UERJ, Cálculo dif e int 2 , 2013/1

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Lista 09 Gradiente e derivada direcional 17/06

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1. Calcule o gradiente das seguintes funções:

   (a) $z = 2\,x^2 + 5\,y^2$

   (b) $z = \dfrac{1}{x^2 + y^2}$

   (c) $w = 3\,x^2 + y^2 - 4\,z^2$

   (d) $w = cos(x\,y) + sen(y\,z)$

   (e) $w = ln(x^2 + y^2 + z^2)$

2. Determine a derivada direcional da função dada na direção $\vec{v}$:

   (a) $z = 2\,x^2 + 5\,y^2$, $\vec{\mathbf{v}} = (cos(\frac{\pi}{2}), sen(\frac{\pi}{2}))$.

   (b) $z = \dfrac{1}{x^2 + y^2}$, $\vec{\mathbf{v}} = (1, 1)$.

   (c) $z = x^2 y^3$, $\vec{\mathbf{v}} = \dfrac{1}{5}(3, -4)$.

   (d) $z = x^2 + x\,y + y^2 + 3\,x - 3\,y + 3$, $\vec{\mathbf{v}} = \dfrac{1}{\sqrt{5}}(1, 2)$.

   (e) $z = y^2\,tg^2(x)$, $\vec{\mathbf{v}} = \dfrac{1}{2}(-\sqrt{3}, 1)$.

   (f) $w = 3\,x^2 + y^2 - 4\,z^2$, $\vec{\mathbf{v}} = (cos(\dfrac{\pi}{3}), cos(\dfrac{\pi}{4}), cos(\dfrac{2\pi}{3}))$.

   (g) $w = cos(x\,y) + sen(y\,z)$, $\vec{\mathbf{v}} = (-\dfrac{1}{3}, \dfrac{2}{3}, \dfrac{2}{3})$.

   (h) $w = ln(x^2 + y^2 + z^2)$, $\vec{\mathbf{v}} = \dfrac{\sqrt{3}}{3}(1, -1, -1)$.

3. Considere f e g diferenciáveis, verifique as seguintes identidades:

   (a) $\nabla(f + g) = \nabla f + \nabla g$

   (b) $\nabla(f\,g) = f\,\nabla g + g\,\nabla f$

Gabarito

1. $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}$

   a) $4\,x, 10\,y, 0$

   b)
   $$-\frac{2\,x}{(x^2+y^2)^2}, -\frac{2\,y}{(x^2+y^2)^2}, 0$$

   c)
   $6\,x, 2\,y, -8\,z$

   d)
   $-\sin(x\,y)\;y + \cos(x\,y)\;y, -\sin(x\,y)\;x + \cos(x\,y)\;x, 0$

   e)
   $$\frac{2\,x}{x^2+y^2+z^2}, \frac{2\,y}{x^2+y^2+z^2}, \frac{2\,z}{x^2+y^2+z^2}$$

2. $\vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$ "" $\|\vec{u}\| = \sqrt{x^2 + y^2 + z^2}$ "" $\hat{u}$ "" $\frac{\partial f}{\partial \vec{u}}$

a)

$4\,x, 10\,y, 0, "", 1, "", 0, 1, 0, "", 10\,y$

b)

$-\frac{2\,x}{\left(x^2+y^2\right)^2}, -\frac{2\,y}{\left(x^2+y^2\right)^2}, 0, "", \sqrt{2}, "", \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}, 0, "",$
$-\frac{\sqrt{2}\,(x+y)}{\left(x^2+y^2\right)^2}$

c)

$2\,x\,y^3, 3\,x^2\,y^2, 0, "", 1, "", \frac{3}{5}, -\frac{4}{5}, 0, "", \frac{6}{5}\,x\,y^3 - \frac{12}{5}\,x^2\,y^2$

d)

$2\,x + y + 3, x + 2\,y - 3, 0, "", 1, "", \frac{1}{5}\sqrt{5}, \frac{2}{5}\sqrt{5}, 0, "", \frac{4}{5}\sqrt{5}\,x$
$+\sqrt{5}\,y - \frac{3}{5}\sqrt{5}$

e)

$2\,y^2\tan(x)\left(1+\tan(x)^2\right), 2\,y\tan(x)^2, 0, "", 1, "", -\frac{1}{2}\sqrt{3}, \frac{1}{2}, 0, "",$
$-\frac{\sin(x)\,y\left(y\sqrt{3} - \sin(x)\cos(x)\right)}{\cos(x)^3}$

Obs: $1+tg^2=sec^2$

f)

$6\,x, 2\,y, -8\,z, "", 1, "", \frac{1}{2}, \frac{1}{2}\sqrt{2}, -\frac{1}{2}, "", 3\,x + y\sqrt{2} + 4\,z$

g)

$-\sin(x\,y)\,y, -\sin(x\,y)\,x + \cos(y\,z)\,z, \cos(y\,z)\,y, "", 1, "", -\frac{1}{3}, \frac{2}{3}, \frac{2}{3},$
$"", \frac{1}{3}\sin(x\,y)\,y - \frac{2}{3}\sin(x\,y)\,x + \frac{2}{3}\cos(y\,z)\,z + \frac{2}{3}\cos(y\,z)\,y$

h)

$\frac{2\,x}{x^2+y^2+z^2}, \frac{2\,y}{x^2+y^2+z^2}, \frac{2\,z}{x^2+y^2+z^2}, "", 1, "", \frac{1}{3}\sqrt{3},$
$-\frac{1}{3}\sqrt{3}, -\frac{1}{3}\sqrt{3}, "", \frac{2}{3}\,\frac{\sqrt{3}\,(x-y-z)}{x^2+y^2+z^2}$