Arithmetic level raising and Bloch–Kato conjecture

Murilo Corato Zanarella

Massachusetts Institute of Technology
Program Associate (Algebraic Cycles, L-Values, and Euler Systems)

Let K be quadratic imaginary, ℓ a prime.

m will denote square-free products of certain *admissible* primes p.

$f = f_E \in S_2(N)$	π cuspidal cohomological rep of $U(2r,0)$
$X_m = \mathrm{Sh}_{N^+}(B_{N^-m})$, Shimura curve $X_{m\infty} = \mathrm{Sh}_{N^+}(B_{N^-m\infty})$, Shimura set	$\mathcal{M}_m = \mathrm{Sh}(U(V_m))$, V_m nonsplit at m, $(2r-1,1)$ $\mathcal{M}_{m\infty} = \mathrm{Sh}(U(V_{m\infty}))$, $V_{m\infty}$ nonsplit at m, $(2r,0)$
$C_m =$ (Heegner points) $\subseteq X_m$ $C_{m\infty} =$ (Heegner points) $\subseteq X_{m\infty}$	$Z_m = \mathrm{Sh}(U(r-1,1)) \times \mathrm{Sh}(U(r,0)) \subseteq \mathcal{M}_m$ $Z_{m\infty} = \mathrm{Sh}(U(r,0)) \times \mathrm{Sh}(U(r,0)) \subseteq \mathcal{M}_{m\infty}$
$X_m(\mathbb{F}_{p^2})^{ss} \cong \bigcup_{X_{mp\infty}}(\mathrm{pt})$ $(X_{mp})_{\mathbb{F}_{p^2}} \cong \bigcup_{X_{m\infty} \sqcup X_{m\infty}} \mathbb{P}^1_{\mathbb{F}_{q^2}}$	$(\mathcal{M}_m)^{ss}_{\mathbb{F}_{p^2}} \cong \bigcup_{\mathcal{M}_{mp\infty}}(\text{DL var})$ $(\mathcal{M}_{mp})^{ss}_{\mathbb{F}_{p^2}} \cong \bigcup_{\mathcal{M}_{m\infty}} \mathbb{P}^{2r-1}_{\mathbb{F}_{p^2}} \cup \bigcup_{\mathcal{M}^\bullet_{m\infty}}(\text{DL var})$
If $\mathfrak{m} = \ker(\mathbb{T} \xrightarrow{\phi_f} \mathbb{F}_\ell)$, then $\Phi(\mathrm{Jac}(X_{mp})_{/K_p})/\mathfrak{m} \cong H^1_{sing}(K_p, \overline{\rho_{f,\ell}})$ $\Cup \qquad \Cup$ $C_{m\infty} \leftrightarrow \partial_{sing}\mathrm{loc}_p AJ(C_{mp})$	If $\mathfrak{m} = \ker(\mathbb{T} \xrightarrow{\phi_\pi} \mathbb{Z}/\ell^n\mathbb{Z})$, then $\mathbb{Z}_\ell[\mathcal{M}_{m\infty}]/\mathfrak{m} \cong H^1_{sing}(K_p, H^{2r-1}_{et}(\mathcal{M}_{mp,\overline{\mathbb{Q}}}, \mathbb{Z}_\ell)/\mathfrak{m})$ $\Cup \qquad \Cup$ $\delta_{Z_{m\infty}} \overset{?}{\leftrightarrow} \partial_{sing}\mathrm{loc}_p AJ(Z_{mp})$
$L(E/K,1) \neq 0 \iff (C_\infty)_f \neq 0$ $(C_\infty)_f \neq 0 \overset{\text{Kol sys}}{\Longrightarrow} \#\mathrm{Sel}_{\ell^\infty}(E/K) < \infty$	$L(\mathrm{BC}(\pi), \frac{1}{2}) \neq 0 \iff (Z_\infty)_\pi \neq 0$ $(Z_\infty)_\pi \neq 0 \overset{\text{Kol sys}}{\Longrightarrow} H^1_f(K, V_{\pi,\ell}) = 0.$