OPNS 523 Assignment: Aguirregabiria (1999)

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Part I

1. Explain the intuition for how K-convex cost functions leads to (s,S) inventory policies.

   Let $G(y)$ be the expected total discounted cost less the fixed ordering cost (K) if we start a period with no on-hand item and we buy y items for the period.

   Suppose that we have x items on hand at the beginning of the period. If we decide to buy $y - x$ items for the period, our expected discounted cost for the period is then $G(y) + K$. If we decide not to buy, our expected discounted cost for the period is then $G(x)$.

   Therefore, compared to doing nothing, it is better to buy $y - x$ items if $G(y) + K < G(x)$; otherwise, it is better not to order.

   The optimality of the (s, S) policy is characterized by the following sufficient conditions (1) and (2) that must hold at the same time: Take S to be the global minimum of $G(y)$. Set $s = min\{u : G(u) = K + G(S)\}$. Then

   $$G(y) \leq K + G(S) \text{ for } s \leq y \leq S. \tag{1}$$

   For any local minimum a of G such that $S < a$,

   $$G(y) \leq K + G(a) \text{ for } S \leq y \leq a. \tag{2}$$

   For $y < s$, the shapes and values (which must be no less than K+G(S)) are irrelevant to the optimality of the (s, S) policy. We can see that the above sufficient conditions are exactly the definition of K-convexity.

2. Aguirregabiria (1999, p. 293-4) explain that

   The decision rule in equations (16) and (17) is the combination of marginal conditions of optimality and optimal discrete choices. In this paper we obtain estimates of the structural parameters which exploit moment conditions associated with the optimal discrete choice, but not moment conditions associated with the marginal conditions of optimality (i.e. Euler equations). It is also important to notice that lump-sum adjustment costs do not appear explicitly in the Euler equations. The identification of these parameters requires one to exploit moment conditions associated with the optimal discrete choice.

What does this mean. Does considering only the discrete choices sacrifice much information?

This means we can still recover the structural estimates by only considering the moment conditions associated with the optimal discrete choices. We can see that even though equations (16) and (17) contain marginal conditions of optimality and optimal discrete choices, (16) and (17) can be represented by equation (19) which only uses information from the optimal discrete choice because at the optimal discrete choices, these decision rules should also be optimal.

3. Aguirregabiria (1999, p. 296) uses Hotz and Miller's (1993) estimation procedure, which is equivalent to Aguirregabiria and Mira's (2002) nested pseudo-likelihood, with K = 1. Can you roughly express Aguirregabiria's (1999) estimation procedure in Aguirregabiria and Mira's (2002) terms?

   Algorithm:

   1. Pre-estimate transition probabilities of the state variables.
   2. Pre-estimate CCPs ($P^d(x)$) using nonparametric kernel estimation and estimate $\Pi_1^d(x) = E(exp\{m + c\}y - exp\{c\}q \mid x, d)$.
   3. Use GMM to obtain θ_π using the following moment conditions:

   $$E(Z_{it}[I(d_{it} = d) - p^{(d)}(\Pi_{it}, \hat{W}_{it}; \theta_\pi)]) = 0 \text{ for } d = 1, 2, ..., 5.$$

   4. Update CCPs using the following equation:

   $$p^{(d)}(\Pi_{it}, \hat{W}_{it}; \theta_\pi) = \frac{exp(\Pi_{it}^{d'}\mu(\theta_\pi) + \hat{W}_{it}^{d'}\lambda(\theta_\pi))}{\sum_{j=1}^{6} exp(\Pi_{it}^{j'}\mu(\theta_\pi) + \hat{W}_{it}^{j'}\lambda(\theta_\pi))}$$

   5. Run for K=1 iterations.

Part II

1. Express double integrated value function $\tilde{V}_i = E_s[\bar{V}_{i,s}]$ as a function of $f_i = E_s[U_{i,s}(1) - log(P_{i,s}(1))$ and $\tilde{V}_Q$.

$$\begin{aligned}
\because \bar{V}_{i,s} &= E_\epsilon[\max_q U_{i,s}(q) + \epsilon + \beta \bar{V}_{i',s}] \text{ where } i' = i - s + q(Q - i + S) \\
&= E_\epsilon[U_{i,s}(1) + \epsilon + \beta \bar{V}_{Q,s} - log(P_{i,s}(1))] \text{ (Gumbel Property 4)} \\
&= U_{i,s}(1) + \beta \bar{V}_{Q,s} - log(P_{i,s}(1))
\end{aligned}$$

$$\begin{aligned}
\therefore \tilde{V}_i &= E_s[\bar{V}_{i,s}] \\
&= E_s[U_{i,s}(1) + \beta \bar{V}_{Q,s} - log(P_{i,s}(1))] \\
&= f_i + \beta \tilde{V}_Q.
\end{aligned}$$

2. Use finite dependence to express $P_{i,s}$ in terms of U and f.

$$\begin{aligned}
P_{i,s}(1) &= \frac{exp(U_{i,s}(1) + \beta \tilde{V}_Q)}{exp(U_{i,s}(1) + \beta \bar{V}_Q) + exp(U_{i,s}(0) + \beta \tilde{V}_{i-s})} \\
&= \frac{1}{1 + exp(U_{i,s}(0) - U_{i,s}(1) + \beta(\tilde{V}_{i-s} - \tilde{V}_Q))} \\
&= \frac{1}{1 + exp(U_{i,s}(0) - U_{i,s}(1) + \beta(f_{i-s} + \beta \tilde{V}_Q - (f_Q + \beta \tilde{V}_Q)))} \\
&= \frac{1}{1 + exp(U_{i,s}(0) - U_{i,s}(1) + \beta(f_{i-s} - f_Q))}.
\end{aligned}$$

3. Roughly explain how we could empirically identify the inventory underage cost, inventory overage cost, and shipping cost.

   First, overage cost can be identified by the specification of the model. The stock-out rate helps identify the underage cost relative to the overage cost. In fact, the news-vendor model suggests that the service level should be around $\frac{\text{overage cost}}{\text{overage cost+underage cost}}$. In addition, the magnitude of orders helps to identify the shipping cost relative to the underage cost.