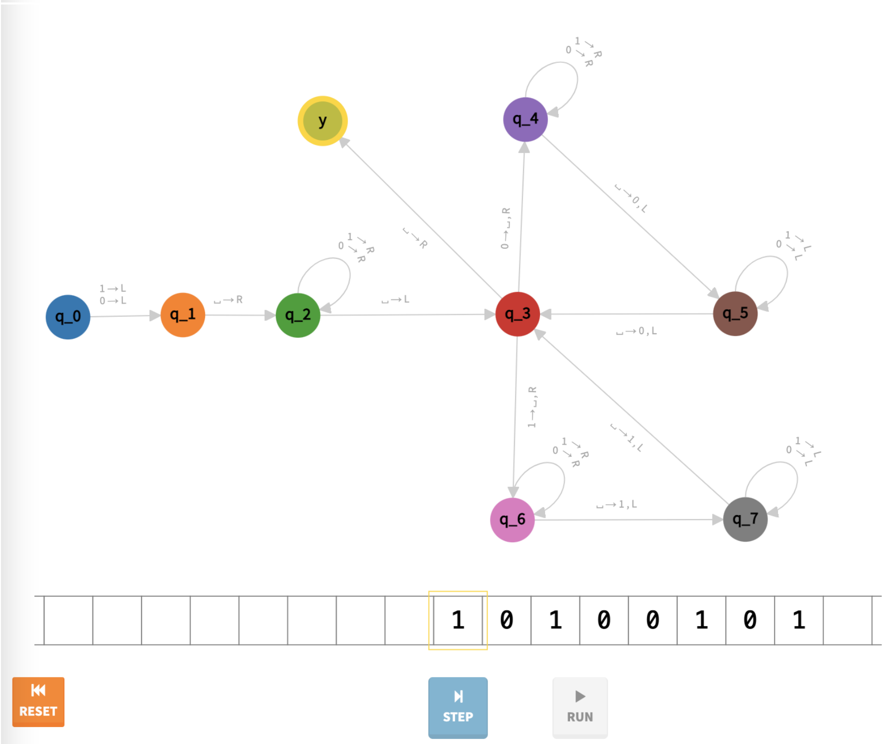

y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 1 1 0
RESET
STEP
RUN

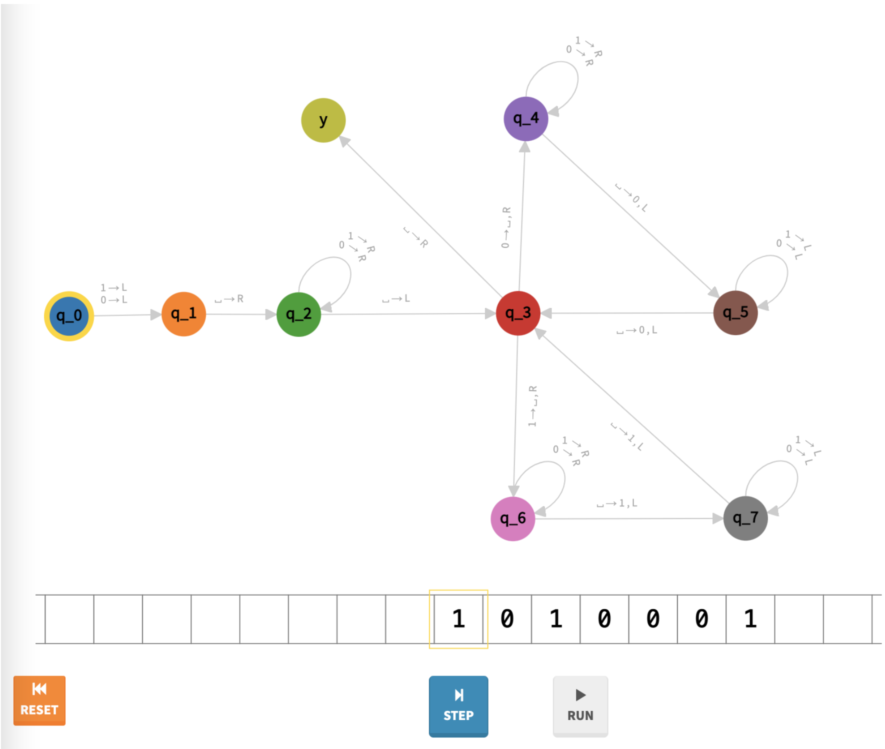

y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 1 1 0 0 1 1 1
RESET
STEP
RUN

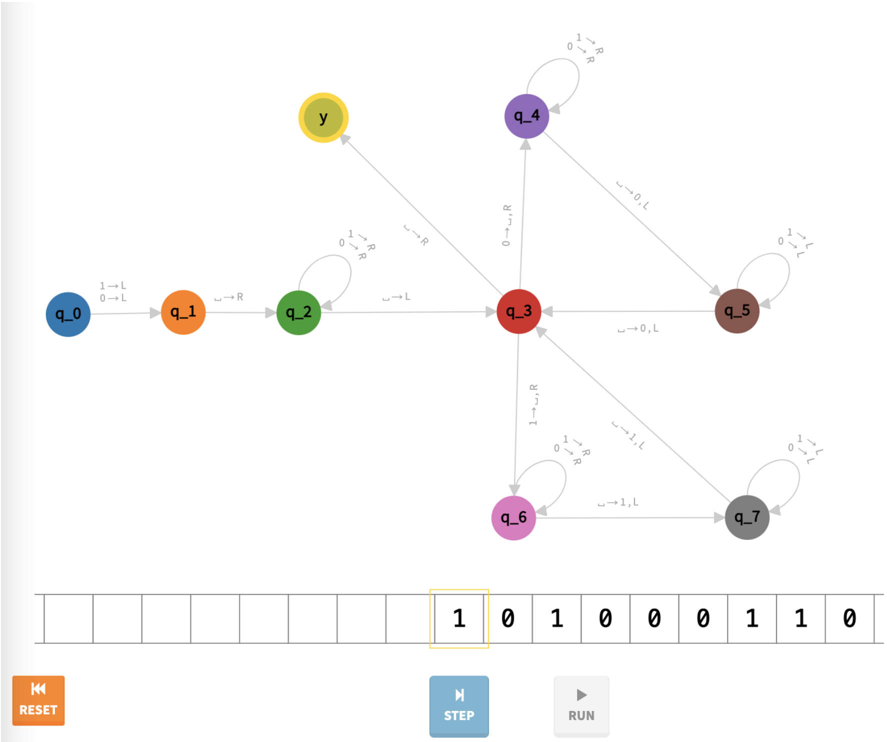

y
q_4
q_0
1 → L
0 → L
q_1
␣ → R
q_2
␣ → L
q_3
␣ → 0, L
q_5
0 → ␣, R
1 → ␣, R
␣ → 1, L
q_6
q_7
0
1
0
1
RESET
STEP
RUN

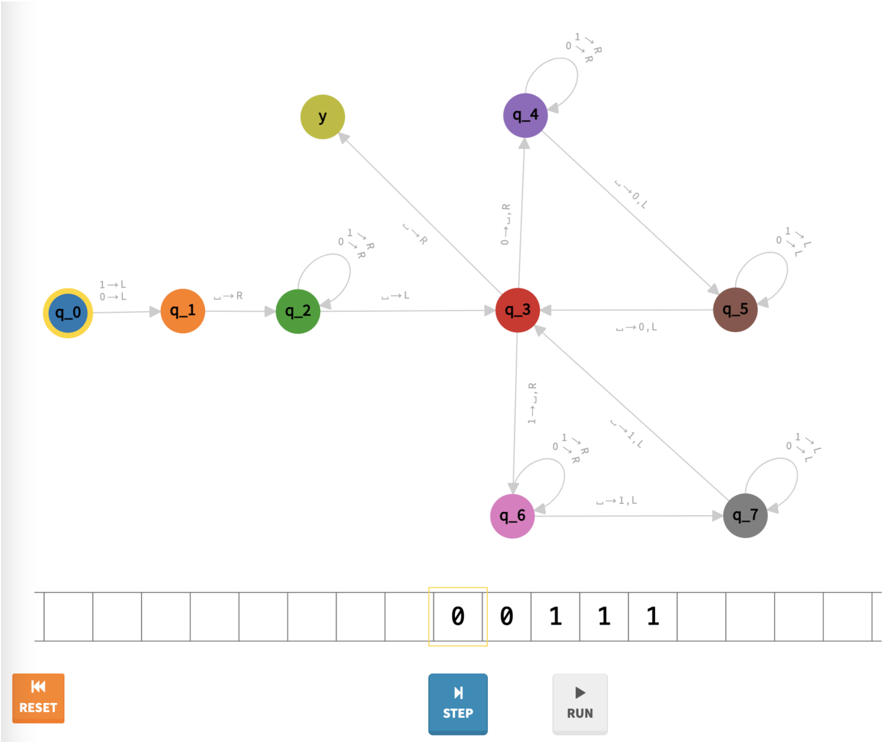

y
q_4
q_0
1 → L
0 → L
q_1
␣ → R
q_2
␣ → L
q_3
␣ → 0, L
q_5
0 → ␣, R
1 → ␣, R
␣ → 1, L
q_6
q_7
0
1
0
1
1
0
1
0
RESET
STEP
RUN

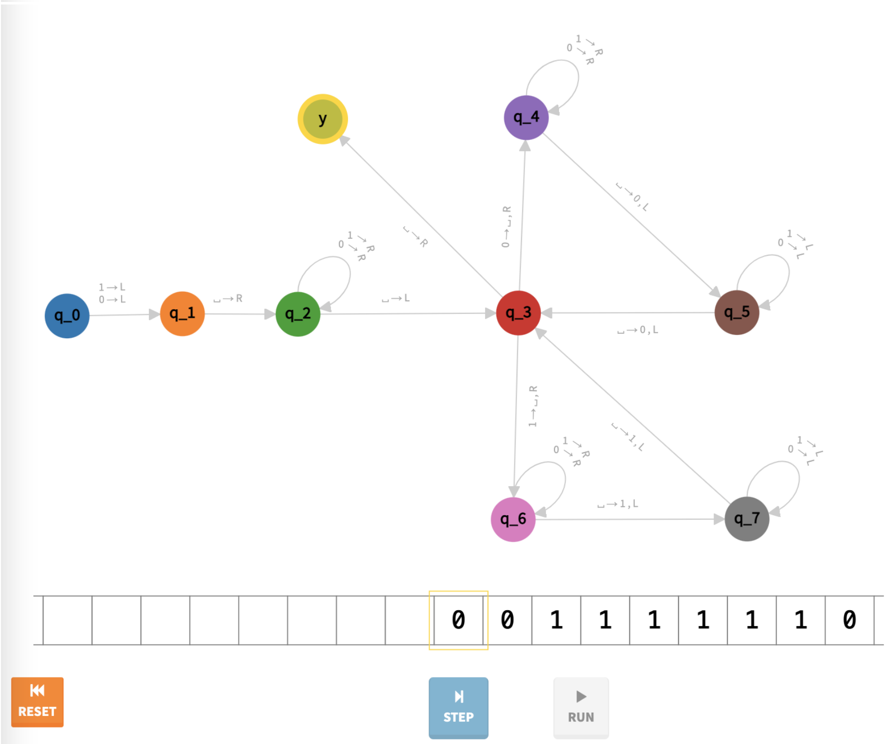

y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 0 1 0
RESET
STEP
RUN


y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 0 1 0 0 1 0 1
RESET
STEP
RUN


y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 0 1 0 0 0 1
RESET
STEP
RUN


y
q_4
q_0
q_1
q_2
q_3
q_5
q_6
q_7
1 0 1 0 0 0 1 1 0
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
q_6
q_7
0 0 1 1 1
RESET
STEP
RUN


y
q_0
q_1
q_2
q_3
q_4
q_5
q_6
q_7
0 0 1 1 1 1 1 1 0
RESET
STEP
RUN

Answer 3

Let M be a Turing machine with a two-dimensional tape formally defined by the quintuple $(K, \Sigma, \delta, s, H)$ where K is a finite set of states, Σ is an alphabet, δ is the transition function, s is the initial state, and H is the set of halting states. Let there be two more arrows, $\uparrow$ and $\downarrow$, which will denote the head moving up or down on the tape. The tape goes to infinity for four sides, its center is marked with $\triangleright$ and it possesses similar properties with the usual $\triangleright$.

$$
\begin{gathered}
\sqcup \in \Sigma,\ \triangleright \in \Sigma,\ \uparrow \notin \Sigma,\ \rightarrow \notin \Sigma,\ \downarrow \notin \Sigma,\ \leftarrow \notin \Sigma \\
\delta :\ (K \setminus H) \times \Sigma\ \rightarrow\ K \times (\Sigma \cup \{\uparrow, \rightarrow, \downarrow, \leftarrow\}) \\
\delta(q_0, \triangleright) = (q_1, p) \Rightarrow p = \rightarrow,\ \forall q_0 \in K \setminus H \\
\delta(q_0, p_0) = (q_1, p_1) \Rightarrow p_1 \neq \triangleright,\ \forall q_0 \in K \setminus H \wedge p_0 \in \Sigma \\
s \in K \\
H \subseteq K
\end{gathered}
$$

Each cell can be represented with a pair of integers. Assume it is similar to a Cartesian coordinate system with pairs (x, y) where x increases in rightward direction and y increases in upward direction. Therefore a configuration for M is an element of $K \times \mathbb{Z} \times \mathbb{Z} \times F$ where K is the set of states, $\mathbb{Z}$ is the set of integers, and F is the set of all functions with the properties $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \Sigma$ and $f(0,0) = \triangleright\ \forall f \in F$. F is defined in a way such that all elements of F are functions with finitely many symbols different from $\sqcup$ in their ranges, more formally:

$$
\begin{gathered}
\exists X \in \mathbb{Z},\ \forall y \in \mathbb{Z},\ \forall f \in F,\ (|a| > X \Rightarrow f(a, y) = \sqcup \wedge f(-a, y) = \sqcup) \\
\exists Y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ \forall f \in F,\ (|a| > Y \Rightarrow f(x, a) = \sqcup \wedge f(x, -a) = \sqcup)
\end{gathered}
$$

K represents the current state, $\mathbb{Z} \times \mathbb{Z}$ represent the position of the head, and F represent the contents of the tape.
A step of computation, denoted with $(q_0, x_0, y_0, f_0) \vdash_M (q_1, x_1, y_1, f_1)$ is valid if and only if $\delta(q_0, f_0(x_0, y_0)) = (q_1, p)$ and

$$
\begin{aligned}
p = \uparrow &\Leftrightarrow y_1 = y_0 + 1 \wedge x_1 = x_0 \wedge f_1 = f_0 \\
p = \rightarrow &\Leftrightarrow x_1 = x_0 + 1 \wedge y_1 = y_0 \wedge f_1 = f_0 \\
p = \downarrow &\Leftrightarrow y_1 = y_0 - 1 \wedge x_1 = x_0 \wedge f_1 = f_0 \\
p = \leftarrow &\Leftrightarrow x_1 = x_0 - 1 \wedge y_1 = y_0 \wedge f_1 = f_0 \\
p \notin \{\uparrow, \rightarrow, \downarrow, \leftarrow\} &\Leftrightarrow x_1 = x_0 \wedge y_1 = y_0 \wedge f_1(x_1, y_1) = p
\end{aligned}
$$

Let $\vdash_M^*$ be the reflexive transitive closure of $\vdash_M$. The language accepted by this machine is as follows. Let the machine M have exactly two halting states, y and n, and any other number of non-halting states. Let s be the starting state. Let the string w start from the position $(2, 0)$ and go leftwards. That is, $\forall i \in \mathbb{N},\ 2 \leq i \leq |w| + 2 \Rightarrow w_{i-2} = f(i, 0)$ where $f \in F$ is the function defining the contents of the tape. Let the head start in the position $(1, 0)$. Let the machine M halt for all inputs, that is, it is a decider, an algorithm. The machine M accepts this string if and only if it halts in the state y and rejects otherwise. More specifically:

$$w \in L(M) \Leftrightarrow (s, 1, 0, F) \vdash_M^* (y, x_n, y_n, F_n)$$

Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{N}$ be a bijection mapping pairs of integers to natural numbers. Let $g(0,0) = 0$. We can map the two-dimensional tape to a semi-infinite one-dimensional tape by using g. Let M_s be the standard Turing machine with semi-infinite tape that will simulate M which has a two-dimensional tape. Let M_s be formally defined by the quintuple $(K_s, \Sigma_s, \delta_s, s_s, H_s)$, where

$$\begin{gathered} K_s = K \\ \Sigma_s = \Sigma \\ \delta_s(q_0, p_0) = \delta(q_0, p_0) = (q_1, p_1) \Leftrightarrow p_1 \notin \{\uparrow, \rightarrow, \downarrow, \leftarrow\} \\ s_s = s \\ H_s = H \end{gathered}$$

If $p_1 \in \{\uparrow, \rightarrow, \downarrow, \leftarrow\}$, then the machine M_s moves the head to the appropriate position given by the function g. Therefore, the machine M can be simulated by the standard Turing machine M_s.