Elif Yılmaz. Heuristic search algorithms to detect collusive opportunities in deregulated electricity markets, 2020.

Appendix A Nash Model

In this appendix, the details of the Nash model introduced in Section 4.2 is demonstrated. In deregulated electricity markets, since generators compete with each other, each generator maximizes its profit given the other generators' bid states as fixed. This selection leads to a Nash state which is regarded as the best solution in its neighborhood. Let us denote $b^{\mathbb{N}} = (b_1^{\mathbb{N}}, ...b_i^{\mathbb{N}}..., b_n^{\mathbb{N}})$ bids at equilibrium, $b_{-i}^{\mathbb{N}} = (b_1^{\mathbb{N}}, ...b_i..., b_n^{\mathbb{N}})$ where all bids but $Genco_i$'s bids are fixed to Nash, $b_{-ik}^{\mathbb{N}} = (b_1^{\mathbb{N}}, ...Bid_{ik}..., b_n^{\mathbb{N}})$ is almost alike $b_{-i}^{\mathbb{N}}$ but $Genco_i$'s bid is set to Bid_{ik} while $r_i^{b^{\mathbb{N}}}$, $r_i^{b_{-i}^{\mathbb{N}}}$ and $r_i^{b_{-ik}^{\mathbb{N}}}$ correspond to payoffs at $Genco_i$, respectively. In order to find the Nash equilibrium, we need to satisfy the following condition.

$$r_i^{b^{\mathbb{N}}} \geq \max_{Bid_{ik}|k \in K_i} \{r_i^{b_{-i}^{\mathbb{N}}}\} \qquad \forall i \in GN \tag{23a}$$

$$\Leftrightarrow r_i^{b^{\mathbb{N}}} \geq r_i^{b_{-ik}^{\mathbb{N}}} \qquad \forall i \in GN, \forall k \in K_i \tag{23b}$$

where the first constraint points out the real Nash condition and the second constraint in (23b) transforms the first complex equation in terms of inequalities over k indices. Yet, this equation cannot be found directly, either. As a consequence, two different constraint sets are defined for the right and the left hand side of constraint (23b). At first, *the left hand side constraints* are shown below:

$$
\begin{aligned}
&(13) - (19) && \forall i \in GN \\
&(2) - (7), \text{RF1 dual feasibility constraints}, \\
&\text{Lower level RF2 constraints or RF1 Strong duality constraint} \\
&\text{Change every variable name with } variablename^{\mathbb{N}} && && (24)\\
&r_i^{\mathbb{N}} = P_i^{\mathbb{N}} LMP_i^{\mathbb{N}} - P_i^{\mathbb{N}} C_i = P_i^{\mathbb{N}} \bar{b}_i^{\mathbb{N}} + P_i^{max} \phi i - P_i^{\mathbb{N}} C_i && \forall i \in GN && (25)
\end{aligned}
$$

Constraint (24) is about changing variable names, such as $\bar{b}_i$ to $\bar{b}_i^{\mathbb{N}}$. Constraint (25) highlights the left hand side of (23b).

Secondly, *the right hand side constraints* are presented as follows:

$$
\begin{aligned}
&(13) - (19) && \forall i \in GN \\
&(2) - (7), \text{RF1 dual feasibility constraints}, \\
&\text{Lower level RF2 constraints or RF1 Strong duality constraint} \\
&\text{Change every variable name with } variablename^{i,k} && (26)\\
&r_i^{b^{\mathbb{N}}_{-ik}} = P_i^{i,k} LMP_i^{i,k} - P_i^{i,k} C_i = P_i^{i,k} \bar{b}_i^{i,k} + P_i^{max} \phi_i - P_i^{i,k} C_i && (27)\\
&b_j = b_j^N \quad j \neq i, \bar{b}_i = Bid_{ik}, B_{ik} = 1 && (28)
\end{aligned}
$$

In this model, constraint (26) is only a bit different from the left hand side constraints by its definition. Constraint (27) refers to the right-hand side of constraint (23b). Constraint (28) defines b^N_{-ik} at a specific i and k. After introducing these constraints, the EPEC model capturing the Nash state can be formulated as follows:

$$\text{The left hand side constraints, (23b)} \tag{29}$$

$$\text{The right hand side constraints} \qquad \forall i \in GN, \forall k \in K_i \tag{30}$$