Foundations of Artificial Intelligence

14. State-Space Search: Analysis of Heuristics

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
  - 13. Heuristics
  - 14. Analysis of Heuristics
  - 15. Best-first Graph Search
  - 16. Greedy Best-first Search, A*, Weighted A*
  - 17. IDA*
  - 18. Properties of A*, Part I
  - 19. Properties of A*, Part II

Reminder: Heuristics

Definition (heuristic)

Let $\mathcal{S}$ be a state space with states S.
A heuristic function or heuristic for $\mathcal{S}$ is a function

$$h : S \to \mathbb{R}_0^+ \cup \{\infty\},$$ infinity = no path to goal

mapping each state to a non-negative number (or ∞).

Properties of Heuristics

Perfect Heuristic

Definition (perfect heuristic)

Let $\mathcal{S}$ be a state space with states S.

The perfect heuristic for $\mathcal{S}$, written h^*, maps each state $s \in S$

- to the cost of an optimal solution for s, or
- to ∞ if no solution for s exists.

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Properties of Heuristics

Definition (safe, goal-aware, admissible, consistent)

Let $\mathcal{S}$ be a state space with states S.

A heuristic h for $\mathcal{S}$ is called

- safe if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$ all predicted inf are actually inf
- goal-aware if $h(s) = 0$ for all goal states s goal state has to be 0
- admissible if $h(s) \leq h^*(s)$ for all states $s \in S$ heuristic has to be underestimation
- consistent if $h(s) \leq cost(a) + h(s')$ for all transitions $s \xrightarrow{a} s'$

s → s' should be s → s1 → s'
cost of first transition should not be less than of both the other transitions, because we end up in the same state in both cases.

= detour should not be cheaper than directly going to the node.

Properties of Heuristics

Definition (safe, goal-aware, admissible, consistent)

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A heuristic h for $\mathcal{S}$ is called

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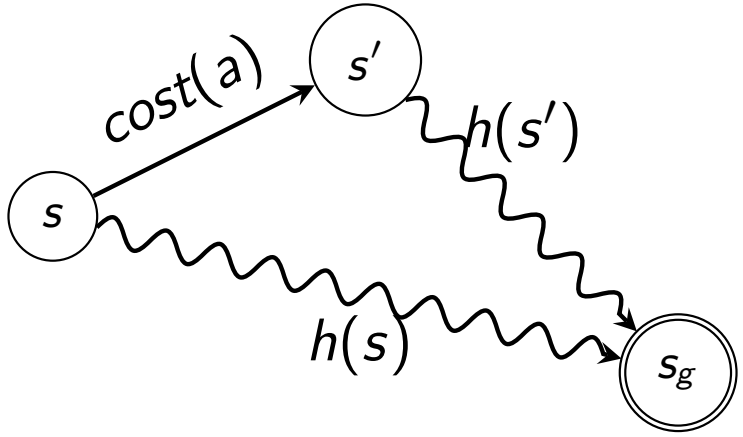
- goal-aware if $h(s) = 0$ for all goal states s
- admissible if $h(s) \leq h^*(s)$ for all states $s \in S$
- consistent if $h(s) \leq cost(a) + h(s')$ for all transitions $s \xrightarrow{a} s'$



Examples

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Route Planning in Romania

straight-line distance:

- safe
- goal-aware
- admissible
- consistent

Why?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Blocks World

misplaced blocks:

- safe?
- goal-aware?
- admissible?
- consistent?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Missionaries and Cannibals

people on wrong river bank:

- safe?
- goal-aware?
- admissible?
- consistent?

Connections

Properties of Heuristics: Connections (1)

Theorem (admissible $\Longrightarrow$ safe + goal-aware)

Let h be an admissible heuristic.

Then h is safe and goal-aware.

Why?

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admissable → we underestimate → goal has to be 0 (lowest number) → we are goal-aware

admissable → ...
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Properties of Heuristics: Connections (2)

Theorem (goal-aware + consistent $\Longrightarrow$ admissible)

Let h be a goal-aware and consistent heuristic.

Then h is admissible.

Why?

`consider h* is infinite → admissable`

Showing All Four Properties

How can one show most easily that a heuristic has all four properties?

you need to show consistency (no other property implies it)

and

goal-aware (easier) or admissable

Summary

Summary

- perfect heuristic h^*: true cost to the goal
- important properties: safe, goal-aware, admissible, consistent
- connections between these properties
  - admissible $\Longrightarrow$ safe and goal-aware
  - goal-aware and consistent $\Longrightarrow$ admissible