IE 313 – ASSIGNMENT 1 – MURAT TUTAR

Q1 – Base Problem

The formulation and solving of the aggregate production planning problem is done by Python PuLP and is attached in submission file (in case of a problem with opening up the Py file, I also uploaded a txt file including my code script). The mathematical model is also below.

Sets

- T: Months {Jun.23, Jul.23, ..., May.24}

Parameters

- d_t^B: Demand for basic caravan in month t
- d_t^P: Demand for pro caravan in month t
- IB_0: Initial basic inventory (8 units for May.23)
- IP_0: Initial pro inventory (4 units for May.23)
- W_0: Initial workers (93 for May.23)

Decision Variables

- B_t: Basic caravan production in month t
- P_t: Pro caravan production in month t
- IB_t: Basic inventory in month t
- IP_t: Pro inventory in month t
- W_t: Workers in month t
- OT_t: Overtime hours in month t
- H_t: Hires in month t
- F_t: Fires in month t

Objective

$$\min \sum_{t \in T} (6000B_t + 9750P_t + 270IB_t + 540IP_t + 2.5 \times 180W_t + 3.5OT_t + 850H_t + 1300F_t)$$

Constraints

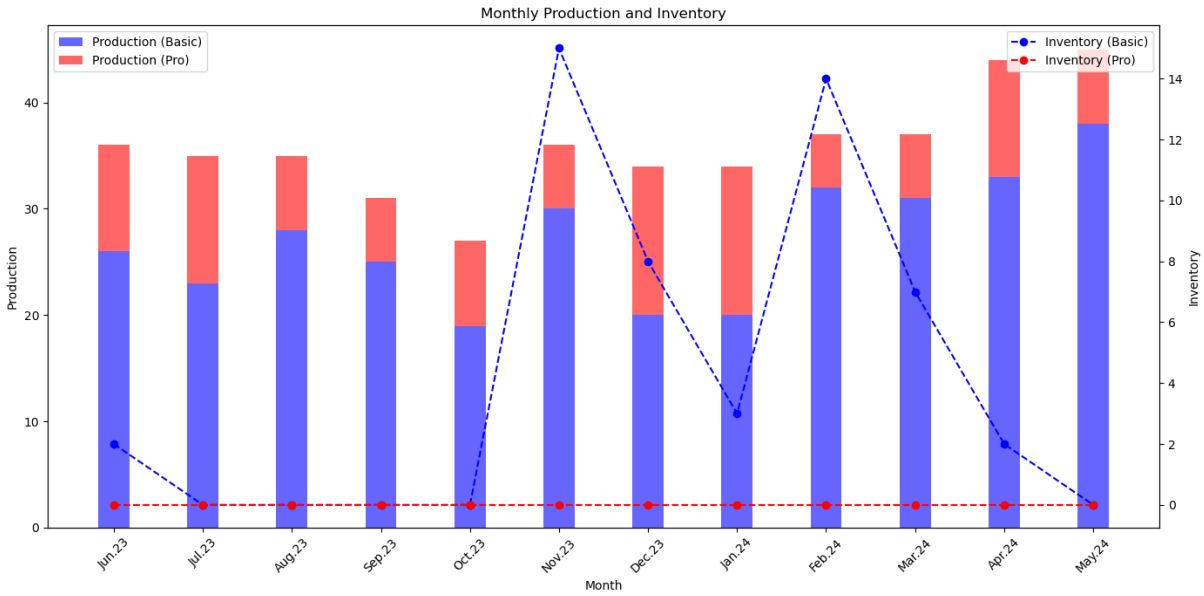
1. $B_t + IB_{t-1} = d_t^B + IB_t$ for all t
2. $P_t + IP_{t-1} = d_t^P + IP_t$ for all t
3. $420B_t + 510P_t \leq 180W_t + OT_t$ for all t
4. $OT_t \leq 40W_t$ for all t
5. $W_{t-1} + H_t - F_t = W_t$ for all t

The output result for optimum production plan is below, where

- B = monthly basic caravan production quantity
- P = monthly pro caravan production quantity
- IB = inventory of basic caravans
- IP = inventory of pro caravans
- W = number of workers per month
- OT = amount of total overtime per month
- H = number of hired employees per month
- F = number of fired employees per month

```
Jun.23: B: 26, P: 10, IB: 2, IP: 0, W: 89, OT: 0.0, H: 0, F: 4
Jul.23: B: 23, P: 12, IB: 0, IP: 0, W: 87, OT: 120.0, H: 0, F: 2
Aug.23: B: 28, P: 7, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Sep.23: B: 25, P: 6, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Oct.23: B: 19, P: 8, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Nov.23: B: 30, P: 6, IB: 15, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Dec.23: B: 20, P: 14, IB: 8, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Jan.24: B: 20, P: 14, IB: 3, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Feb.24: B: 32, P: 5, IB: 14, IP: 0, W: 89, OT: 0.0, H: 2, F: 0
Mar.24: B: 31, P: 6, IB: 7, IP: 0, W: 89, OT: 60.0, H: 0, F: 0
Apr.24: B: 33, P: 11, IB: 2, IP: 0, W: 89, OT: 3450.0, H: 0, F: 0
May.24: B: 38, P: 7, IB: 0, IP: 0, W: 89, OT: 3510.0, H: 0, F: 0
Total Cost: 3506060.0
```

The optimum total cost is $3,506,060. The bar-chart of monthly production and inventory is below.



The total production satisfies the total demand based on this plan. Before the months with high demand forecasts, production is made upfront in the previous months and kept in inventory.

Extra workforce is fired in the first two periods, as the demand seems to be relatively low from Aug23 until Mar24. Before the last three months (Mar24, Apr24, May24) periods where the demand is expected to be very high, it is planned to hire two employees during Feb24 where the demand is relatively low to the next months; so that the production for coming months can be made upfront to satisfy the demand.

In the last two months, a huge amount of overtime is made, which is expected as the demand is very high in these months.

Since it is more costly to keep pro caravans in the inventory than basic ones, they are always produced and sold in the same month without inventory.

In summary, the model provides a production and workforce plan that aims to meet demand while minimizing costs. It adjusts production, labor, and inventory levels to cope with fluctuations in demand. However, the extensive use of overtime and hiring in some months may lead to increased costs. The total cost, as reported by the model, represents the trade-off between meeting demand and managing production-related expenses.

Q2-) The new model is below.

Sets

- T: Months {Jun.23, Jul.23, ..., May.24}
- T_{extra}: Months with extra hours {Dec.23, Jan.24}

Parameters

- d_t^B: Demand for basic caravan in month t
- d_t^P: Demand for pro caravan in month t
- IB_0: Initial basic inventory (8 units for May.23)
- IP_0: Initial pro inventory (4 units for May.23)
- W_0: Initial workers (93 for May.23)

Decision Variables

- B_t: Basic caravan production in month t
- P_t: Pro caravan production in month t
- IB_t: Basic inventory in month t
- IP_t: Pro inventory in month t
- W_t: Workers in month t
- OT_t: Overtime hours in month t
- H_t: Hires in month t
- F_t: Fires in month t
- EH_t: Extra hours in month t for t in T_{extra}

Objective Function

$\min \sum_{t \in T}(6000B_t + 9750P_t + 270IB_t + 540IP_t + 2.5 \times 180W_t + 3.5OT_t + 850H_t + 1300F_t) + \sum_{t \in T_{extra}} 1.85EH_t$

Constraints

1. $B_t + IB_{t-1} = d_t^B + IB_t$ for all t in T
2. $P_t + IP_{t-1} = d_t^P + IP_t$ for all t in T
3. $420B_t + 510P_t \leq 180W_t + OT_t$ for all t in T and t not in T_{extra}
4. $420B_t + 510P_t \leq 180W_t + OT_t + EH_t$ for all t in T_{extra}
5. $OT_t \leq 40W_t$ for all t in T
6. $W_{t-1} + H_t - F_t = W_t$ for all t in T

The output of this new model is below.

```
Jun.23: B: 26, P: 10, IB: 2, IP: 0, W: 89, OT: 0.0, H: 0, F: 4
Jul.23: B: 23, P: 12, IB: 0, IP: 0, W: 87, OT: 120.0, H: 0, F: 2
Aug.23: B: 28, P: 7, IB: 0, IP: 0, W: 86, OT: 0.0, H: 0, F: 1
Sep.23: B: 25, P: 6, IB: 0, IP: 0, W: 86, OT: 0.0, H: 0, F: 0
Oct.23: B: 19, P: 8, IB: 0, IP: 0, W: 86, OT: 0.0, H: 0, F: 0
Nov.23: B: 29, P: 6, IB: 14, IP: 0, W: 86, OT: 0.0, H: 0, F: 0
Dec.23: B: 20, P: 14, IB: 7, IP: 0, W: 86, OT: 0.0, H: 0, F: 0
Extra Hours in Dec.23: 60.0
Jan.24: B: 25, P: 14, IB: 7, IP: 0, W: 86, OT: 0.0, H: 0, F: 0
Extra Hours in Jan.24: 2160.0
Feb.24: B: 31, P: 5, IB: 17, IP: 0, W: 86, OT: 90.0, H: 0, F: 0
Mar.24: B: 30, P: 6, IB: 9, IP: 0, W: 87, OT: 0.0, H: 1, F: 0
Apr.24: B: 32, P: 11, IB: 3, IP: 0, W: 87, OT: 3390.0, H: 0, F: 0
May.24: B: 37, P: 7, IB: 0, IP: 0, W: 87, OT: 3450.0, H: 0, F: 0
Total Cost: 3505712.0
```

As can be seen from the output, I would go for that option in both months, with 60 hours and 2160 hours of extra workforce in Dec23 and Jan24, respectively. Thanks to that opportunity, we decreased the workforce once more in Aug23 and we did not have to hire 2 people again before the last high-demand months, instead we produced upfront from Dec23 and Jan24 with extra workforce where there is no hiring cost just the salary per hour.

As can be seen, the total cost decreased to $3,505,712

Q3-) The new model is below.

Sets

- T: Months {Jun.23, Jul.23, ..., May.24}

Parameters

- d_t^B: Demand for basic caravan in month t
- d_t^P: Demand for pro caravan in month t
- IB_0: Initial basic inventory (8 units for May.23)
- IP_0: Initial pro inventory (4 units for May.23)
- W_0: Initial workers (93 for May.23)

Decision Variables

- B_t: Basic caravan production in month t
- P_t: Pro caravan production in month t
- IB_t: Basic inventory in month t
- IP_t: Pro inventory in month t
- W_t: Workers in month t
- OT_t: Overtime hours in month t
- H_t: Hires in month t
- F_t: Fires in month t
- $Rental_t$: Amount of parking space rented in month t

Objective Function

$$\min \sum_{t \in T} (6000B_t + 9750P_t + 270IB_t + 540IP_t + 2.5 \times 180W_t + 3.5OT_t + 850H_t + 1300F_t + Rental_t)$$

Constraints

1. $B_t + IB_{t-1} = d_t^B + IB_t$ for all t in T
2. $P_t + IP_{t-1} = d_t^P + IP_t$ for all t in T
3. $420B_t + 510P_t \leq 180W_t + OT_t$ for all t in T
4. $OT_t \leq 40W_t$ for all t in T
5. $W_{t-1} + H_t - F_t = W_t$ for all t in T
6. $40IB_t + 60IP_t \leq 500 + Rental_t$ for all t in T (Storage constraint)

The new output is below.

```
Jun.23: B: 26, P: 10, IB: 2, IP: 0, W: 89, OT: 0.0, H: 0, F: 4, Rental: 0.0
Jul.23: B: 23, P: 12, IB: 0, IP: 0, W: 87, OT: 120.0, H: 0, F: 2, Rental: 0.0
Aug.23: B: 28, P: 7, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 0.0
Sep.23: B: 25, P: 6, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 0.0
Oct.23: B: 19, P: 8, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 0.0
Nov.23: B: 30, P: 6, IB: 15, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 100.0
Dec.23: B: 20, P: 14, IB: 8, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 0.0
Jan.24: B: 20, P: 14, IB: 3, IP: 0, W: 87, OT: 0.0, H: 0, F: 0, Rental: 0.0
Feb.24: B: 32, P: 5, IB: 14, IP: 0, W: 89, OT: 0.0, H: 2, F: 0, Rental: 60.0
Mar.24: B: 31, P: 6, IB: 7, IP: 0, W: 89, OT: 60.0, H: 0, F: 0, Rental: 0.0
Apr.24: B: 33, P: 11, IB: 2, IP: 0, W: 89, OT: 3450.0, H: 0, F: 0, Rental: 0.0
May.24: B: 38, P: 7, IB: 0, IP: 0, W: 89, OT: 3510.0, H: 0, F: 0, Rental: 0.0
Total Cost: 3506220.0
```

As can be seen, the production plan in the base problem is kept again. This means that changing the production plan by hiring new employees, keeping less inventories etc is still more costly than original plan even when it includes rental costs. It can be seen that in Nov23 and Feb24 the rental costs occur, which is expected because those are the months with highest inventory.

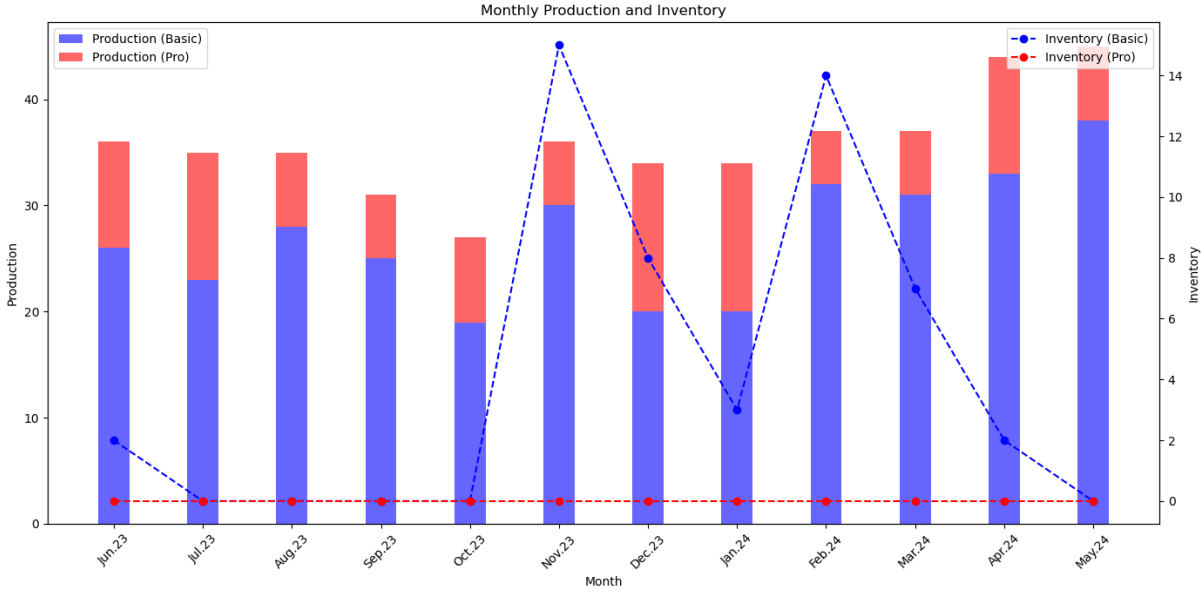
The total cost is in this case $3,506,220

Overall, in every three model; we mostly aimed to keep the total workforce amount low and instead keep upfront inventory or have the existing workforce work overtime. This shows that the salary costs are more impactful than the inventory costs or overtime costs. That's why for this company keeping up inventory is preferred to having more workers and keeping low inventory.

Q4-) As explained in the previous questions and also can be again seen below, it is more beneficial to keep basic caravan inventory than to keep pro caravan inventory. Along with pro caravans having a higher inventory cost, another reason for that situation is that the demand for pro caravans is lower and with the existing workforce it is possible to satisfy the demand during each month, although pro caravans require longer manhours than basic ones (510 vs 420). So, since it is possible to meet their demand at each period and they are more costly to keep in the inventory; it is optimal to keep the basic version in the inventory than the pro version. If the demand forecasts for the next months were pro caravans were high, or if it had different parameters indicating a lower inventory cost; then it could be optimum to keep them in the inventory, however in this model this is not the case. If the demand for the prospective months were so high for both pro and basic, and if they are not so high for the current month, then we could produce lots of them and keep both of them in the inventory; if the hire-fire and salary costs are still disadvantageous compared to inventory holding costs.

The inventory projection and bar chart (see next page) for the base model are below. (Also provided in Q1)

```
Jun.23: B: 26, P: 10, IB: 2, IP: 0, W: 89, OT: 0.0, H: 0, F: 4
Jul.23: B: 23, P: 12, IB: 0, IP: 0, W: 87, OT: 120.0, H: 0, F: 2
Aug.23: B: 28, P: 7, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Sep.23: B: 25, P: 6, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Oct.23: B: 19, P: 8, IB: 0, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Nov.23: B: 30, P: 6, IB: 15, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Dec.23: B: 20, P: 14, IB: 8, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Jan.24: B: 20, P: 14, IB: 3, IP: 0, W: 87, OT: 0.0, H: 0, F: 0
Feb.24: B: 32, P: 5, IB: 14, IP: 0, W: 89, OT: 0.0, H: 2, F: 0
Mar.24: B: 31, P: 6, IB: 7, IP: 0, W: 89, OT: 60.0, H: 0, F: 0
Apr.24: B: 33, P: 11, IB: 2, IP: 0, W: 89, OT: 3450.0, H: 0, F: 0
May.24: B: 38, P: 7, IB: 0, IP: 0, W: 89, OT: 3510.0, H: 0, F: 0
Total Cost: 3506060.0
```


Monthly Production and Inventory
Production (Basic)
Production (Pro)
Inventory (Basic)
Inventory (Pro)
Production
Inventory
Month
0
10
20
30
40
0
2
4
6
8
10
12
14
Jun.23
Jul.23
Aug.23
Sep.23
Oct.23
Nov.23
Dec.23
Jan.24
Feb.24
Mar.24
Apr.24
May.24