

①

- I) a) V b) F c) V d) F e) V f) V
 g) V h) V i) V j) V k) V l) F
 m) F n) F o) F

II) a) $\vec{AO} + \vec{FO} = \vec{AO} = AE$ e) $\vec{AF} + \vec{AE} = \vec{AF} + \vec{BG} = \vec{AO}$
 b) $\vec{EG} = \vec{AB}$ f) $\vec{AF} - \vec{AE} = \vec{AO}$
 c) $\vec{FH} = \vec{AD}$ ~~g) $\vec{AD} + \vec{AB} = \vec{AC}$~~
 d) \vec{AG} h) $\vec{AB} + \vec{AC} = \vec{AD}$
 i) $\vec{AF} + \vec{FO} = \vec{AO} + \vec{AO} = \vec{AC}$

- ② a) V b) F c) F d) V e) F f) V

- g) F h) V i) F j) V k) V

6) $\vec{v} = (-1, 2, 2)$
 $\vec{w} = (2, -1, 3)$

$$3 \cdot (\vec{u} - \vec{v}) + \frac{1}{2} \vec{w} = 4\vec{u} - \vec{w}$$

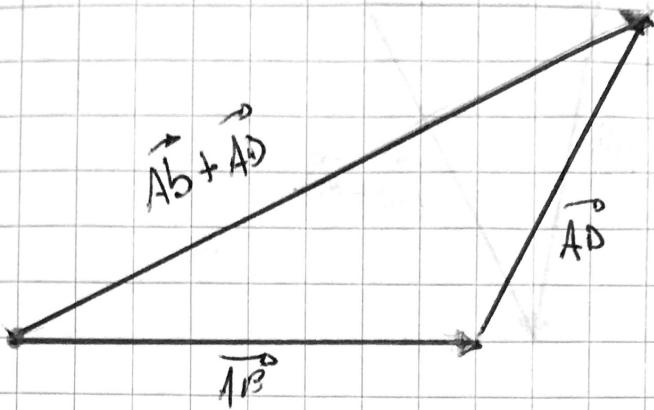
$$3 \cdot (2, -1, 3) - (-1, 2, 2)$$

$$3 \cdot (3, -3, 4) + \frac{1}{2} w = (9, -9, 12) - w$$

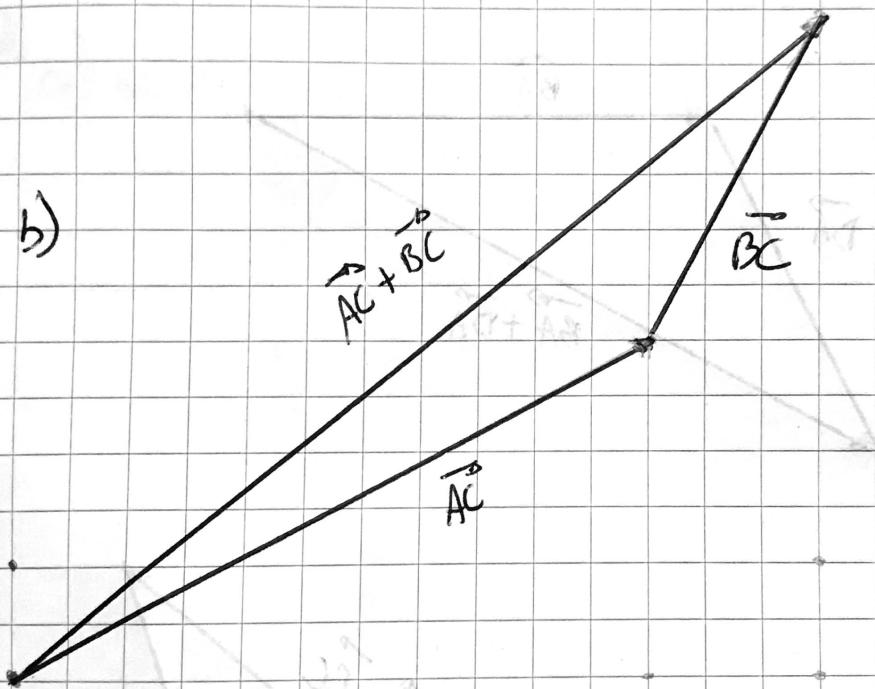
$$(9, -9, 12) + \frac{1}{2} w = (0, -4, 12) - w$$

$$w = \left(-\frac{7}{3}, \frac{10}{3}, 6 \right)$$

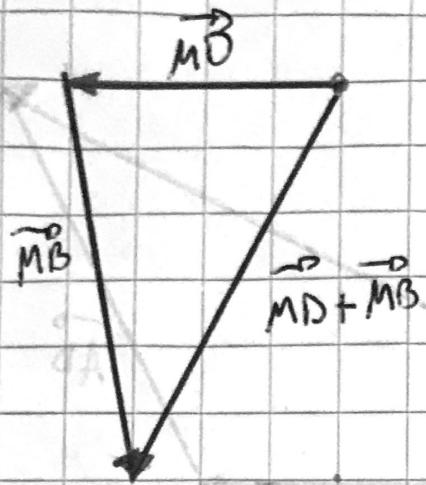
3) a)



b)

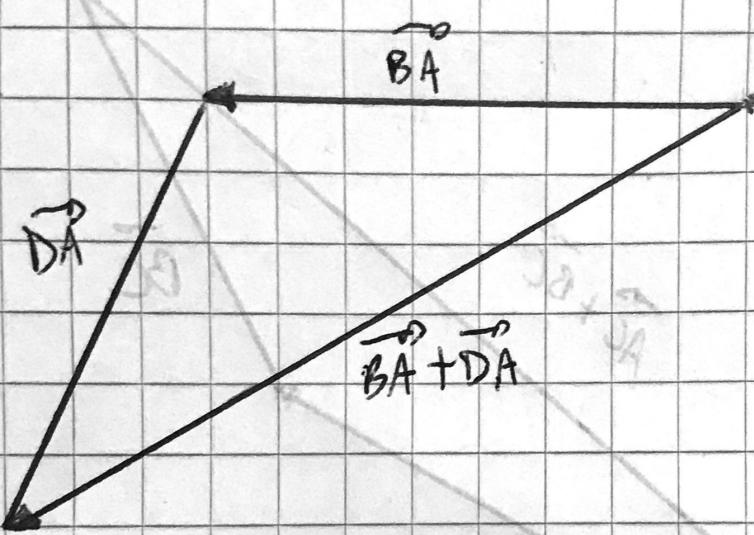


3c)



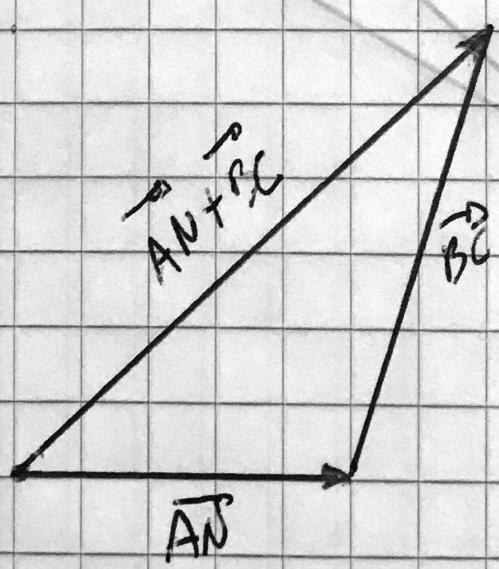
3d)

c)

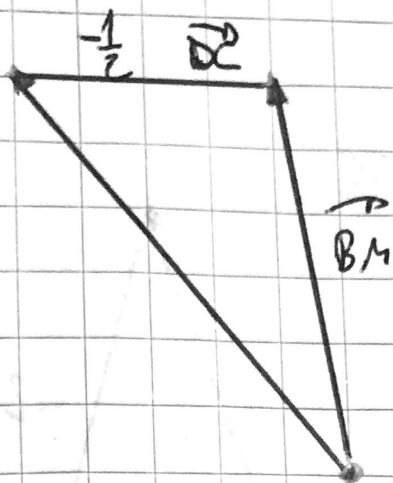


4)

e)

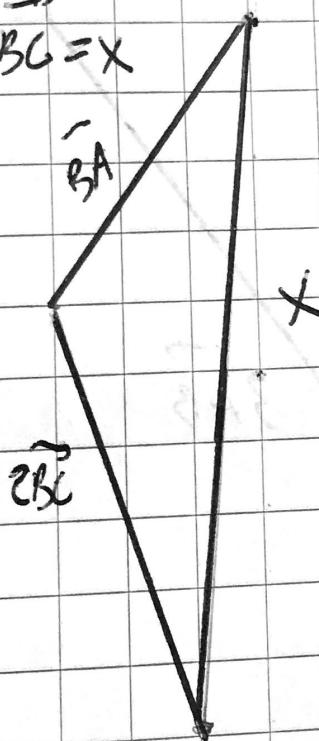


3f)

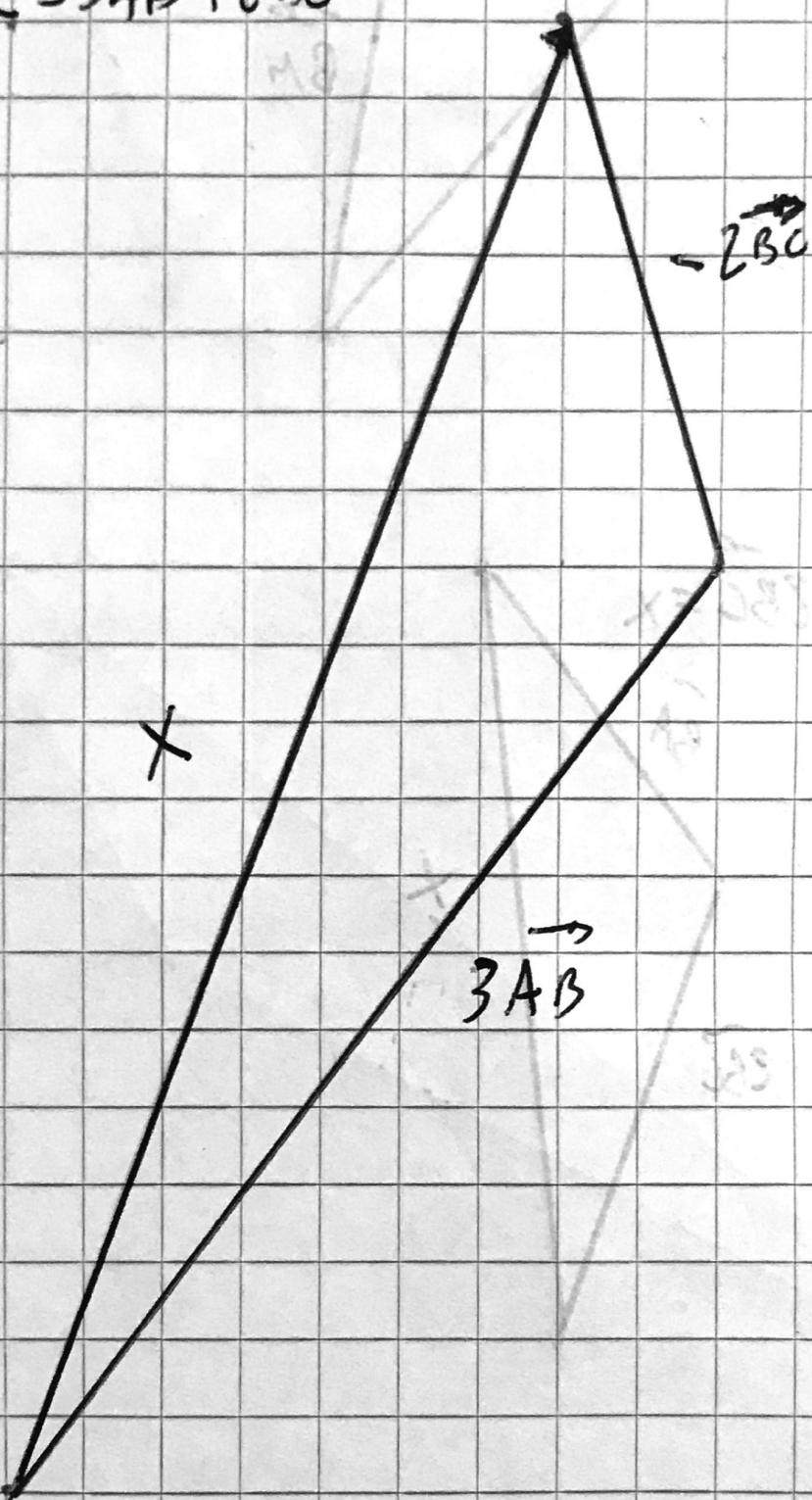


4)

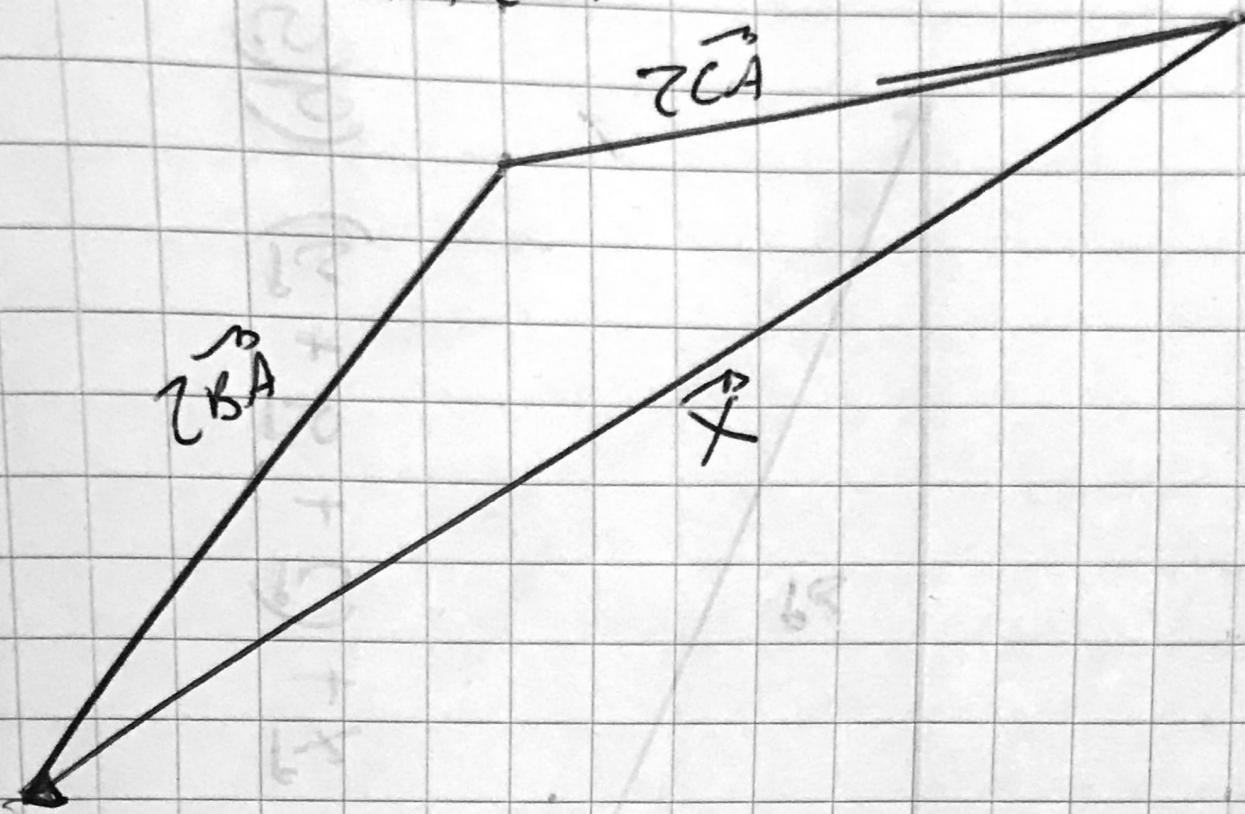
a) $\vec{BA} + \vec{BC} = x$



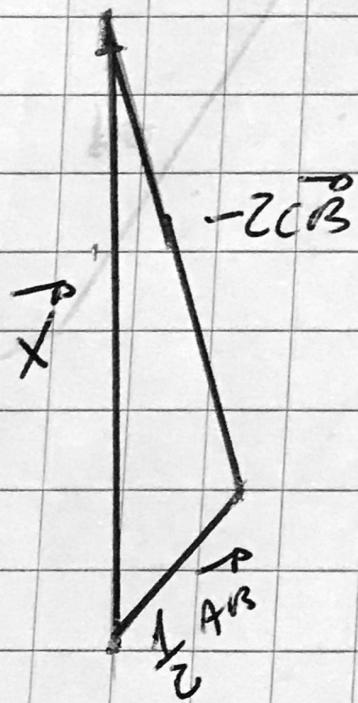
$$4b) \vec{x} = 3\vec{AB} + 2\vec{AC}$$



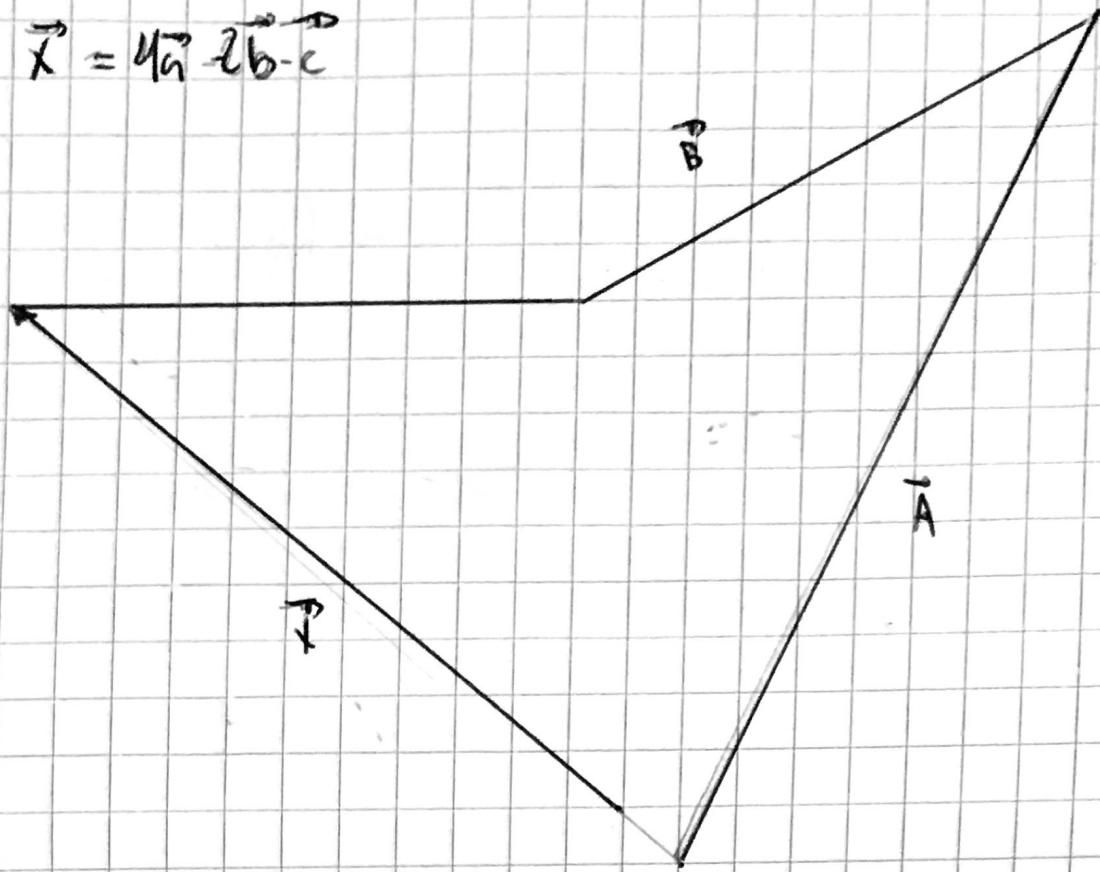
$$hc) \quad x = 2\vec{CA} + 2\vec{BA}$$



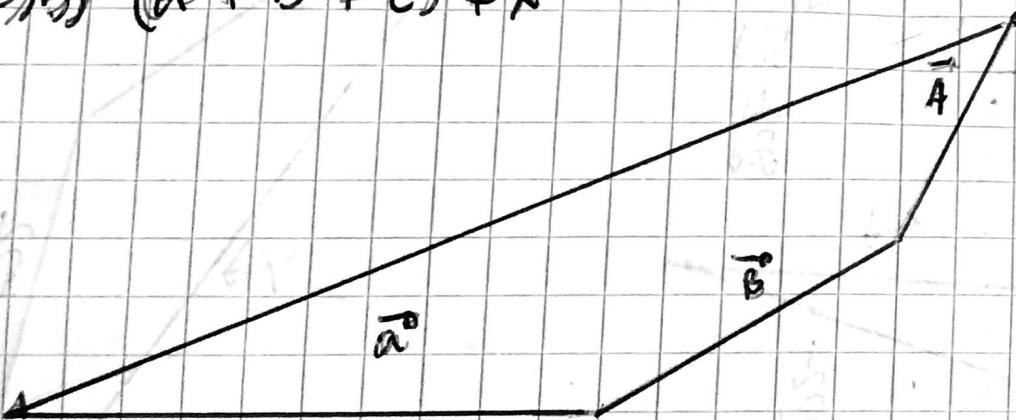
$$hd) \quad x = \frac{1}{2}\vec{AB} - 2\vec{CB}$$



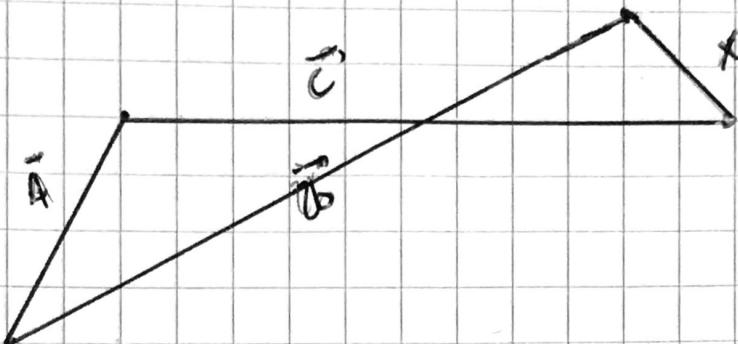
$$5) a) \vec{x} = 4\vec{a} - 2\vec{b} - \vec{c}$$



$$5)b) (\vec{a} + \vec{b} + \vec{c}) + \vec{x}$$



$$5) d) \vec{a} + \vec{c} - \vec{x} = 2\vec{b}$$



$$\textcircled{7} \quad \vec{u} = (-4, 3) \quad \vec{w} = a\vec{u} + b\vec{v}$$

$$\vec{v} = (1, 5) \quad (-2, 6) = a(-4, 3) + b(1, 5)$$

$$\vec{u} = (-2, 6) \quad (-2, 6) = (-4a, 3a) + (b, 5b)$$

$$(-2, 6) = (-4a+b, 3a+5b)$$

$$\begin{cases} -4a+b = -2 & (-5) \\ 3a+5b = 6 \end{cases} \Rightarrow \begin{cases} 20a+(-5b) = 10 \\ 3a+5b = 6 \end{cases} \Rightarrow \begin{cases} 23a = 16 \\ a = \frac{16}{23} \end{cases}$$

$$115b = 90 \quad b = \frac{90}{115} = \frac{18}{23}$$

$$\textcircled{8} \quad \vec{u} = (3, 0, 1)$$

$$\vec{v} = (-2, 1, -1)$$

$$\vec{w} = (1, 2, 3)$$

$$\vec{t} = (4, 1, 0)$$

$$\vec{t} = (4, 1, 0) = (3a - 2b + c, b + 2c, a - b + 3c)$$

$$3a - 2b + c = 4$$

$$\begin{aligned} b + 2c &= 1 \\ a - b + 3c &= 0 \end{aligned}$$

$$\begin{aligned} a &= 3a - 2b + c = 4 \\ -a + b + c &= 1 \\ \hline 2a &= 5 \quad a = \frac{5}{2} \end{aligned}$$

$$b = b - \frac{6}{10} = 1 \quad c = a + 5c = 1$$

$$10b - 6 = 10$$

$$5c + \frac{5}{2} = 1$$

$$\begin{cases} 10b = 16 \\ b = \frac{8}{5} \end{cases}$$

$$\frac{10c + 5}{2} = 1$$

$$10c = -3$$

$$c = -\frac{3}{10}$$

$$\textcircled{9} \quad A(1, 2, 3) \quad 3BA - 2BC$$

$$B(-6, -2, 3)$$

$$C(1, 2, 1)$$

$$3(-7, -4, 0) - 2(-7, -4, 2)$$

$$(-21, -12, 0) - (-14, -8, 4)$$

$$V = (7, 4, 4) \quad \text{Volumen} = 7, 4, 4$$

$$|W| = \sqrt{7^2 + 4^2 + 4^2}$$

$$|V| = \sqrt{49 + 16 + 16} = \sqrt{81}$$

55,5

$$⑩ u \cdot (3, -1, 4) \quad \vec{w} \parallel \vec{v}$$

$$v(4, 3, 6) \quad (3, -1, 4) = (4t, 6t, 6t)$$

$$t = \frac{1}{5} \quad -\frac{1}{5}a = 3 \quad \boxed{a = -15} \quad -\frac{1}{5}b = 4 \quad \boxed{b = -20}$$

$$⑪ B-A = (15, 2) + (2, -1, -3) = (3, 4, -1)$$

$$x = -2 + 3t \quad a = -2 + 3t$$

$$y = 1 + 4t \quad \Rightarrow \quad b = 1 + 4t$$

$$z = 3 - 1t$$

$$c = 3 - 1t$$

$$a = -2 + 3 \cdot -1 \Rightarrow \boxed{5}$$

$$b = +1 + 4 \cdot -1 \Rightarrow \boxed{-3}$$

$$⑫ a) (2, 1, m)$$

$$b) (m+2, -5, 2) \quad \vec{w} = (2, 1, m) + (m+2, -5, 2) = (m+4, -4, m+2)$$

$$c) (2m, 8, m) \quad \vec{v} = (2m, 8, m) + (2, -4, -m) = (2m-2, 2, 0)$$

$$(m+4, -4, m+2) \cdot (2m-2, 2, 0) = 0$$

$$(2m^2 + 6m - 8 - 28) = 0$$

$$\Delta = 9 + 72 = 81$$

$$2m^2 + 6m - 36 = 0$$

$$\Delta = 3^2 - 4(1)(-16)$$

$$4m^2 + 3m - 18 = 0$$

$$m = \frac{-3 \pm \sqrt{81}}{2}$$

$$m = \frac{-3 \pm 9}{2} \quad m_1 = \frac{6}{2} = 3$$

$$m_2 = \frac{-12}{2} = -6$$

(13)

$$C = \begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix} \quad P_M = \begin{pmatrix} 5 \\ 2 \\ 1 \\ 3 \end{pmatrix}$$

(14)

$$t = (x, y, z)$$

$$\frac{0 = -5 + xt}{2} \quad -\frac{1 = 2 + yt}{2} \Rightarrow -2 = 2 + yt$$

$$x, t = -5$$

$$yt = -4$$

$$3 = \frac{-1 + zt}{2} \quad 6 = -1 + zt$$

~~$t \neq 0$~~

~~$\vec{f} = (-5, -4, 2)$~~

(15)

$$(1, a, -2a-1) \cdot (a, a-1, 1) =$$

$$= [(1, a, -2a-1) + (a, a-1, 1)] \cdot (a, -1, 1)$$

$$1 \cdot a + a \cdot (a-1) + (-2a-1) \cdot 1 = (1+a, a+a-1, -2a-1+1) \cdot (a, -1, 1)$$

$$a + a^2 - a - 1 = (1+a, 2a-1, -2a) \cdot (a, -1, 1)$$

$$a^2 - 2a - 1 = (1+a) \cdot a + (2a-1) \cdot (-1) + (-2a) \cdot 1$$

$$a^2 - 2a - 1 = a + a^2 - 2a + 1 - 2a$$

$$-1 = a + 1 - 2a$$

~~$a = 1$~~

$$u = (1, 2, -5)$$

$$v = (2, 1, 1)$$

$$w = (2, -1, 1)$$

$$16) \quad d = \sqrt{(n)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$$

$$d = n^2 + \frac{4}{25} + \frac{4}{25}$$

$$d = \frac{25n^2 + 8}{25} \Rightarrow 25 = 25n^2 + 8$$

$$25n^2 = 17$$

$$n = \sqrt{\frac{17}{25}} \Rightarrow n = \frac{\sqrt{17}}{5}$$

$$17) \quad D_{AP} = \sqrt{(x-2)^2 + (3)^2 + (-1)^2} = \sqrt{(x-2)^2 + 10}$$

$$D_{BP} = \sqrt{(x+2)^2 + (-1)^2 + (1)^2} = \sqrt{(x+2)^2 + 2}$$

$$\sqrt{(x-2)^2 + 10} = \sqrt{(x+2)^2 + 2}$$

$$(x-2)^2 + 10 = (x+2)^2 + 2$$

$$x^2 - 4x + 4 + 10 = x^2 + 4x + 4 + 2$$

$$-4x + 14 = 4x + 6$$

$$8x = 8 \Rightarrow x = \boxed{1}$$

18)

$$B.A = (-1+4, -2+2, 4-0) = (3, 0, 4)$$

$$B.C = (3+4, -2+2, 1-0) = (7, 0, 1)$$

$$|BA| = \sqrt{(9+0+16)} = \sqrt{25} = 5$$

$$|BC| = \sqrt{49+0+18} = \sqrt{50} = 5\sqrt{2}$$

$$\cos \theta = \frac{(3, 0, 4) \cdot (7, 0, 1)}{5 \cdot 5\sqrt{2}}$$

$$\cos \theta = \frac{21}{25\sqrt{2}} = \frac{1}{5} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \underline{45^\circ}$$

$$\textcircled{19} \quad (v)(w) = (2, 1, -1) \cdot (1, 1, m+2) = 2 \cdot 1 - 1 \cdot m - 1 \cdot (-1-m)$$

$$|v|^2 = 4 + 1 + 1 \quad |w|^2 = 1 + 1 + m^2 + 2 \cdot 2m + 4$$

$$|v|^2 = 6 \quad |w|^2 = m^2 + 4m + 6$$

$$|v| = \sqrt{6} \quad w = \sqrt{m^2 + 4m + 6}$$

$$\cos \frac{\pi}{3} = \frac{1}{2} = \frac{-1-m}{\sqrt{6} \cdot \sqrt{m^2+4m+6}} = \frac{1}{2} \quad \frac{1 = 2(-1-m)}{6m^2 + 24m + 36}$$

$$\sqrt{(6m^2 + 24m + 36)} = (2(-1-m))^2$$

$$6m^2 + 24m + 36 = 4(-1-m)^2 \quad (0)^2 - 4 \cdot 1(16) = 0$$

$$6m^2 + 24m + 36 = 4(1 + 2m + m^2) \quad \Delta = 64 - 64$$

$$2m^2 + 16m + 32 = 0 \quad (\div 2)$$

$$m^2 + 8m + 16 = 0$$

$$\Delta = 0 \quad \boxed{m = -4}$$

\textcircled{20}

$$\vec{v} = n \cdot \vec{w}$$

$$\vec{v} \cdot \vec{w} = 12$$

$$n(-4, 2, 6) \cdot (-1, 4, 2) = 12$$

$$w \cdot (4 + 8 + 12) = -12$$

$$24n = -12$$

$$\boxed{n = \frac{1}{2}}$$

$$v = \underline{(2, 1, -3)}$$

$$\textcircled{21} \quad \vec{v} = (x, 4, 0)$$

$$\vec{w} = (0, 4, 3)$$

$$\vec{v} \cdot \vec{w} = 6$$

$$\vec{w} = 5$$

$$\vec{v} \cdot \vec{w} = 6$$

$$(x, 4, 0) \cdot (0, 2, 3) = 6$$

$$0 + 8 + 0 = 6$$

$$4 = 3$$

$$x^2 + 9 + 0 = 25 \quad \vec{v}_1 = (-4, 3, 0)$$

$$x^2 = 25 - 9$$

$$x^2 = 16$$

$$\vec{v}_2 = (4, 3, 0)$$

$$\boxed{x = 4}$$

$$\textcircled{22} \quad |\vec{v}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$$

$$\frac{1}{\sqrt{6}} \cdot (1, -1, 2) = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right) \cdot 5 =$$

$$= \left(\frac{5}{\sqrt{6}}, -\frac{5}{\sqrt{6}}, \frac{10}{\sqrt{6}} \right)$$

$$(23) \quad \begin{aligned} a+3b=10 &\rightarrow a \cdot 2 + b \cdot 3 + 0 \cdot (-1) = 10 \\ a-b=-5 &\rightarrow a \cdot 1 + b \cdot (-1) + 0 \cdot 2 = -5 \end{aligned}$$

$$2(b-5) + 3b = 10$$

$$\begin{aligned} 2b-10+3b &= 10 \\ 5b-20 &= 10 \quad |+20 \\ 5b &= 30 \quad |:5 \\ b &= 6 \\ a &= -1 \end{aligned}$$

$$\vec{v} = (-1, 6, 0)$$

$$(24) \quad \frac{(1, 2, -3) \cdot (2, 1, -2)}{(2, 1, -2) \cdot (2, 1, -2)} \cdot (2, 1, -2)$$

$$\frac{1 \cdot 2 + 2 \cdot 1 + (-3) \cdot (-2)}{2 \cdot 2 + 1 \cdot 1 + (-2) \cdot (-2)} \cdot (2, 1, -2) = \frac{2+2+6}{4+1+4} \cdot (2, 1, -2) = \frac{10}{9} \cdot (2, 1, -2)$$

$$(2, 1, -2) =$$

$$= \left(\frac{20}{9}, \frac{10}{9}, -\frac{20}{9} \right)$$

$$x \cdot v \quad \begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

~~$$12i + 6j + 18k - i + 20j = 11i + 26j + 23k$$~~

$$12i + 6j + 18k - i + 20j = 11i + 26j + 23k$$

$$|x \cdot v| = \sqrt{121 + 6^2 + 529} = \sqrt{1376}$$

$$\frac{\vec{u} + \vec{v}}{|\vec{u} + \vec{v}|} = \left(\frac{11}{\sqrt{1376}}, \frac{26}{\sqrt{1376}}, \frac{23}{\sqrt{1376}} \right) \cdot s = \frac{5s}{\sqrt{1376}}, \frac{130}{\sqrt{1376}}, \frac{11s}{\sqrt{1376}}$$

$$\textcircled{2} \quad \vec{w} \cdot \vec{u} = 0$$

$$(1, 1, m) \cdot (2, -1, 0) = 0$$

$$2 - 2 = 0$$

$$(1, 2, m) \cdot (2, -3, -1) = 0$$

$$1 - 6 - m = 0$$

$$-m = 5 \rightarrow \boxed{m = -5}$$