Macroeconomics III - Problem Set 4

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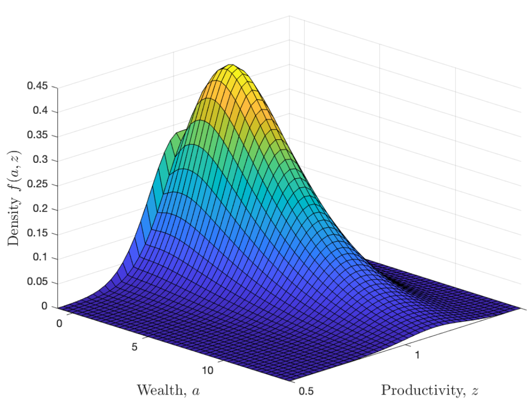
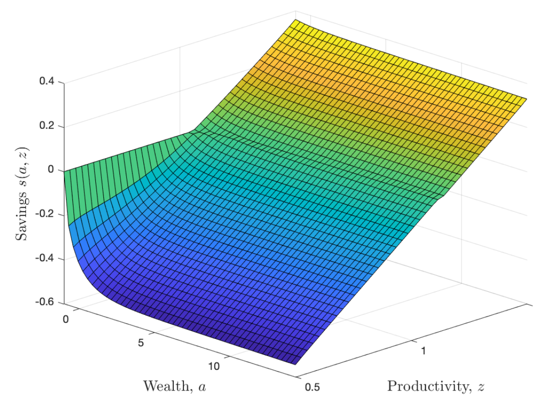
Question 1

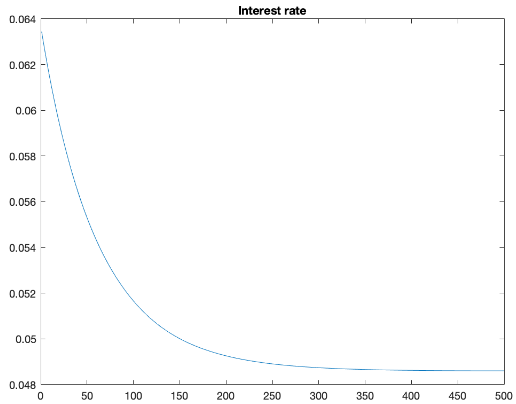
a. The code simulates an Aiyagari economy with idiosyncratic brownian motion. The value function is found every round by solving the HJB equation through an upwind finite differences scheme. The distribution is found by also solving using finite differences the Fokker-Planck (Kolmogorov forward) equation. The problem considers a a system of partial diffenntial equations. First, the Hamilton-Jacobi-Bellman equation for the optimal choices of a single individual that take prices as given. Second, the Kolmogorov Forward equation capturing the evolution of the distribution. The Hamilton-Jacobi-Bellman considers optimal individual's consumption and saving taking into account a stochastic income process and the Kolmogorov Forward equation cracterizes the evolution of the joint distribution of income and wealth. After parameters definitions, the algorithm solve the HJB equation for a given time path of prices. Then, it solves the KF equation for the evolution of the joint distribution of income and wealth. After having solved the HJB equation, we can obtain the time path of the distribution essentially with a a few number of code lines, this comes from the fact that the differential operator in the KF equation is the adjoint of the differential operator in the HJB equation. Finally, the algorithm consist of iterate until an equilibrium fixed point for the time path of prices is found.

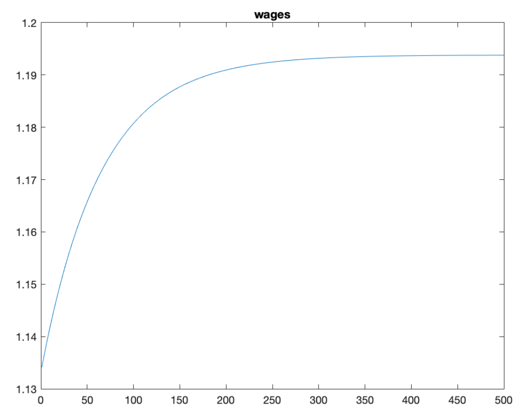
b. Considering a permanent increase of 10% in TFP, we can get the following figures. Note that woth a higher TFP, for a given level of wealth there is an increment in savings.

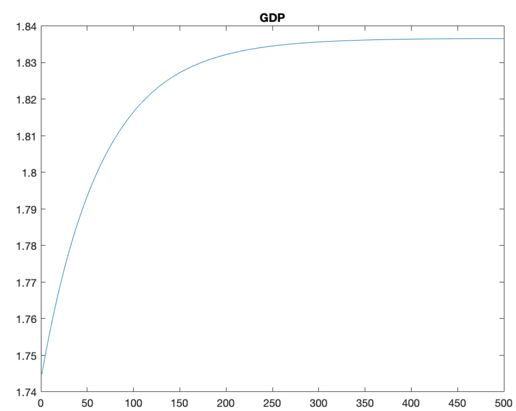


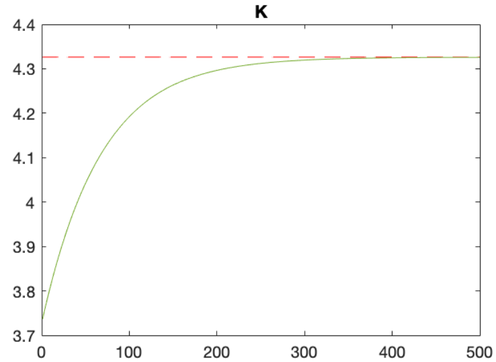


c. The code is the file attached. As we can see in the figures below, when there is a 10% increase in TFP, naturally we can expect that GDP goes up from the productivity gain. Since we have now a higher productivity and a higher demand for workers, there is pressure on wages, therefore, it increases. At the same time, capital increases to a new level and interest rate decreases.

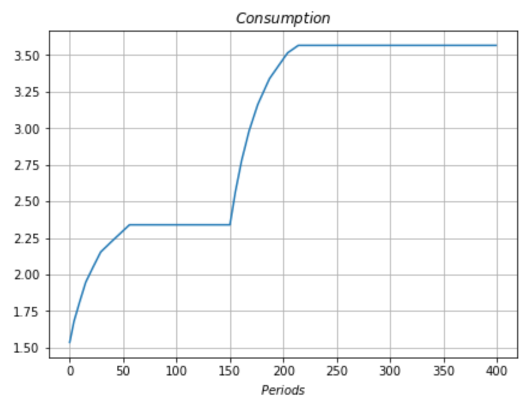

Interest rate
0.064
0.062
0.06
0.058
0.056
0.054
0.052
0.05
0.048
0
50
100
150
200
250
300
350
400
450
500


wages
1.2
1.19
1.18
1.17
1.16
1.15
1.14
1.13
0
50
100
150
200
250
300
350
400
450
500


GDP
1.84
1.83
1.82
1.81
1.8
1.79
1.78
1.77
1.76
1.75
1.74
0
50
100
150
200
250
300
350
400
450
500

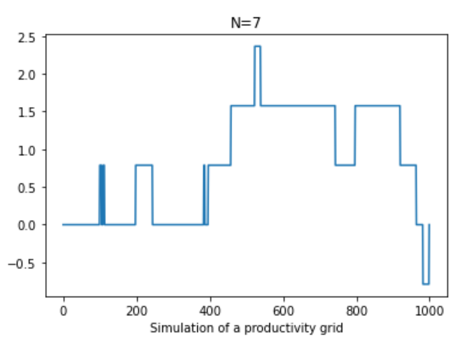


d. In the standard Ramsey model, when there is no idiosyncratic shocks, we can observe some changes relative to the model in the previous items. Considering the same technology and preference parameters as before, also assuming that labor productivity is similar to the average productivity in the Aiyagari model I have adapted the code from question 3 Problem Set 3 to observe some changes. Since there is no idiosyncratic shocks, after the 10% increase in TFP the distribution of wealth must keep the same.



Question 2

a. Simulation with N=7.



b. The household problem is

$$max_{c_l, l_t, a_{t+1}} E_0 \left[\sum_{t=0}^{\infty} u(c_t, l_t) \right]$$

subject to

$$c_t + (a^l_{t+1}) = (1 + r^l_t)a^l_t + w_t l_t z_t$$

or

$$c_t + (a^l_{t+1} + a^h_{t+1}) = (1 + r^l_t)a^l_t + (1 + r^h_t)a^h_t + w_t l_t z_t - d\psi$$

where a is positive, since agents cannot borrow and d is a dummy considering that if the agent invest in high or low return asset.

As the problem repeats every period we can write it in a recursive way

$$\begin{aligned} V(a, z_i) &= max_{c,l,a'} \left\{ u(c, l) + \beta \sum_{zj} Pr(z' = z_j | z = z_i)V(a', z_j) \right\} \\ &= max_{l,a'} \left\{ u((1 + r_t)a + wzl - a', l) + \beta \sum_{zj} Pr(z' = z_j | z = z_i)V(a', z_j) \right\} \end{aligned}$$

c. The way I have tried to solve this problem is following. First I considered $\psi = 1$. Then, I tried to solve the problem separately. Using two different values functions. When maximizing, if one agent have a level of income that guarantees that she can pay the cost for the high asset, she only invest in the high asset. Therefore, I built a map for each of the value functions to solve the problem. In this map we have a value one whenever the value function of the high asset is greater than that of the low asset. Fot the rest o this question, I had some problems and could not finish on time.