Rosenbaum

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1 The model

- Stage 1: Supplier firms choose the minimum expected cost Perfect Bayes Nash Equilibrium (PBNE) plant locations, where each firm chooses locations that maximize its expected profit given the locations of other firms, with expectation taken over the distributions of contract/firm specific costs η and contract/plant specific costs ϵ
- Stage 2: The contract/firm specific costs η are realized and assemblers award contracts using Second Price Auction (SPA) with each eligible firm bidding its expected cost for the contract, with expectation taken over the distribution of contract/plant specific costs ϵ
- Stage 3 : The contract/plant specific costs ϵ are realized and the supplier firms incur the actual costs and profits

1.1 Working out the mathematics (backwards)

- Stage 3: For every contract $r \in R$, a firm f incurs variable cost:
  $C_{r,f}^{actual} = \begin{cases} \bar{C}_r - \max_{i\in(x_f,y_f)} \{-\bar{C}_{r,i} - \epsilon_{r,i}\} + \eta_{r,f}; C_{r,f} \leq C_{r,f'} \forall f' \in F_{p(r)} \\ 0, otherwise \end{cases}$
  and makes total profit:
  $\pi_{r,f}^{actual} = \begin{cases} \psi(r)(M_{r,f} - C_{r,f}^{actual}); C_{r,f} \leq C_{r,f'} \forall f' \in F_{p(r)} \\ 0, otherwise \end{cases}$
- Stage 2: For its plant locations (x_f, y_f), the firm f incurs expected cost $C_{r,f} = \bar{C}_r - E_\epsilon[\max_{i\in(x_f,y_f)} \{-\bar{C}_{r,i} - \epsilon_{r,i}\}] + \eta_{r,f}$
  $= \bar{C}_r - log(\sum_{i\in(x_f,y_f)} exp(-C_{r,i})) - \gamma + \eta_{r,f}$
  for contract r, and is awarded the contract if $-C_{r,f} \geq -C_{r,f'} \forall f' \in F_{p(r)}$.
  The assembler then pays the amount $M_{f,r} = -\max_{(f'\in F_{p(r)}-\{f\})} (-C_{f',r})$
- Let $N_r(x,y) = \min_{f\in F_{p(r)}} C_{f,r} = -\max_{f\in F_{p(r)}} -C_{f,r}$.
  The cost minimizing PBNE plant locations are solutions to
  $\max_{(x,y)} \{\sum_r -E_\eta[N_r(x,y)] - \sum_f \phi(x_f, y_f)\}$

2 Homework Assignment (Short Answers)

What is $\mathbf{E}[\mathbf{N_r}(\mathbf{x},\mathbf{y})]$?

It is the expectation (over the distributions of contract/firm specific costs)of the minimum (over all the firms f that manufacture parts $P(r)$) expected (expecation taken over contract/plant specific costs) minimum (over all possible plant locations for each firm)cost for contract r and plant locations specified by (x,y).

$E[N_r(x,y)] = E_\eta[min_{f\in F_{P(r)}} C_{f,r}] = -E_\eta[-max_{f\in F_{P(r)}} -\bar{C}_r - \bar{C}_{r,f} - \eta_{r,f}] = \bar{C}_r - log(\sum_{f\in F_{P(r)}} exp(-\bar{C}_{r,f})) - \gamma$. This assuming that $-\eta_{r,f} \sim Gumbel(0)$, and therefore $-\bar{C}_{r,f} - \eta_{r,f} \sim Gumbel(-\bar{C}_{r,f})$

Why does the author use a nested logit structure with two error terms?

Suppose we consider a discrete choice set with each choice being a (firm,plant) pair. In this case Independence of Irrelevant Alternatives may not hold as the choices of the different plants corresponding to the same firm may be interdependent. However if we consider the firms as nests, and plants as elements of a nest, and first choose a nest and then choose a plant, then IIA holds at both the steps.

Table 1: Optimal Locations

x	y
0.677	0.435
0.417	0.050
0.112	0.981
0.838	0.860
0.232	0.408
0.736	0.552

3 Homework Assignment (Coding Assignment)

3.1 Formulating the Optimization Problem

Assumption: $-\eta_{r,f}, -\epsilon_{r,i} \sim Gumbel(0)\ \forall r \in R, f \in F_{p(r)}, i \in \mathcal{I}_f$
For the given locations (x_{-f}, y_{-f}) of the competitor, the firm f chooses locations (x_f, y_f) that solve

$\max_{x_f,y_f}(\sum_r[-E[N_r(x_f,y_f;x_{-f},y_{-f})]] - \sum\phi(x_f,y_f) - \sum\phi(x_{-f},y_{-f}))$, or :

$\max_{x_f,y_f}(\sum_r[log(exp(-\bar{C}_{r,f}) + exp(-\bar{C}_{r,-f}))] - \sum\phi(x_f,y_f) - \sum\phi(x_{-f},y_{-f}))$, or:

$\max_{x_f,y_f}(\sum_r[log(exp(log(\sum_{i\in(x_f,y_f)} exp(-C_{r,i})) + \gamma) + exp((log(\sum_{i\in(x_{-f},y_{-f})} exp(-C_{r,i})) + \gamma)] - \sum\phi(x_f,y_f) - \sum\phi(x_{-f},y_{-f}))$, or:

$\max_{x_f,y_f}(\sum_r[log(\sum_{i\in(x_f,y_f)} exp(-C_{r,i}) + \sum_{i\in(x_{-f},y_{-f})} exp(-C_{r,i}))] - \sum\phi(x_f,y_f) - \sum\phi(x_{-f},y_{-f}))$

For the linear cost problem with no fixed costs, the optimization problem is

$$\max_{x_f,y_f}(\sum_r[log(\sum_{i\in(x_f,y_f)} exp(-\beta^d||i-r|| - \beta^u u_t(i)) + \sum_{j\in(x_{-f},y_{-f})} exp(-\beta^d||j-r|| - \beta^u u_t(j)))])$$

where r is the assembler location and number of optimization variables (x_f, y_f) is equal to number of sites

3.2 Coding and Results

Please refer to the assignment-1-sim-bin.R document for code details. We ran the optimization 30 times from different starting values and we picked the one with lowest optimum value. The final results are listed in Table 1 above.