Evidential Deep Learning to Quantify Classification Uncertainty

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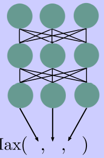
1 Motivation

Vanilla Pipeline

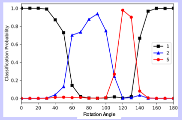


$\mathbf{p} \sim \mathrm{SoftMax}(\ ,\ ,\ )$

models only empirical risk

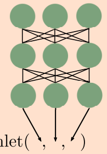
$\mathcal{L} = \mathrm{CrossEntropy}(\mathbf{y}, \mathbf{p})$

generates only point estimates



leads to over-confidence

Our Method

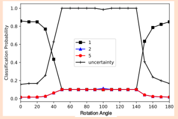


$\mathbf{p} \sim \mathrm{Dirichlet}(\ ,\ ,\ )$

generates a distribution

$\mathcal{L} = \mathbb{E}_{\mathbf{p}}\left[||\mathbf{y} - \mathbf{p}||_2^2\right]$

models Bayes risk



learns to capture uncertainty

2 The Subjective Logic Interpretation

- Consider a frame of K mutually exclusive singletons (e.g., class labels)
- Assign a belief mass $b_k \geq 0$ on each singleton $k = 1, \ldots, K$ with and define an uncertainty score $u \geq 0$ such that

$$u + \sum_{k=1}^{K} b_k = 1.$$

- Let $e_k \geq 0$ be the evidence derived for the k^{th} singleton, then the belief b_k and the uncertainty u are computed as

$$b_k = \frac{e_k}{S} \quad \text{and} \quad u = \frac{K}{S},$$

where $S = \sum_{i=1}^{K}(e_i + 1)$.

- This way, a subjective opinion can be derived easily from a Dirichlet distribution with parameters α_k such that

$$b_k = (\alpha_k - 1)/S.$$

3 The Loss Design

As our method provides a distribution on class probabilities for a given input, we need to minimize the Bayes risk with respect to a loss:

$$\mathcal{L}_i(\Theta) = \int ||\mathbf{y}_i - \mathbf{p}_i||_2^2 \frac{1}{B(\boldsymbol{\alpha}_i)} \prod_{i=1}^{K} p_{ij}^{\alpha_{ij}-1} d\mathbf{p}_i$$

$$= \sum_{j=1}^{K} \mathbb{E}\left[y_{ij}^2 - 2y_{ij}p_{ij} + p_{ij}^2\right]$$

$$= \sum_{j=1}^{K} \left(y_{ij}^2 - 2y_{ij}\mathbb{E}[p_{ij}] + \mathbb{E}[p_{ij}^2]\right).$$

Regularize the loss against unjustified evidence prediction with an absolutely uncertain predictor:

$$\mathcal{L}(\Theta) = \sum_{i=1}^{N} \mathcal{L}_i(\Theta)$$

$$+ \lambda_t \sum_{i=1}^{N} KL[D(\mathbf{p_i}|\tilde{\boldsymbol{\alpha}}_i) \ || \ D(\mathbf{p}_i|\langle 1, \ldots, 1 \rangle)].$$

4 Theoretical Properties

Our loss can be expressed in the following easily interpretable form

$$\mathcal{L}_i(\Theta) = \sum_{j=1}^{K} (y_{ij} - \mathbb{E}[p_{ij}])^2 + \mathrm{Var}(p_{ij})$$

$$= \sum_{j=1}^{K} \underbrace{(y_{ij} - \alpha_{ij}/S_i)^2}_{\mathcal{L}_{ij}^{err}} + \underbrace{\frac{\alpha_{ij}(S_i - \alpha_{ij})}{S_i^2(S_i + 1)}}_{\mathcal{L}_{ij}^{var}}$$

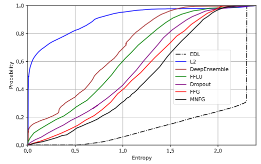
which satisfies the following three propositions.

- **Proposition 1.** For any $\alpha_{ij} \geq 1$, the inequality $\mathcal{L}_{ij}^{var} < \mathcal{L}_{ij}^{err}$ is satisfied.
  i.e. The loss prioritizes data fit over variance estimation.
- **Proposition 2.** For a given sample i with the correct label j, L_i^{err} decreases when new evidence is added to α_{ij} and increases when evidence is removed from α_{ij}.
  i.e. The loss has a tendency to fit to the data.
- **Proposition 3.** For a given sample i with the correct class label j, L_i^{err} decreases when some evidence is removed from the biggest Dirichlet parameter α_{il} such that $l \neq j$.
  i.e. The loss performs learned loss attenuation.

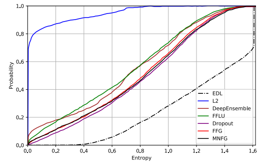
5 Experiments

Detection of Out-of-Distribution Samples

notMNIST

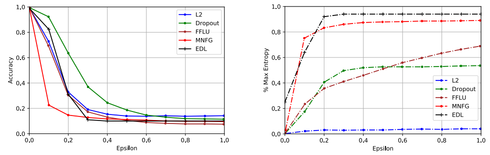


CIFAR5

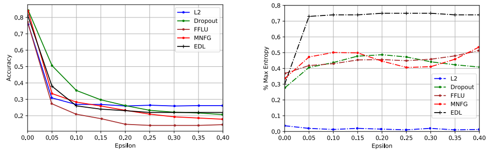


Detection of White-Box Adversarial Attacks

MNIST



CIFAR5



6 Take Homes

- Replace the SoftMax-generated class probabilities with a Dirichlet distribution.
- Minimize Gibbs risk in addition to the empirical risk.
- Draw links to and get inspiration from opinion modeling.
- Outperform state of the art in detection of out-of-distribution samples and white-box attacks without any security-specific design.

Paper

Code