ECE 271A HW5 Quiz

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1 5 Mixtures of 8 Components for each Class

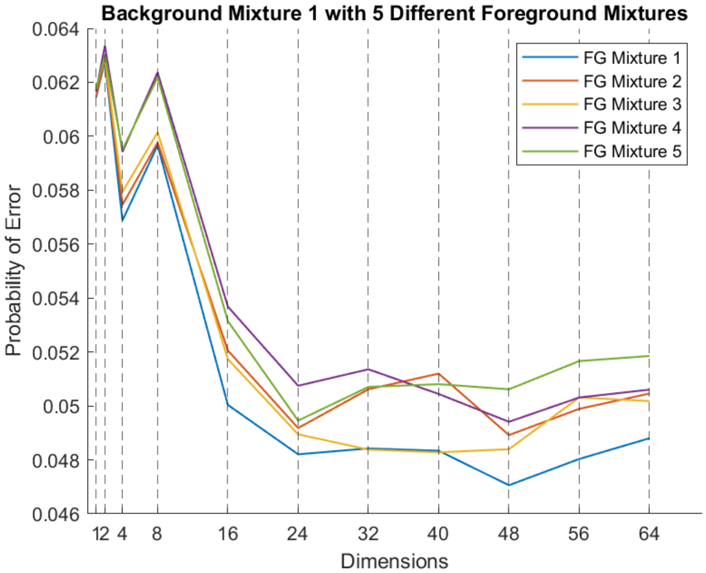
For each class (background and foreground), we use the Expectation Maximization algorithm to learn 5 mixtures of 8 components, with a random initialization. Inside each EM loop, we track the value of the log-likelihood and use it to determine whether the iteration should stop, that is to stop the iteration when the increment of the log-likelihood is less than a threshold (a very small value). After we obtain the parameters of the total 5 Gaussian mixture models for each class, we then select one of the 5 models for each class to conduct the classification task, which ends up in 25 classifiers in total. We fix the model of the background data and compare it with the other 5 models of the foreground data, and the plot the probabilities of error with regard to the dimensions of space, i.e. $d \in \{1, 2, 4, 8, 16, 24, 32, 40, 48, 56, 64\}$. The results are shown in Figure 1a-1e.

As shown in the results, we can see that different Background-Foreground mixture combinations lead to different classification results, which is caused by random initialization of the parameters. Even though, these curves still share similar trends. As the dimension of the data increases, the probabilities of error will eventually converge to a small interval (approximately from 0.048 to 0.052 in our 25 classifiers), since with more features provided, the impact of the initialization become less significant. It is also clear to see that in all classifiers, in general, the probabilities of error become lower when more dimensions of the data are provided. But this doesn't mean that more dimensions provided necessarily lead to better performances, since we can find that the classifier has a lower probability of error with $d \in \{24, 32, 40, 48\}$ than with $d \in \{56, 64\}$. This result points out the importance of the selection of features. In this case, we can know that the not all dimensions of the data are good features, and we can obtain quite good performance using the selected good features, while adding more dimensions could even be worse. This is consistent with the conclusion we draw from Quiz 2, where using the 8 best features strongly beat using all 64 features.

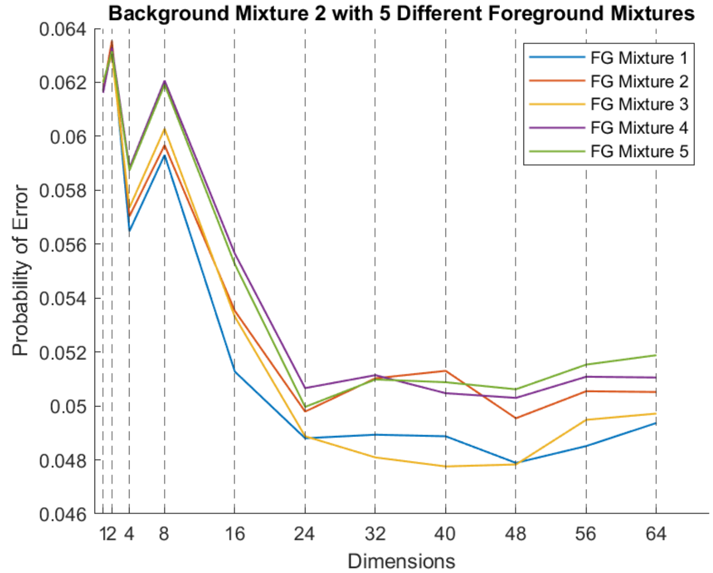
2 Mixtures of Different Numbers of Components

Then, we change the number of mixture components, i.e. $C \in \{1, 2, 4, 8, 16, 32\}$, and learn one mixture for each class. Again we plot the probabilities of error with regard to dimensions under different assumptions of the number components, and the result is shown in Figure 1f.

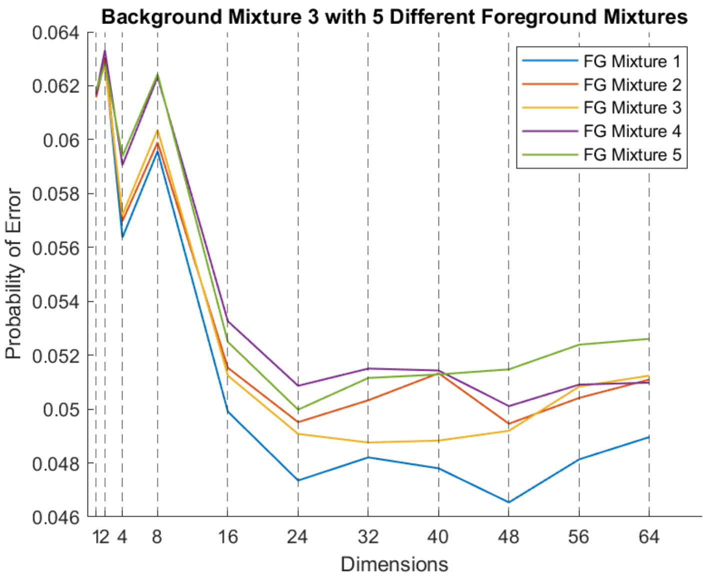
Obviously, when we assume the Gaussian mixture model contains only one component, i.e. it is just a regular multivariate Gaussian distribution, the performance are much worse than other assumptions, which indicates that such assumptions underestimate the complexity of our data distributions. And the differences become larger as the dimension increases, since as we provide more dimensions, the data becomes more complex and much less can the data be viewed as a single Gaussian distribution. Likewise, the results show that assuming more components doesn't necessarily lead to better performances. In our case, when we assume the model contains 4 components, the classifier reaches the best performance.



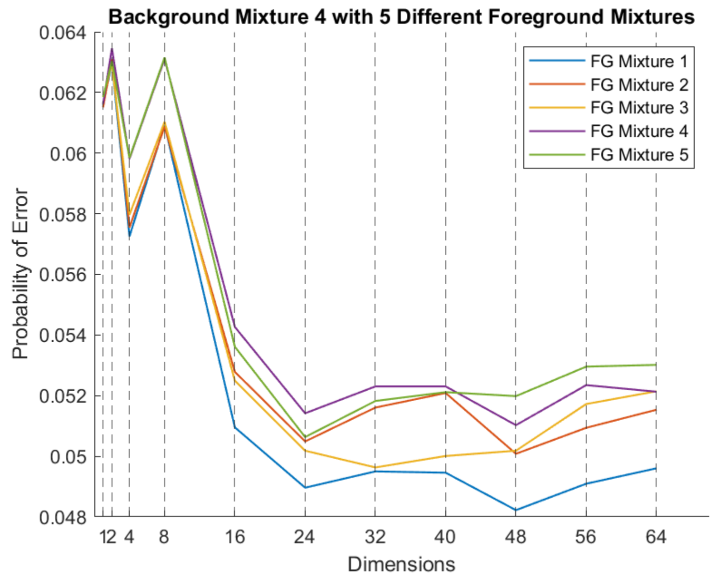
(a) Background Mixture 1



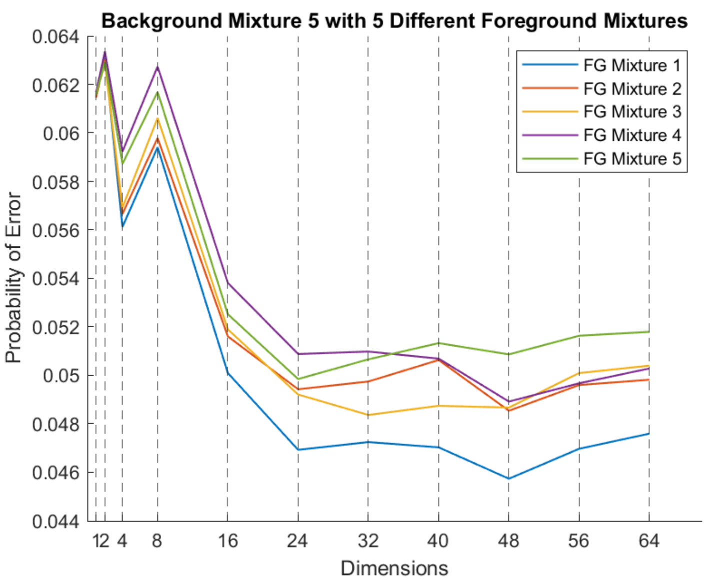
(b) Background Mixture 2



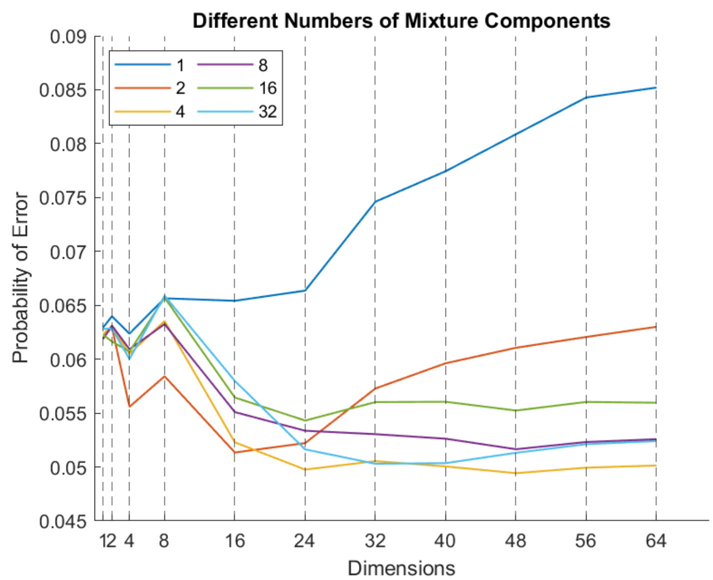
(c) Background Mixture 3



(d) Background Mixture 4



(e) Background Mixture 5



(f) Different Numbers of Mixture Components

Figure 1: Results

3 Code

```
load("TrainingSamplesDCT_8_new.mat")
BG = TrainsampleDCT_BG;
FG = TrainsampleDCT_FG;

% Compute priors using MLE
c_fg = size(FG, 1);
c_bg = size(BG, 1);
n = c_fg + c_bg;
prob_bg = c_bg / n;
prob_fg = c_fg / n;

% Load ZigZag Pattern
ZigZagPattern = readmatrix("Zig-Zag Pattern.txt");
ZigZagPattern = ZigZagPattern + 1;
ZigZagPattern = int8(ZigZagPattern);

% Load cheetah image
img = imread("cheetah.bmp");
img = im2double(img);

% Compute DCT and ZigZag Scan
img_dct = dct_8(img);
img_scan = blockproc(img_dct, [8 8], @(block_struct)
    ZigZagScan(block_struct.data, ZigZagPattern));

% Load ground truth image
ground_truth = imread("cheetah_mask.bmp");
ground_truth = im2double(ground_truth);

n_mixtures = 5;
dims = [1 2 4 8 16 24 32 40 48 56 64];
n_componets = [1 2 4 8 16 32];

% Create dictionaries containing parameters
mu_bg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
```

```
sigma_bg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
pi_bg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
mu_fg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
sigma_fg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
pi_fg_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');
poe_list_dict = containers.Map('KeyType', 'uint32', '
    ValueType','any');

% generate GMM parameters using EM
for c = n_componets
    mu_bg_c = zeros(c, 64, n_mixtures);
    sigma_bg_c = zeros(64, 64, c, n_mixtures);
    pi_bg_c = zeros(n_mixtures, c);
    mu_fg_c = zeros(c, 64, n_mixtures);
    sigma_fg_c = zeros(64, 64, c, n_mixtures);
    pi_fg_c = zeros(n_mixtures, c);
    % BG parameters
    for i_BG = 1:n_mixtures
        [mu_bg, sigma_bg, pi_bg] = EM(BG, c, 200);
        mu_bg_c(:, :, i_BG) = mu_bg;
        sigma_bg_c(:, :, :, i_BG) = sigma_bg;
        pi_bg_c(i_BG, :) = pi_bg;
    end
    mu_bg_dict(c) = mu_bg_c;
    sigma_bg_dict(c) = sigma_bg_c;
    pi_bg_dict(c) = pi_bg_c;
    % FG parameters
    for i_FG = 1:n_mixtures
        [mu_fg, sigma_fg, pi_fg] = EM(FG, c, 200);
        mu_fg_c(:, :, i_FG) = mu_fg;
        sigma_fg_c(:, :, :, i_FG) = sigma_fg;
        pi_fg_c(i_FG, :) = pi_fg;
    end
```

```
    mu_fg_dict(c) = mu_fg_c;
    sigma_fg_dict(c) = sigma_fg_c;
    pi_fg_dict(c) = pi_fg_c;
end

% Classification
for c = n_componets
    mu_bg_all = mu_bg_dict(c);
    sigma_bg_all = sigma_bg_dict(c);
    pi_bg_all = pi_bg_dict(c);
    mu_fg_all = mu_fg_dict(c);
    sigma_fg_all = sigma_fg_dict(c);
    pi_fg_all = pi_fg_dict(c);
    poe_list = zeros(n_mixtures, size(dims, 2), n_mixtures);
    for i_BG = 1:n_mixtures
        for i_FG = 1:n_mixtures
            mu_bg = mu_bg_all(:, :, i_BG);
            sigma_bg = sigma_bg_all(:, :, :, i_BG);
            pi_bg = pi_bg_all(i_BG, :);
            mu_fg = mu_fg_all(:, :, i_FG);
            sigma_fg = sigma_fg_all(:, :, :, i_FG);
            pi_fg = pi_fg_all(i_FG, :);
            for i_dim = 1:size(dims, 2)
                dim = dims(i_dim);
                mu_bg_d = mu_bg(:, 1:dim);
                sigma_bg_d = sigma_bg(1:dim, 1:dim, :);
                mu_fg_d = mu_fg(:, 1:dim);
                sigma_fg_d = sigma_fg(1:dim, 1:dim, :);

                mask = blockproc(img_scan, [1, 64], @(block_struct) mixBDR(block_struct.data(1, 1:dim), ...,
                    c, mu_bg_d, mu_fg_d, sigma_bg_d, sigma_fg_d, pi_bg, pi_fg, prob_bg, prob_fg));
                % Zero Padding
```

```
                mask = [[mask zeros(248, 7)]; zeros(7, 270)];
                poe_list(i_BG, i_dim, i_FG) = P_Error(ground_truth, mask, prob_bg, prob_fg);
            end
        end
    end
    poe_list_dict(c) = poe_list;
end

% Plot the 25 classifiers
for c = n_componets(1, 4)
    poe_list = poe_list_dict(c);
    for i_bg = 1:n_mixtures
        f = figure(i_bg);
        clf;
        hold on;
        l1 = plot(dims, poe_list(i_bg, :, 1), 'r', 'LineWidth', 1);
        l2 = plot(dims, poe_list(i_bg, :, 2), 'g', 'LineWidth', 1);
        l3 = plot(dims, poe_list(i_bg, :, 3), 'b', 'LineWidth', 1);
        l4 = plot(dims, poe_list(i_bg, :, 4), 'y', 'LineWidth', 1);
        l5 = plot(dims, poe_list(i_bg, :, 5), 'm', 'LineWidth', 1);

        for i = 1:length(dims)
            xline(dims(i), '--')
        end
        xticks(dims);
        title(join(["Background Mixture", int2str(i_bg), " with 5 Different Foreground Mixtures"]))
        xlabel("Dimensions")
        ylabel("Probability of Error")
        legend([l1, l2, l3, l4, l5], 'FG Mixture 1', 'FG Mixture 2', 'FG Mixture 3', ...,
```

```
            'FG Mixture 4', 'FG Mixture 5', 'Location', 'southeast')
        exportgraphics(f, append('5-1-', int2str(i_bg), '.png'));
    end
end

% Plot the 6 classifiers
colors = ['y', 'c', 'r', 'b', 'g', 'm'];
f = figure;
for c = n_componets
    i = find(n_componets==c);
    poe_list = poe_list_dict(c);
    plot(dims, poe_list, colors(1, i), 'LineWidth', 1);
    hold on;
    legend_str{i} = int2str(c);
end

for j = 1:length(dims)
    xline(dims(j), '--')
end
xticks(dims);
title("Different Numbers of Mixture Components");
xlabel("Dimensions")
ylabel("Probability of Error")
lgd = legend(legend_str, 'Location', 'northwest');
lgd.NumColumns = 2;
exportgraphics(f, '5-2.png');

function [mu, sigma, pi_z] = EM(sample, c, n_iter)
    % initialize mu, sigma, pi
    mu = sample(randperm(size(sample, 1), c), :);
    sigma = zeros(64, 64, c);
    sigma_diag = rand(c, 64);
    for i = 1:c
        sigma(:, :, i) = diag(sigma_diag(i, :));
    end
    pi_z = ones(1, c) / c;
```

```
    H = zeros(size(sample, 1), c);
    epsilon = diag(1e-6*ones(1, 64));

    % track the log-likelihood for stopping the iteration
    ll = loglikelihood(sample, mu, sigma, pi_z);
    ll_track = [ll];
    threshold = 1e-4;
    iters = [0];

    for iter = 1:n_iter
        iters = [iters, iter];
        % E-step
        for i = 1:size(sample, 1)
            for k = 1:c
                H(i, k) = mvnpdf(sample(i, :), mu(k, :), sigma(:, :, k)) * pi_z(k);
            end
            H(i, :) = H(i, :) / sum(H(i, :));
        end
        % M-step
        sigma_new = zeros(64, 64, c);
        for j = 1:c
            mu(j, :) = sum(H(:, j).*sample) / sum(H(:, j));
            for i = 1:size(sample, 1)
                tmp = (sample(i, :) - mu(j, :)).'*(sample(i, :) - mu(j, :));
                sigma_new(:, :, j) = sigma_new(:, :, j) + H(i, j) * diag(diag(tmp));
            end
            sigma_new(:, :, j) = sigma_new(:, :, j) / sum(H(:, j)) + epsilon;
            pi_z(j) = sum(H(:, j)) / size(sample, 1);
        end
        sigma = sigma_new;

        ll_new = loglikelihood(sample, mu, sigma, pi_z);
        ll_track = [ll_track, ll_new];
```

```
        % stop the iteration if the increment is less than the threshold
        if (ll_new - ll) < threshold
            break
        end
        ll = ll_new;
        plot(iters, ll_track)
        drawnow();
    end
end

function vector = ZigZagScan(matrix, pattern)
    vector = zeros(1, size(matrix, 1) * size(matrix, 2));
    for i = 1:size(matrix, 1)
        for j = 1:size(matrix, 2)
            position = pattern(i, j);
            vector(1, position) = matrix(i, j);
        end
    end
end

function dct = dct_8(img)
    dct = zeros((size(img, 1) - 7) * 8, (size(img, 2) -7) * 8);
    for i = 1:(size(img, 1)-7)
        for j = 1:(size(img, 2)-7)
            dct((8*i-7):(8*i), (8*j-7):(8*j)) = dct2(img(i:i+7, j:j+7));
        end
    end
end

function density = mvnpdf(x, mu, sigma)
    k = size(x, 2);
    density = (2*pi).^(-k/2) / sqrt(det(sigma)) * exp(-(x-mu)*inv(sigma)*(x-mu).'/2);
end
```

```
function ll = loglikelihood(sample, mu, sigma, pi)
    ll = 0;
    for i = 1:size(sample, 1)
        ll_x = 0;
        for j = 1:size(mu, 1)
            ll_x = ll_x + pi(1, j) * mvnpdf(sample(i, :),
               mu(j, :), sigma(:, :, j));
        end
        ll = ll + log(ll_x);
    end
end

function mask = mixBDR(feature, c, mu_bg, mu_fg, sigma_bg,
    sigma_fg, pi_bg, pi_fg, P_bg, P_fg)
    p_x_bg = 0;
    p_x_fg = 0;
    for i = 1:c
        p_x_bg = p_x_bg + pi_bg(1, i) * mvnpdf(feature,
           mu_bg(i, :), sigma_bg(:, :, i));
        p_x_fg = p_x_fg + pi_fg(1, i) * mvnpdf(feature,
           mu_fg(i, :), sigma_fg(:, :, i));
    end
    if p_x_bg * P_bg > p_x_fg * P_fg
        mask = 0;
    else
        mask = 1;
    end
end

function p = P_Error(gt, mask, prob_bg, prob_fg)
    gt = int8(gt);
    mask = int8(mask);
    diff = gt - mask;
    detect = 1 - sum(sum(diff==1))/sum(sum(gt==1));
    fAlarm = sum(sum(diff==-1))/sum(sum(gt==0));
    p = fAlarm * prob_bg + (1 - detect) * prob_fg;
end
```