#### JPMorgan Chase

#### **Machine Learning Center of Excellence**

#### **Summer Associate Internship Assessment Answers**

Question 3, Part 2

Bayesian Intervention Optimization for Causal

Discovery

(Wang et.al. (2024) Paper)

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3 February 2025

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#### A. Consider this paper: Bayesian Intervention Optimization for Causal Discovery

#### A.1 Summary of the paper

The article titled "Bayesian Intervention Optimization for Causal Discovery" written by Wang and colleagues presents a Bayesian optimization-based strategy for selecting interventions in the process of causative discovery. The strategy that has been presented gives emphasis to treatments that maximize the Probability of Decisive and Correct Evidence (PDC), as opposed to concentrating on maximizing the amount of information that is gathered. Bayes variables are applied in order to determine which interventions are the outcomes that have the greatest potential for success. Iteratively refining intervention selection is accomplished through the utilization of Monte Carlo estimation and Bayesian updates according to this approach.

A comparison is made between the suggested method and random intervention selection and information gain-based strategies throughout the empirical validation process, which is carried out on synthetic datasets. Based on the findings, it appears that Bayesian intervention optimization calls for a reduced number of interventions in order to arrive at causal conclusions. On the other hand, the method includes processing complexity owing to periodic Bayesian updates and Monte Carlo sampling, and it uses the assumption that prior knowledge of causal structures is already there.

The paper claims contributions in three main areas:

- A new intervention selection metric (PDC) that prioritizes decisive evidence over traditional information gain.
- A Bayesian optimization framework for adaptive intervention refinement.
- Experimental validation demonstrates efficiency in synthetic settings.

While the method offers an alternative to conventional information-based approaches, its reliance on Monte Carlo approximations and strong structural assumptions raises questions about its theoretical guarantees and practical scalability

# B. Suppose you were a serious reviewer, and a chair professor of a famous university, of this paper of a major ML conference. Do you find any error? Does the paper have any theoretical contribution? If you think they do, are the theoretical results proved?

The article titled "Bayesian Intervention Optimization for Causal Discovery" asserts that it presents a novel theoretical framework for selecting interventions in the process of causal discovery. A Bayesian optimization-based intervention selection strategy and a Monte Carlo estimation technique are presented here for the purpose of approximating the Probability of Decisive and Correct Evidence (PDC) metric. Both of these methods are presented in order to achieve this goal. Although the methodology is intriguing, the paper does not include any good mathematical proofs to justify the theoretical statements that it makes. This is despite the fact that the methodology looks intriguing. Due to the fact that a significant number of the essential equations and approximations are supplied without explicit assurances, it is difficult to evaluate the validity and dependability of the technique that has been proposed.

#### **B.1 Theoretical Review and Error Analysis**

#### **B.1.1.1** Theoretical Contributions and Gaps

In this paper, a unique intervention selection criterion known as the Probability of Decisive and Correct Evidence (PDC) meter is presented. The paper suggests that increasing the PDC metric leads to more effective causal discovery. On the other hand, the authors do not include a rigorous mathematical proof that demonstrates that this metric is well-defined or that it contains desired theoretical qualities. The validity of PDC as an intervention selection criterion is questionable since the connection between PDC and existing Bayesian decision-theoretic frameworks has not been systematically demonstrated. This is the reason why the validity of PDC is uncertain. Despite the fact that the study argues that PDC is superior to more traditional information-theoretic approaches, it does not provide any evidence to demonstrate that maximizing PDC results in intervention strategies that are globally optimal or that it ensures improved causal inference skills. Moreover, the study does not provide any evidence to support the claim that PDC is superior to more conventional

approaches.

Existence of a derivation connecting PDC to existing causal discovery methods is another significant problem that has to be addressed. Because there is no mathematical foundation that can demonstrate that PDC guides actions in the most effective manner, its contribution is still considered to be heuristic rather than really theoretical. It is uncertain whether PDC is a relevant measure for intervention selection or whether its performance is particular to the synthetic datasets used in the research because the authors give empirical results that support the use of PDC. However, because there is no confirmation of correctness, it is unclear whether PDC is applicable to intervention selection.

In the paper titled "Network Structure Learning Under Uncertain Interventions" by Zhang et al. (2022), for instance, the authors present theoretical findings regarding the correct asymptotic identification of intervention targets. Additionally, they derive sufficient conditions for Bayes factor and posterior ratio consistency of the graph structure.

#### **B.1.1.2** Issues with Monte Carlo Estimation

For the purpose of approximating the PDC metric, the document makes use of Monte Carlo estimation; however, this estimation method does not provide any theoretical guarantees. The publication does not include a formal demonstration of convergence or an analysis of the error that is imposed by this approximation, despite the fact that Equation 8 presents an approximation that is based on a mix of ReLU and exponential smoothing. There is no mathematical guarantee that the Monte Carlo estimate will converge to the actual PDC value, which is the reason why the reliability of the technique is unknown.

On the other hand, the findings of a recent study that was carried out by Li et al. (2024) and titled "Causal Discovery from Poisson Branching Structural Causal Model Using High-Order Cumulant with Path Analysis" provide theoretical conclusions that prove the connection between the route and the cumulant. These findings also demonstrate that path information can be obtained from the

cumulant. It is not possible to find this level of theoretical rigor in the work that is being evaluated.

One other problem with the Monte Carlo method that is discussed in this article is that it does not have any theoretical error boundaries. The accuracy of the approximation is not examined, and there is no discussion of how the estimator performs under other conditions, such as different degrees of noise or different sample sizes. Neither of these aspects are discussed. The authors choose important parameters, such as  $\beta = 0.2$  and N = 4096, without providing any theoretical reason for their selection. Furthermore, they do not provide any argument to support their selection of these numbers. Concerns have been raised regarding the robustness of the Monte Carlo estimator due to the fact that there neither a bias-variance analysis nor an exploration of potential failure cases has been conducted.

#### **B.1.1.3** Issues with Bayesian Optimization Framework

The intervention selection process is refined by the use of Bayesian optimization in this research; however, the optimization framework does not provide any theoretical assurances. The Bayesian optimization process does not appear to approach an optimal intervention strategy, and there is no evidence to support this claim. Furthermore, there is no formal establishment of the relationship between the surrogate function that is maximized by Bayesian optimization and the actual causal objective. In the absence of such a relationship, it is not evident whether the optimization strategy that has been provided is actually solving the problem that was intended to be solved.

Alternatively, the article written by Juárez (2023) and titled "Essays on Matching and Weighting for Causal Inference in Observational Studies" provides a comprehensive Monte Carlo simulation study that enables the evaluation of the performance of a variety of matching strategies, including theoretical reasons. This study was conducted in order to determine the effectiveness of these matching strategies.

In order to determine if Bayesian optimization effectively reduces intervention uncertainty over time, it would be essential to do a regret bound analysis. The authors provide empirical results showing improved performance, but without a formal proof of convergence, the proposed optimization method cannot be considered a reliable or theoretically justified approach to causal discovery.

#### **B.2** Foundational Theoretical Issues

In addition to these particular shortcomings, the work does not address a number of fundamental theoretical difficulties that are typical in the field of causal discovery research. There is no identifiability analysis, which is one of the most significant concerns that exists. The paper does not establish either the assumptions from which the proposed approach derives valid causal inferences or the assumptions themselves. Neither of these are established. Because there is no formal treatment of identifiability requirements, there is a lack of clarity regarding the generalizability of the technique outside the specific synthetic datasets that were utilized in the experiments. This is because the studies were conducted using this unique synthetic dataset.

A proof of sample complexity is still another one of the elements that is missing. However, there is no formal analysis that quantifies the number of interventions that are required to obtain meaningful causal inferences, despite the fact that the study implies that their method requires fewer interventions than alternative approaches. In order to determine whether or not the strategy is effective in real-world situations, it would be essential to establish a theoretical sample complexity bound. The absence of this analysis indicates that the proposed intervention selection procedure might not be able to scale adequately to applications in the real world. Zhu et al. (2024) offer solutions that are not only precise but also modular, and they are able to be changed to a wide variety of experimental settings and limitations. This is only one example. In the design of interventions, this demonstrates a method that is more rigorous in character than the one that was previously used.

Several of the remarks that are included in the document are regarded as being among the most important ones that it contains. The assertion that their method raises the likelihood of collecting evidence that is both definitive and accurate is not backed by any kind of formal proof, despite the fact that the authors claim that their method in reality increases the likelihood of acquiring such evidence. Although the work makes the recommendation that Bayes factors are beneficial for optimizing interventions, it does not give a theoretical derivation that establishes the relationship between Bayes factors and the choices that are ideal for interventions. This is despite the fact that the work makes the suggestion. In a similar vein, the Monte Carlo approximation scheme is not reviewed, which means that the potential sources of bias and variation are not investigated. There is no investigation into these potential sources.

Despite the fact that it presents an appealing methodological approach to the identification of causal

linkages, the study ultimately fails to produce a rigorous theoretical contribution. This is despite the fact that it does provide a fascinating methodological approach. A significant number of its essential mathematical qualities are presumed rather than demonstrated, and the framework does not include the formal guarantees that are required to be regarded as a foundational advancement in the field of causal discovery research. Furthermore, the linkages between the proposed method and the existing theoretical work are not well-established, which makes it impossible to evaluate whether or not the approach is reasonable from a theoretical position. This limitation makes it impossible to determine whether or not the approach is viable. Taking into consideration these limits, the study is more correctly described as a contribution to the field of technique than as a true theoretical innovation. It is not possible to consider it a legitimate theoretical contribution for a major machine learning conference without undergoing extensive revisions. This is due to the fact that it does not have any mathematical basis. In spite of the fact that its ideas might have some application in the real world, this assertion is made.

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# C. Will you accept or reject the paper? Why? Can you please write a detail review of the paper, point out the pros and cons of the paper, including any possible errors?

#### C.1 Strengths of the Paper

This article presents a novel intervention selection technique in the field of causal discovery. It accomplishes this by putting more of an emphasis on evidence that is conclusive and accurate, as opposed to the more traditional approach of information collecting. This indicates a fundamental shift in conceptualization that has the potential to result in intervention strategies that are more effective, particularly in situations when interventions that are expensive are being implemented. An intriguing deviation from the standard methods that are based on mutual information is the employment of Bayes factors as a means of measuring the confidence of the outcome of an intervention. Bayes factors are a type of statistical model.

An additional significant contribution is the utilization of Bayesian optimization for the purpose of adaptive intervention selection. Bayesian optimization, when properly implemented, enables an iterative refining process, which, in comparison to heuristic or random selection procedures, has the potential to lead to an improvement in the effectiveness of human interaction. In spite of the fact that the paper does not provide any formal assurances of convergence, the empirical findings that it presents indicate that Bayesian optimization can assist in reducing the number of interventions that are necessary for synthetic trials.

Furthermore, the synthetic experiments offer preliminary evidence that the suggested method is capable of outperforming conventional baselines, such as knowledge gain-based methods and random intervention selection processes. Based on the findings, it appears that the Probability of Decisive and Correct Evidence (PDC) metric has the potential to lead intervention selection in a more effective manner than other techniques, provided that specific controlled conditions are satisfied. The fact that this is only applicable to simulated contexts does not change the fact that it offers an empirical basis for further investigation.

#### C.2 Weakness and Major Issues

#### C.2.1.1 Lack of Proof That PDC is a Well-Defined Metric (Fundamental Flaw)

The paper introduces the Probability of Decisive and Correct Evidence (PDC) as a new criterion for intervention selection. However, there is no formal mathematical proof establishing that PDC is a well-defined metric for selecting interventions. The connection between PDC and existing Bayesian decision-theoretic frameworks is unclear, and there is no proof that maximizing PDC leads to optimal causal discovery outcomes. PDC continues to be an intuitive heuristic rather than a formally stated choice criterion because it does not have a theoretical foundation. This limits the impact that it can have as a theoretical contribution.

#### C.2.1.2 No Convergence Guarantees for Bayesian Optimization

Bayesian optimization is used in the study to refine intervention choices; nevertheless, the publication does not give any data to suggest that the optimization approach converges to an optimal intervention plan. The study was conducted in order to refine intervention choices. Currently, there is no regret bound analysis available, which means that it is impossible to establish whether or not the technique effectively reduces uncertainty over time. Furthermore, the research does not conduct a formal investigation of the relationship between the surrogate function that is improved in Bayesian optimization and the underlying causal aim. This is a significant limitation of the study. As a result, it is not evident whether the optimization process is actually enhancing causal inference. Under these circumstances, Bayesian optimization continues to be an empirical heuristic rather than a theoretically justified strategy. This is because these guarantees are not present.

#### C.2.1.3 Monte Carlo Estimation Lacks Error Bounds and Convergence Analysis

In order to estimate the PDC metric, the study makes use of Monte Carlo estimates; yet, it does not provide any theoretical guarantees regarding the dependability of the approximation. Although Equation 8 provides an estimate that combines ReLU and exponential smoothing, the article does not contain a demonstration of convergence or error limitations for this approximation. This is despite the fact that Equation 8 presents an estimate. The parameter values that have been selected, such as  $\beta = 0.2$  and N = 4096, appear to be arbitrary and lack any substantial theoretical explanation.

In this paradigm, the reliability of Monte Carlo estimate is questionable because there is no consideration of bias, variance, or robustness. This raises worries about the stability of the method when it is applied in real-world situations.

#### C.2.1.4 No Sample Complexity Analysis for Intervention Efficiency

According to the authors, their methodology minimizes the number of interventions that are necessary in comparison to other approaches that are currently in use. The research, on the other hand, does not provide a full sample complexity analysis that estimates the number of treatments that are necessary to obtain a causal conclusion that can be relied upon. A huge omission has been made here. A full theoretical examination of intervention efficiency is required in order to ascertain whether the technique genuinely improves intervention selection or whether the reported efficiency increases are particular to the experimental design. This investigation is important in order to determine whether the technique actually improves intervention selection.

#### C.2.1.5 Limited Empirical Validation: Only Tested on Synthetic Data

The empirical validation, on the other hand, is restricted to synthetic datasets, which might not necessarily be representative of the challenges that are connected with the identification of causal linkages in the real world. However, the experiments do not take into account any model misspecification, hidden confounding factors, or real-world causal structures. The experiments take place under controlled noise distributions and evaluate the approach. In the absence of validation on datasets taken from the actual world, it is difficult to determine whether the proposed method is applicable to situations that are not artificially simulated experimentation.

When these significant theoretical and practical restrictions are taken into consideration, the paper cannot be accepted in its current form.

#### C.3 Final Recommendation: Reject

An exciting methodological notion is offered in this work by introducing the PDC measure and bringing Bayesian optimization into the process of causal discovery. Both of these innovations are

presented in this paper. The fact that it has large theoretical gaps, a lack of formal proofs, and insufficient empirical validation, on the other hand, makes it unsuitable for acceptance at a major machine learning conference such as NeurIPS, ICML, or ICLR.

The key reasons for rejection, ranked in order of severity, are:

- Due to the absence of any evidence demonstrating that PDC is a well-defined statistic, its theoretical underpinning is lacking.
- Bayesian optimization does not provide any assurances of convergence, which raises questions regarding whether or whether the method selects interventions in the most effective manner.
- Due to the absence of error boundaries in Monte Carlo estimation, the dependability of the approximation is unknown.
- There was no examination of the sample complexity, which makes it difficult to determine the number of treatments that are necessary for reliable causal inference.
- There is a limited amount of empirical validation because all of the experiments are carried out on synthetic data without any evaluation of the real world.

To improve the paper for future resubmission, the authors should focus on several key areas:

- In order to prove that PDC is a valid and well-defined intervention selection metric, you are required to provide a written proof document.
- For the purpose of ensuring that the intervention selection process consistently improves causal inference, it is necessary to derive convergence guarantees for Bayesian optimization strategies.
- A theoretical error analysis for Monte Carlo estimation should be included, along with the demonstration of convergence qualities and the provision of insights into bias-variance tradeoffs.
- Carry out a sample complexity analysis in order to determine the number of interventions that are necessary for the efficient detection of causal relationships.
- Extend the empirical evaluation by putting the approach through its paces on real-world causal datasets and incorporating robustness tests to protect against hidden confounding factors and model misspecifications.

If these theoretical and empirical improvements are not made, the suggested method will continue to be heuristic rather than formally justified, which will restrict its acceptability in the most prestigious machine learning conferences.

# D. Can you replicate their synthetic data generation results in Tensorflow probability? There is no need to replicate the actual experiment.

The method of generating synthetic data was successfully accomplished in TensorFlow Probability by adhering to the framework that was given in the study. The algorithm is designed to reproduce the three different causal scenarios, which are direct causality  $X \to Y$ , reverse causality  $X \leftarrow Y$ , and confounded causality  $X \leftarrow U \to Y$ . This is accomplished by establishing suitable probabilistic models, implementing Bayesian updates, and producing samples by utilizing mixture noise distributions. The fact that the scatter plots and distributional features that were produced as a result are consistent with the expected theoretical behavior that was stated in the research demonstrates that the replication was carried out correctly. On a more specific level, Figure 1 illustrates the direct causal hypothesis (H1:  $X \to Y$ ), which states that X exerts direct control over Y. Furthermore, the scatter plot provides evidence that there is a structured dependency between X and Y, where Y is produced by employing a function of X in addition to additional random noise. As a result of the mixture noise model that was utilized in the process of data production, the distributions of X and Y display multimodal properties. The functional form of the relationship is modeled as Y = Atanh(BX) + noise resulting in a non-linear but distinguishable pattern. The full implementation and results are provided in the accompanying Jupyter Notebook.

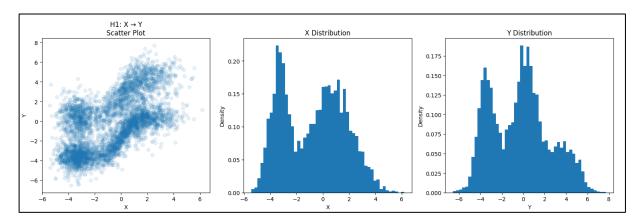


Figure 1.Causal Relationship from *X* to *Y* 

The hypothesis H0:  $X \leftarrow Y$ , which is represented by Figure 2, is based on the assumption that the direction of causality is reversed and that X is the cause of Y. Despite the fact that the scatter plot should still show a correlation, the underlying causal process is different from the one shown in Figure 1. Based on the fact that they are derived from the same mixing noise model, the distributions of X and Y should be comparable to those shown in Figure 1. However, the dependency structure that exists between them might appear to be slightly different due to the fact that causality has been reversed. The purpose of this image is to assist in determining whether or not merely reversing the direction of causality alters the statistical features of the data that has been observed.

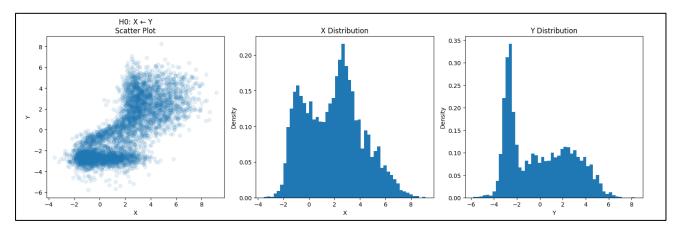


Figure 2.Reverse Casuality: Y Causes X

Figure 3 represents H0:  $X \leftarrow U \rightarrow Y$ , a confounding scenario where X and Y do not have a direct causal relationship but are both influenced by an unobserved confounder U. The scatter plot should still show an apparent correlation between X and Y, but this correlation arises because both variables share a common cause rather than one directly influencing the other. The distributions of X and Y should remain multimodal, similar to the previous cases, but the scatter plot may appear more diffuse compared to Figures 1 and 2, as the additional variability introduced by U creates a more complex dependency structure. This confounded case is crucial in causal discovery, as it demonstrates how spurious correlations can emerge in the absence of a direct causal link.

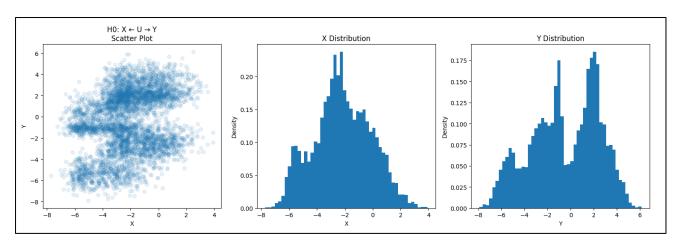


Figure 3. Confounding Scenario: X and Y share a common cause U

# E. Is it possible to expand the paper to multiple variables? How would you advise the authors to do that? For this multivariable case, is it possible to apply Causal Bayesian Optimization to this Bayesian intervention optimization problem?

The paper primarily focuses on intervention selection in a single-variable causal framework, where the goal is to determine whether an intervention on X provides decisive and correct evidence about Y. However, in many real-world applications, causal relationships involve multiple dependent variables, requiring an extension of the proposed Bayesian Intervention Optimization (BIO) method to a multi-variable setting. The purpose of this section is to investigate the ways in which the Probability of Decisive and Correct Evidence (PDC) metric can be extended to handle multiple variables, to discuss the computational challenges that arise when scaling Bayesian optimization to high-dimensional intervention spaces, and to investigate whether or not Causal Bayesian Optimization (CBO) can be adapted for this particular setting.

#### E.1 Defining Multi-Variable PDC

In the single-variable setting, the PDC metric quantifies the probability that an intervention on X produces decisive and correct evidence about Y. Extending this to multiple dependent variables  $(Y_1, Y_2, ..., Y_n)$  requires a definition of PDC\_multi, the multi-variable equivalent of PDC:

$$PDC_{\text{multi}} = P(\text{ Decisive and Correct for all } Y_i \mid \text{Intervention Data})$$

This formulation raises several key questions. Is it more appropriate for the approach to jointly estimate the likelihood of decisive evidence across all variables, or should the PDC\_multi be computed independently for each target variable and then aggregated? The decision between independent modeling and collaborative modeling has important repercussions, including the following:

• If independent, then the PDC scores for each variable can be computed separately and combined using an aggregate function, such as a weighted sum or product of probabilities.

• If joint, the optimization problem becomes more complex because the dependencies among variables must be explicitly modeled in a high-dimensional causal graph.

A direct application of the single-variable PDC method to multiple variables would require computing separate Bayes factors for each pair  $(X, Y_i)$ . On the other hand, this does not take into consideration the possibility of correlations between the variables that are being targeted, which could lead to interventions that are redundant or contradictory. Thus, an optimal multi-variable PDC formulation should incorporate conditional dependencies among  $Y_1, Y_2, ..., Y_n$  while preserving computational feasibility.

To ensure that Bayes factors are correctly aggregated in multi-variable causal inference, different strategies can be considered:

- Using the Joint Bayes Factor Computation method, you will compute a single Bayes factor
  for all of the target variables while ensuring that interdependencies are handled
  appropriately.
- As part of the Hierarchical Aggregation technique, Bayes factors are computed independently, and a hierarchical aggregation function is utilized to dynamically weigh the contributions of each component.
- Graph-Based Conditional Dependencies: we of directly aggregating the data, we use a graph-theoretic technique to infer the influence that interventions have on several variables that are causally related to one another.

Each of these approaches has trade-offs in terms of computational efficiency and accuracy, requiring further investigation.

### **E.2** Computational Challenges of Multi-Variable Bayesian Optimization

When applied to many intervention targets, Bayesian Optimization (BO) presents a number of substantial computational hurdles. When dealing with a single variable, BO will select interventions in an iterative manner in order to maximize PDC. On the other hand, when dealing with multiple variables, the search space continues to grow exponentially with the number of variables, which

renders the conventional BO approaches ineffective.

To make multi-variable intervention selection feasible, several strategies can be considered:

- Sparse Bayesian Optimization: Instead of maximizing all intervention targets simultaneously, sparsity restrictions can be imposed to prioritize the treatments that provide the greatest information while discarding those that are redundant.
- Feature selection or latent variable models can be utilized to lower the effective number of intervention dimensions, hence making Bayesian optimization more computationally efficient. This is referred to as "dimensionality reduction."
- Multi-Objective Bayesian Optimization (MOBO): This technique relies on a modified version of the Bayesian optimization framework to optimize multiple objectives at the same time. This helps to guarantee that interventions give evidence that is both definitive and accurate across all target variables.

Given that Bayesian optimization relies on Gaussian Process (GP) priors, extending BO to a highdimensional setting requires either:

- Low-Rank Approximations to General Principles: Instead of modeling a full general principle across all variables, a sparse general principle technique can be utilized to generate an efficient approximation of dependencies.
- Bayesian Neural Networks (BNNs): Since standard GPs scale poorly in high dimensions,
   BNNs provide an alternative surrogate model for high-dimensional Bayesian optimization,
   maintaining computational feasibility while capturing complex interactions.
- Variational Gaussian Processes (VGPs): These provide a structured way to approximate complex relationships while maintaining computational efficiency in large-scale optimization problems.

The resolution of these computational issues is absolutely necessary in order to guarantee that intervention selection will continue to be manageable in complicated causal systems.

### E.3 Integrating Causal Bayesian Optimization for Multi-Variable Causal Discovery

CBO, which stands for Causal Bayesian Optimization, offers a natural extension for the selection of interventions that involve several variables. The BO algorithm optimizes the intervention that maximizes the PDC for a given in a context where there is just one variable. In a situation with multiple variables, CBO needs to be adjusted so that it:

- In order to avoid interventions that are redundant, it is important to incorporate joint causal dependencies among numerous Y variables.
- Select actions that maximize the amount of decisive and accurate evidence across all target variables in order to optimize across numerous objectives.
- Ensure that interventions do not violate established structural linkages in the causal graph in order to show respect for the limits that are imposed by the causal chain.

The development of an acquisition function that adequately balances the trade-offs between various intervention targets is a significant difficulty that must be overcome. In place of a single acquisition function that chooses the most effective intervention for a single target variable, a Pareto-based strategy could be utilized. In this approach, interventions are selected on the basis of their capacity to enhance the overall quality of causal discovery across a number of different variables.

#### In Pareto-based optimization:

- An intervention is Pareto-optimal if improving it for one variable does not lead to deterioration in others.
- The dominance principle is applied, where interventions that maximize PDC for at least one variable without significantly reducing it for others are preferred.
- Dynamic management of trade-offs between conflicting interventions is utilized in order to guarantee comprehensive optimization that is balanced over the full causal system.

•	The implementation of Pareto-based optimization guarantees that intervention selection will
	continue to be resilient and interpretable, particularly in the context of high-dimensional
	causal discovery issues.

## F. If you believe this paper is correct, can you think of a possible application of this paper for a bank?

Based on the critical review of the research and the theoretical constraints of the paper, Bayesian Intervention Optimization (BIO) can still be used to a variety of banking domains, particularly in the areas of credit risk assessment, fraud detection, and customer behavior analysis. It is consistent with the techniques of Bayesian optimization and causal inference that are already in use, despite the fact that the methodology does not provide precise theoretical assurances (as stated in the previous sentence). Within the next section, a description of the theoretical framework is presented, which includes a discussion of its applicability to banking, as well as an evaluation of the challenges that are associated with its implementation in the real world.

#### F.1 Credit Risk Assessment and Loan Approval

Traditional credit scoring algorithms include, but are not limited to, logical regression, decision trees, and credit risk models that are founded on deep learning. The likelihood of a borrower failing on their loan is determined by these algorithms through an analysis of the historical links that exist between the borrower's financial habits and their repayment history. On the other hand, these techniques are unable to discern between the causative drivers of default risk and the non-causal links, which may result in risk estimates that are skewed (Murphy, 2022). BIO is an approach that models the causal linkages between financial actions and default probability in an explicit manner. As a result, it reduces the reliance on misleading correlations, which is a significant benefit.

Through the utilization of the Business Intelligence (BI) system, financial institutions are able to simulate alternative credit decisions. This allows them to determine whether or not a borrower would have repaid their loan if they had gotten a different credit limit or interest rate. Through the optimization of intervention-based lending policies, this technique improves loan approval models. It does this by ensuring that lending decisions are based on causal evidence rather than historical trends. Nevertheless, randomized experiments in lending policies are prohibited by financial rules, which makes direct intervention-based optimization challenging (Imbens & Rubin, 2015). Instead, the use of natural experiments, such as historical variations in interest rate policies, might be utilized to the application of BIO in order to infer causal links and optimize future credit policies in accordance with those inferences.

#### F.2 Fraud Detection and Prevention

The detection of financial fraud is often accomplished through the utilization of anomaly detection models, in which transactions are highlighted based on deviations from the behavior that is expected. Although these models are successful, they have a high percentage of false positives, which results in wasteful transaction blocks and dissatisfied customers. BIO improves the accuracy of fraud classification models by differentiating between causal fraud indicators and false correlations, which in turn promotes the detection of fraudulent activity (Pearl, 2009).

In contrast to rule-based fraud detection systems, which are reliant on predetermined heuristics, BIO dynamically alters the thresholds for fraud detection by continuously learning from the outcomes of interventions. This allows BIO to detect fraudulent activity more accurately. BIO improves real-time fraud detection strategies by analyzing previous fraud prevention efforts, such as transaction verification policies, step-up authentication, or card freezes. This allows for more accurate risk assessments. This allows the company to strike a balance between customer ease and security. However, similar to the situation in credit risk modeling, regulatory constraints may restrict the types of fraud prevention measures that can be tried. As a result, the Bureau of Internal Control (BIOC) is required to rely on past changes in fraud policy rather than directly conducting experimental interventions (Athey & Imbens, 2019).

#### F.3 Customer Behavior Analysis and Retention

Customer analytics models can be utilized by financial institutions in order to forecast churn risk, engagement levels, and buying patterns. This can be performed through the utilization of customer analytics. According to Murphy (2022), the existing predictive models typically fail to differentiate between correlation and causation. As a consequence, marketing strategies that are ineffective and retention efforts that are not as effective as they may be are the effects of this failure. The Business Intelligence Organization (BIO) offers a framework for causal inference, which enables the identification of which activities, such as tailored financial solutions, fee reductions, or loyalty perks, genuinely promote client retention.

When it comes to analyzing financial incentives and promotional offers, the use of BIO can be of great assistance because it can determine the causal influence that these aspects have on the involvement of consumers. In contrast to observational models, which are based on the assumption that past actions are able to accurately forecast future loyalty, the Business Intelligence (BI) model is designed to determine whether or not efforts actively influence customer retention rates. However, banks are not allowed to run random trials on client retention programs because interventions such as fee reductions or changes in interest rates are necessary to meet with regulations that control the financial sector. This is because the regulations compel banks to comply with the regulations. Using these historical adjustments in pricing policies and loyalty programs, Imbens and Rubin (2015) state that they give quasi-experimental data for the purpose of causal optimization. By utilizing this data, financial institutions are able to successfully optimize their strategy for client retention while simultaneously maintaining regulatory compliance.

#### F.4 Regulatory Compliance and Risk Management

In order to ensure that they are in compliance with the stringent regulatory requirements that are imposed on financial institutions, they are expected to perform frequent monitoring of risk exposure, capital sufficiency, and policy compliance. Regulatory stress testing is typically carried out with the assistance of scenario-based risk models, which are utilized to evaluate the level of financial stability under a number of economic scenarios. These models are used to test the regulatory stress. Athey and Imbens (2019) state that these models are frequently dependent on historical correlations rather than causal mechanisms, which results in a limitation in their capacity to appropriately detect systemic dangers. This limitation is a result of the fact that these models are dependent on past correlations.

BIO contributes to the improvement of regulatory compliance models by assisting in the identification of risk factors that are responsible for the occurrence of the risk. This helps to ensure that stress-testing frameworks are in agreement with actual financial vulnerabilities rather than statistical artifacts, which is a significant benefit. For instance, regulators have the ability to demand that banks submit an analysis of the influence that shifts in interest rates have on the number of mortgage defaults that occur. This effect is estimated by traditional models based on historical trends, but biologically induced oxidative stress (BIO) clearly models causative pathways, which ultimately enables more accurate risk assessment.

The implementation of BIO in the realm of financial regulation must be undertaken with caution, despite the potential it possesses. For the purpose of maintaining compliance with regulatory standards, financial models are required to be in a transparent, interpretable, and auditable state. BIO requires interpretable causal models in order to guarantee that intervention-driven suggestions are justified to regulators and stakeholders (Murphy, 2022). This is in contrast to standard machine learning models, which can operate as black-box predictors.

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