Distribution of the Number of Triangles in an Erdös-Rényi RG

Muriel Pérez

November 3, 2016

output: md_document variant: markdown_github

Distribution of the Number of Triangles in an Erdös-Rényi RG

Here I investigate numericaly the limiting distribution of the number of triangles in a $ER(n, p = \lambda/n)$ graph. I want to confirm that as the number of nodes $n \to \infty$, the number of triangles $\Delta_n \to Pois(\lambda^3/6)$ in distribution.

First, I show the R package that I used, igraph.

igraph

There is a nice R package to work with graphs called igraph.

```
library(igraph)
```

igraph is a collection of network analysis tools. Among them, there are functions to easily sample $ER(n, p = \lambda/n)$ graphs, plot them, and count the number of triangles, which is what we need. Let's generate a graph with $n = 100$, $\lambda = 3$, and count the triangles with the function count_triangles, which returns how many triangles each vertex is part of.

```
set.seed(2016)
rg <- sample_gnp(n = 100, p = 3/100)
count_triangles(rg) %>%
    sum/3
```

```
## [1] 6
```

we can also plot rg (See Figure 1).

```
plot(rg, vertex.label = NA, vertex.size = 2)
```

Random sample from $ER(n, p = \lambda/n)$

Now, we want to confirm that as the number of nodes $n \to \infty$, the number of triangles $\Delta_n \to Poiss(\lambda^3/6)$ in distribution. For that, I generate a random sample of $N = 10000$ random graphs for $\lambda = 0.1, 1, 2, 5$, and $n = 100, 200, 500, 1000, 5000$, and then analyze the empirical distribution of Δ_n in each case.

We can compare the empirical distribution with the theoretical probability mass function.

The qqplots look like this, but they are not very promising

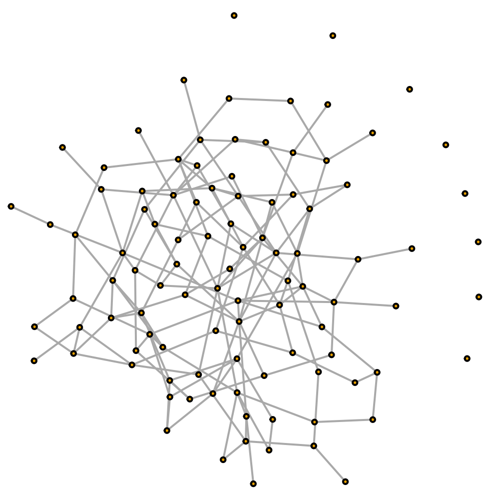

Figure 1: A realization of $ER(n = 100, p = 3/100)$

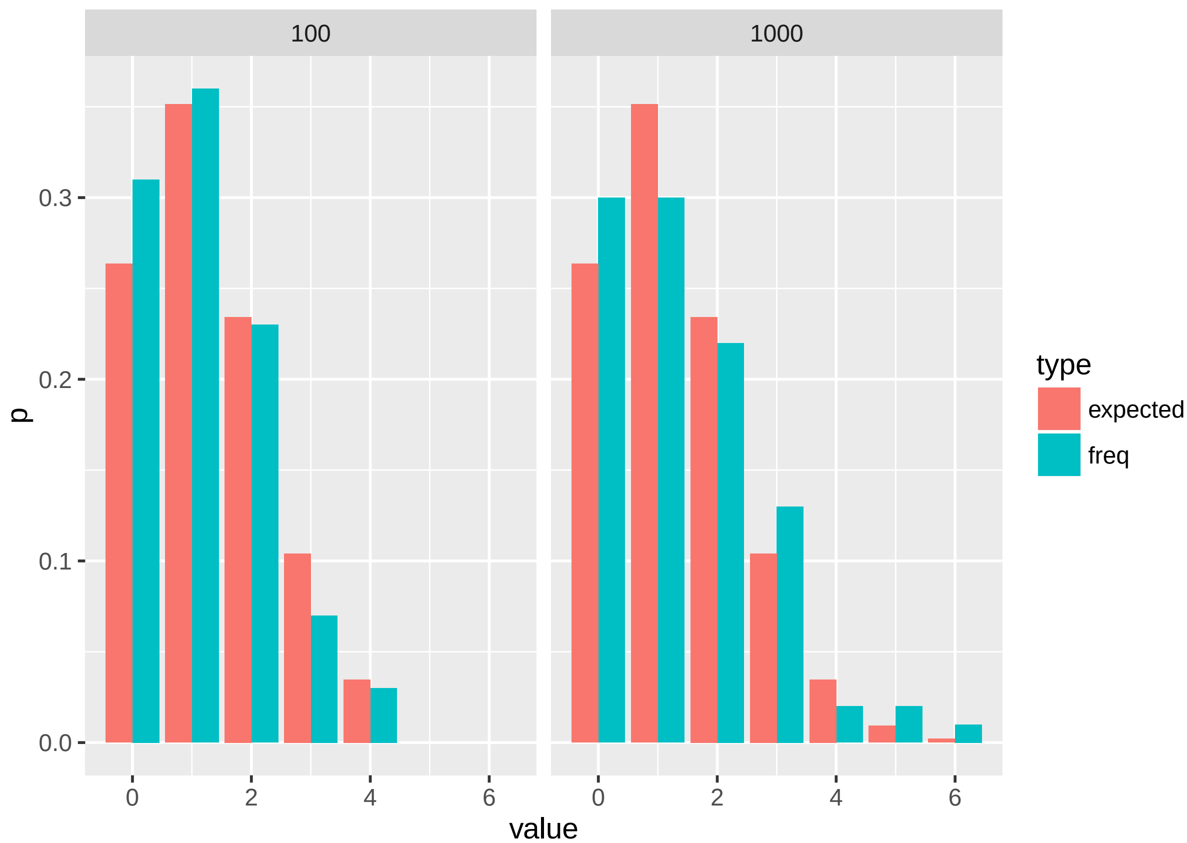

Figure 2: Observed frequency distribution and expected Poisson distribution

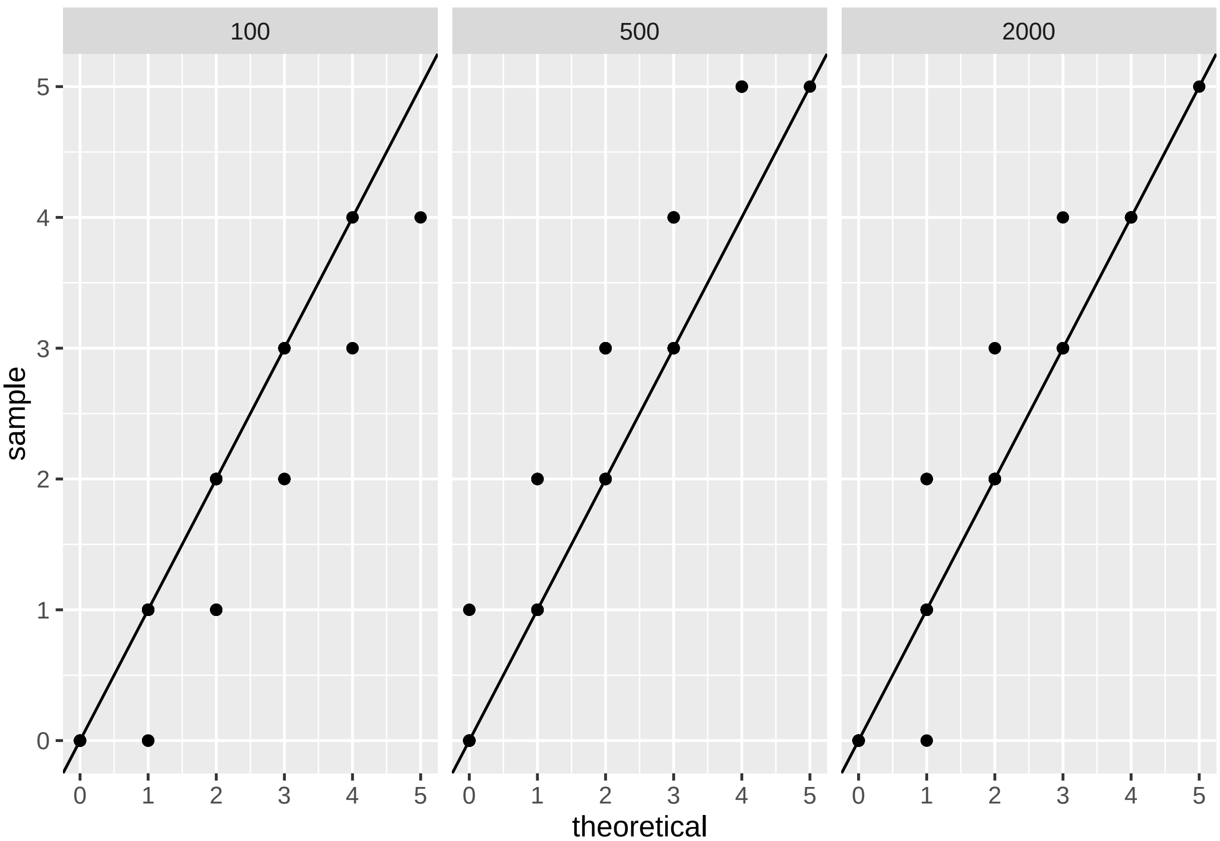

Figure 3: QQ-plots of the sample versus the Poisson distribution quantiles