Project - Bayesian Macroeconometrics

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1 Introduction

The pandemic triggered by COVID-19 has affected the world economy and generated large variations in macroeconomic indicators in a short period of time. These extreme variations in macroeconomic data pose additional challenges to estimate time-series models. During March and April of 2021 macroeconomic indicators, such as unemployment, industrial production and consumption expenditures, experienced changes of two-orders of magnitude larger than standard month-to-month changes.

How to deal with these observations and the potential impact of them in the transmission mechanism of shocks is not exactly clear. Furthermore, abnormal variations may "contaminate" historical data. The recent paper from Michele Lenza and Giorgio E. Primiceri named "How to Estimate a VAR after March 2020" address how to handle extreme observations when estimating Vector Autoregression. The authors are interested in the transmission mechanism of shocks given the large variations in data recorded during the pandemic. They have presented two empirical applications for the model proposed. The first consist in analyse the Impulse Response Functions considering a positive shock in unemployment. In the second application, they have predicted the future path of macroeconomic variables conditional on the average forecasts for unemployment from the Blue Chip Forecasts.

The main objective of this project is to replicate the model proposed in Lenza and Primiceri [2020], updating with more recent data, given that the paper only consider observations from the beginning of the COVID-19 pandemic. As I will present below, the inclusion of more recent data affect the posterior distributions of the parameters. In addition, I will also present the results generated by the simple mechanism to consider a dummy variable for the period of extreme observations.

2 Model

The solution to handle extreme observations proposed by Lenza and Primiceri [2020] consist of explicitly modeling the change in shock volatility. Since the researcher knows the exactly time when the volatility increases, the model simply rescale the standard deviation of the shocks by an unknown parameter. The standard deviation from March are rescale by $\bar{s}_0$ and those from April and May by $\bar{s}_1$ and $\bar{s}_2$, respectively. These parameters will be estimated using Bayesian approach, similar to Giannone et al. [2015]. To model the evolution of the residual variance after May 2020, the authors assume that it decays at a constant rate, ρ, the persistence effect.

The modified VAR can be represented such that

$$y_t = C + B_1 y_{t-1} + \ldots + B_p y_{t-p} + s_t \varepsilon_t$$

$$\varepsilon_t \sim N(0, \Sigma)$$

where y_t is an $n \times 1$ vector of variables, modeled as a function of a constant term, their own past values, and an $n \times 1$ vector of forecast errors ϵ_t. s_t is equal to 1 before the time period in which the epidemic begins (March/2021), denoted by t^*. Also, $s_{t^*} = \bar{s}_0, s_{t^*+1} = \bar{s}_1, s_{t^*+2} = \bar{s}_2$, and $s_{t^*+j} = 1 + (\bar{s}_2 - 1)\,\rho^{j-2}$, where $\theta \equiv [\bar{s}_0, \bar{s}_1, \bar{s}_2, \rho]$ is a vector of unknown coefficients.

The estimation procedure works as follow, assuming we know s_t, we can write the VAR such that

$$y_t = X_t \beta + s_t \varepsilon_t$$

where $X_t \equiv I_n \otimes x_t', x_t \equiv \left[1, y_{t-1}', \ldots, y_{t-p}'\right]$ and $\beta \equiv \operatorname{vec}\left([C, B_1, \ldots, B_p]'\right)$. Dividing by s_t both sides we have

$$\tilde{y}_t = \tilde{X}_t \beta + \varepsilon_t$$

where $\tilde{y}_t \equiv y_t / s_t$ and $\tilde{X}_t \equiv X_t / s_t$.

In order to estimated β and Σ the authors consider a prior distribution belonging to the conjugate Normal-Inverse Wishart family

$$\Sigma \sim IW(\Psi, d)$$

$$\beta \mid \Sigma \sim N(b, \Sigma \otimes \Omega)$$

Similar to Giannone et al. [2015], the posterior of the parameter θ that governs s_t can be evaluated like the posterior of γ in

$$p(\gamma, \theta \mid y) \propto p(y \mid \gamma, \theta) \cdot p(\gamma, \theta),$$

and the marginal likelihood represent by the first element in right-hand side of the expression above has analytical form. The prior for $\bar{s}_0$, $\bar{s}_1$ and $\bar{s}_2$ is a Pareto distribution with scale and shape equal to one. A Beta prior is considered for ρ with mode and standard deviation equal to 0.8 and 0.2.

3 Empirical Results

In the first exercise I have estimated a VAR considering the modelling approach presented with data until May/2021. I have considered the following U.S macroeconomic variables: unemployment, employment, PCE, PCE: services, PCE (price), PCE: services (price), core PCE. Similar to Lenza and Primiceri [2020], the VAR has 13 lags.

The first panel in figure 1 presents the posterior distribution of the hyperparameters of the model, the parameter λ is from the Minnesota prior and it determines the degree of shrinkage of the coefficients. The other panels shows the posterior of the volatility scaling factors and the rate of decay, ρ. The posterior distribution of ρ in Lenza and Primiceri [2020] is centered around 0.45, which indicates that volatility has been halving each month since June. Interestingly, the last panel of figure 1 presented in this project shows that the posterior of the decay parameter is centered around 0.77, indicating a much more persistent effect of the extreme observations on the volatility of the variables on subsequent months than originally estimated in Lenza and Primiceri [2020]. Therefore, as commented bellow, the uncertainty is underestimated not only in the case the estimation period ends in February/2020 but also in the main model. This fact confirm the importance to carefully model the extreme variations observed due to COVID-19.

Figure 2 presents the Impulse Response Functions with posterior credible regions of the variables in the VAR to a one standard deviation positive shock in unemployment, when it is ordered first in a Cholesky identification scheme. The main objective of this IRFs is to use then as summary statistics of the estimated dynamics. They do not have any structural interpretation. In general, the results are similar to the one estimated in the original paper with some slightly changes. After the positive unemployment shock, employment and consumption decreases and then partially recover. In our setting, prices also decreases but the recovery seems to be faster than in the IRFs from Lenza and Primiceri [2020]. The IRFs reported here have the same dynamic of the IRFs before March/2020, the main difference concerns the estimated credible regions. In the appendix, I present figures with the posterior distribution of λ and the Impulse Response Function in the case when the observations from the pandemic period are considered without any special treatment. The histogram of λ is centered in a higher point (around 0.5) to compensate the extreme observations shrinking the β. Furthermore, the IRFs becomes explosive and misleading.

In this project I also present the results of the simple procedure that consist to put dummy variables for the pandemic period. The Impulse Response Functions are presented below in figure 3. The results are quite similar to the case when the observations are considered without any special treatment (see appendix). The IRFs have a tendency to become explosive and unreasonable. The fact that in this exercise the observations are treated equally within

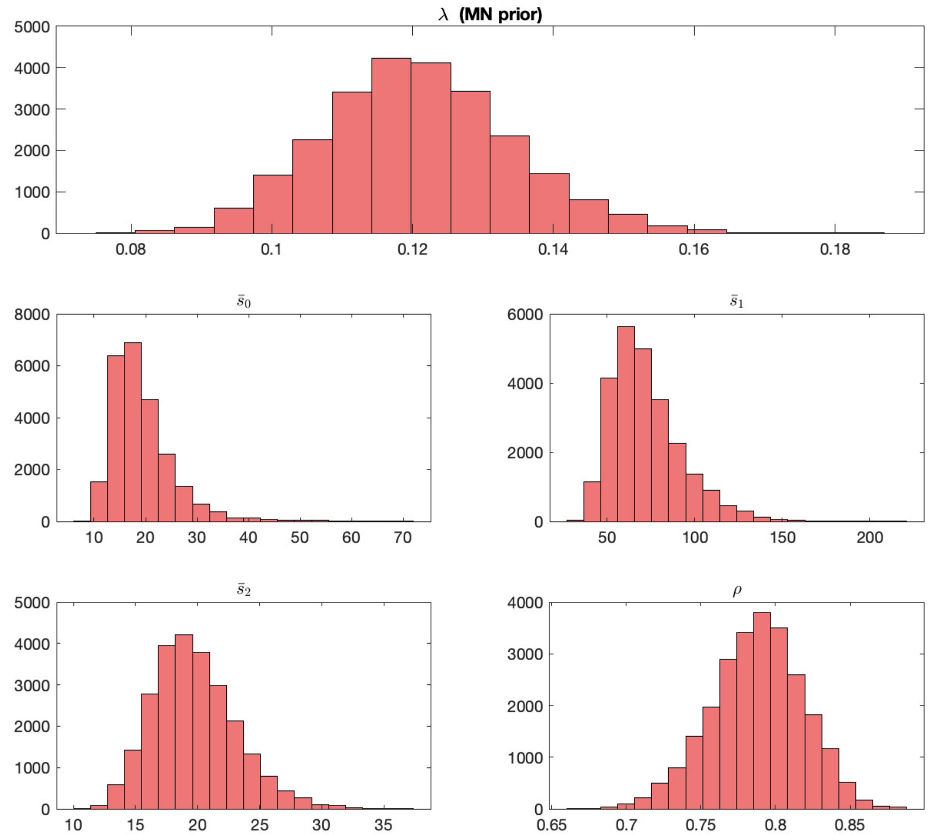

Figure 1: Posterior distribution of the overall standard deviation of the Minnesota prior, the March, April and May 2020 volatility scaling factors, and the post-May 2020 volatility decay parameter.

the pandemic period may have contributed to this result since during the pandemic period there are large differences in macroeconomic indicators volatility month-to-month. The method proposed by Lenza and Primiceri [2020] seems to better capture this condition than the simple dummy strategy.

In the second empirical application, I present the results of the most likely evolution of the macroeconomic variables and the uncertainty associate with these forecasts. The forecasts are computed conditional on the average unemployment forecasts release on the Blue Chip Forecasts from May/2021. The average for the unemployment forecast from this survey was 4.2% for the end of 2022. The last observation considered in the estimation of the model was from May/2021, the unemployment at that period was 5.8%.

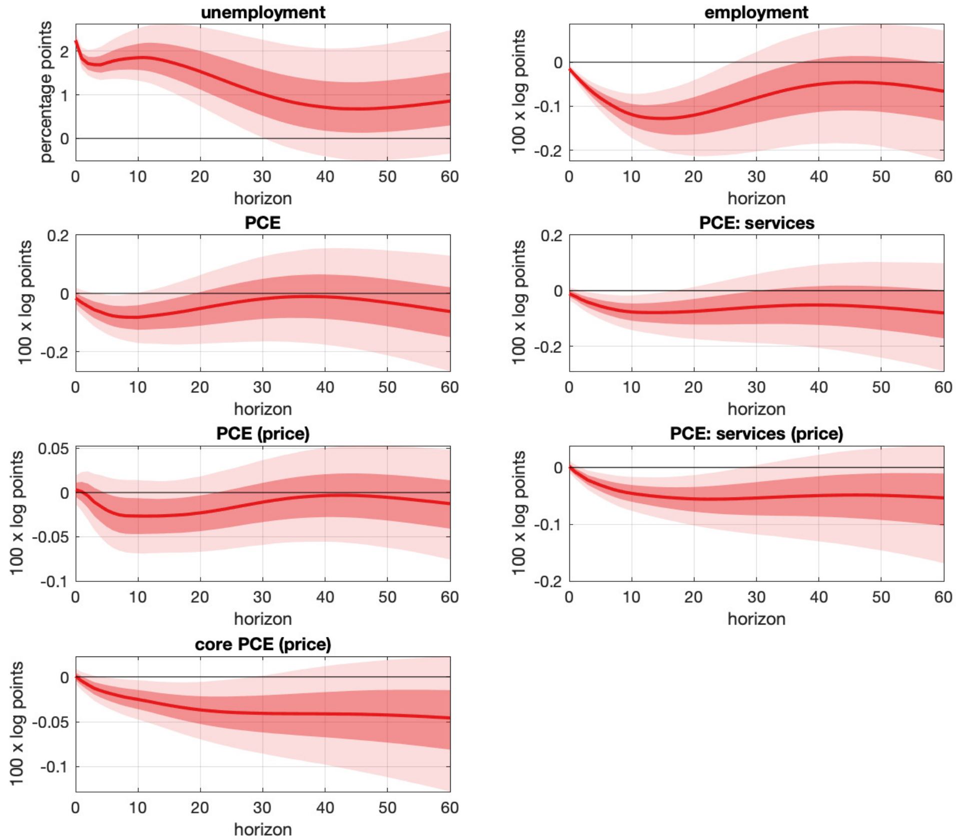

Figure 2: The solid lines are posterior medians, while the shaded areas correspond to 68- and 95-percent posterior credible regions.

Figure 4 shows the density forecasts of employment, consumption and prices obtained using the model that accounts for the change in shock volatility after February 2020. Different from Lenza and Primiceri [2020], our updated results predict an employment level close to the pre-pandemic period by the end of 2022. In general, all measures of personal consumption recovery faster than expected with the model estimated with data until September/2020, highlight for the PCE that recovery it's level from before the pandemic period in January/2021. Comparing this model with the standard way to estimate a VAR, if the last observations from the pandemic period is included, then the results are in line with the Impulse Response Functions presented in the appendix and any inference is impossible. On the other hand, the exclusion of the observations from the pandemic period generate forecasts that does not take into account the higher uncertainty after February/2020.

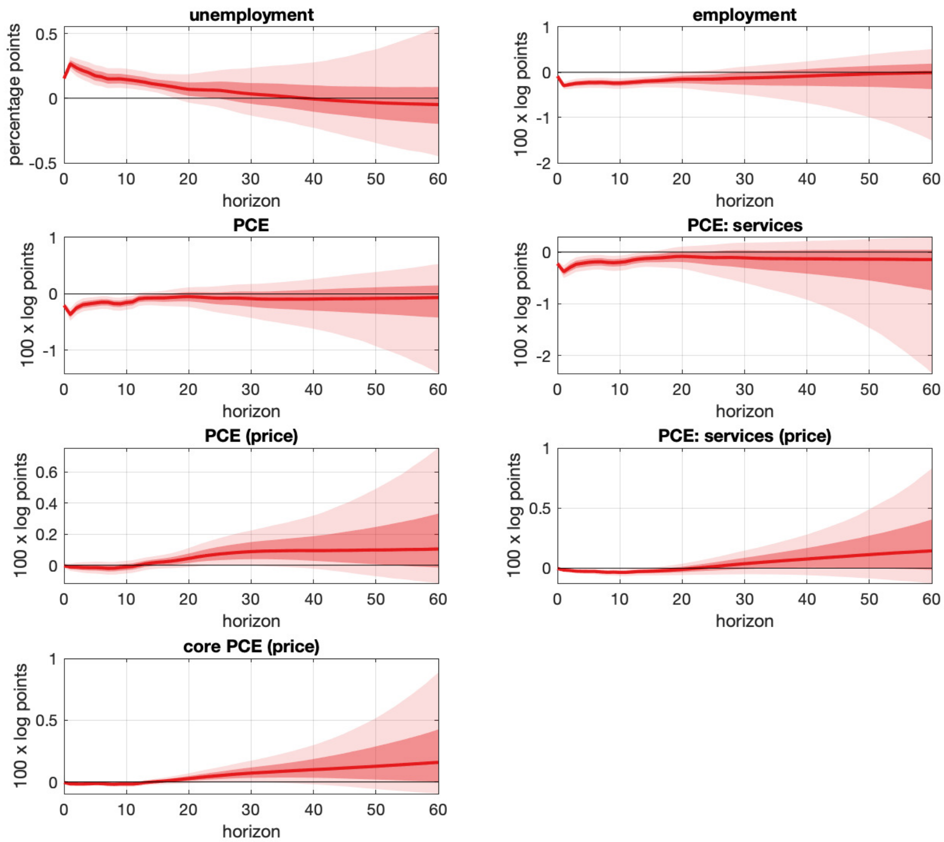

Figure 3: The solid lines are posterior medians, while the shaded areas correspond to 68- and 95-percent posterior credible regions.

4 Concluding Remarks

In this piece, I replicate the work from Lenza and Primiceri [2020], updating the model forecasts. Considering recent data, I have highlighted some differences between the projections made in the original paper and the true values of the indicators. In general, the forecasts generated by their proposed model are reasonable, although uncertainty is underestimated.

The inclusion of more recent data affect the posterior distributions of the parameters. Notably, the persistence effect of the higher volatility is much larger than the one considered in Lenza and Primiceri [2020]. This fact confirms the importance to assign a special treatment to extreme observations in order to credible regions to be reasonable. In addition, I have also presented the results from a simple BVAR considering dummy variables for the pandemic period.

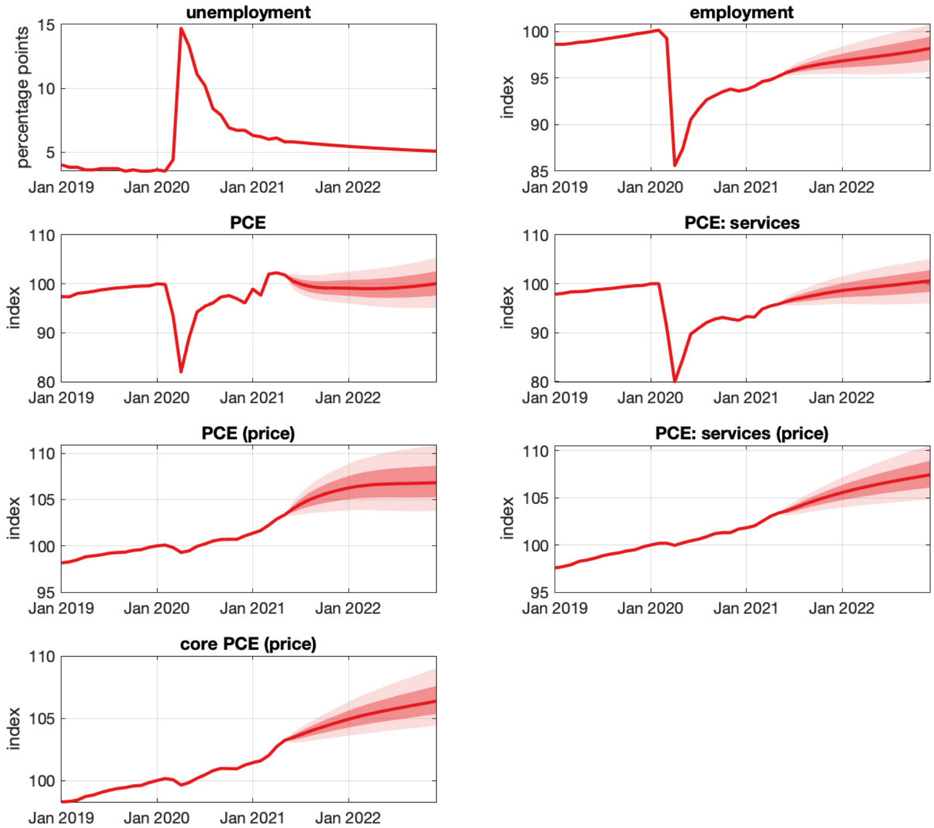

Figure 4: The solid lines are posterior medians, while the shaded areas correspond to 68- and 95-percent posterior credible regions.

References

M. Lenza and Giorgio E. Primiceri. How to estimate a var after march 2020. *Risk Management & Analysis in Financial Institutions eJournal*, 2020.

Domenico Giannone, Michele Lenza, and Giorgio E. Primiceri. Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451, May 2015.

Appendix

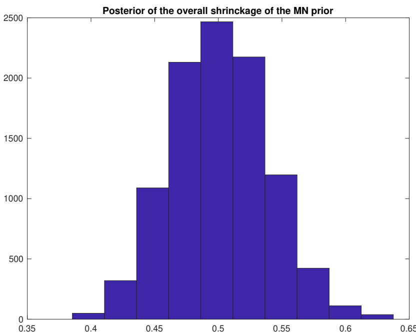

Figure 5

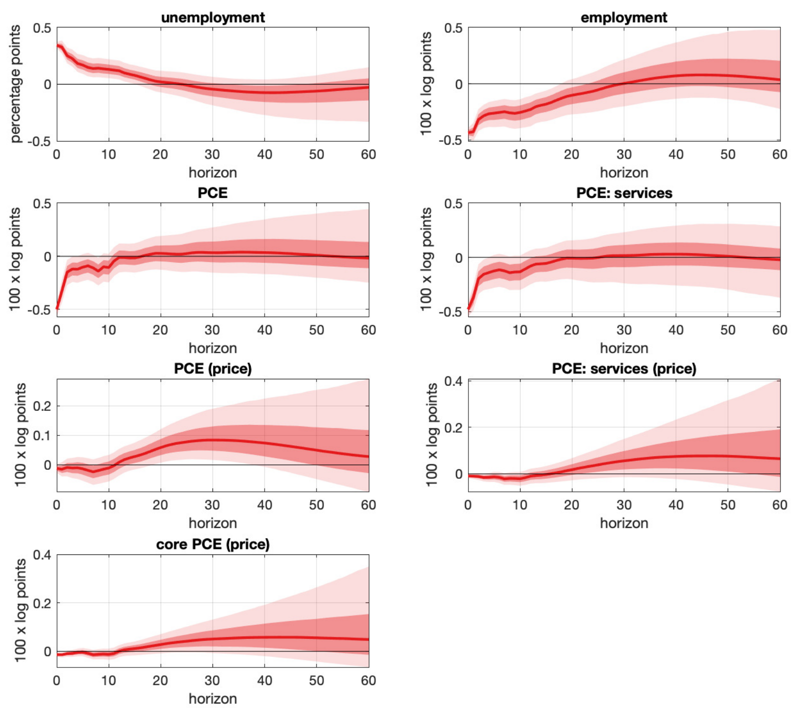

Figure 6: IRFs considering all observations with a simple VAR - The solid lines are posterior medians, while the shaded areas correspond to 68- and 95-percent posterior credible regions.

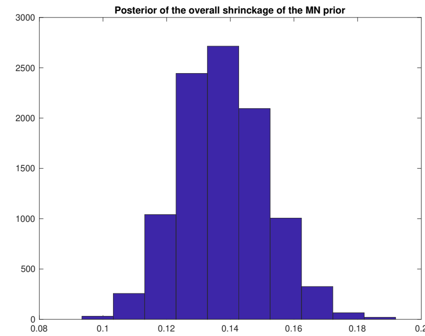

Figure 7

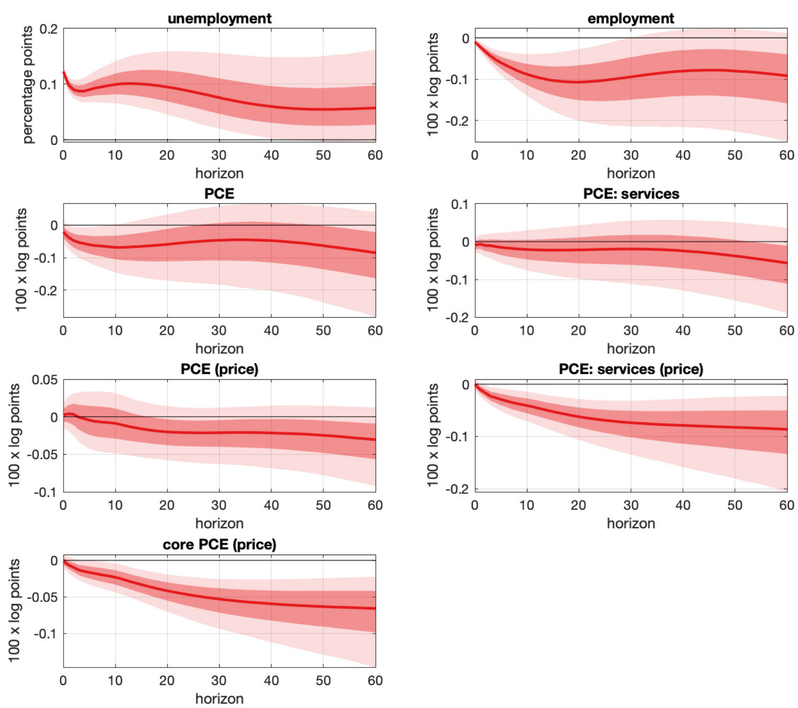

Figure 8: IRFs considering observations until Feb/2020 with a simple VAR - The solid lines are posterior medians, while the shaded areas correspond to 68- and 95-percent posterior credible regions.