Business Analytics and Emerging Trends

Instructor: Dr. Murat Tunc

Social Network Analysis

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Module 6

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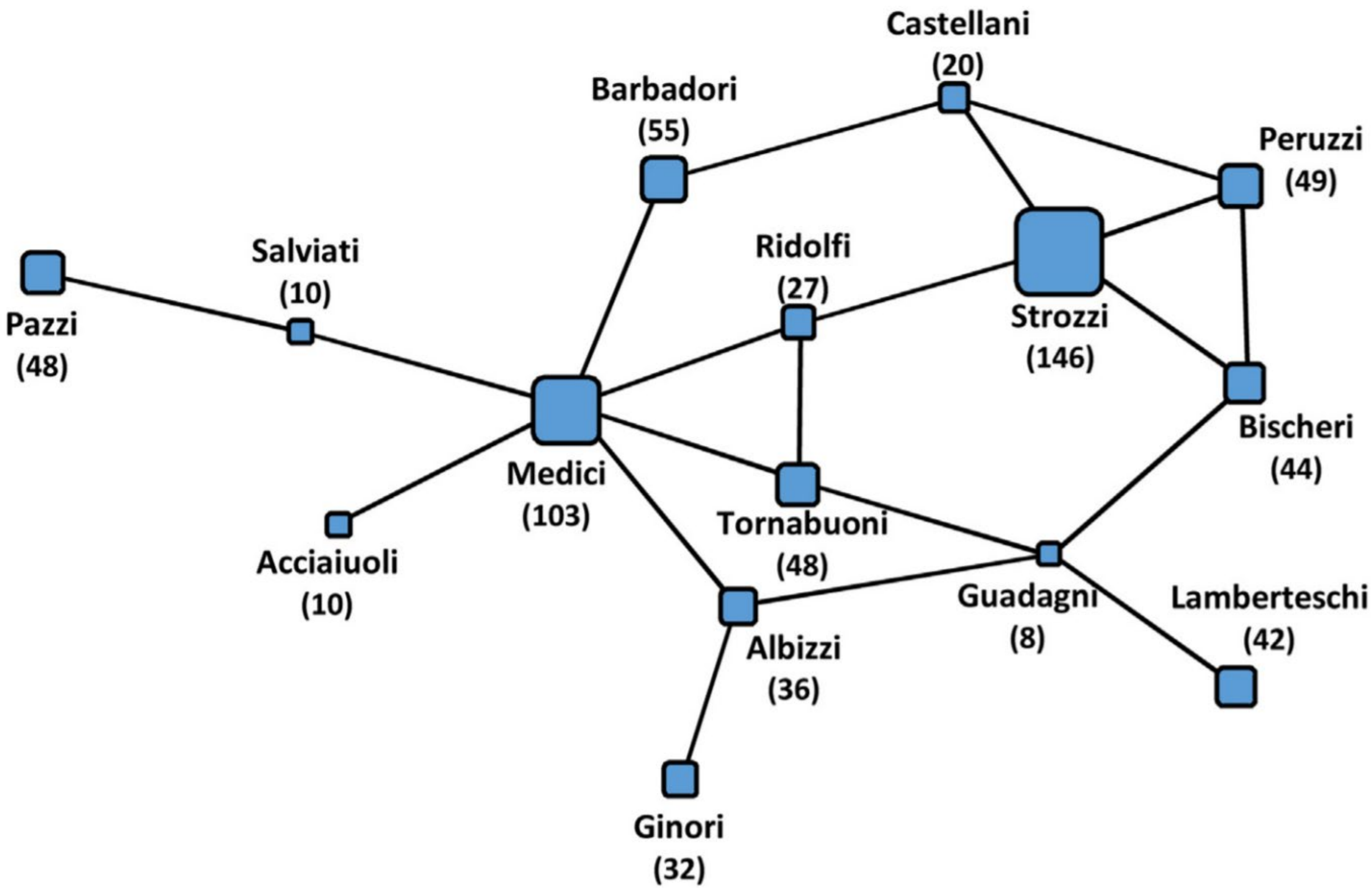
Why analyze social networks?

- Network influences **behavior**
- Labor markets
  - **Textile** workers
    - 62% found their **first job through a contact**
    - 23% by direct applications
  - Chicago labor market: First job through a contact?
    - 37% of typists
    - 65% of janitors
    - 23% of accountants
    - 57% of electricians
    - 73% of material handlers

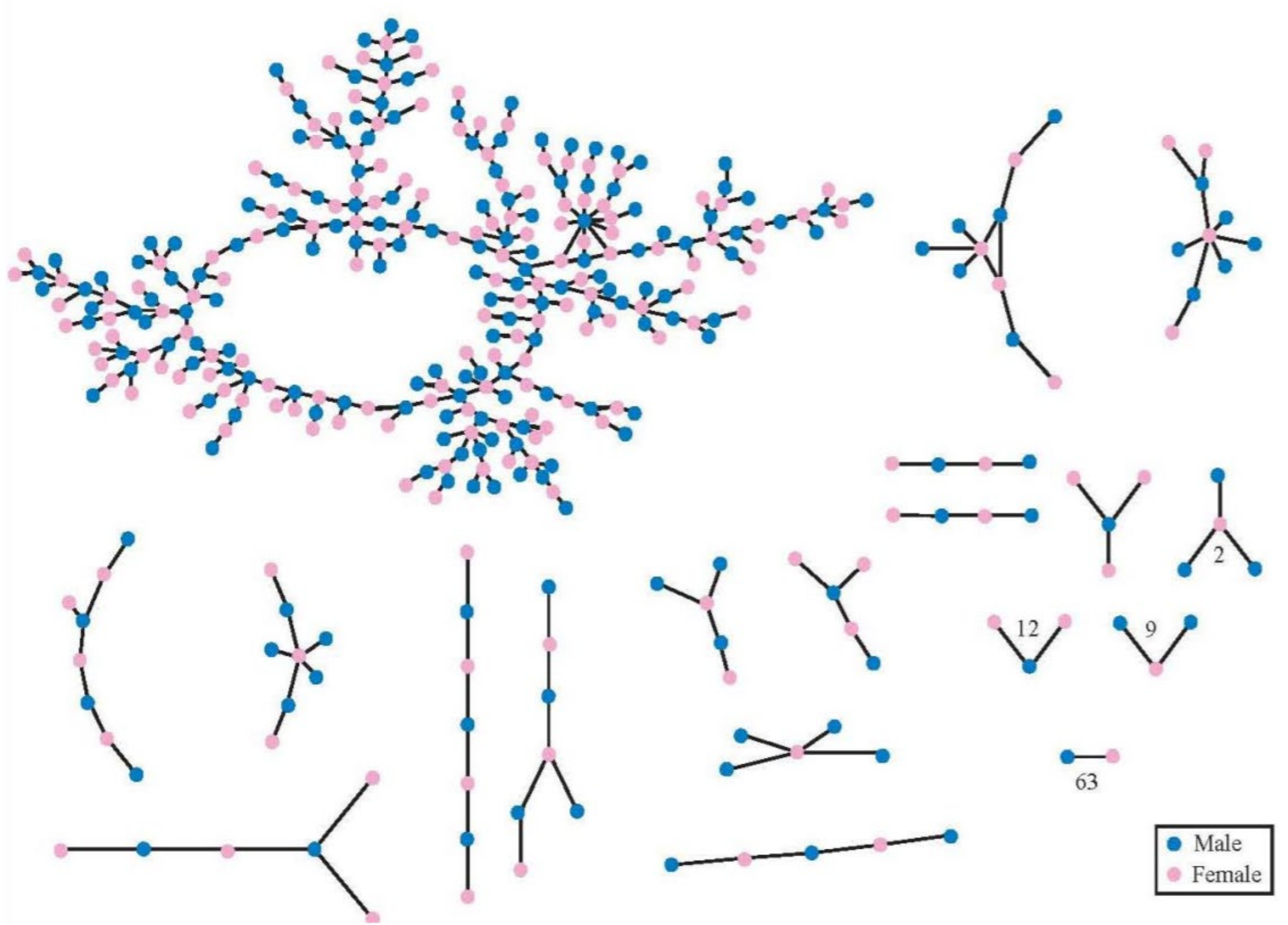
Why analyze social networks?

- If a person had **a friend who became obese**
  - Chances of becoming obese increased by 57%
- Social interactions in youth
  - Determine who are **likely to commit a crime**
- Diffusion of new products
  - Prescription of a new drug
    - Depends on doctor networks
  - How long it takes to become a mainstream product?
- Aids transmission, immigration, voting behavior, epidemics…

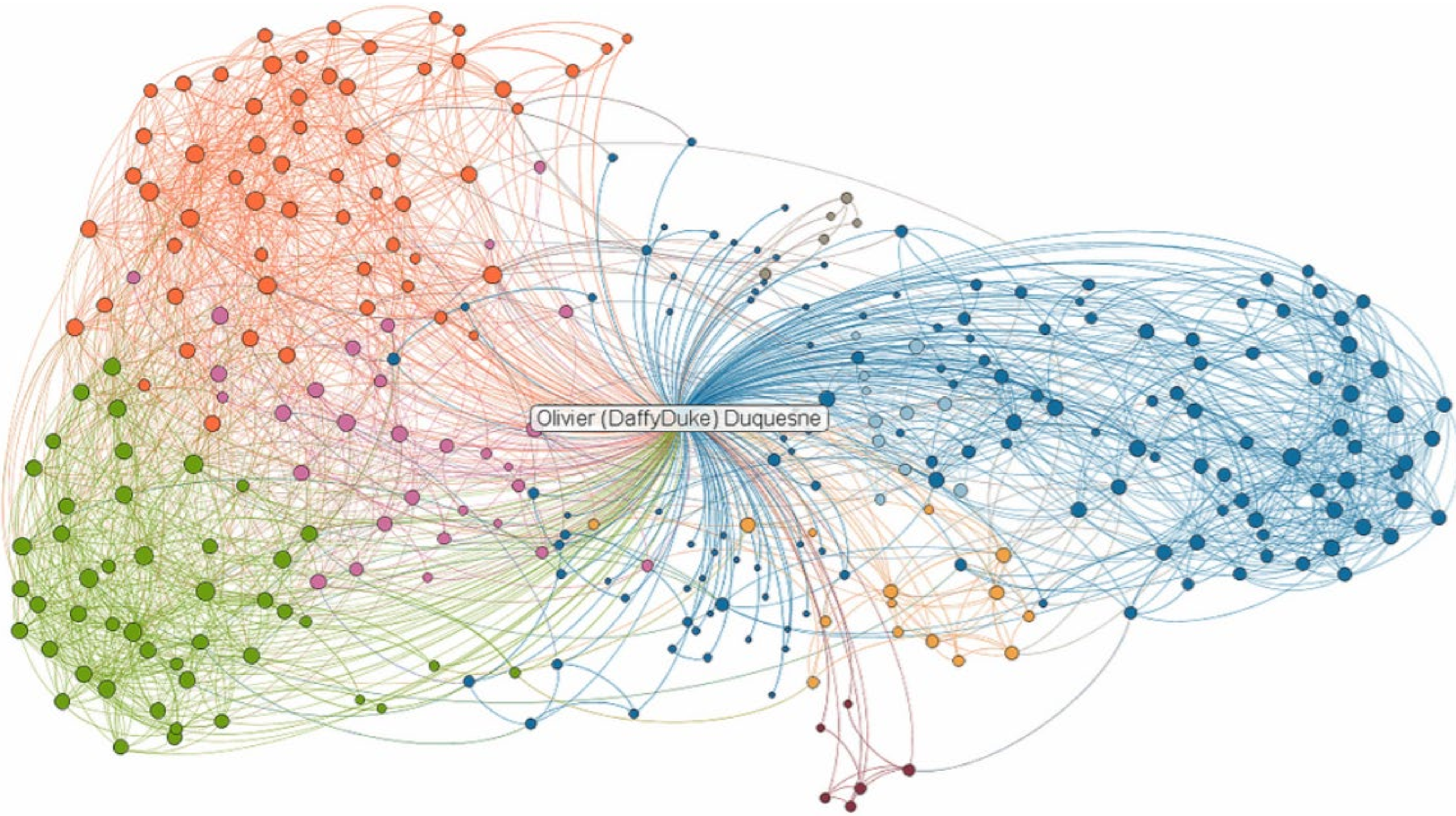
15th Century Florentine Marriages



Romantic relationships in a high school



Linkedin Friendship Maps



Analysis of Social Networks

Basic Definitions

Measures of Connectivity

Network Types

Information Flows

Basic Definitions

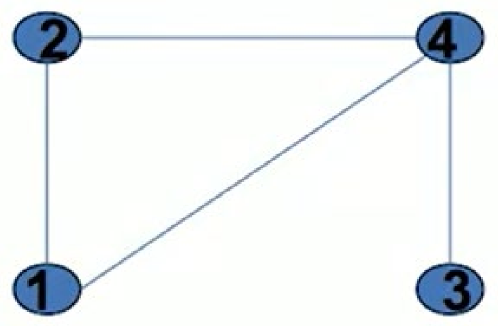
- **Nodes**, vertices
  - Objects, individuals, players
- **Edges**, links, ties
  - Connections between nodes
- **Weighted** edges
  - The intensity of a link
  - How many hours do two people spend together?
- **Unweighted** edges
  - 0 or 1
- **Directed** edges
  - One way relationship
- **Undirected** edges
  - Mutual relationships

Neighborhood and Degree

- **Degree** of a node
  - Number of edges connected to a node
- Neighborhood
  - Two nodes are **neighbors** if they share an edge in-between
- **Density** of a network
  - Average degree of all the nodes in the network
  - Only tells a partial story
- **Degree distribution** of a network
  - Explains more characteristics

Adjacency matrix

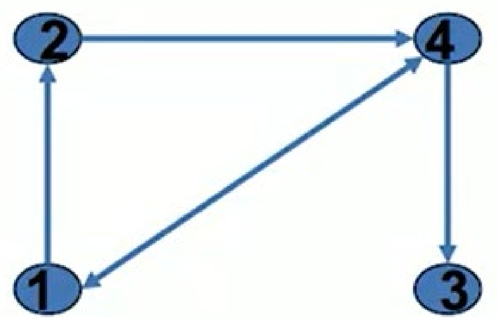
- An undirected network



Adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

- A directed network



Adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

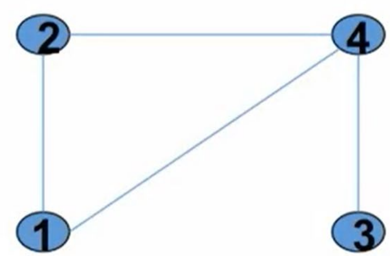
Walks and Paths

- Walk
  - Sequence of links connecting two nodes
- Cycle
  - A walk that starts and ends at the same node
- **Path**
  - A walk where a node appears at most once
- Geodesic
  - The **shortest path** between two nodes

Adjacency matrix and walks

- The n^{th} power of an adjacency matrix
  - Number of walks of length n

$$g = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$



$$g^2 = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \end{pmatrix}$$

number of walks of length 2 from i to j

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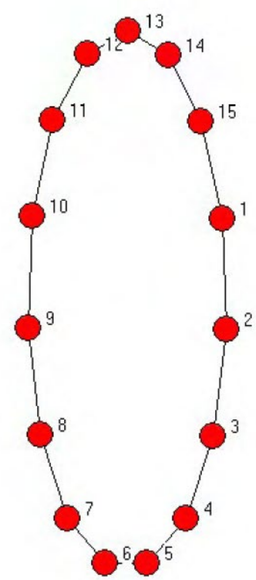
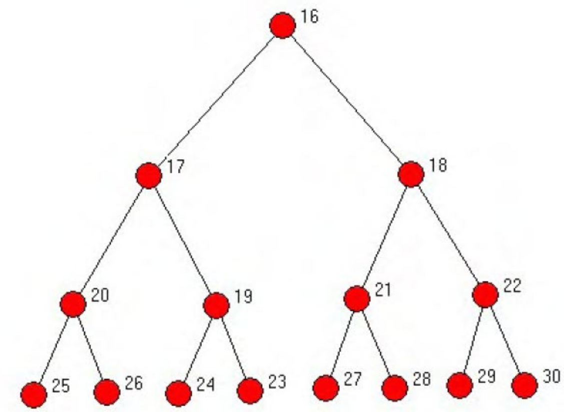
Connectedness

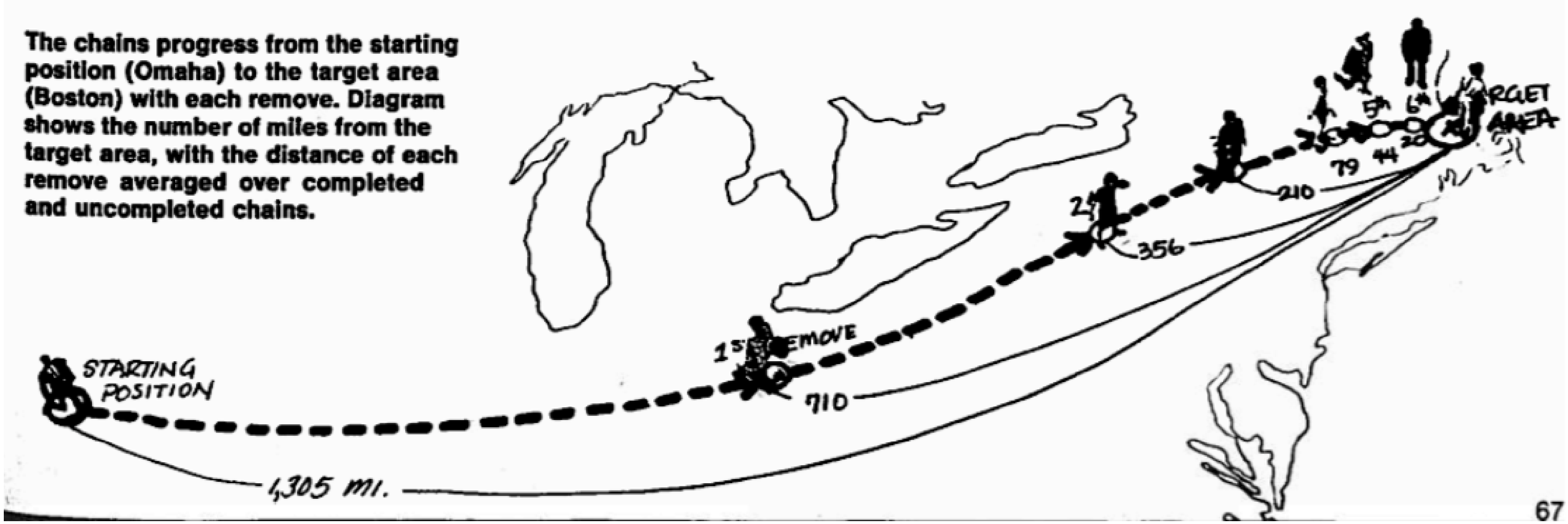
- A network is **connected** if
  - There is a path between **every** two nodes
- **Diameter** of a network
  - The largest geodesic (the maximum length of shortest paths)
- Average path length
  - What is the most likely distance between any nodes?





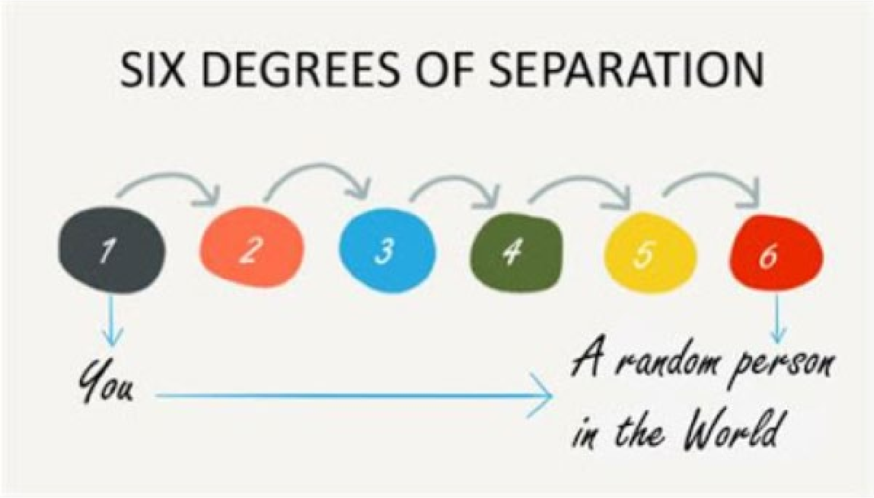
Average Path Length in real world

- Milgram (1967) – **letter experiment**
  - **Please send this letter to someone you know**
    - Starts from the Midwest (Nebraska)
    - Destination: An address in the Northeast (Massachusetts)
  - Median number of steps is 6 out of 25% of letters made it



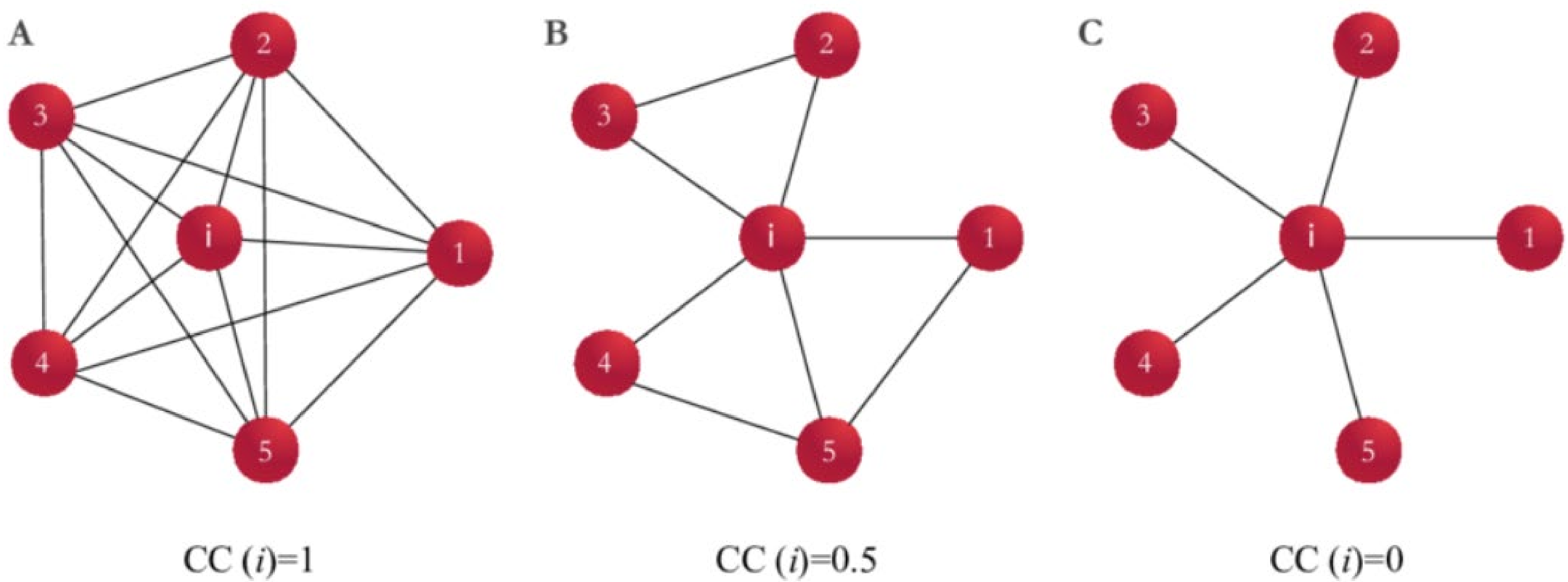
Six degrees of separation

- Academic **co-authorship** networks
  - Math: mean 7.6
  - Physics: mean 5.9
  - Economics: mean 9.5
- **Facebook friendship** network
  - Mean 4.74 (721 million users)



Clustering Coefficient

- How many of your friends know each other?

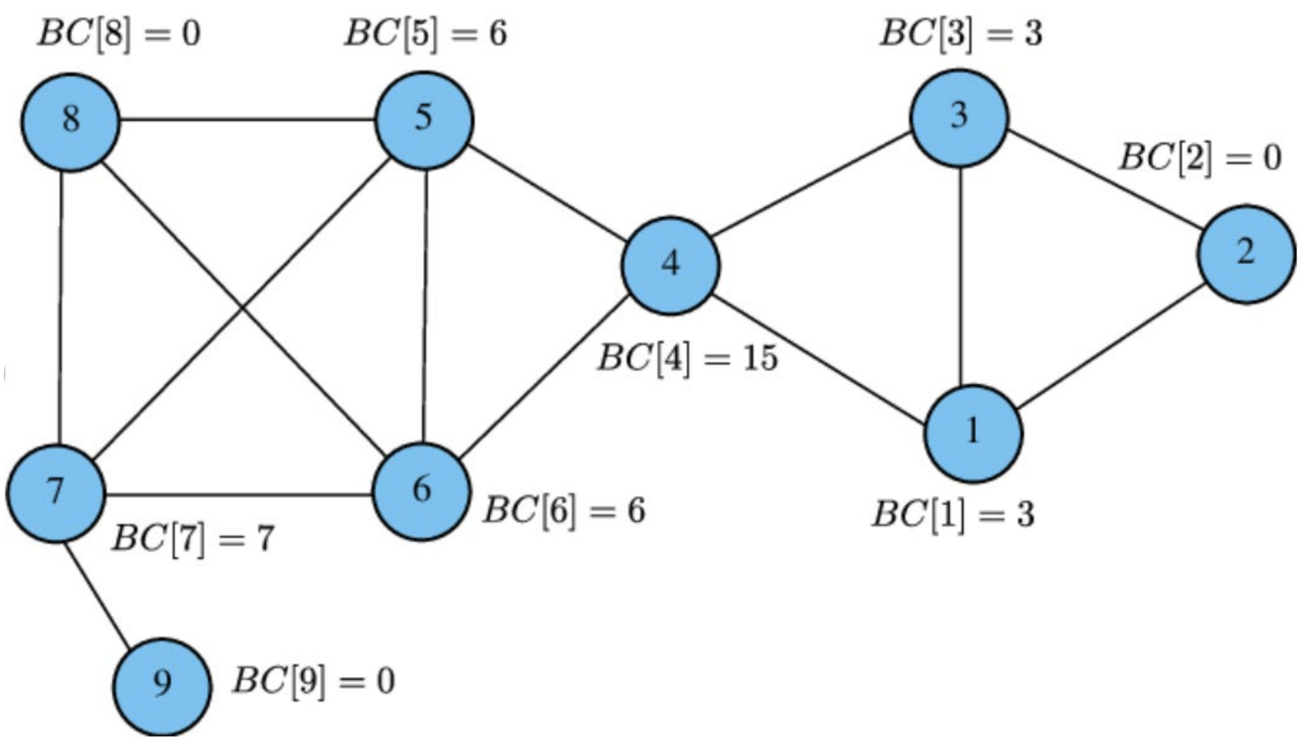


Clustering Coefficient in real world

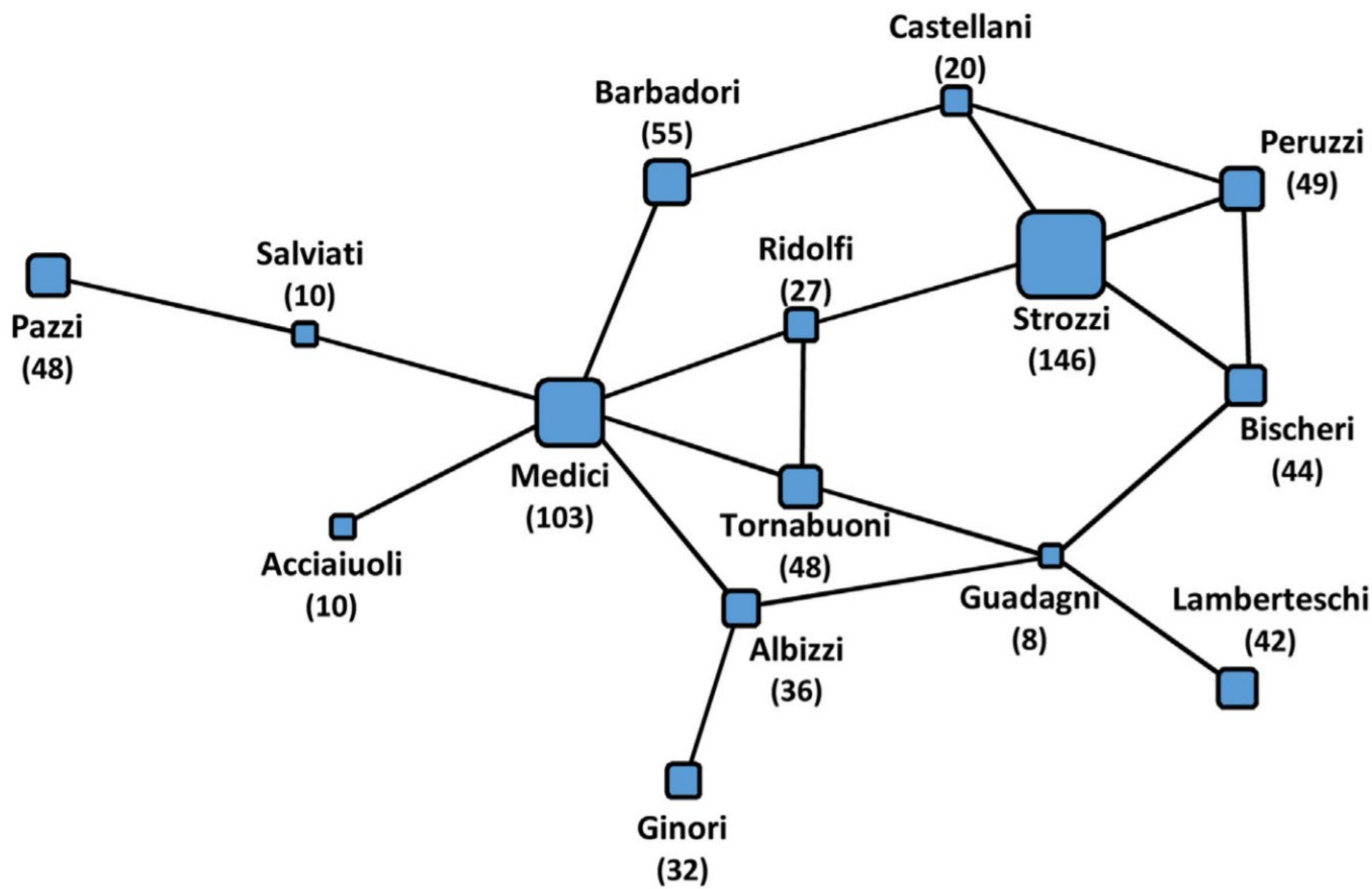
- **Prison friendship**
  - 0.31 (MacRae 1960)
- Academic **co-authorship networks**
  - 0.15 (Math)
  - 0.09 (Biology)
  - 0.19 (Econ)
- www
  - 0.11 (Web links)
- Real world networks are **highly clustered**

Betweenness Centrality

- Number of shortest path that **passes through** a node
  - Represents the influence of a node for information flows



Medici family became the wealthiest



Analysis of Social Networks

Basic Definitions

Measures of Connectivity

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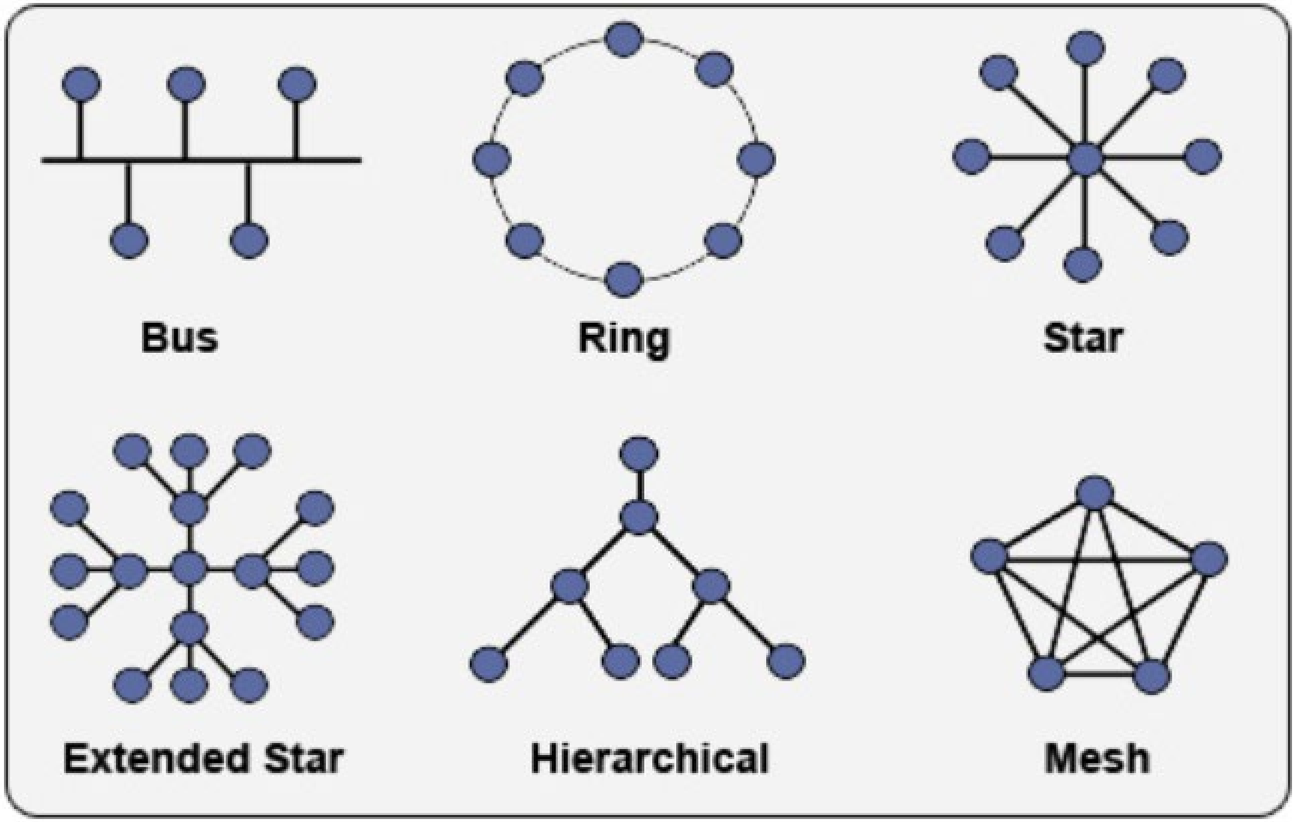
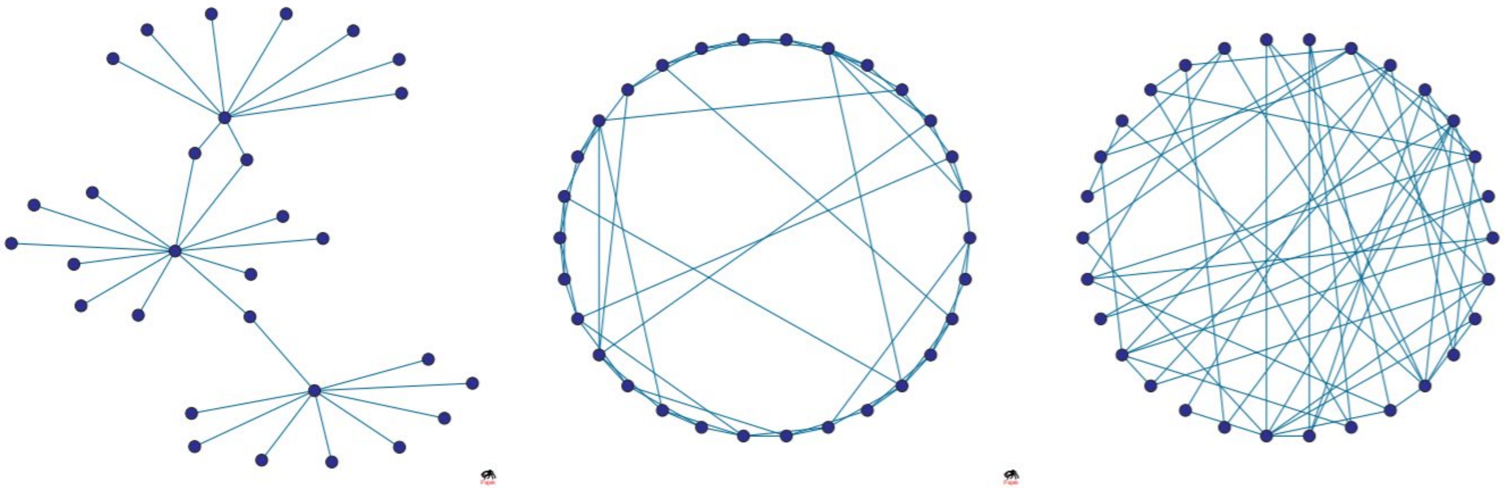
Information Flows

Network Topologies



Network Types to model real world cases

- Random networks
  - Erdos & Renyi 1959
- Small world networks
  - Watts & Strogatz 1998
- Scale free networks
  - Barabasi & Alert 1999



(a) Scale-free network (b) Small-world network (c) Erdős–Rényi network

Random Networks - Generation

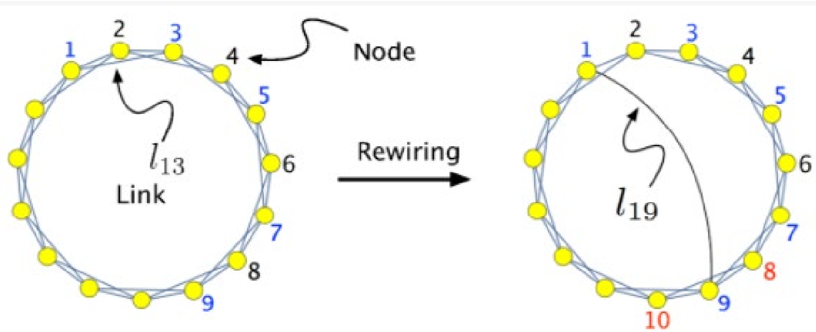
- Links between each node is random
  - With equal probability
- Start with **n** nodes
  - **Connect each pair** of nodes with a probability **p**
  - All nodes have approximately the **same degree**: **k**
- Random networks
  - # of nodes: n
  - Probability of an edge between any two nodes: p
  - **Notation:** G(n, p)

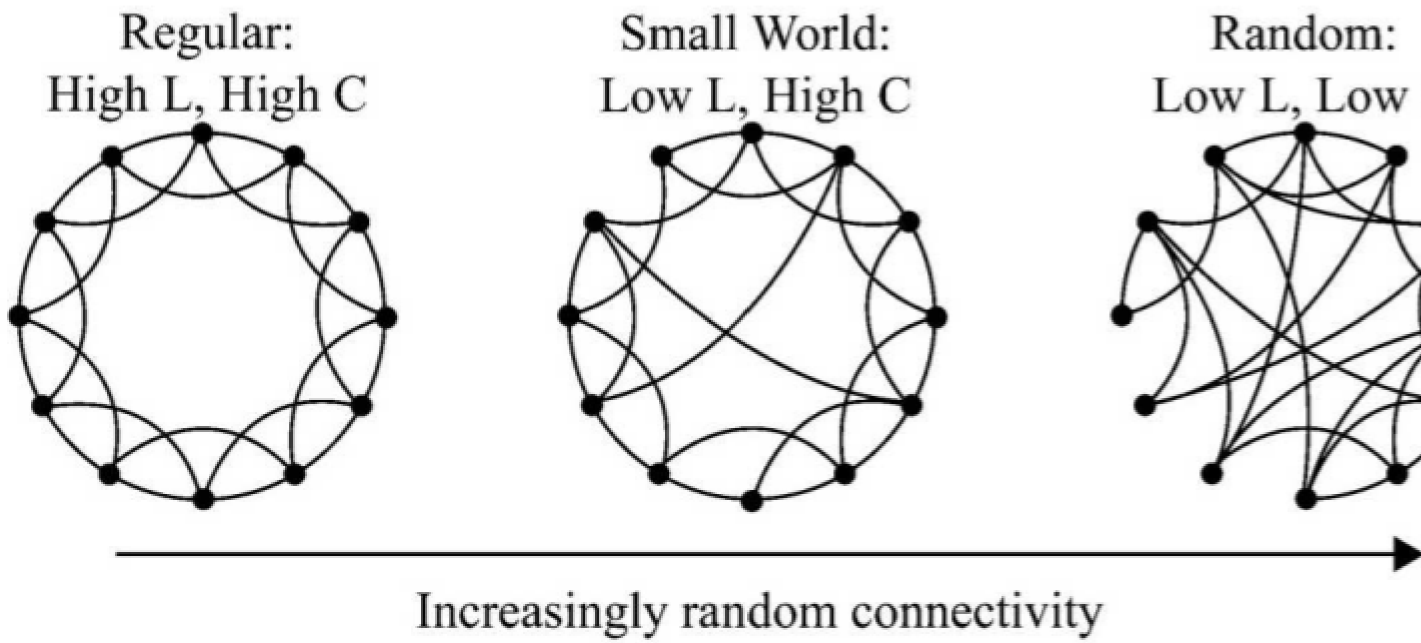
Random Networks - Properties

- Average path length: $\frac{\ln(n)}{\ln(k)}$
- Clustering coefficient: $\frac{k}{n} = p$
- Degree distribution:
    - Binomial distribution for small n
    - Poisson distribution for large n

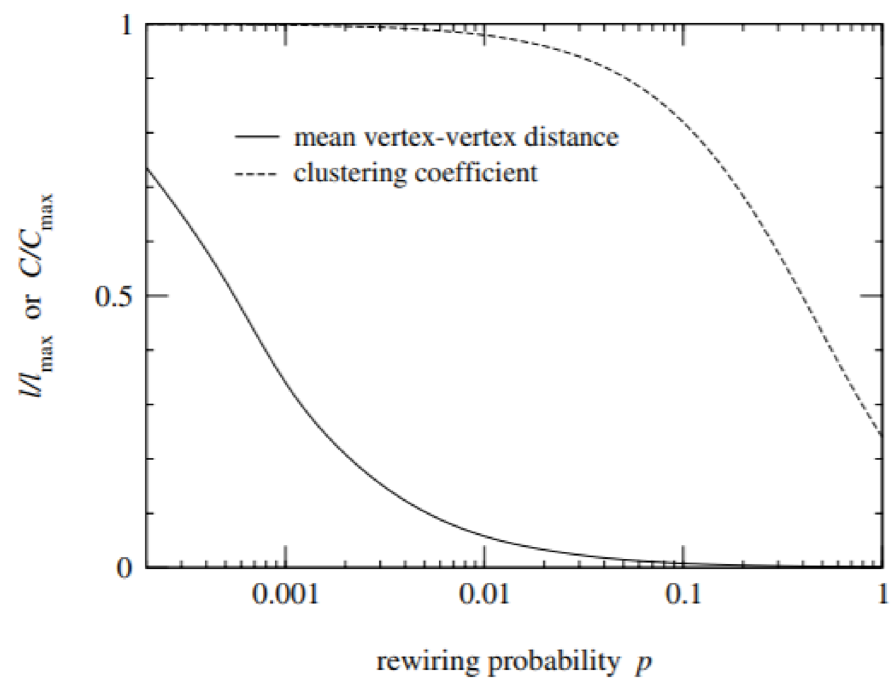
Small World Networks - Generation

- Similar to social networks in real world
  - A group of people are **closely related**
  - A few people have **far reaching connections**
- Generation
  - Start with a ring lattice of **n** nodes
  - Each node connected to its closest **k** neighbors
  - **Rewire the edges**
    - Delete edges with probability **p**
    - Create a random edge such that the number of links remain the same





Regular:
High L, High C
Small World:
Low L, High C
Random:
Low L, Low C
Increasingly random connectivity



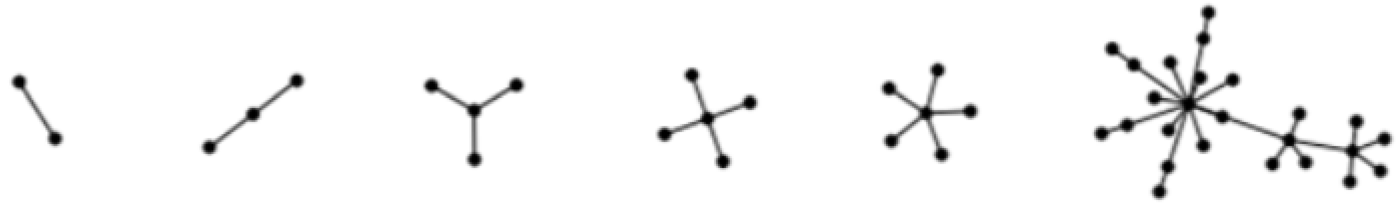
mean vertex-vertex distance
clustering coefficient
l/l_{max} or C/C_{max}
rewiring probability p
1
0.5
0
0.001
0.01
0.1
1

Small World Network - Properties

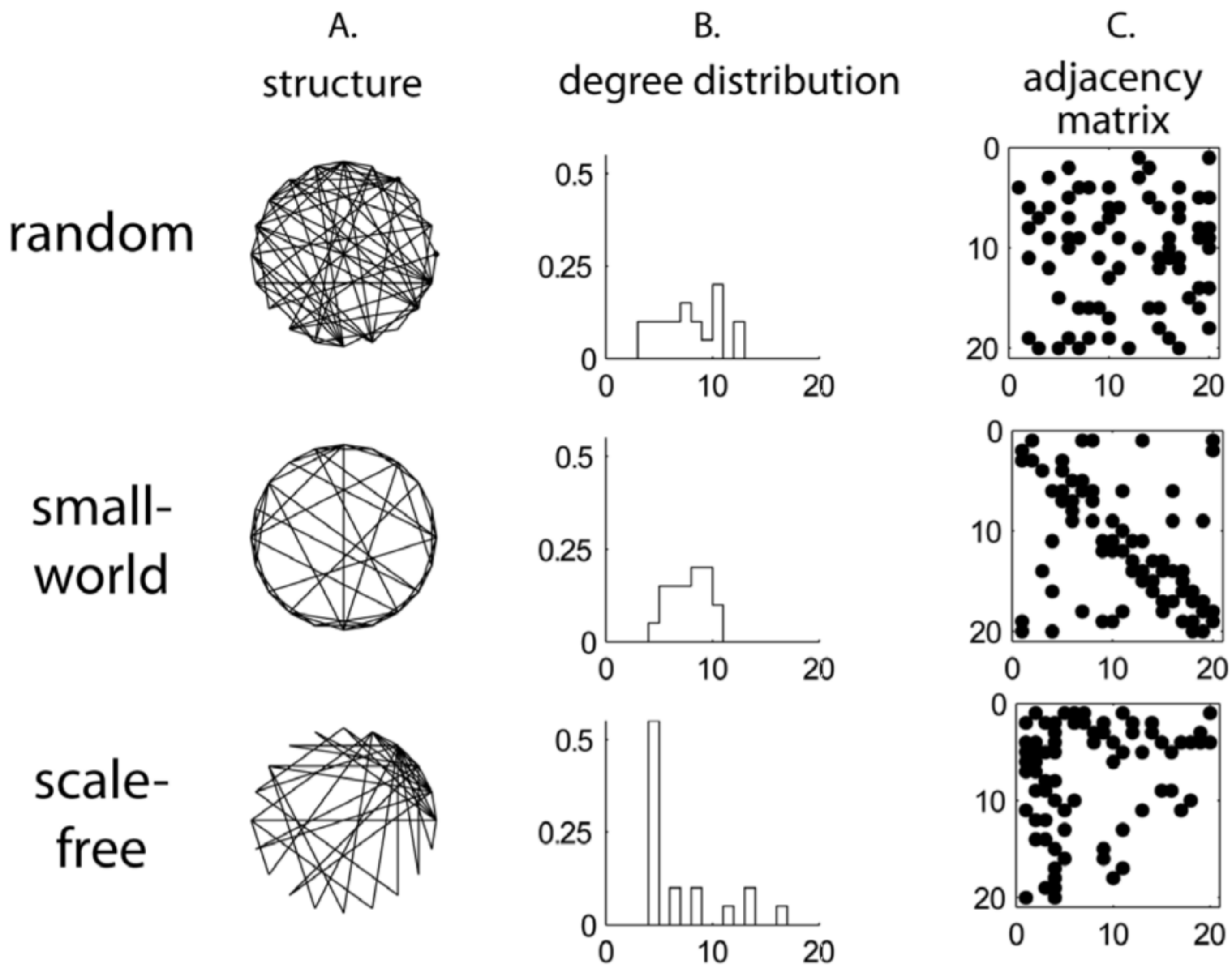
- Average path length:
  - Proportional to $\ln(n)$
- Clustering coefficient:
  - **Highly clustered**
  - Compared to random networks, $CC_{SW} \gg CC_{RN}$
- Degree distribution
  - Similar to random networks
  - Binomial distribution for small n
  - Poisson distribution for large n

Scale Free Networks - Generation

- Power-law network
  - Hub networks
- Preferential attachment
  - Start with 1 node
  - Add a new node via a link
  - **Which node** will be linked in the existing network?
  - **Depends on the degree** of the node
  - Rich get richer phenomenon



Comparison of the 3 Network Types



D.
Description

73 connections among 20 nodes assigned randomly

High local clustering and short average path lengths. 'Hub-and-spoke' architecture.

'Hub-and-spoke' architecture is maintained at multiple spatial scales.

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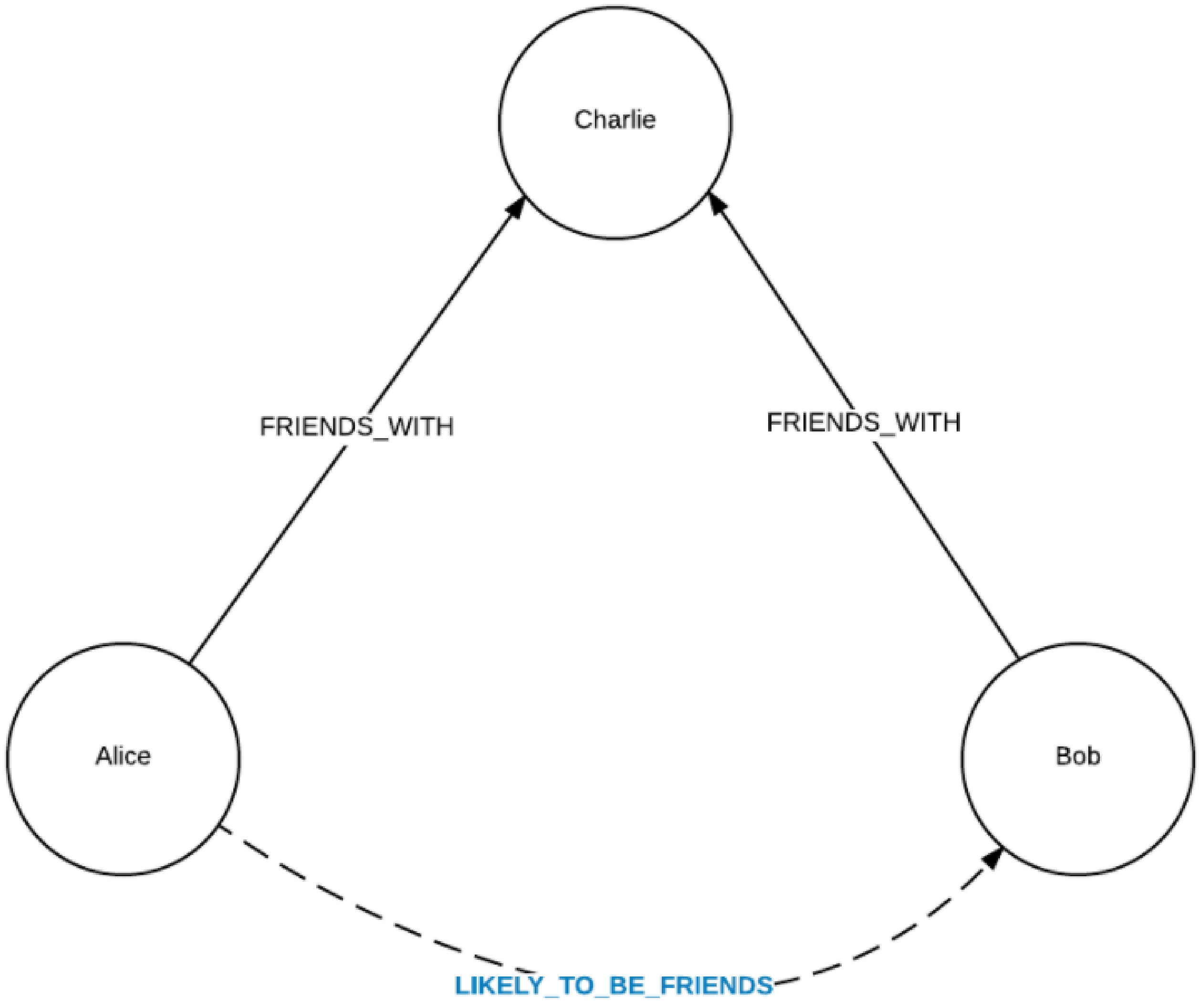
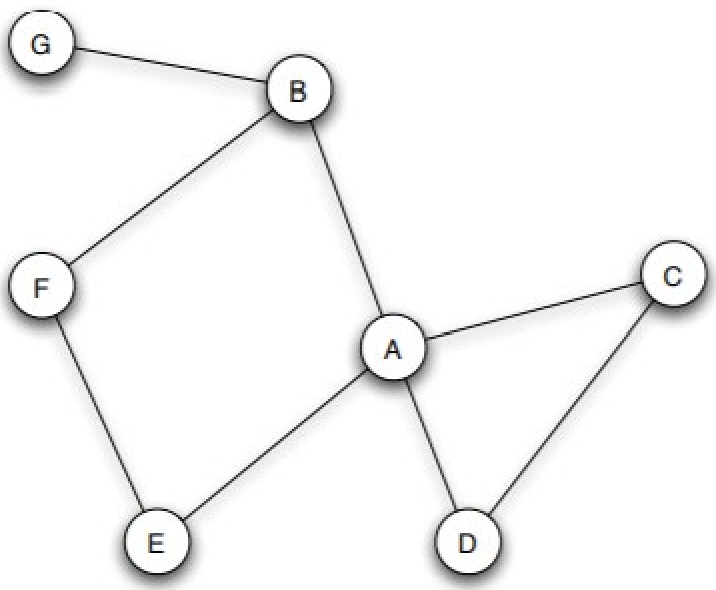
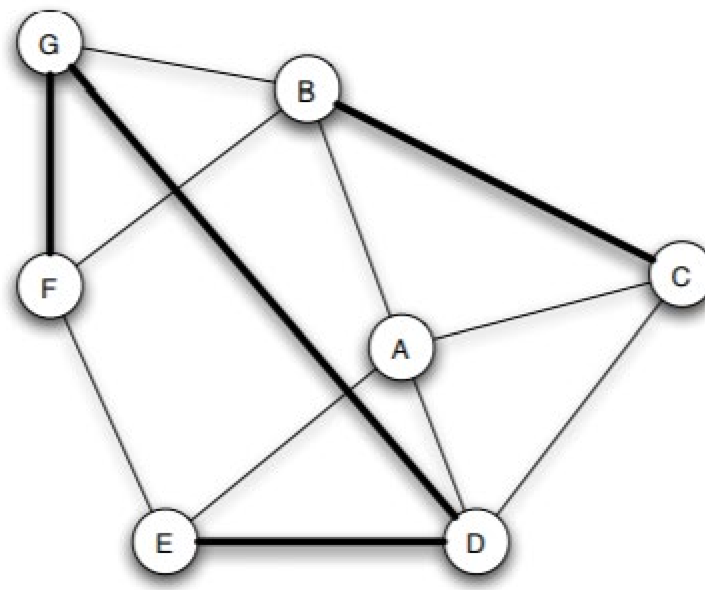
Triadic Closure



Evolution of Networks via Triadic Closure

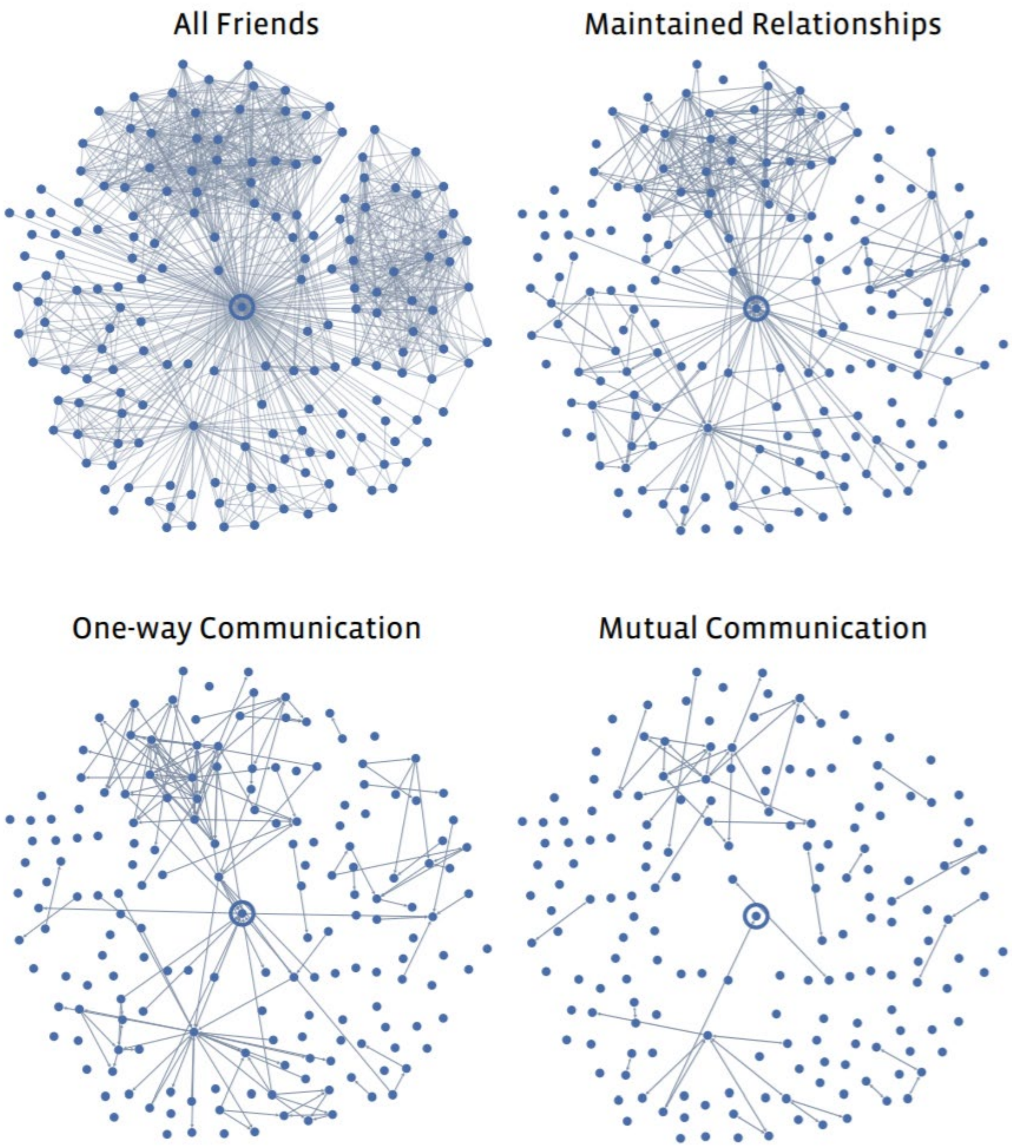


(a) *Before new edges form.*

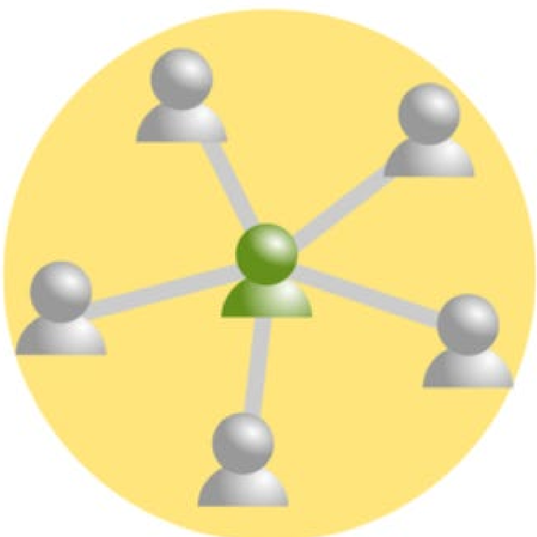


(b) *After new edges form.*

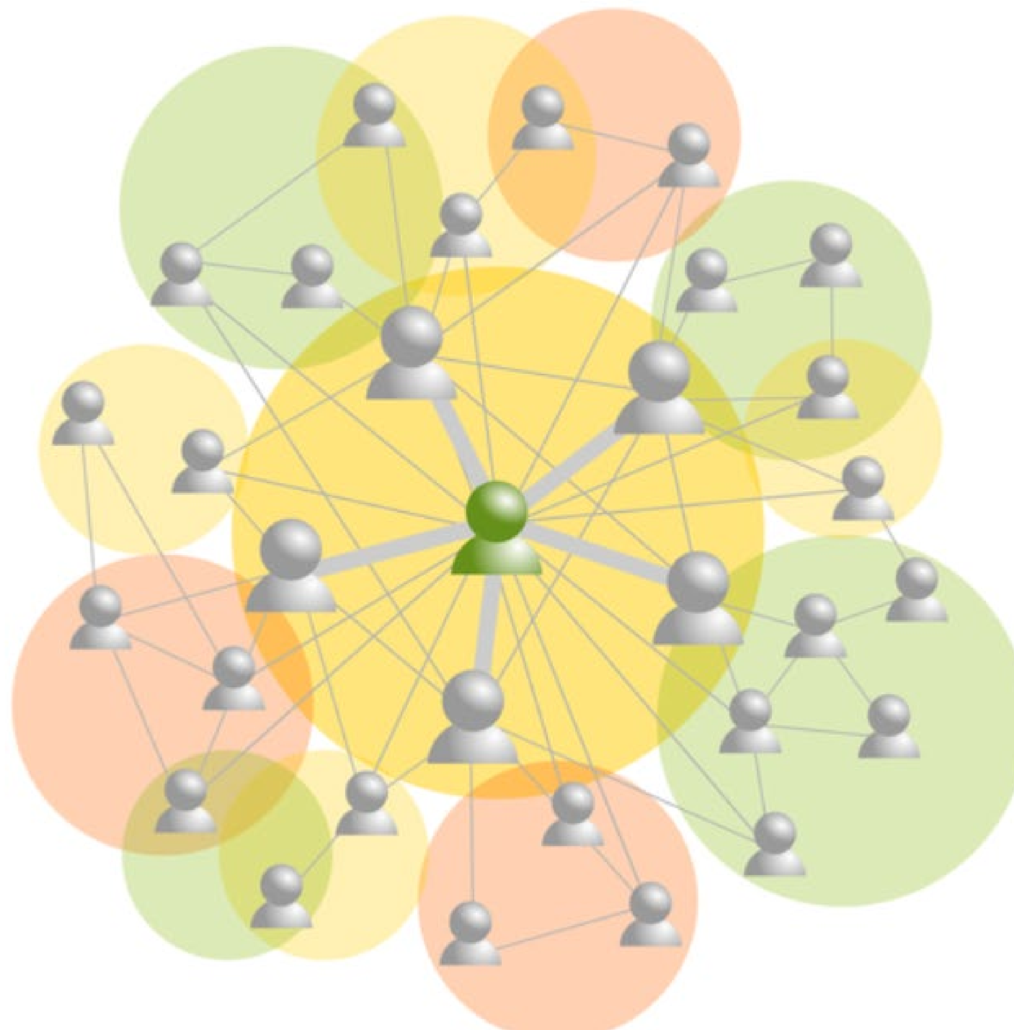
Tie Strength – Facebook users network



Strength of weak ties



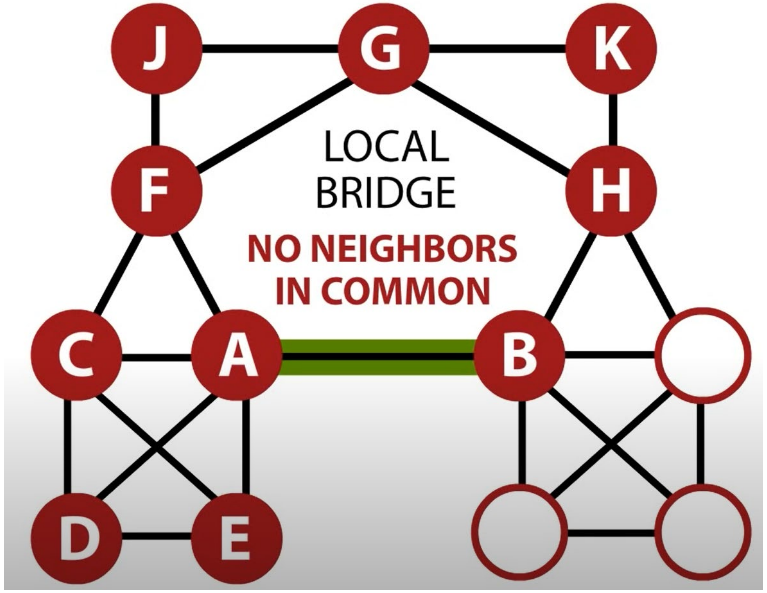
CONNECTIONS THROUGH STRONG TIES



CONNECTIONS THROUGH WEAK TIES

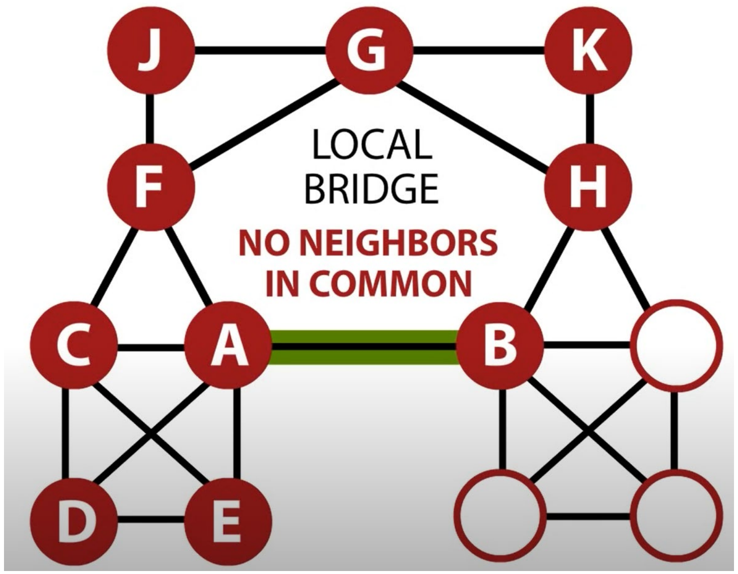
Strong Ties and Weak Ties

- Strong ties
  - Close friendship
  - Your **friends are also friends** with each other
- Weak ties
  - A distant friend
  - Your **friends do not know each other**

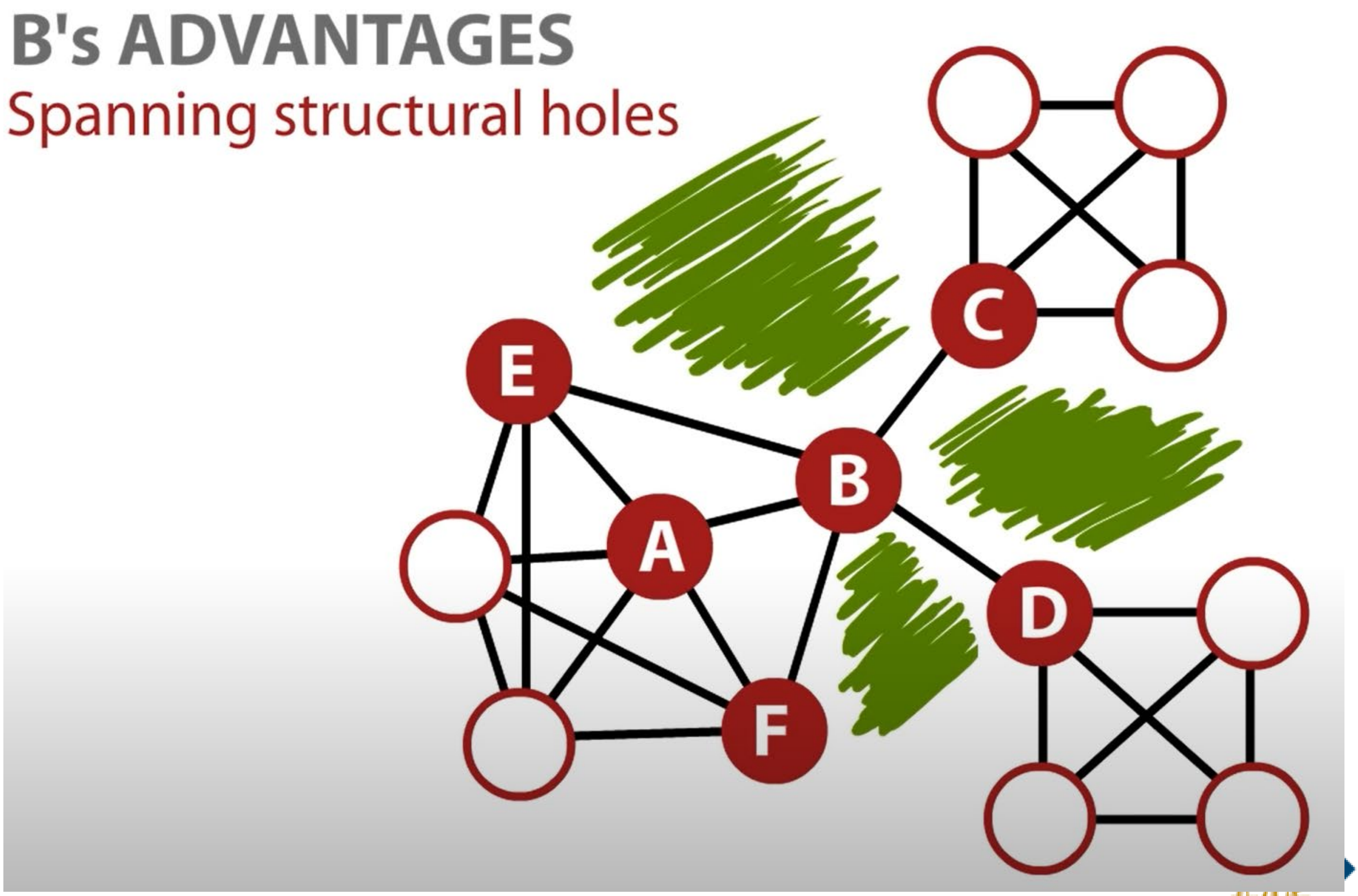


Strength of Weak Ties

- A is looking for a job
  - C, D, E and F are close friends and want to help A
  - But **what they know is similar to what A knows**
- B has access to a bunch of information
  - That A cannot directly perceive
- Job leads, novel information, etc.



Structural Holes



Structural Holes and Good Ideas

- Burt (2004)
  - People **connected across groups**
    - More familiar with **alternative ways** of thinking
    - More options to select and **synthesize**
- Nodes spanning structural holes
  - Information advantage
    - Information across groups are **more additive than overlapping**
  - Control advantage
    - Third-party opportunities
    - Brokerage
    - Entrepreneurship

Readings

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