**Nested Logit Assignment**

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**Results**

**Choice Probabilities:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bucket | Choice | Theoretical | Nested Logit (Sequential) | One Shot |
| 1 | A | 0.363 | 0.354 | 0.347 |
| 1 | B | 0.044 | 0.002 | 0.001 |
| 1 | C | 0.002 | 0.043 | 0.04 |
| 2 | A | 0.03 | 0.034 | 0.03 |
| 2 | B | 0.18 | 0.18 | 0.178 |
| 2 | C | 0.116 | 0.124 | 0.121 |
| 3 | A | 0.061 | 0.03 | 0.031 |
| 3 | B | 0.03 | 0.058 | 0.063 |
| 3 | C | 0.175 | 0.175 | 0.189 |

**Wald Tests:**

Null Hypothesis: Theoretical choice probabilities and simulated Nested Logit choice probabilities are equal

Chi-squared test:

X2 = 1.9, df = 8, P(> X2) = 0.98

Null Hypothesis: Theoretical choice probabilities and simulated one shot choice probabilities are equal

Chi-squared test:

X2 = 5.0, df = 8, P(> X2) = 0.75

Covariance Matrices for Choice Probabilities (Nested Logit)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0.0131 | -0.0047 | -0.0032 | -0.0026 | -0.0013 | -0.0008 | -0.0003 | -0.0001 | 0.0000 |
| -0.0047 | 0.0123 | -0.0022 | -0.0023 | -0.0016 | -0.0011 | -0.0003 | -0.0001 | 0.0000 |
| -0.0032 | -0.0022 | 0.0104 | -0.0016 | -0.0015 | -0.0011 | -0.0005 | -0.0002 | 0.0000 |
| -0.0026 | -0.0023 | -0.0016 | 0.0092 | -0.0015 | -0.0006 | -0.0004 | -0.0002 | 0.0000 |
| -0.0013 | -0.0016 | -0.0015 | -0.0015 | 0.0068 | -0.0007 | -0.0002 | 0.0001 | -0.0001 |
| -0.0008 | -0.0011 | -0.0011 | -0.0006 | -0.0007 | 0.0047 | -0.0002 | -0.0001 | 0.0000 |
| -0.0003 | -0.0003 | -0.0005 | -0.0004 | -0.0002 | -0.0002 | 0.0020 | 0.0000 | 0.0000 |
| -0.0001 | -0.0001 | -0.0002 | -0.0002 | 0.0001 | -0.0001 | 0.0000 | 0.0007 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | -0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0002 |

Covariance Matrices for Choice Probabilities (One Shot)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0.0123 | -0.0039 | -0.0025 | -0.0023 | -0.0018 | -0.0010 | -0.0005 | -0.0003 | 0.0000 |
| -0.0039 | 0.0119 | -0.0025 | -0.0023 | -0.0022 | -0.0006 | -0.0004 | -0.0001 | 0.0000 |
| -0.0025 | -0.0025 | 0.0099 | -0.0011 | -0.0020 | -0.0011 | -0.0005 | 0.0000 | 0.0000 |
| -0.0023 | -0.0023 | -0.0011 | 0.0089 | -0.0012 | -0.0013 | -0.0005 | -0.0001 | 0.0000 |
| -0.0018 | -0.0022 | -0.0020 | -0.0012 | 0.0084 | -0.0006 | -0.0003 | -0.0001 | 0.0000 |
| -0.0010 | -0.0006 | -0.0011 | -0.0013 | -0.0006 | 0.0048 | -0.0001 | -0.0001 | 0.0000 |
| -0.0005 | -0.0004 | -0.0005 | -0.0005 | -0.0003 | -0.0001 | 0.0024 | 0.0000 | 0.0000 |
| -0.0003 | -0.0001 | 0.0000 | -0.0001 | -0.0001 | -0.0001 | 0.0000 | 0.0008 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

**Methods**

**Step 1: Sample 10000 Gumbel RVs εk, ηi,k and the choices**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sample Number | ε1 | ε2 | ε3 | η1,A | η2,A | η3,A | η1,B | η2,B | η3,B | η1,C | η2,C | η3,C | Optimal Bucket | Optimal Choice | Optimal Choice Index |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ….. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10000 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

**One Shot Problem**

For each sampled value calculate Vi,k+ εk+λkηi,k, k=1,2,3, i = A,B,C , and choose the bucket and choice with the greatest value. The probabilities are : P1,A = (number of times we have 1,A)/10000. We can also assign each possible choice an index number from 1 to 9

**Two Shot Problem (Sequential Nested Logit)**

First, for each sampled value, we calculate the expected value from a bucket k i.e. choose bucket with the greatest expected value of Vi,k+ εk+λkηi,k i.e bucket with greatest value of λkLSE(Vi,k/λk)+ εk, then for

Step 1: First we choose the bucket k with the largest λkLSE(Vi,k/λk)+ εk

Step 2: we choose i with the greatest value of Vi,k+ λkηi,k

The probabilities are : P1,A = (number of times we have 1,A)/10000

**Step 2: Generate 1000 resamples using bootstrap i.e. create a 1000\*10000 data frame, row is the bootstrap number and columns are the row numbers of the original sample**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bootstrap # | Row number of the original data sampled | | | |
| 1 | **1** | **1** | **…….** | **99998** |
| 2 |  |  |  |  |
| …. |  |  |  |  |
| 1000 |  |  |  |  |

**1000\*10000 Table**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bootstrap # | Row number of the original data sampled | | | |
| 1 | **Choice Index(1)** | **Choice Index(1)** | **…….** | **Choice Index(99998)** |
| 2 |  |  |  |  |
| …. |  |  |  |  |
| 1000 |  |  |  |  |

**Next, we get the choice index for each row number of original matrix**

**For each boot strap, we therefore calculate the frequency estimate for each probability**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bootstrap # | Row number of the original data sampled | | | |
| 1 | **Choice Index(1)** | **Choice Index(1)** | **…….** | **Choice Index(99998)** |
| 2 |  |  |  |  |
| …. |  |  |  |  |
| 1000 |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bootstrap # | Row number of the original data sampled | | | |
| 1 | **Choice Index(1)** | **Choice Index(1)** | **…….** | **Choice Index(99998)** |
| 2 |  |  |  |  |
| …. |  |  |  |  |
| 1000 |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Bootstrap # | P1,A | P2,A | P3,A | .. |  |  |  |  | P3,C |
| 1 |  |  |  |  |  |  |  |  |  |
| 2…. |  |  |  |  |  |  |  |  |  |
| 1000 |  |  |  |  |  |  |  |  |  |

Using this, we generate the Covariance Matrix

Theoretical Probability of Choosing Bucket k and choice i is:

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