UERJ, Cálculo dif e int 2 , 2013/1

Prof. Thiago Daboit

Lista complementar 10/05

Método de Substituição Trigonométrica

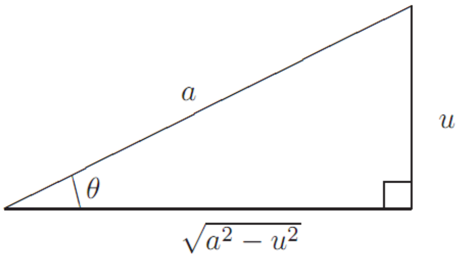
Este método é usado quando a expressão a integrar envolve alguns dos seguintes tipos de radicais:

$$\sqrt{a^2-u^2},\ \sqrt{a^2+u^2},\ \sqrt{u^2-a^2},$$

Onde a>0.

Caso 1: $\sqrt{a^2-u^2}$

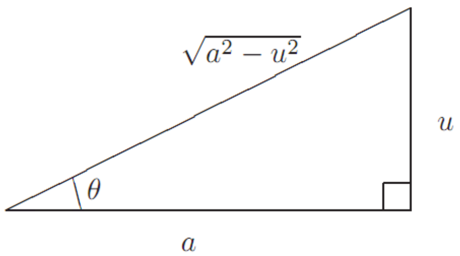
Para $-\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$, seja $u = a\,sen(\theta)$; então, $du = a\,cos(\theta)\,d\theta$. Logo $\sqrt{a^2-u^2} = a\,cos(\theta)$.



$$u = a\,\mathrm{sen}(\theta)$$
$$du = a\cos(\theta)\,d\theta$$
$$\sqrt{a^2-u^2} = a\ \cos(\theta)$$

Caso 2: $\sqrt{a^2+u^2}$

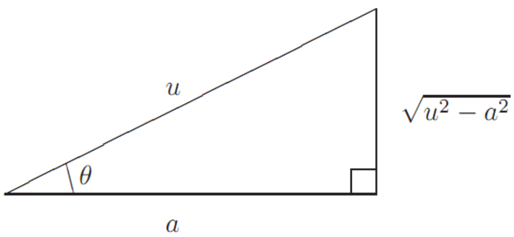
Para $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, seja $u = a\,tg(\theta)$; então, $du = a\,sec^2(\theta)\,d\theta$. Logo $\sqrt{a^2+u^2} = a\,sec(\theta)$.



$$u = a\,\mathrm{tg}(\theta)$$
$$du = a\sec^2(\theta)\,d\theta$$
$$\sqrt{a^2+u^2} = a\ \sec(\theta)$$

Caso 3: $\sqrt{u^2-a^2}$

Para $0 \leq \theta < \dfrac{\pi}{2}$ ou $\pi \leq \theta < \dfrac{3\pi}{2}$, seja $u = a\,sec(\theta)$; então, $du = a\,sec(\theta)\,tg(\theta)\,d\theta$. Logo $\sqrt{u^2-a^2} = a\ tg(\theta)$.



$$u = a\sec(\theta)$$
$$du = a\sec(\theta)\,\mathrm{tg}(\theta)\,d\theta$$
$$\sqrt{u^2-a^2} = a\ \mathrm{tg}(\theta)$$

1. Calcule as seguintes integrais usando o método de substituição:

(a) $\int \frac{x}{\sqrt[5]{x^2-1}}\,dx$

(b) $\int \frac{3x}{x^2+1}\,dx$

(c) $\int \sqrt{x+5}\,dx$

(d) $\int \frac{dy}{\sqrt{b-ay}}$

(e) $\int y(b-ay^2)\,dy$

(f) $\int \frac{4x^2}{\sqrt{x^3+8}}\,dx$

(g) $\int \frac{6x}{(5-3x^2)^2}\,dx$

(h) $\int \frac{dy}{(b+ay)^3}$

(i) $\int x^3\sqrt{a+bx^4}\,dx$

(j) $\int \frac{ln(x)+2}{x}\,dx$

(k) $\int sen(2x)\,cos^2(2x)\,dx$

(l) $\int tg(\frac{x}{2})\,sec^2(\frac{x}{2})\,dx$

(m) $\int \frac{cos(ax)dx}{\sqrt{b+sen(ax)}}$

(n) $\int \frac{1}{x(ln(x))^2}\,dx$

(o) $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

(p) $\int x^2e^{x^3}\,dx$

(q) $\int \frac{arcsen(y)}{2\sqrt{1-y^2}}\,dy$

(r) $\int \frac{e^x}{e^{2x}+16}\,dx$

(s) $\int \frac{sen(\theta)}{(5-cos(\theta))^3}\,d\theta$

(t) $\int \frac{x+3}{(x^2+6x)^2}\,dx$

(u) $\int \frac{dx}{x\,ln(x)}$

(v) $\int \frac{e^{arcsen(x)}}{\sqrt{1-x^2}}\,dx$

(w) $\int \frac{sen(ln(x))}{x}\,dx$

(x) $\int \frac{cos(\sqrt{x+1})}{\sqrt{1+x}}\,dx$

(y) $\int \frac{x^5}{\sqrt[3]{x^6+4}}\,dx$

(z) $\int 3^x\,cos(3^x)\,dx$

2. Calcule as seguintes integrais, usando as substituições indicadas:

(a) $\int \frac{dx}{x\sqrt{x^2-2}}$, use $x=\sqrt{2}\,sec(t)$

(b) $\int \frac{dx}{e^x+1}$, use $x=-ln(t)$

(c) $\int \frac{x\,dx}{\sqrt{x+1}}$, use $t=\sqrt{x+1}$

(d) $\int \frac{x\,dx}{\sqrt{1-x^2}}$, use $x=sen(t)$

(e) $\int \frac{dx}{1+\sqrt{x}}$, use $z=1+\sqrt{x}$

(f) $\int \frac{dx}{\sqrt{1+x^{\frac{1}{3}}}}$, use $z=1+\sqrt[3]{x}$

3. Calcule as seguintes integrais usando o método de integração por partes:

(a) $\int x\,e^x\,dx$

(b) $\int x^2\,sen(x)\,dx$

(c) $\int \frac{x\,e^x}{(1+x)^2}\,dx$

(d) $\int e^{-t}\,cos(\pi t)\,dt$

(e) $\int sen(ln(x))\,dx$

(f) $\int arccos(2\,x)\,dx$

(j) $\int (x-1)\,e^{-x}\,dx$

(k) $\int \frac{e^{\frac{1}{x}}}{x^3}\,dx$

(l) $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

(m) $\int x\,cosec^2(x)\,dx$

(n) $\int x\,sec(x)\,tg(x)\,dx$

(o) $\int x^3\,sen(5\,x)\,dx$

(s) $\int x^2\,senh(x)\,dx$

(t) $\int x\,argsenh(2\,x)\,dx$

(u) $\int x^4e^{-x}\,dx$

(v) $\int \frac{x\,arcsen(x)}{\sqrt{1-x^2}}\,dx$

(w) $\int x\,sec^2(x)\,dx$

(x) $\int ln^3(x)\,dx$

(g) $\int 3^x \cos(x)\, dx$

(h) $\int x\, arctg(x)\, dx$

(i) $\int sec^3(x)\, dx$

(p) $\int x^4 \cos(2x)\, dx$

(q) $\int x^4 e^x\, dx$

(r) $\int (x^5 - x^3 + x)\, e^{-x}\, dx$

(y) $\int \sqrt{x}\, ln(x)\, dx$

(z) $\int x\, \sqrt{x+1}\, dx$

4. Calcule as seguintes integrais usando primeiramente o método de substituição e depois, integração por partes:

(a) $\int \sqrt{1+x^2}\, dx$

(b) $\int x^{11} cos(x^4)\, dx$

(c) $\int cos(ln(x))\, dx$

(d) $\int e^{\sqrt{x}}\, dx$

(e) $\int sen(\sqrt{x})\, dx$

(f) $\int x^5\, e^{x^2}\, dx$

5. Calcule as seguintes integrais que envolvem potências de funções trigonométricas:

(a) $\int \frac{sen^2(x)}{cos^4(x)}\, dx$

(b) $\int tg^5(x) sec^3(x)\, dx$

(c) $\int sen^2(x) cos^2(x)\, dx$

(d) $\int \frac{sen^5(x)}{\sqrt{cos(x)}}\, dx$

(e) $\int \frac{sen(x)}{tg^2(x)}\, dx$

(f) $\int (cotg^2(2x) + cotg^4(2x))\, dx$

(g) $\int \frac{cos^4(x)}{sen^6(x)}\, dx$

(h) $\int sen^4(ax)\, dx$

(i) $\int sen^3(y)\, cos^4(y)\, dy$

(j) $\int \frac{sen^4(x)}{cos^6(x)}\, dx$

6. Calcule as seguintes integrais, usando substituição trigonométrica:

(a) $\int \frac{\sqrt{16-x^2}}{x^2}\, dx$

(b) $\int \frac{dx}{x^3\sqrt{x^2-9}}$

(c) $\int \frac{dx}{x^2\sqrt{5-x^2}}$

(d) $\int \frac{dx}{\sqrt{x^2-7}}$

(e) $\int \frac{dx}{x\sqrt{25-x^2}}$

(f) $\int \frac{x^2}{\sqrt{2\,x-x^2}}\, dx$

(g) $\int \frac{(16-9\,x^2)^{\frac{3}{2}}}{x^6}\, dx$

(h) $\int \frac{dx}{(4x-x^2)^{\frac{3}{2}}}$

(i) $\int \sqrt{x^2+2}\, dx$

(j) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

(k) $\int \frac{dx}{(1-x^2)\sqrt{1+x^2}}$

(l) $\int \frac{dx}{x^2\sqrt{x^2-4}}$

(m) $\int \frac{7x^3}{(4x^2+9)^{\frac{3}{2}}}\, dx$

(n) $\int (\sqrt{1+x^2} + 2x)\, dx$

(o) $\int \frac{e^x}{\sqrt{e^x+1}}\, dx$

(p) $\int \frac{x+1}{\sqrt{x^2-1}}\, dx$

(q) $\int \frac{dx}{x^2\,\sqrt{x^2+4}}$

7. Usando primeiramente o método de substituição simples, seguido do método de substituição trigonométrica, calcule as seguintes integrais.

$(a)\ \displaystyle\int \frac{sen(x)}{(25-cos^2(x))^{\frac{3}{2}}}\,dx \quad (b)\ \int \frac{dx}{x((ln(x))^2-4)^{\frac{3}{2}}} \quad (c)\ \int \frac{cos(x)}{\sqrt{4+sen^2(x)}}\,dx$

8. Completando os quadrados e usando substituição trigonométrica, calcule as seguintes integrais:

(a) $\displaystyle\int \frac{dx}{\sqrt{-3+8x-4x^2}}$

(b) $\displaystyle\int \frac{x}{\sqrt{1-x+3x^2}}\,dx$

(c) $\displaystyle\int \frac{2x}{(x^2+3x+4)^2}\,dx$

(d) $\displaystyle\int \frac{dx}{\sqrt{x^2+3x+5}}$

(e) $\displaystyle\int \frac{dx}{\sqrt{x^2-x-1}}$

(f) $\displaystyle\int \frac{5x+3}{\sqrt{4x^2+3x+1}}\,dx$

(g) $\displaystyle\int \frac{dx}{\sqrt{4x-x^2-3}}$

(h) $\displaystyle\int \frac{1-2x}{\sqrt{2x-x^2+3}}\,dx$

(i) $\displaystyle\int \frac{x}{\sqrt{x^2-3x+4}}\,dx$

(j) $\displaystyle\int \frac{x+2}{\sqrt{x^2+6x+34}}\,dx$

9. Calcule as seguintes integrais, usando frações parciais:

(a) $\displaystyle\int \frac{dx}{x^3+8}$

(b) $\displaystyle\int \frac{4dx}{x^4-1}$

(c) $\displaystyle\int \frac{x^5+4x^3}{(x^2+2)^3}\,dx$

(d) $\displaystyle\int \frac{x^3+3x}{(x^2+1)^2}\,dx$

(e) $\displaystyle\int \frac{dx}{x^4+x^2}$

(f) $\displaystyle\int \frac{x^3+x-1}{(x^2+1)^2}\,dx$

(g) $\displaystyle\int \frac{x^4+8x^3-x^2+2x+1}{(x^2+x)(x^3+1)}\,dx$

(h) $\displaystyle\int \frac{dx}{x^3(x^2+1)}$

(i) $\displaystyle\int \frac{x+1}{(x^2+4x+5)^2}\,dx$

(j) $\displaystyle\int \frac{x^3+x+1}{x(1+x^2)}\,dx$

(k) $\displaystyle\int \frac{x^3+1}{(x^2-4x+5)^2}\,dx$

(l) $\displaystyle\int \frac{dx}{(x+1)(x^2+x+1)^2}$

(m) $\displaystyle\int \frac{dx}{x^8+x^6}$

(n) $\displaystyle\int \frac{3x+1}{x^2-x+1}\,dx$

(o) $\displaystyle\int \frac{dx}{x^4-3x^3+3x^2-x}$

(p) $\displaystyle\int \frac{x}{x^4-1}\,dx$

(q) $\displaystyle\int \frac{5x^3-3x^2+2x-1}{x^4+9x^2}\,dx$

(r) $\displaystyle\int \frac{x^5+4x^3+3x^2-x+2}{x^5+4x^3+4x}\,d$

(s) $\displaystyle\int \frac{2x+2}{x(x^2+2x+2)^2}\,dx$

(t) $\displaystyle\int \frac{dx}{x^3+3x^2+7x+5}$

(u) $\displaystyle\int \frac{x^2-3x+2}{x^3+6x^2+5x}\,dx$

(v) $\displaystyle\int \frac{3x^3+x^2+x-1}{x^4-1}\,dx$

10. Calcule:

(a) $\displaystyle\int cos(x)\, ln(sen(x))\, dx$

(b) $\displaystyle\int x\, 5^x\, dx$

(c) $\displaystyle\int x^5\, cos(x^3)\, dx$

(d) $\displaystyle\int tg(x)\, sec^3(x)\, dx$

(e) $\displaystyle\int cos(3\,x)\, cos(4\,x)\, dx$

(f) $\displaystyle\int \frac{x}{\sqrt{(x^2+4)^5}}\, dx$

(g) $\displaystyle\int \frac{dx}{\sqrt{x^2+4\,x+8}}$

(h) $\displaystyle\int e^t\, \sqrt{9-e^{2t}}\, dt$

(i) $\displaystyle\int \frac{x^2+2\,x}{x^3+3\,x^2+4}\, dx$

(j) $\displaystyle\int \frac{x-3}{(x^2+2\,x+4)^2}\, dx$

(k) $\displaystyle\int \frac{x^4+1}{x\,(x^2+1)}\, dx$

(l) $\displaystyle\int \frac{sen(x)\, cos^2(x)}{5+cos^2(x)}\, dx$

(m) $\displaystyle\int \frac{x^2}{(x+1)^3}\, dx$

(n) $\displaystyle\int \frac{dx}{4\,x^2+12\,x-7}$

(o) $\displaystyle\int \frac{2\,x+3}{x^3+3\,x}\, dx$

(p) $\displaystyle\int \frac{3\,x^2-4\,x+5}{(x-1)\,(x^2+1)}\, dx$

(q) $\displaystyle\int \frac{x^3}{\sqrt[3]{x^2+1}}\, dx$

(r) $\displaystyle\int \frac{\sqrt{x}}{x+1}\, dx$

(s) $\displaystyle\int \frac{dx}{(x^2+9)\,\sqrt{x^2+4}}$

(t) $\displaystyle\int \frac{dx}{(x-1)\,\sqrt{x^2+2\,x-2}}$

(u) $\displaystyle\int \frac{dx}{1+2\,sen(x)\,cos(x)+sen^2(x)}$

(v) $\displaystyle\int \frac{2\,cos^2(\frac{x}{2})}{x+sen(x)}\, dx$

(w) $\displaystyle\int \frac{1-tg^2(x)}{sec^2(x)+tg(x)}\, dx$

(x) $\displaystyle\int \frac{dx}{(x+3)\,\sqrt{x-1}}\, dx$