Macroeconomics III - Problem Set 3

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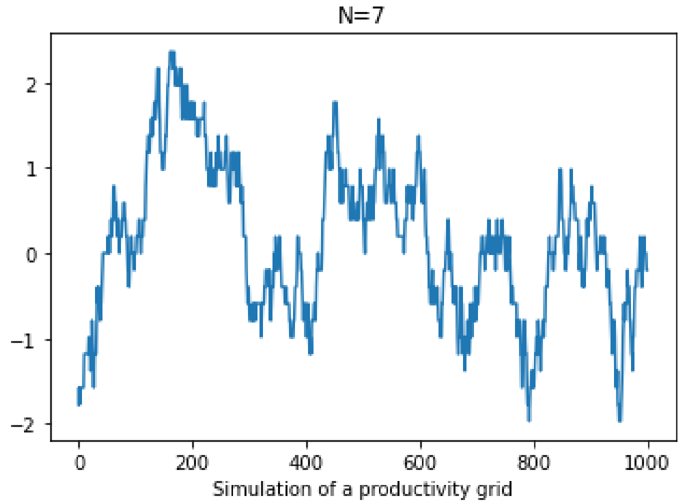
Question 1 - Aiyagari Model

a. Simulated shock with N=7 and $\sigma_z^2 = 0.621$.



b. The household problem is

$$max_{c_l, l_t, a_{t+1}} E_0 \left[\sum_{t=0}^{\infty} u(c_t, l_t) \right]$$

subject to

$$c_t + a_{t+1} = (1 + r_t)a_t + w_t l_t z_t$$

$$a_{t+1} \geq -\frac{w\underline{z}}{r}$$

where the last constraint in the equations above is the natural borrowing limit.

As the problem repeats every period we can write it in a recursive way

$$\begin{aligned} V(a, z_i) &= max_{c,l,a'} \left\{ u(c,l) + \beta \sum_{zj} Pr(z' = z_j | z = z_i) V(a', z_j) \right\} \\ &= max_{l,a'} \left\{ u((1+r_t)a + wzl - a', l) + \beta \sum_{zj} Pr(z' = z_j | z = z_i) V(a', z_j) \right\} \end{aligned}$$

c. Consider now the representative firm's problem

$$\begin{aligned} & maxK_t, N_t\{K_t^\alpha N_t^{1-\alpha} - w_t N_t - \tilde{r}_t K_t\} \\ & \text{subject to} \\ & K = \sum_{z_i} \sum_{a_k} a_k \lambda(a_k, z_i) \\ & N = \sum_{z_i} \sum_{a_k} l_i z_i \lambda(a_k, z_i) \\ & w = (1-\alpha) \left(\frac{K}{N}\right)^\alpha \\ & \tilde{r} = \alpha \left(\frac{K}{N}\right)^{\alpha - 1} \end{aligned}$$

where K denotes aggregate capital, N denotes effective labor and $\tilde{r} = r + \delta$ denotes the capital price paid by the firm.

d. The recursive stationary competitive equilibrium is this economy is a list of

$$\begin{aligned} [w, r] &: \text{prices} \\ [c(a, z_i), l(a, z_i), a'(a, z_i)] &: \text{policy functions} \\ [K, N] &: \text{aggregate capital and labor} \\ [\lambda(a, z_i)] &: \text{distribution function} \end{aligned}$$

where

1. Prices are in equilibrium and the policy functions solve the household problem.

$$\begin{aligned} w &= \partial F(K, N)/\partial N \\ r &= \partial F(K, N)/\partial K - \delta \end{aligned}$$

2. Prices in equilibrium, capital and labor solve the firm's problem.
3. Markets clear.
4. The stationary distribution $\lambda(a, z_i)$ is induced by policy functions and the markov process

$$\lambda(a', z,) = \sum_z \sum_{a:a'=a'(a,z)} \lambda(a, z) \mathcal{P}(z, z')$$

e. The code for this problem is in the file attached. Solving further, calculating the first order conditions for consumption and labor we have

$$\begin{aligned} [c']: &\quad c^{-\sigma} = \beta \frac{\partial}{\partial a'} \int V(a', z') f(z'|z) dz \\ [l']: &\quad \frac{\gamma(1-l)^{-\sigma}}{wz} = \beta \frac{\partial}{\partial a'} \int V(a', z') f(z'|z) dz \end{aligned}$$

Rearranging

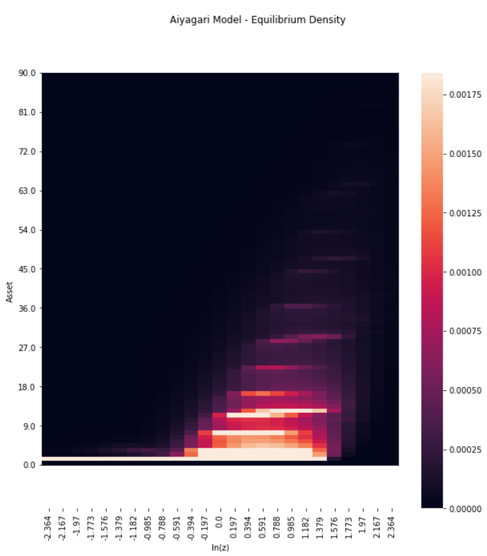
$$c = (1-l) \left(\frac{wz}{r}\right)^{\dfrac{1}{\sigma}}$$

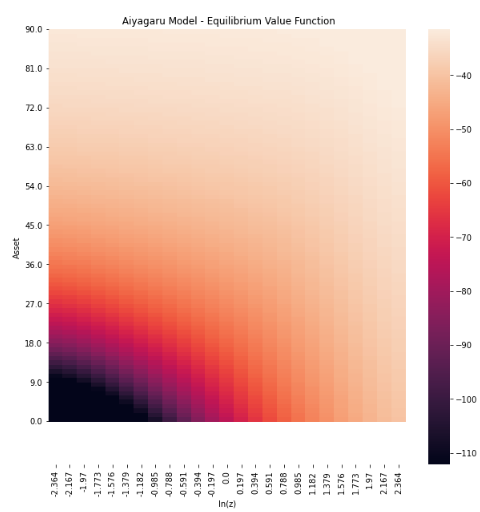
Substituting c in the budget constraint, $c_t + a_{t+1} = (1 + r_t)a_t + w_t l_t z_t$ we have

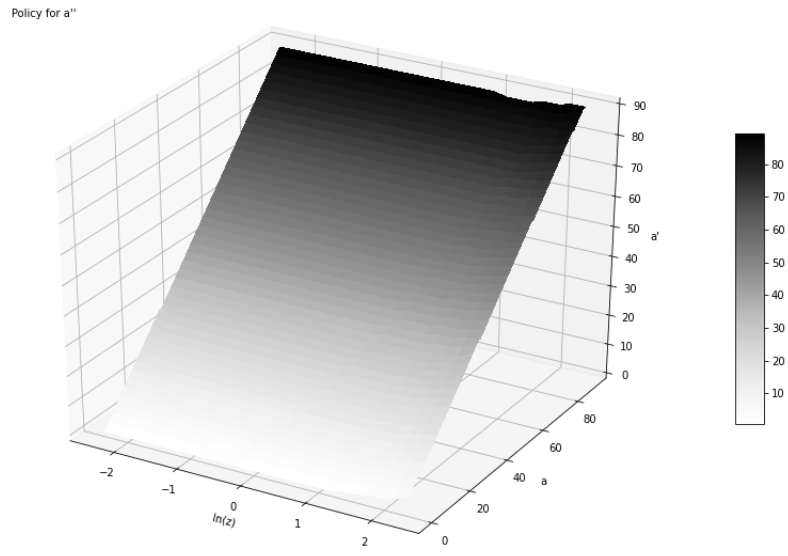
$$l(a, z, a') = \frac{(wz/\gamma)^{\dfrac{1}{\sigma}} + a' - a(1+r)}{wz + (wz/\gamma)^{\dfrac{1}{\sigma}}}$$

solving for c

$$c(a, z, a') = (1 - l(a, z, a')) \left(\frac{wz}{\gamma}\right)^{\dfrac{1}{\sigma}}$$

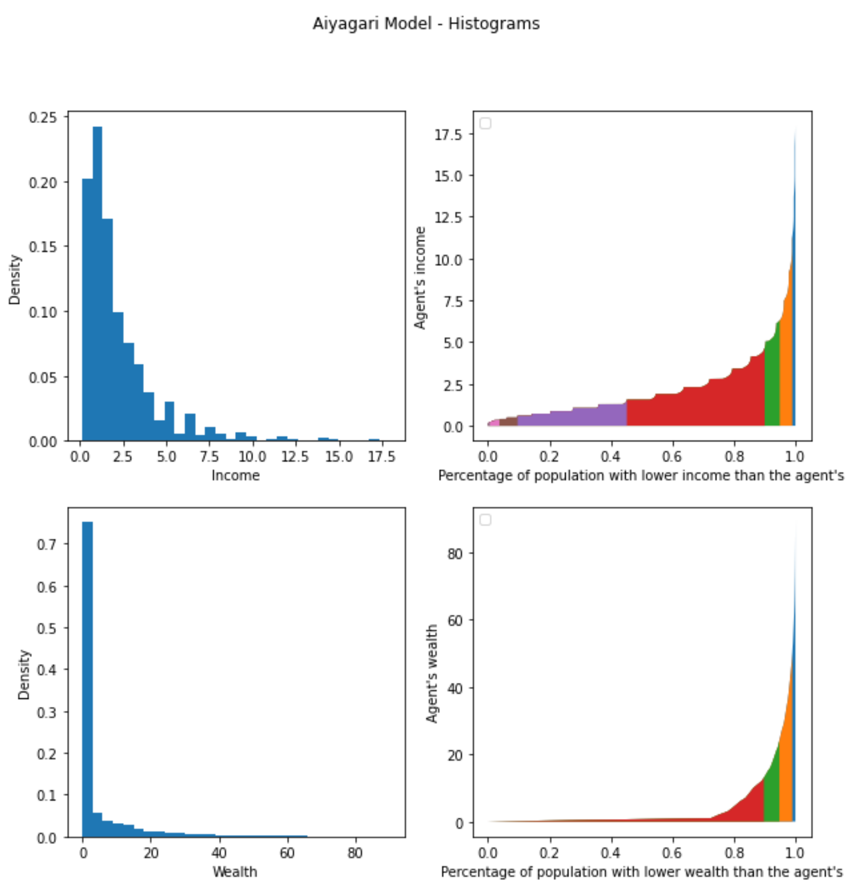






Policy for a''
ln(z)
a
a'

f.



Aiyagari Model - Histograms
Density
Income
Agent's income
Percentage of population with lower income than the agent's
Density
Wealth
Agent's wealth
Percentage of population with lower wealth than the agent's

Summary of statistics for this economy	
Interest rate:	1,691%
Capital-to-Labor ratio:	10.6209
Output:	1.165
Capital-to-Output ratio:	4.1276

Reporting Wealth	Quantiles:
Quantiles	Wealth
1%	1.0
5%	1.0
10%	1.0
50%	1.0
90%	14.0
95%	25.0
99%	51.0

Reporting Income	Quantiles:
Quantiles	Income
1%	0.232
5%	0.406
10%	0.593
50%	1.561
90%	4.659
95%	6.334
99%	11.342

Question 2 - Hopenhayn model

a. The profit function in this model is

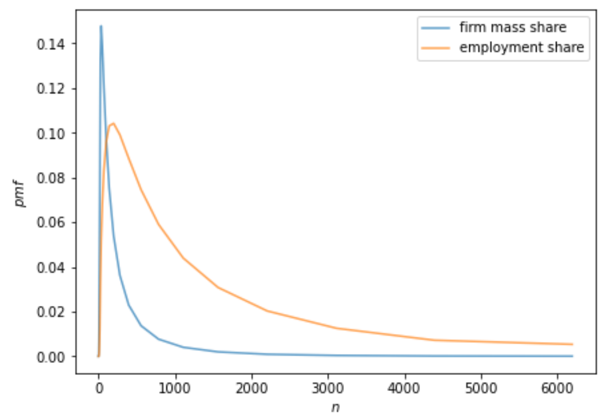
$$\pi = pzn^{\alpha} - wn - \kappa$$

where κ is the fixed operational cost paid by the firm

b. The code for this problem is in the file attached.

Summary of results	
price	1.08413
cut off	3.61572
entry rate	0.07052
exit rate	0.07052

In steady state, entry and exit rates are equal, and the same is job creation and destruction at aggregate level. All the aggregates including price level, exit reservation level, employment, output remain constant over time. Also, note that in equilibrium there exist a set of firms leaving and entering in the market. Follow below the stationary distributions



c. Assuming that the demand curve shifts, with $\bar{D}$ increasing to 120, there is a slightly upward change in equilibrium prices and also the exit and entry rate goes up. When the demand increases since there is an unlimited number of potential entrants, more firms will participate in this market.

Ramsey model in continuous time

a. In this problem households solve

$$max \int_0^\infty e^{-\rho t} u(c) dt \quad s.t \quad \dot{a} = w + ra - c$$

where c is consumption and a denotes assets. ρ is the subjective discount rate, r is the interest rate and w is the wage rate. Firms maximize

$$max AK^\alpha N^{1-\alpha} - wN - (r+\delta)K$$

where δ is the depreciation rate and A is a productivity factor.

The Hamilton-Jacobian-Bellman Equation associated with the households problem is

$$\rho V(a) = max_c\{u(c) + V'(a)(w + ar - c)$$

where a is the state variable and c is the control variable.

b. The code for this item is in the file attached

c. From $\rho V(a) = max_c\{u(c) + V'(a)(w + ar - c)$ we can get the following first order condition $u'(c) = V'(a)$. Then,

$$\rho V'(a) = V'(a)(w + ar - c) + V'(a)a$$

Therefore, substituting we can find that

$$u''(c)c'(a)(w + ar - c) = u'(c)(r - \rho)$$

Also, we must have in equilibrium

$$\dot{a} = w + ra - c$$

In addition, from firms maximization problem

$$w = (1-\alpha)Ak^{\alpha}$$
$$r + \delta = \alpha A k^{\alpha-1}$$

and the aggregate capital accumulation

$$\dot{K} = Y - C - \delta K$$

e. The code for this item is in the file attached. Consider the equation derived above $u''(c)c'(a)(w + ar - c) = u'(c)(r - \rho)$, using the utility function in the question $u(c) = c^{1-\gamma}/(1-\gamma)$ we have that

$$\gamma \frac{c'(a)}{c}\dot{a} = (r - \rho)$$

where we use the fact that

$$\frac{u''(c)c}{u(c)} = -\gamma$$

and also that

$$\dot{a} = w + ra - c$$

and we can use $\dot{c} = c'(a)\dot{k}$ to write

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}(r - \rho)$$

Hence,

$$\dot{c} = \frac{c}{\gamma}(r - \rho)$$
$$\dot{a} = w + ra - c$$