

Unidade II - Imagem Digital

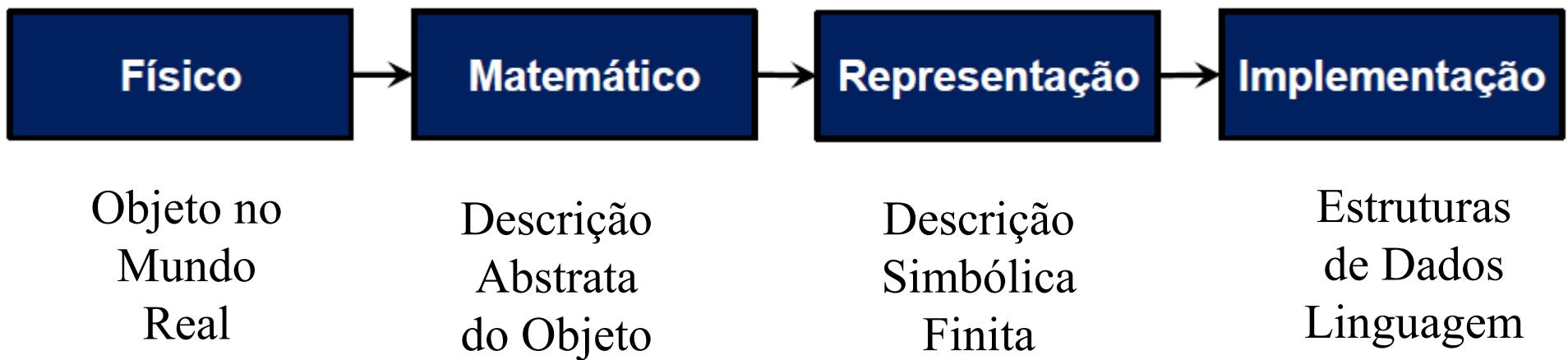


IME 04-10842
Computação Gráfica
Professor Guilherme Mota
Professor Gilson Costa

Definições Formais

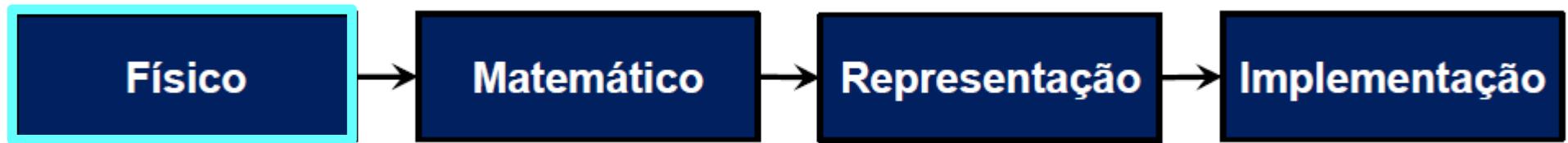
Paradigma dos 4 Universos

Hierarquia de Abstrações

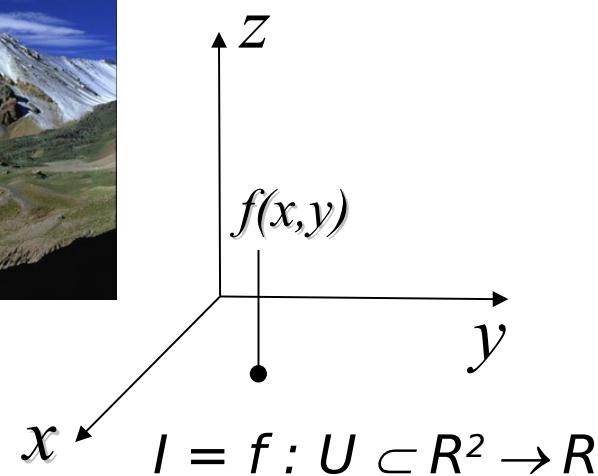
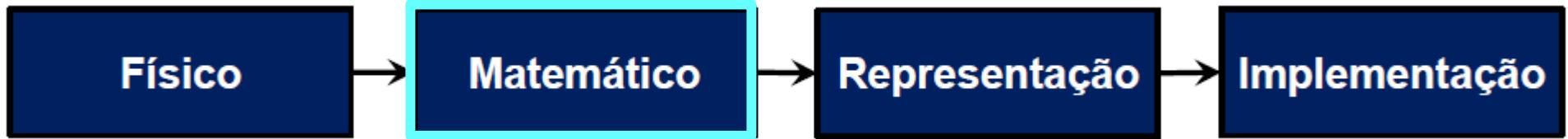


Exemplo Modelagem de Terreno

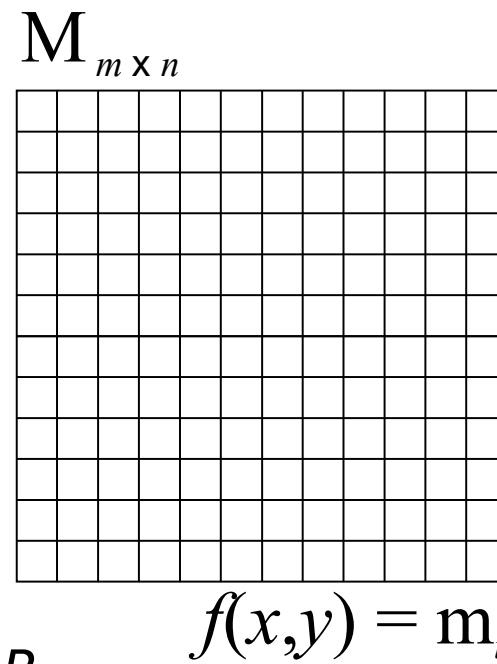
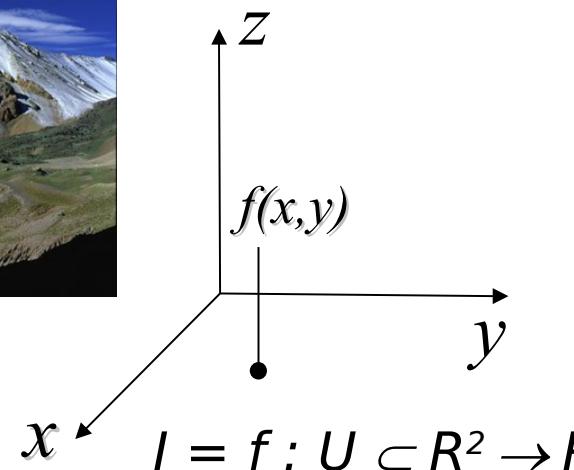
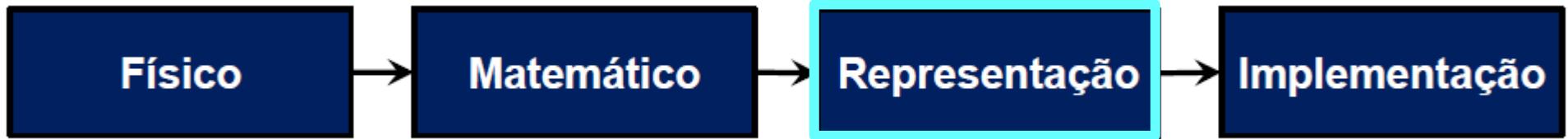
Paradigma dos 4 Universos



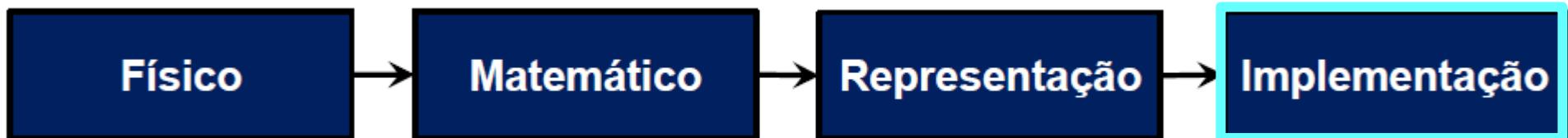
Paradigma dos 4 Universos



Paradigma dos 4 Universos



Paradigma dos 4 Universos



A diagram illustrating a function $f: U \subset \mathbb{R}^2 \rightarrow \mathbb{R}$. The vertical axis is labeled z , and the horizontal axis is labeled y . A point on the z -axis is labeled $f(x, y)$. A curved arrow at the bottom points from the text $f: U \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ to the z -axis.

```
float terreno [M] [N];
```

Níveis de Abstração na Representação de Imagens

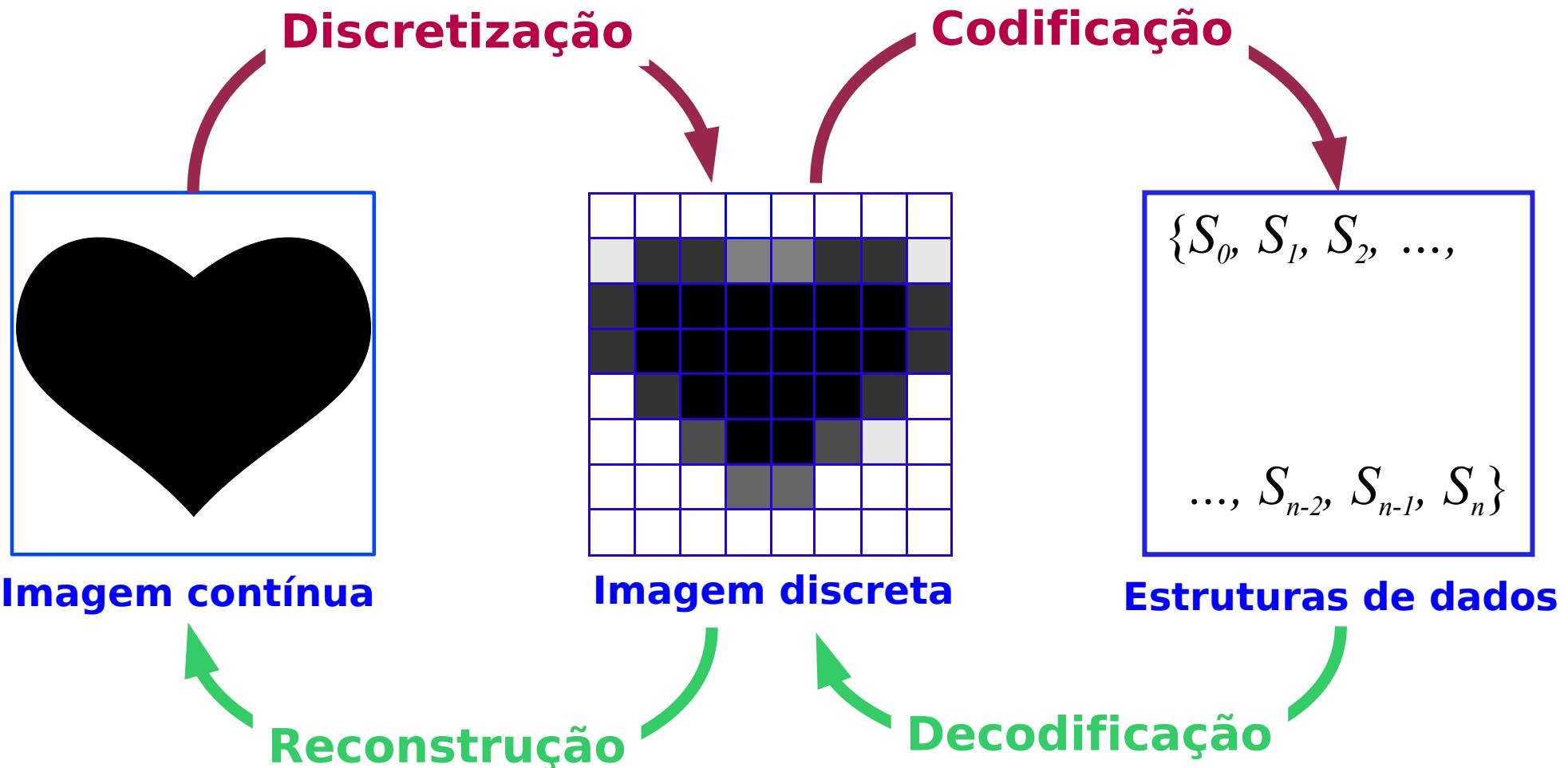
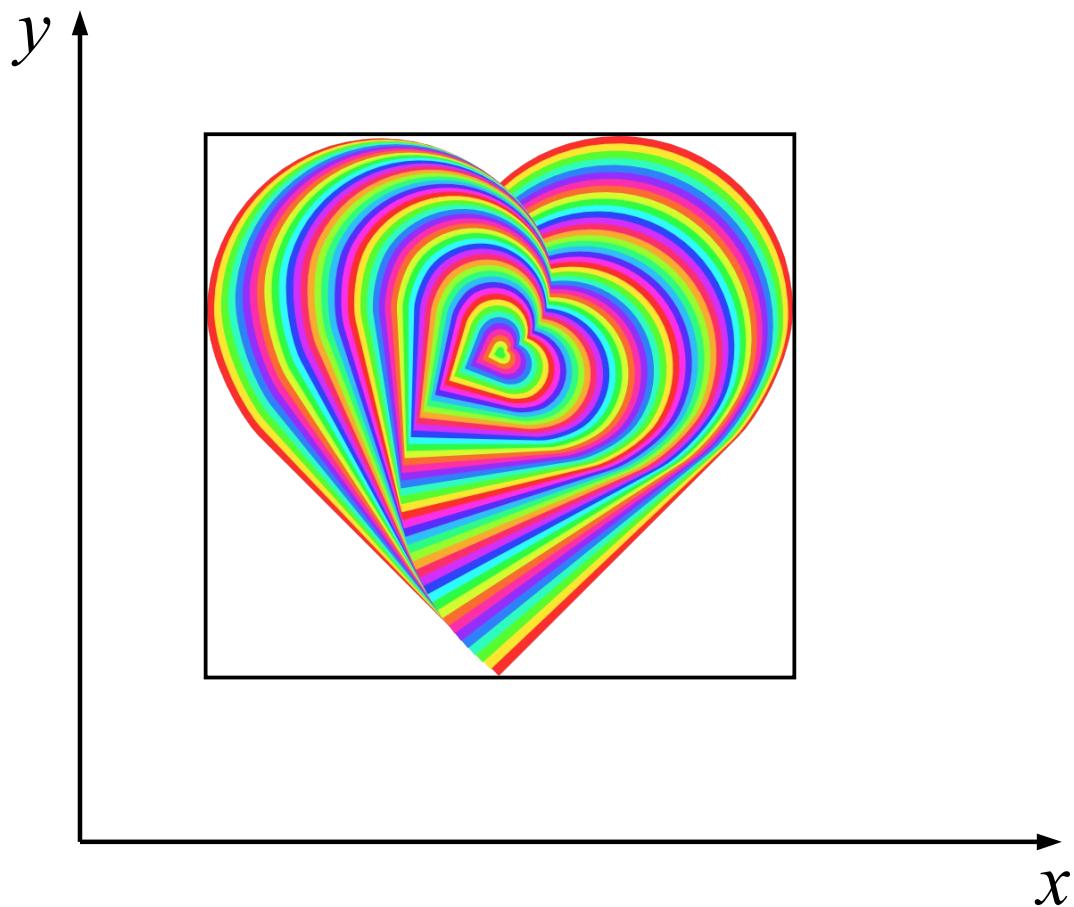


Imagen Contínua

$$f : U \subset \mathbb{R}^2 \rightarrow C$$

$f \rightarrow$ função imagem



$C \rightarrow$ espaço de cor, $C = \mathbb{R}^n$

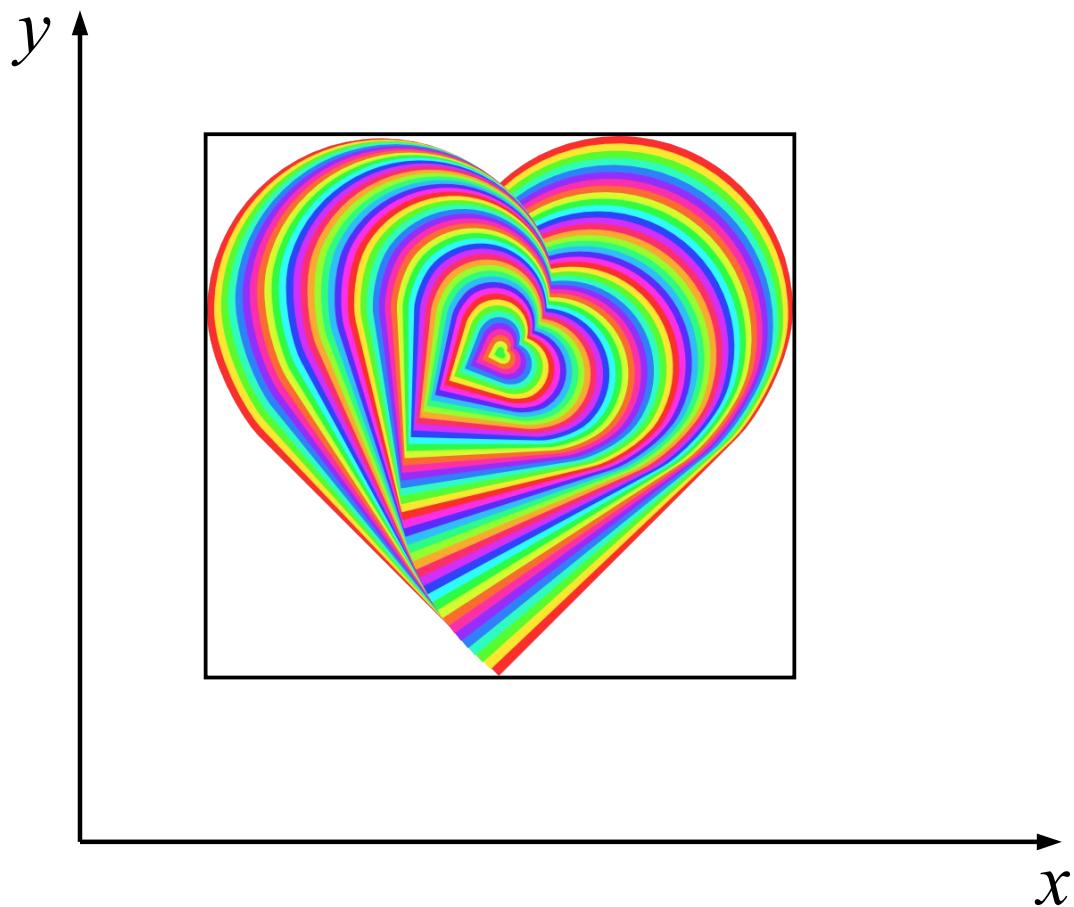
$U \rightarrow$ suporte da imagem

$f(U) \subset C \rightarrow$ gamute

Imagen Contínua

$$f : U \subset \mathbb{R}^2 \rightarrow C$$

$f \rightarrow$ função imagem



$C \rightarrow$ espaço de cor, $C = \mathbb{R}^3$
(imagem tricromática, RGB)

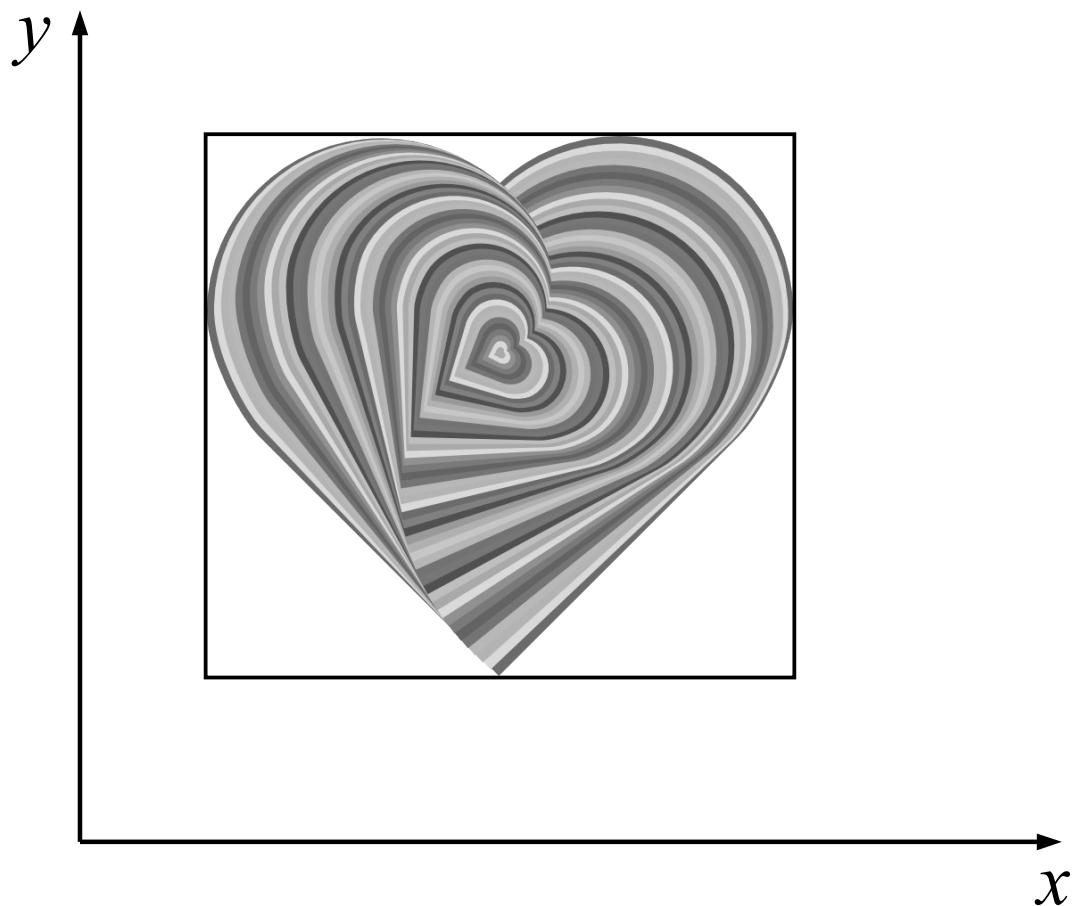
$U \rightarrow$ suporte da imagem

$f(U) \subset C \rightarrow$ gamute

Imagen Contínua

$$f : U \subset \mathbb{R}^2 \rightarrow C$$

$f \rightarrow$ função imagem



$C \rightarrow$ espaço de cor, $C = \mathbb{R}$
(imagem monocromática)

$U \rightarrow$ suporte da imagem

$f(U) \subset C \rightarrow$ gamute

Imagen Continua

Quando $f: U \rightarrow \mathbb{R}^3$ podemos escrever

$$f(x, y) = (f_1(x, y), f_2(x, y), f_3(x, y)), \text{ onde } f: U \rightarrow \mathbb{R}$$

Uma imagem colorida é formada por três imagens monocromáticas: os **componentes de cor** de f .



$f(x, y)$



$f_1(x, y)$



$f_2(x, y)$



$f_3(x, y)$

Qual componente $f_i(x, y)$ está associado a qual cor primária (R, G B)?

Discretização

Imagen continua

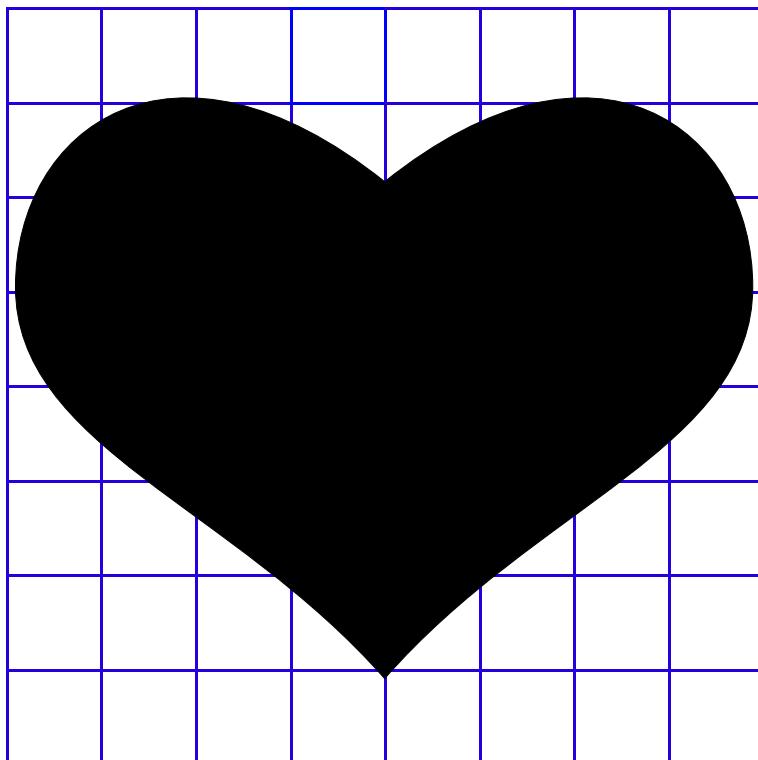
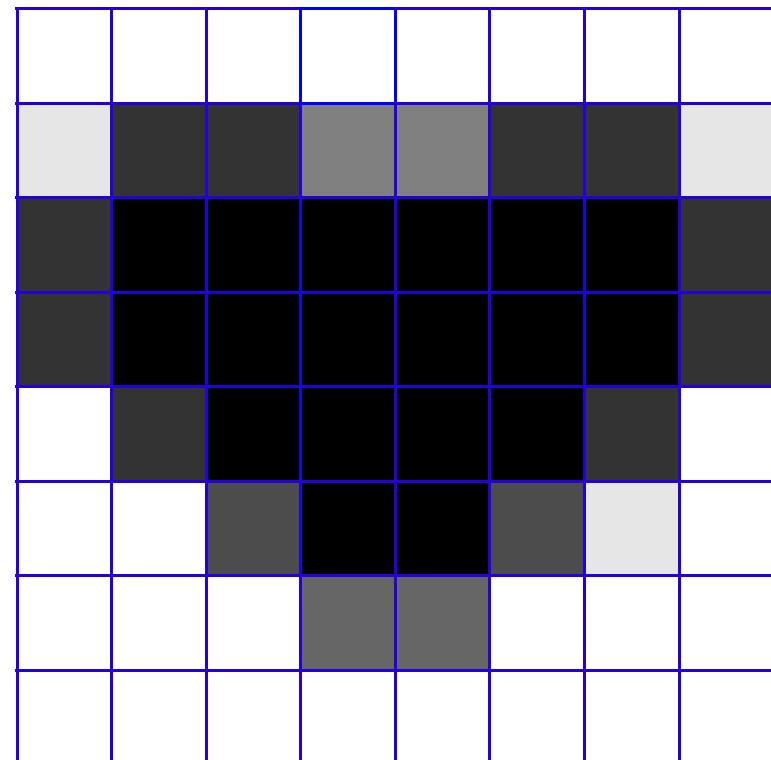
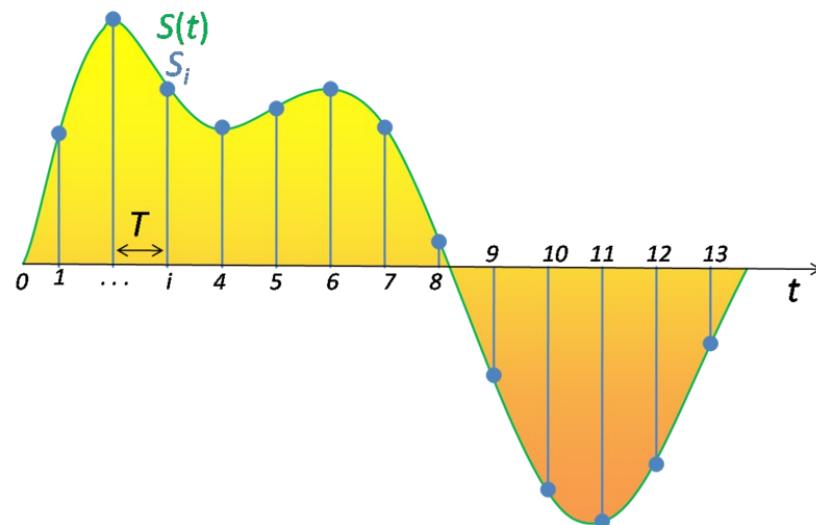


Imagen discreta



Processo de Amostragem

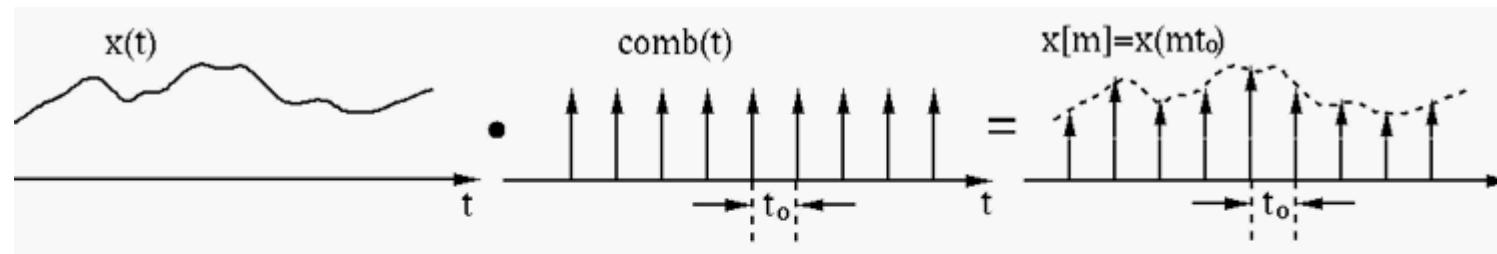
Sinal de entrada $S(t)$



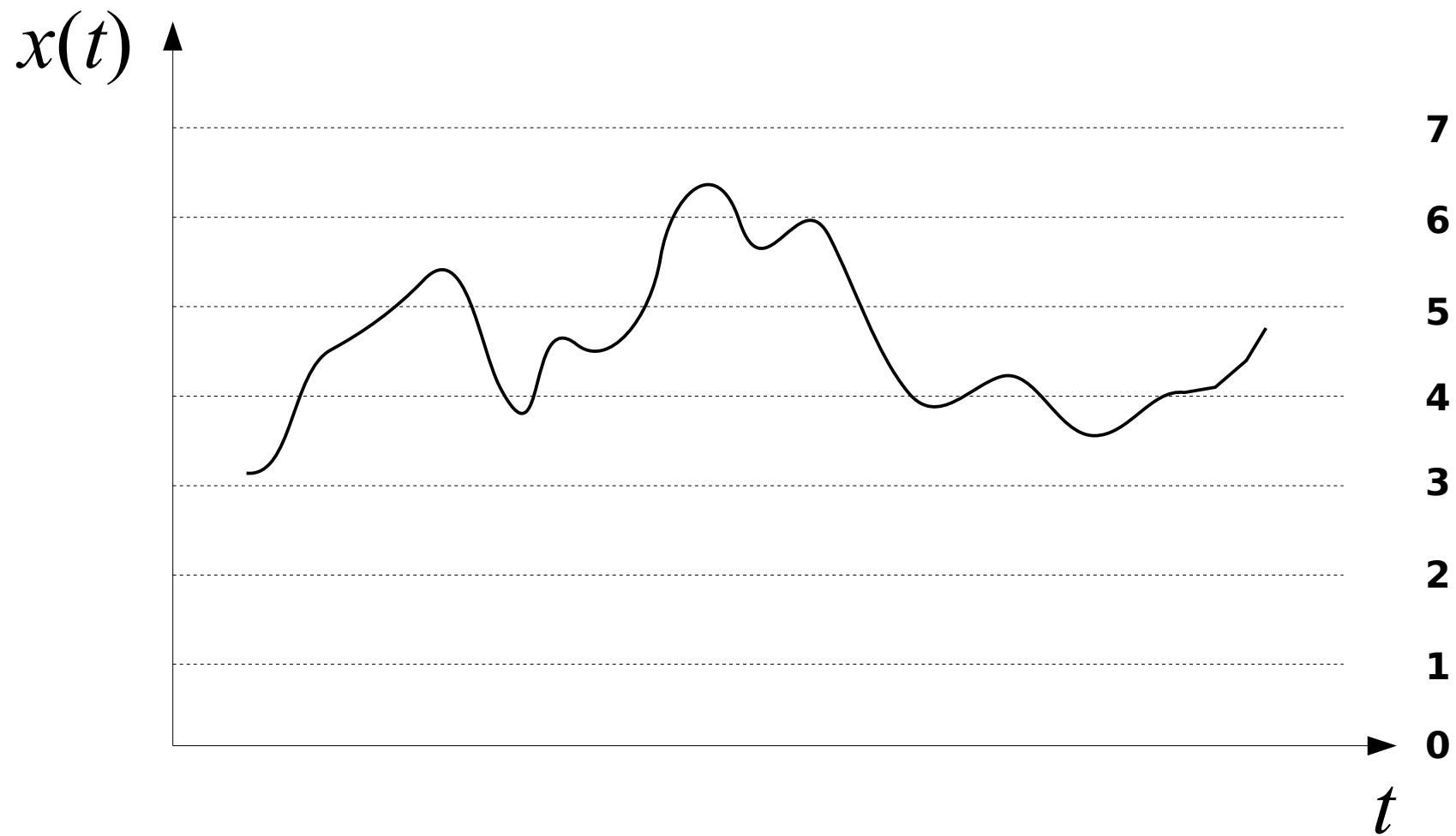
Sinal de saída $S[t]$

Sinal contínuo

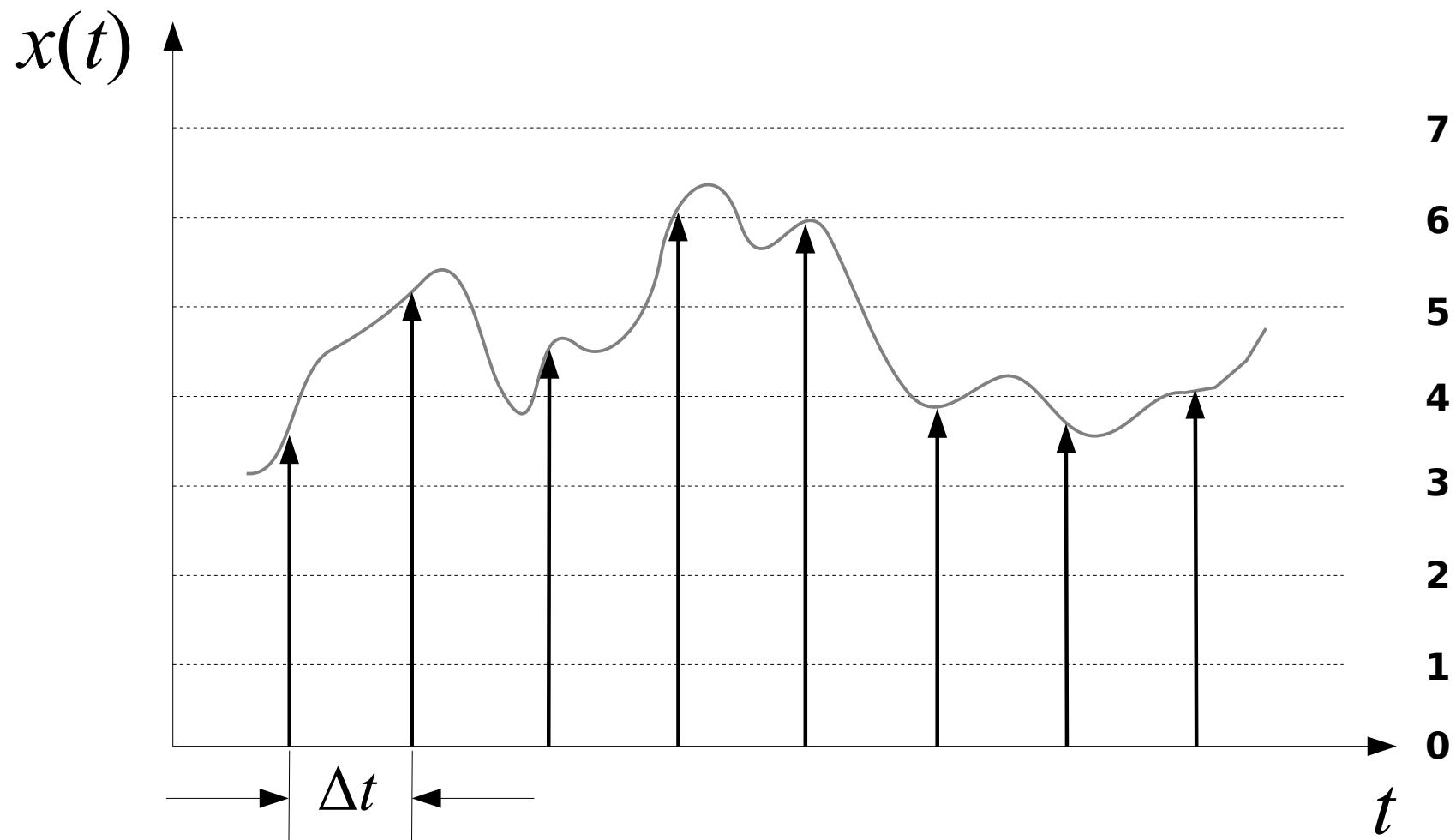
Sinal discreto



Sinal Contínuo

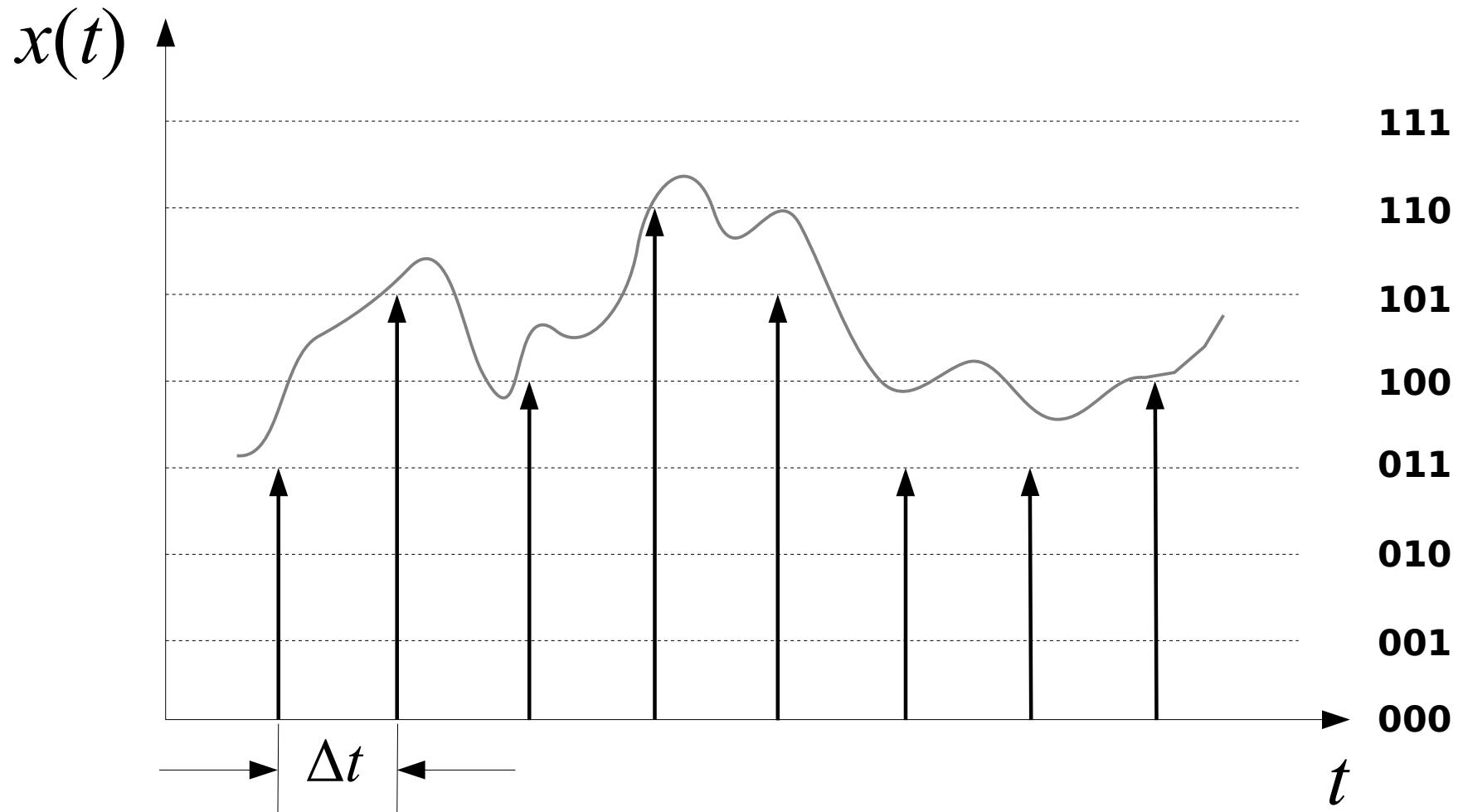


Sinal Discreto



$$x(t) = \{ \dots, 3.4, 5.1, 4.3, 6.0, 5.9, 3.8, 3.6, 4.1, \dots \}$$

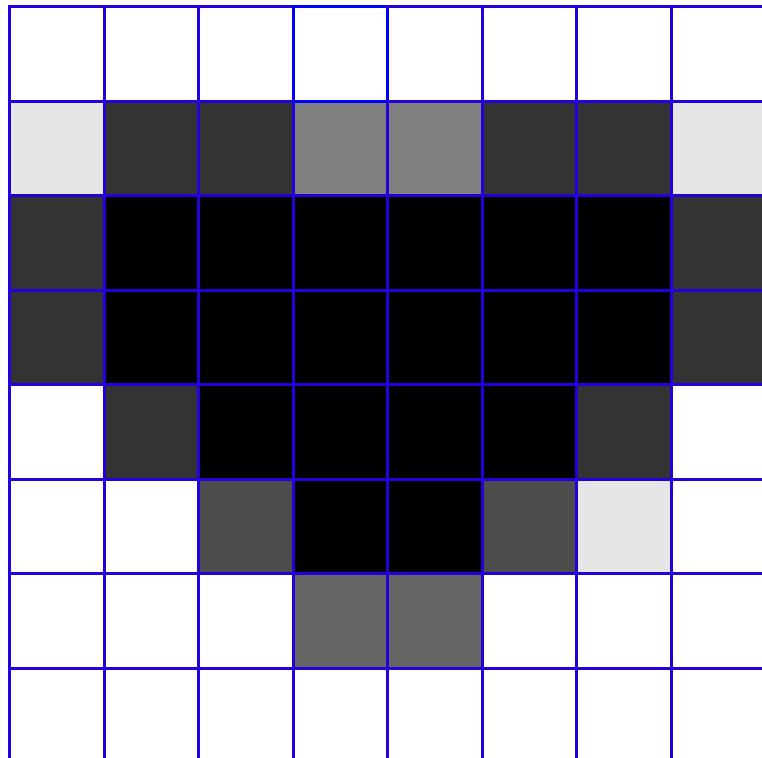
Sinal Digital



$$x(t) = \{\dots, 011, 101, 100, 110, 101, 011, 011, 100, \dots\}$$

Imagen Digital

$$f : U \subset \mathbb{R}^2 \rightarrow C$$



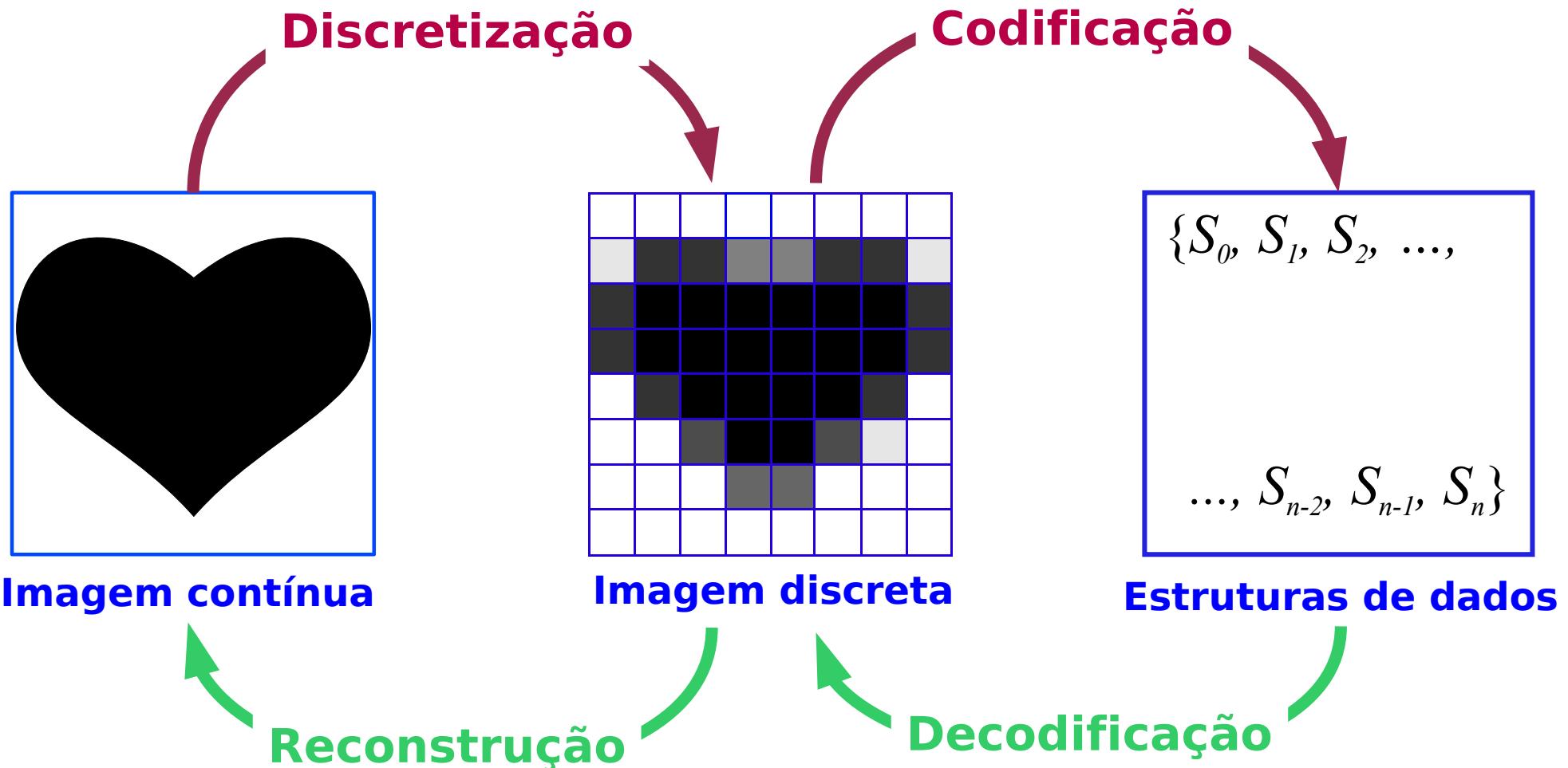
$f \rightarrow$ função imagem

$C \rightarrow$ espaço de cor

$U \rightarrow$ suporte da imagem

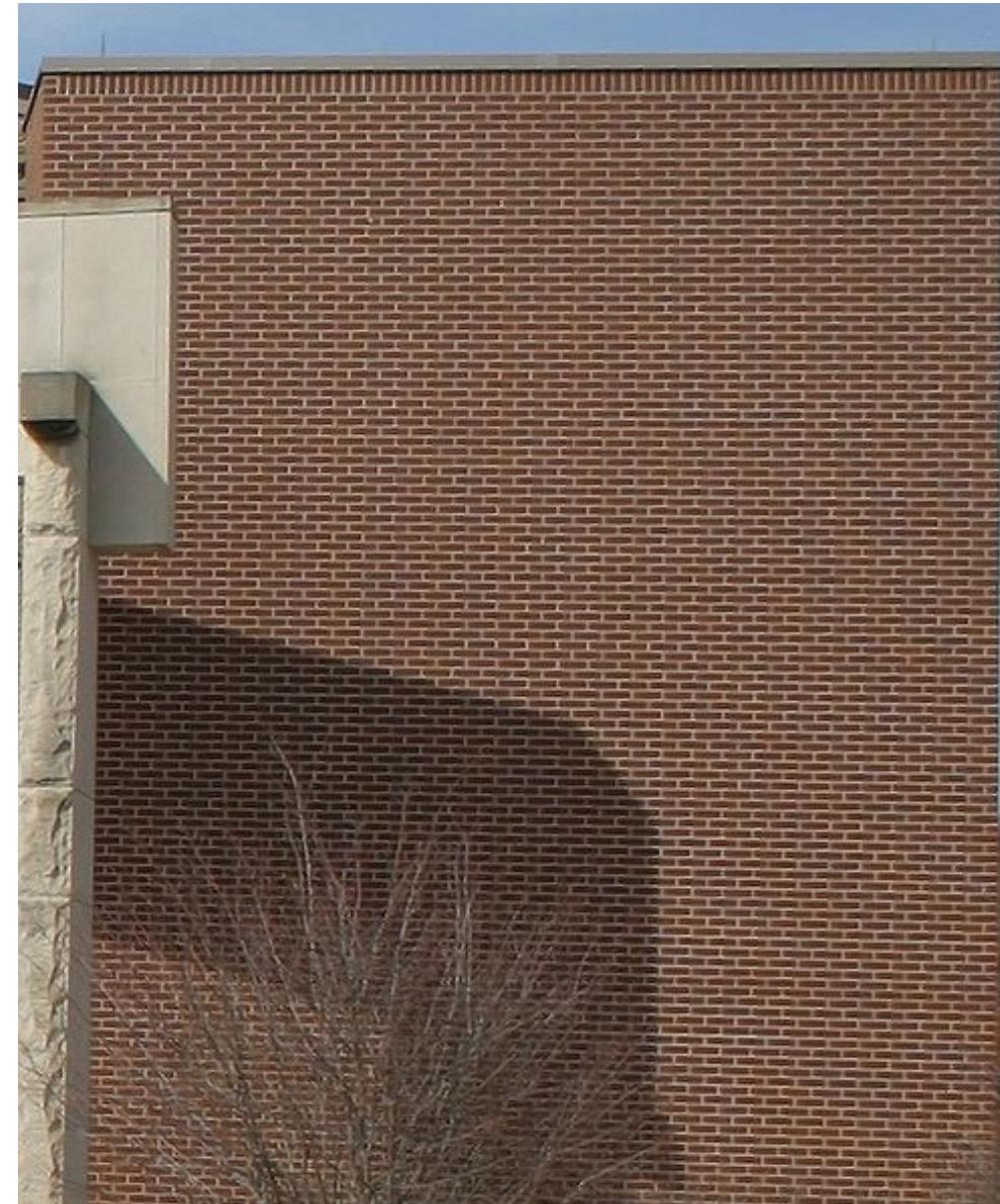
U e C são discretizados

Níveis de Abstração na Representação de Imagens



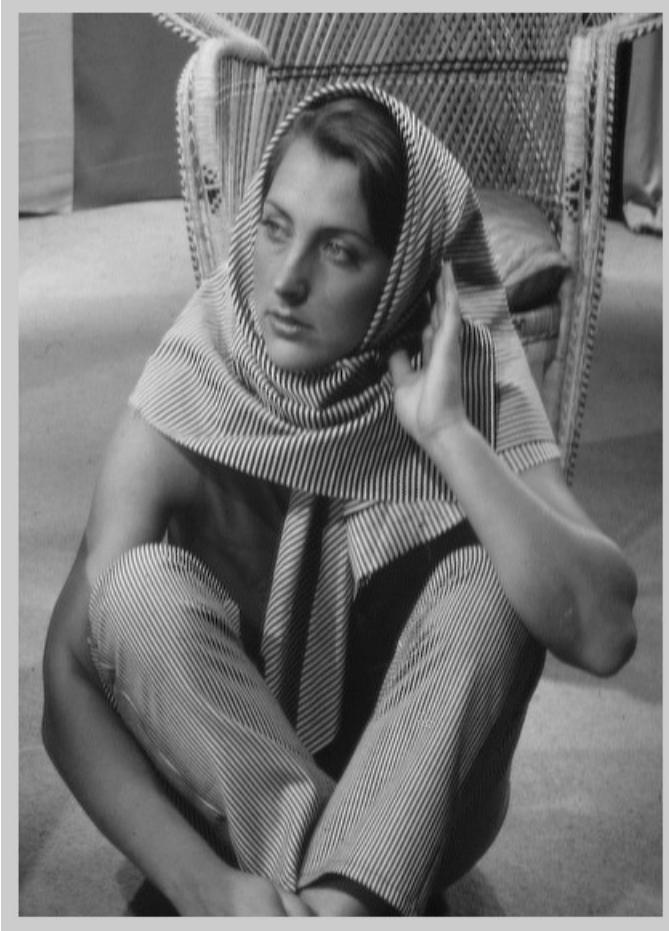
Limitações da Discretização

Limite de Representação



Limite de Representação

Original

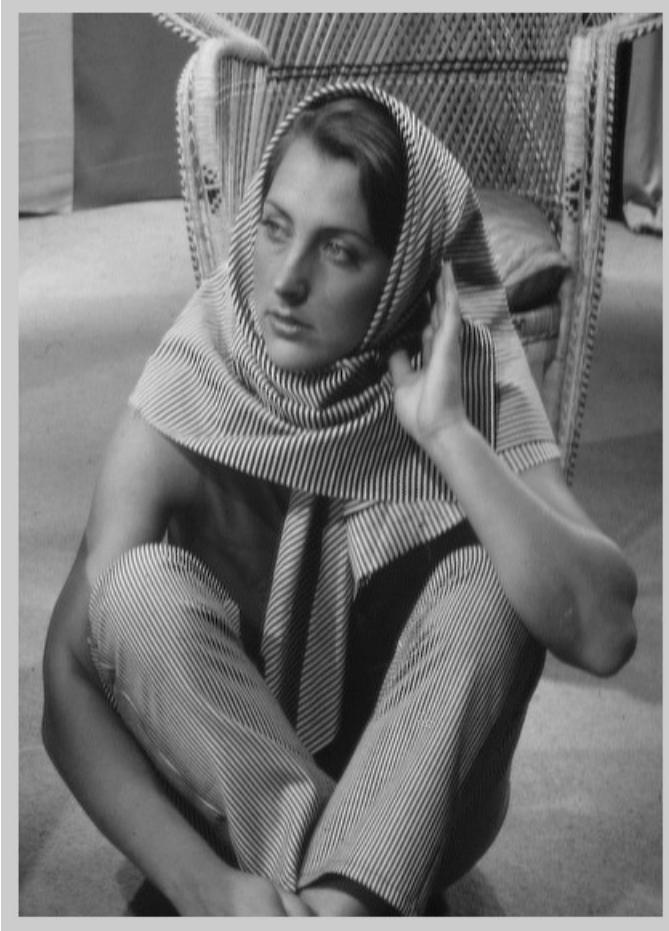


Reduzida em 50%



Limite de Representação

Original

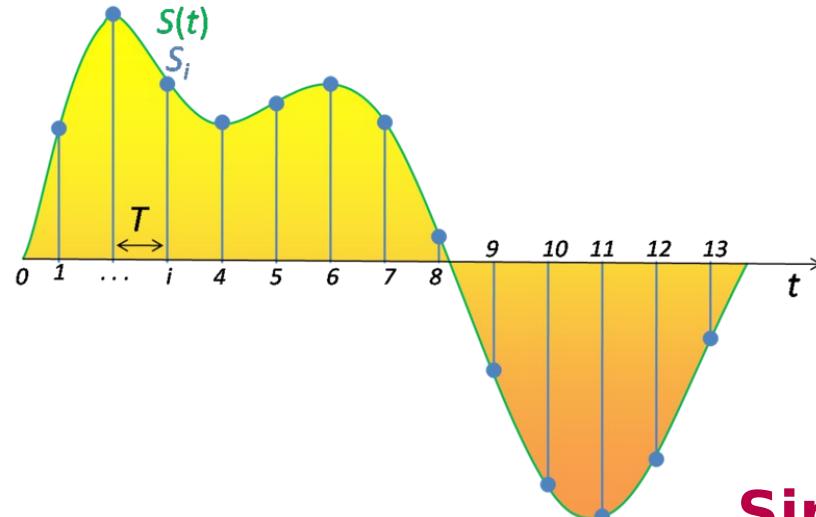


Reduzida em 50%



Processo de Amostragem

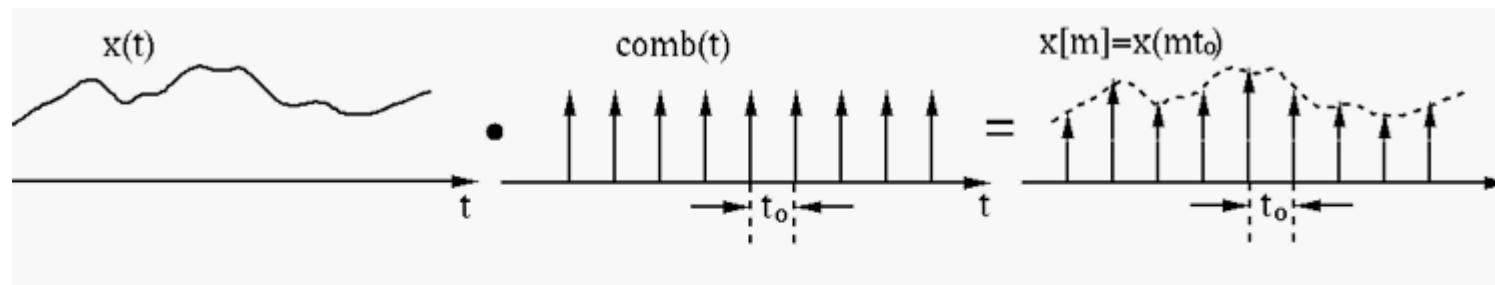
Sinal de entrada $S(t)$



Sinal de saída $S[t]$

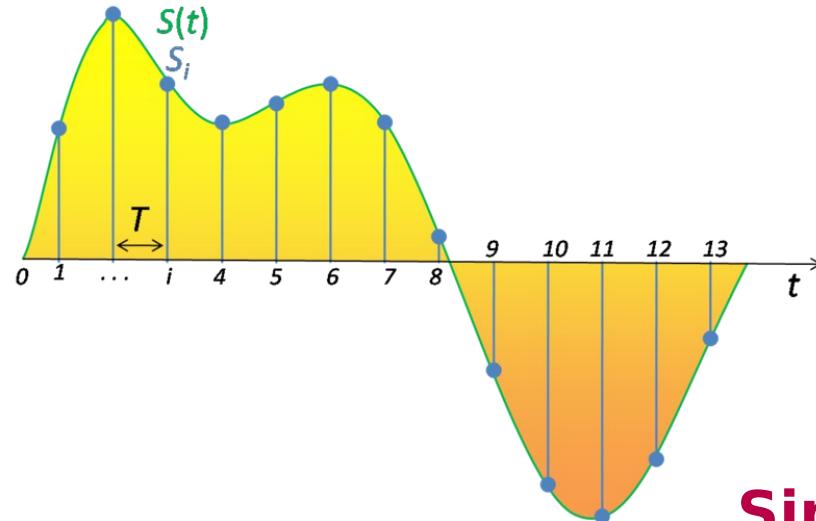
Sinal contínuo

Sinal discreto



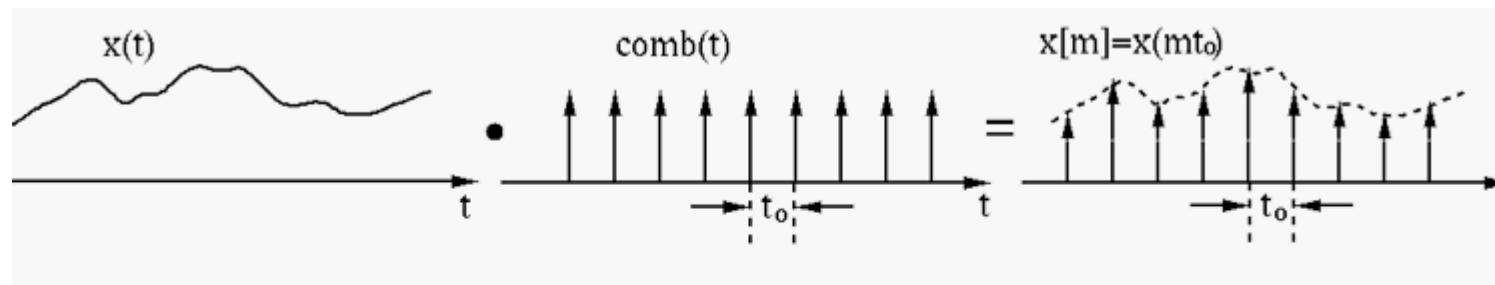
Processo de Amostragem

Sinal de entrada $S(t)$



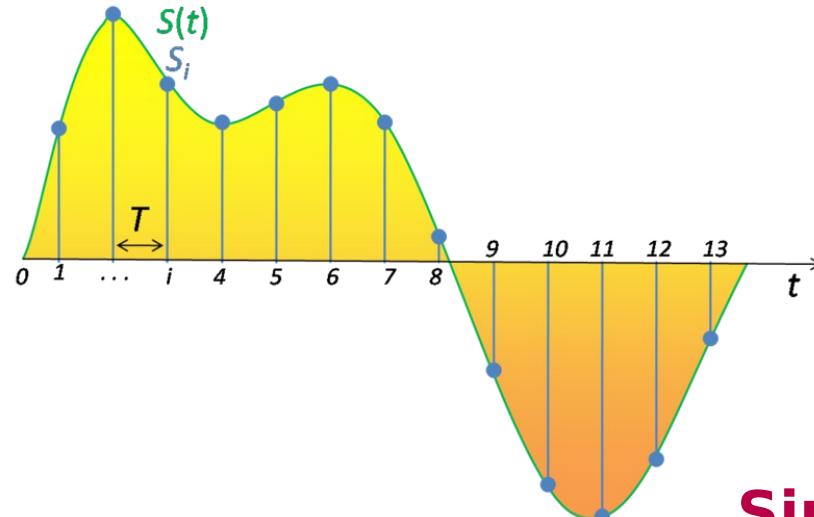
Sinal contínuo

Sinal de saída $S[t]$



Processo de Amostragem

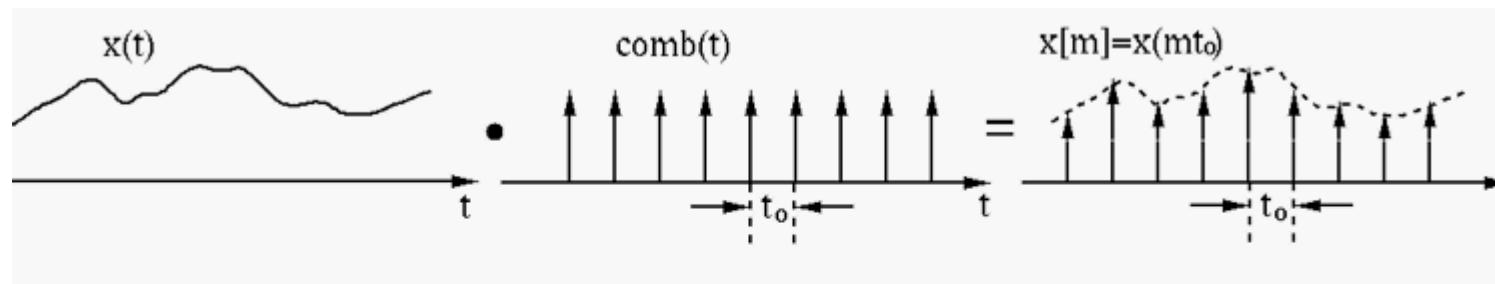
Sinal de entrada $S(t)$



Sinal de saída $S[t]$

Sinal contínuo

Sinal discreto

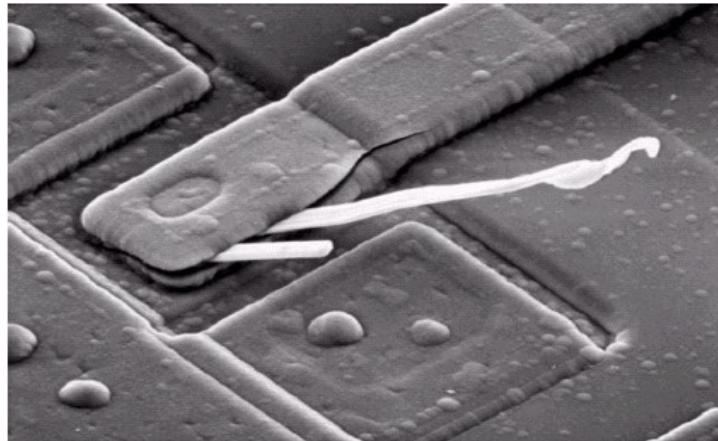


Transformada de Fourier: Domínio da Frequência

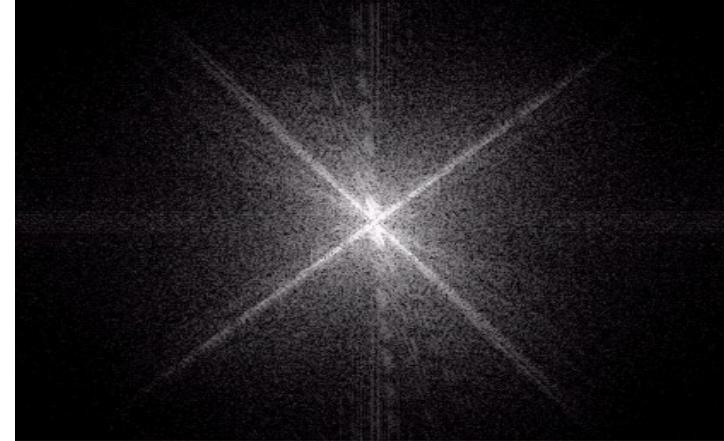
$$\mathfrak{J}\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt, \text{ onde } j = \sqrt{-1}$$

Processo de Amostragem

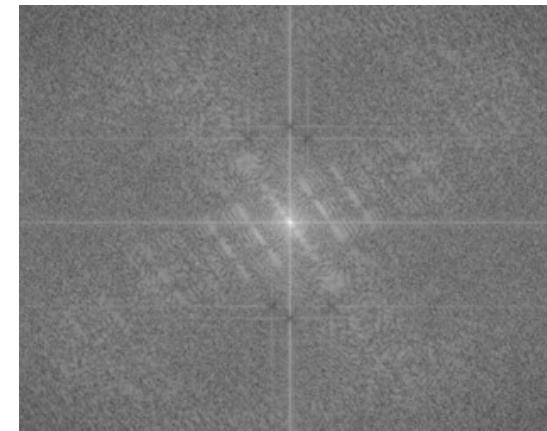
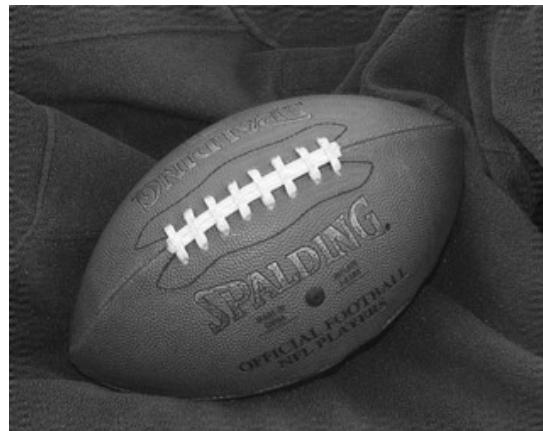
$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$
$$F(u, v) = R(u, v) + jI(u, v)$$



$f(x, y)$

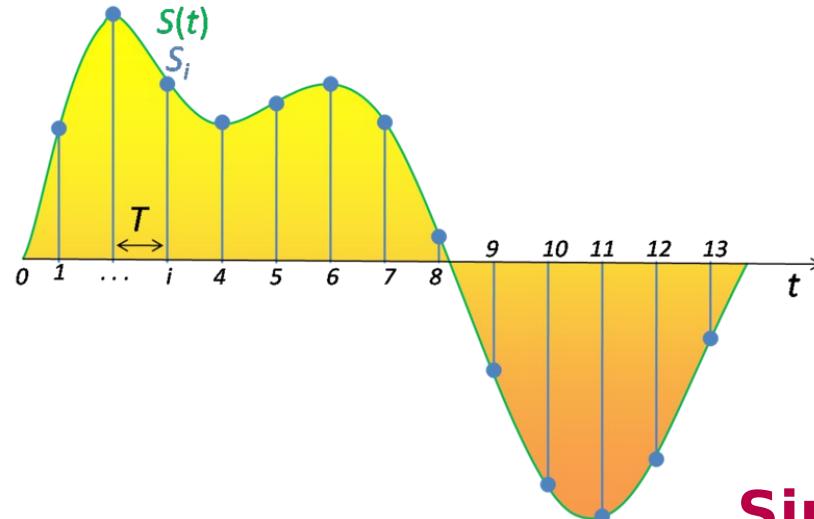


$: R(u, v)$



Processo de Amostragem

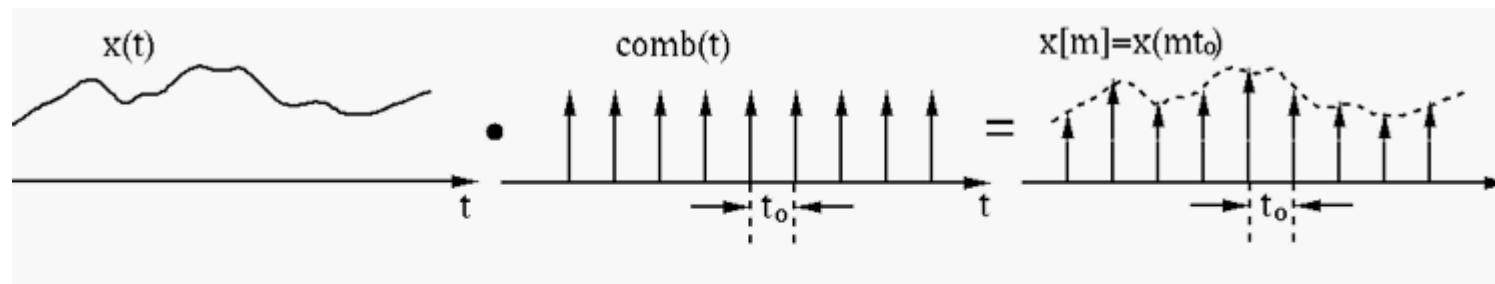
Sinal de entrada $S(t)$



Sinal de saída $S[t]$

Sinal contínuo

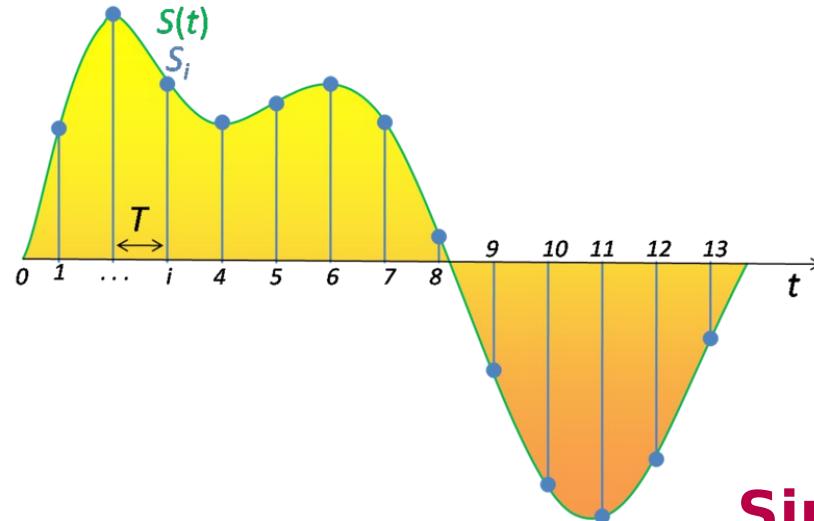
Sinal discreto



$$\mathfrak{I}\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt, \text{ onde } j = \sqrt{-1}$$

Processo de Amostragem

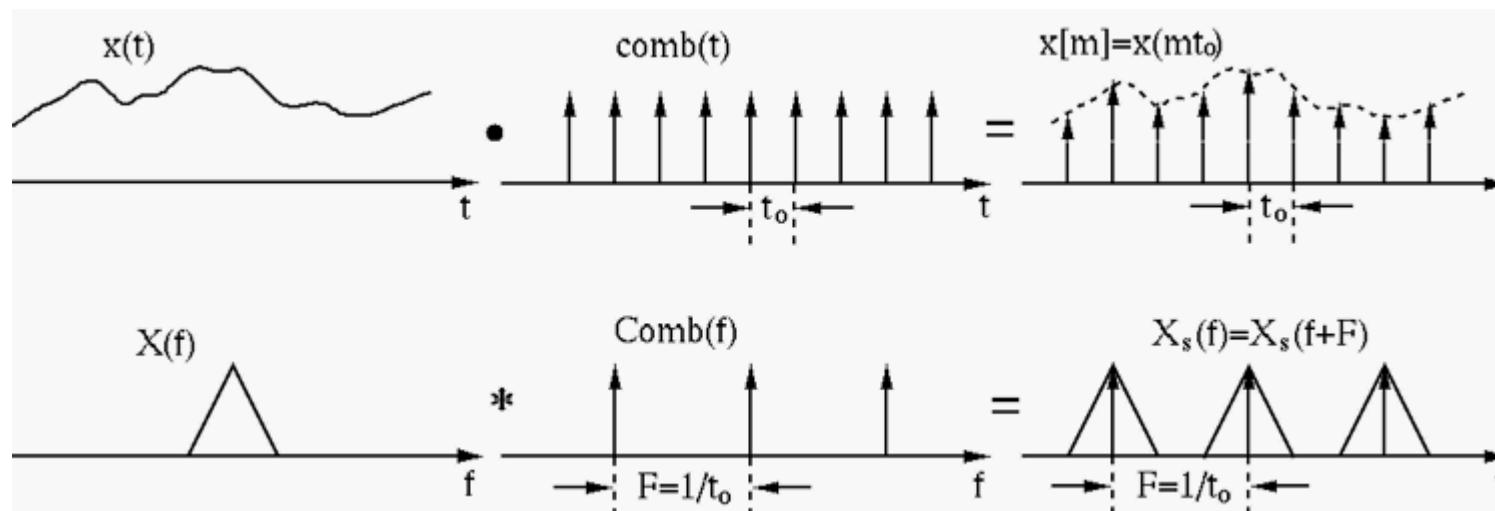
Sinal de entrada $S(t)$



Sinal de saída $S[t]$

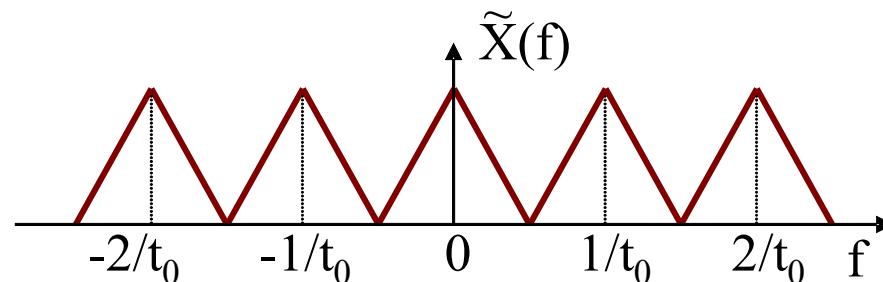
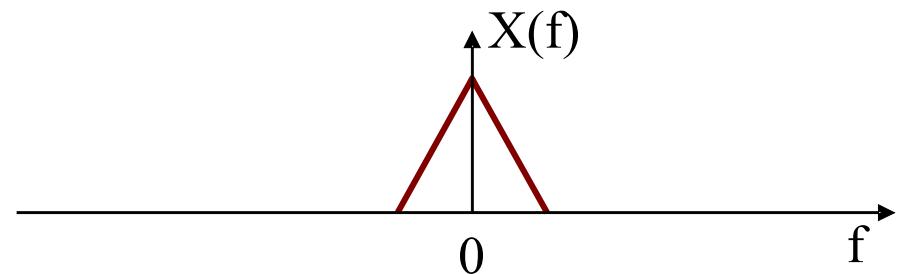
Sinal contínuo

Sinal discreto



Amostragem - Teorema de Nyquist $f_a > 2f_{\max}$

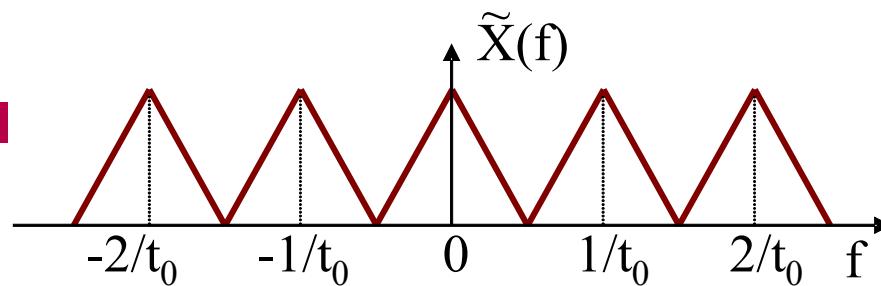
Amostragem no domínio da frequência: acrescentar cópias deslocadas (pela frequência de amostragem) da transformada de Fourier do sinal de entrada.



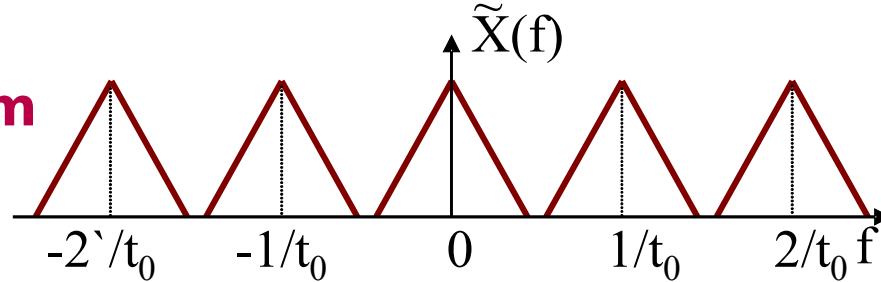
Amostragem - Teorema de Nyquist $f_a > 2f_{\max}$

Amostragem no domínio da frequência: acrescentar cópias deslocadas (pela frequência de amostragem) da transformada de Fourier do sinal de entrada.

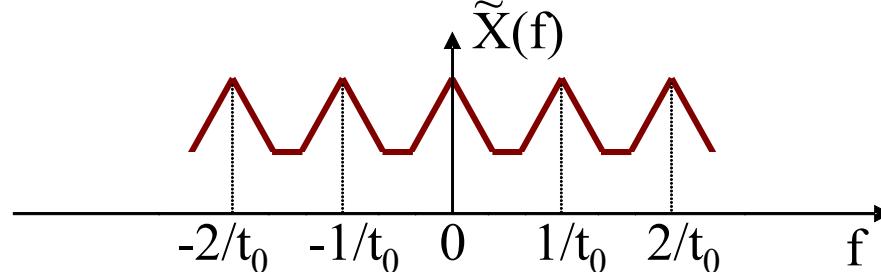
Amostragem ideal



Super amostragem

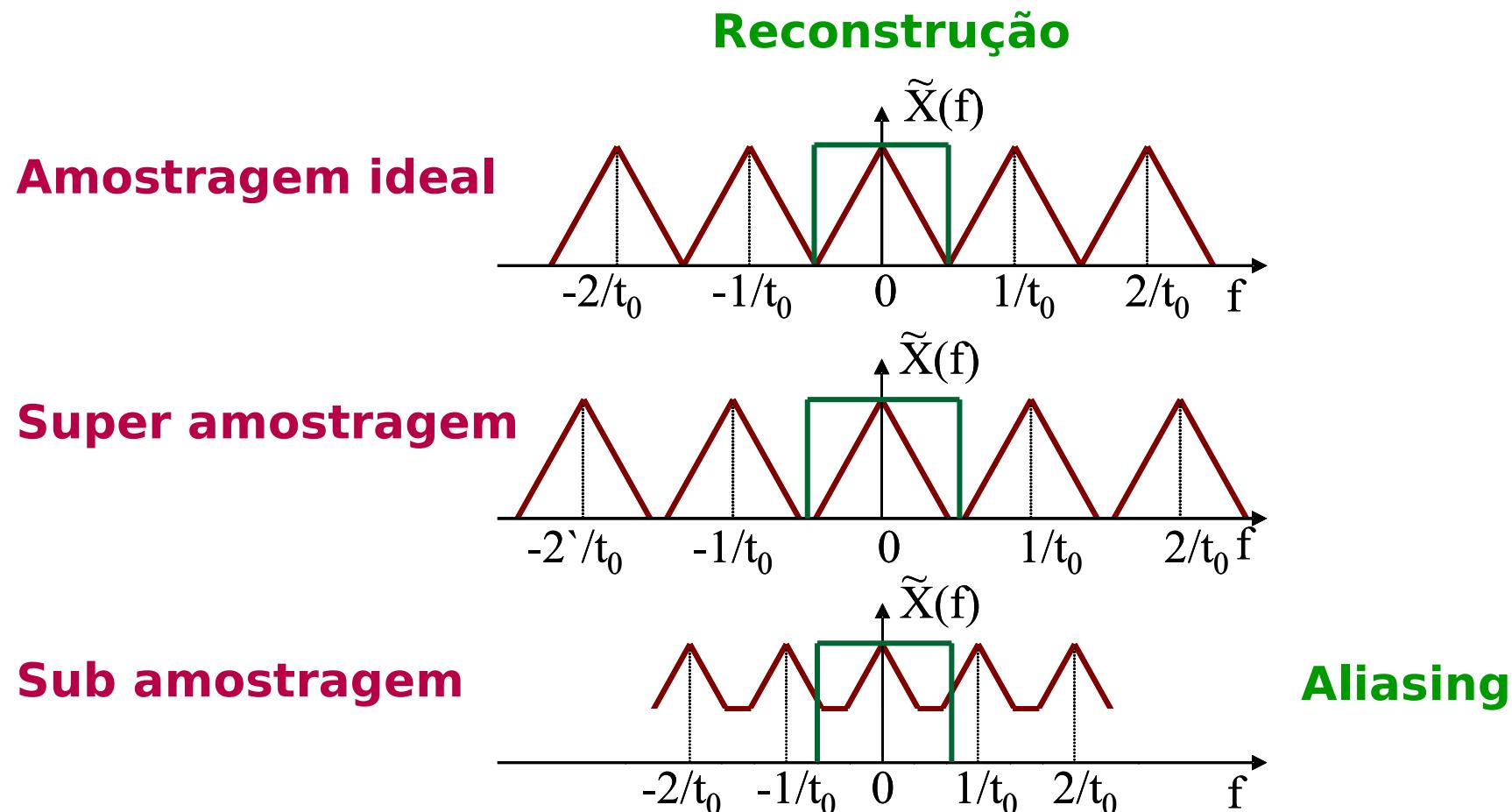


Sub amostragem



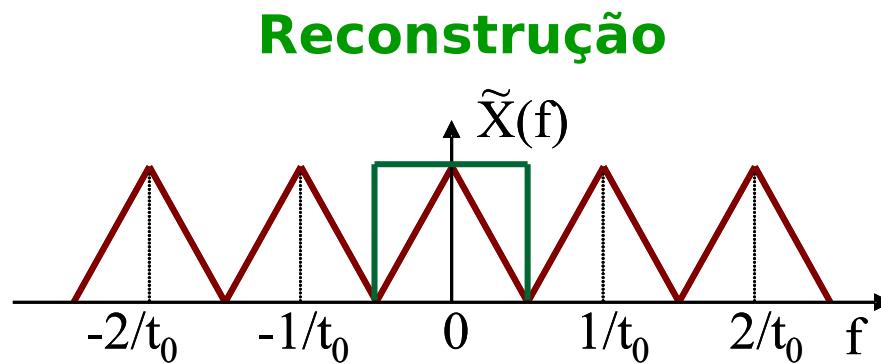
Amostragem - Teorema de Nyquist $f_a > 2f_{\max}$

Amostragem no domínio da frequência: acrescentar cópias deslocadas (pela frequência de amostragem) da transformada de Fourier do sinal de entrada.



Amostragem - Teorema de Nyquist $f_a > 2f_{\max}$

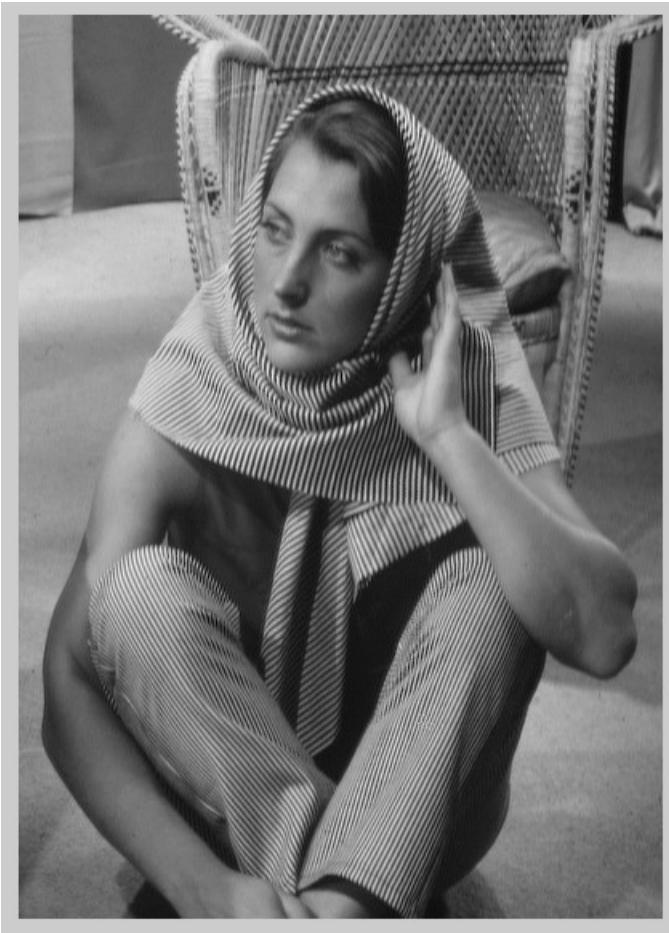
Amostragem no domínio da frequência: acrescentar cópias deslocadas (pela frequência de amostragem) da transformada de Fourier do sinal de entrada.



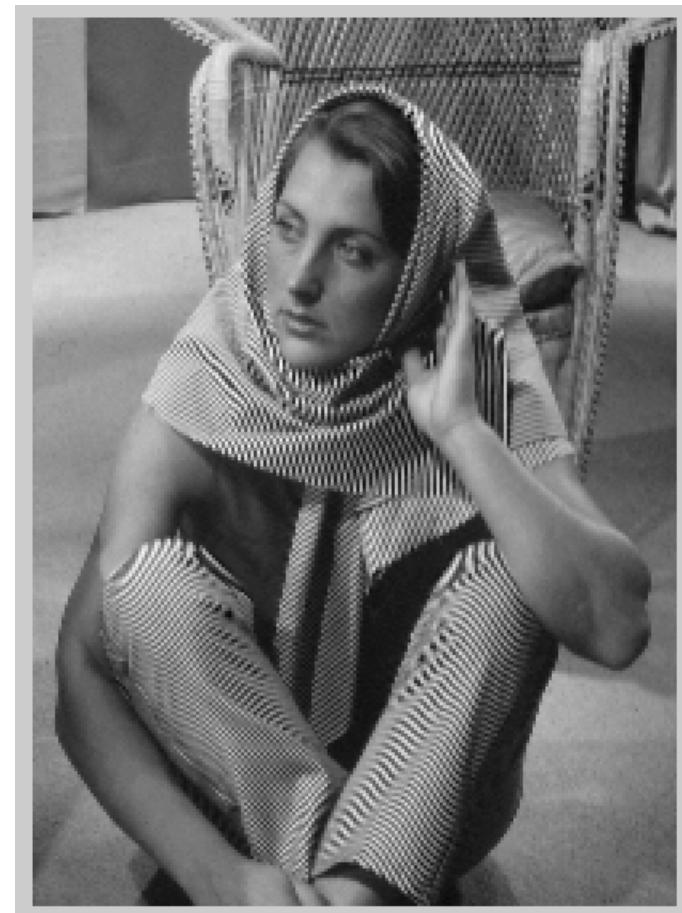
Teorema de Nyquist: uma função pode ser reconstruída perfeitamente a partir de uma amostragem se a frequência de amostragem for maior que o dobro do maior componente de frequência da função!

Limite de Representação

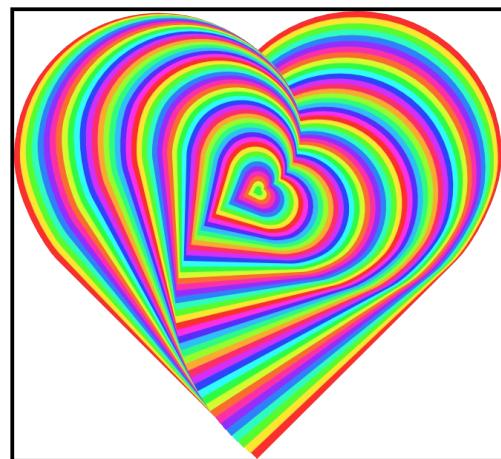
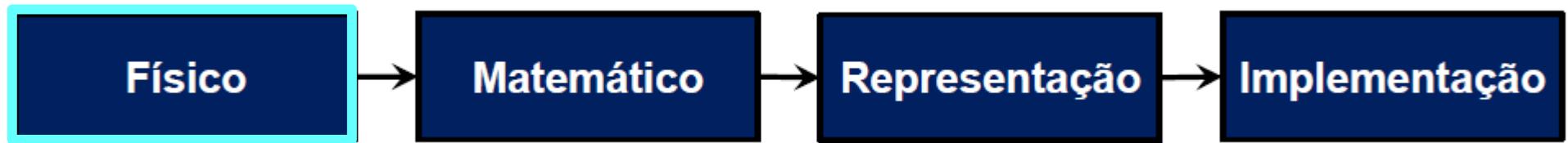
Original



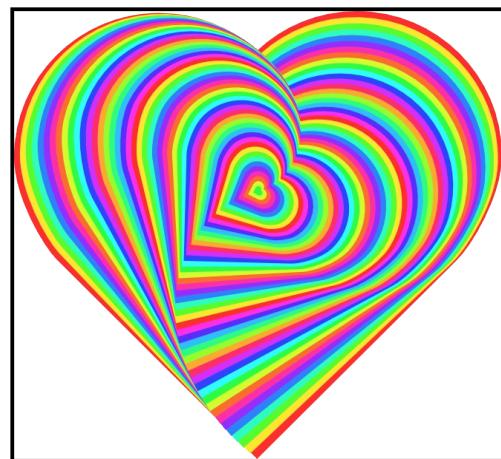
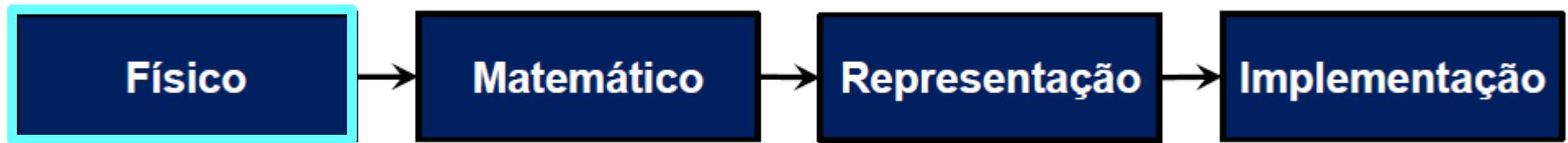
Baixa freq. amostragem



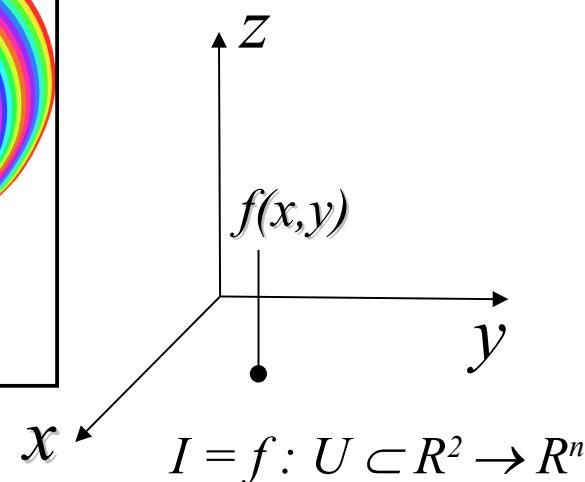
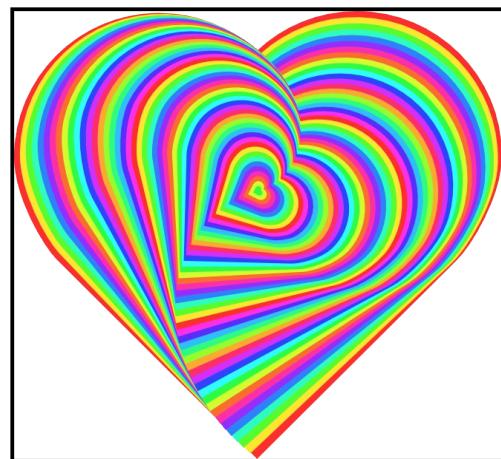
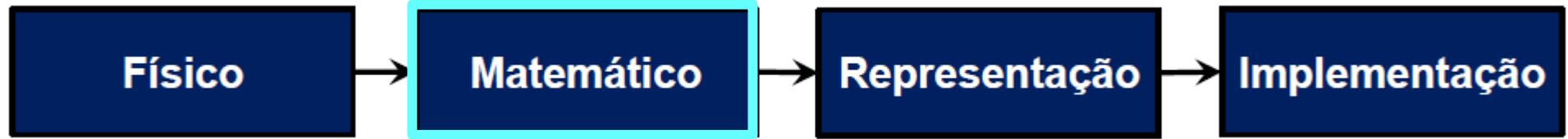
Paradigma dos 4 Universos



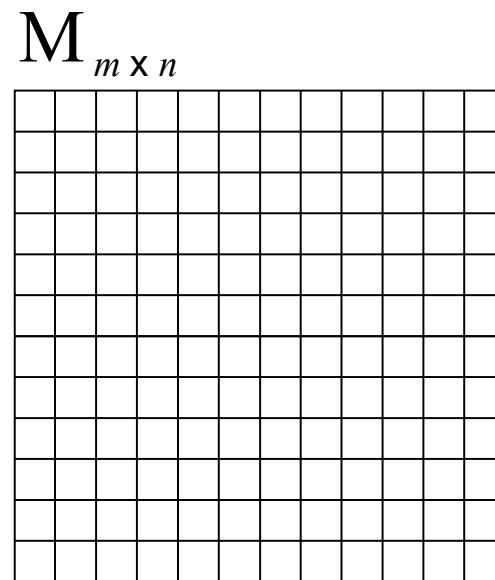
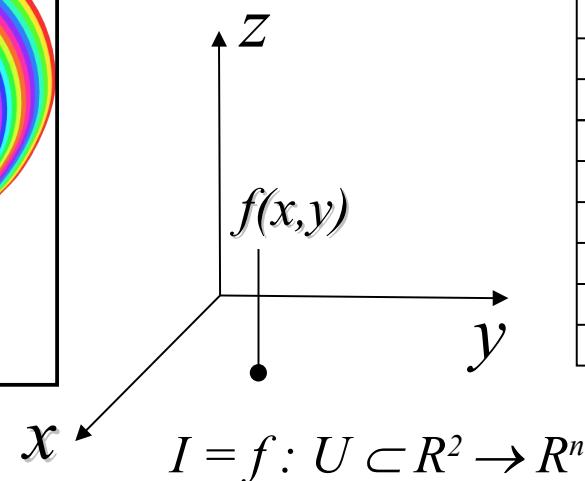
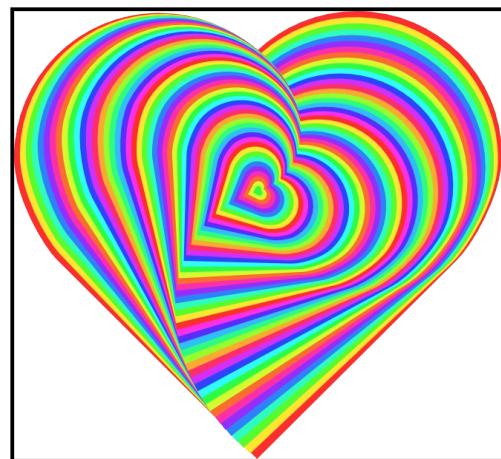
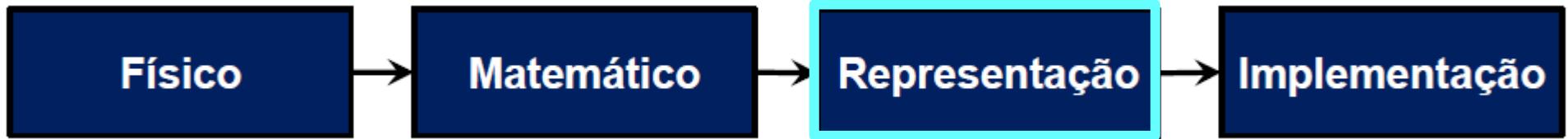
Paradigma dos 4 Universos



Paradigma dos 4 Universos

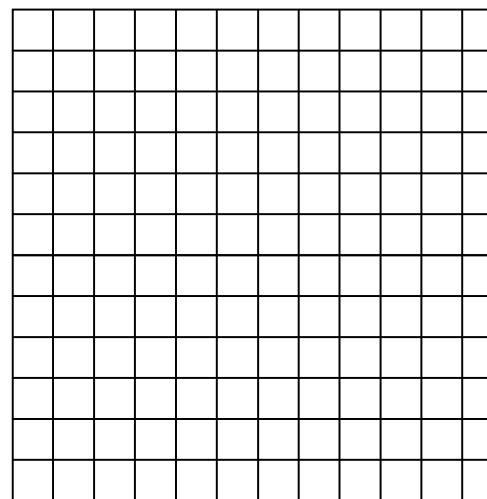
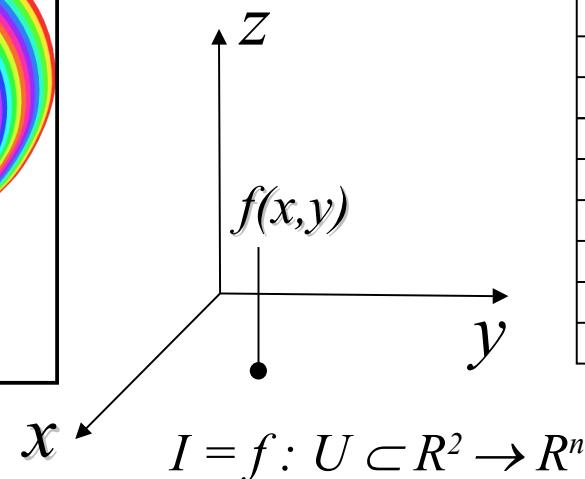
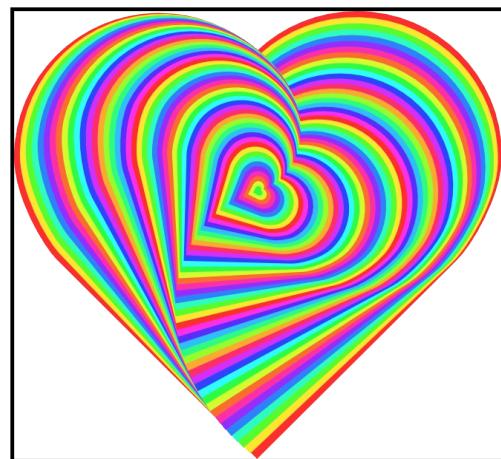
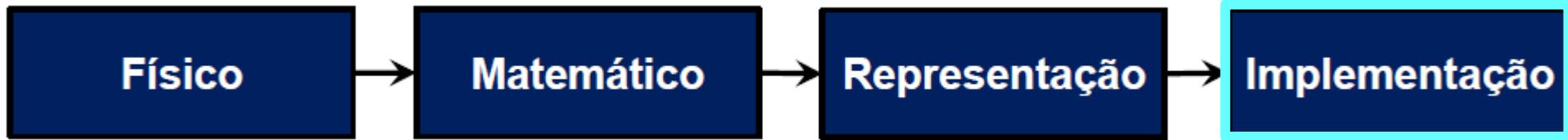


Paradigma dos 4 Universos



$$f_k(x,y) = m_{ij}$$

Paradigma dos 4 Universos



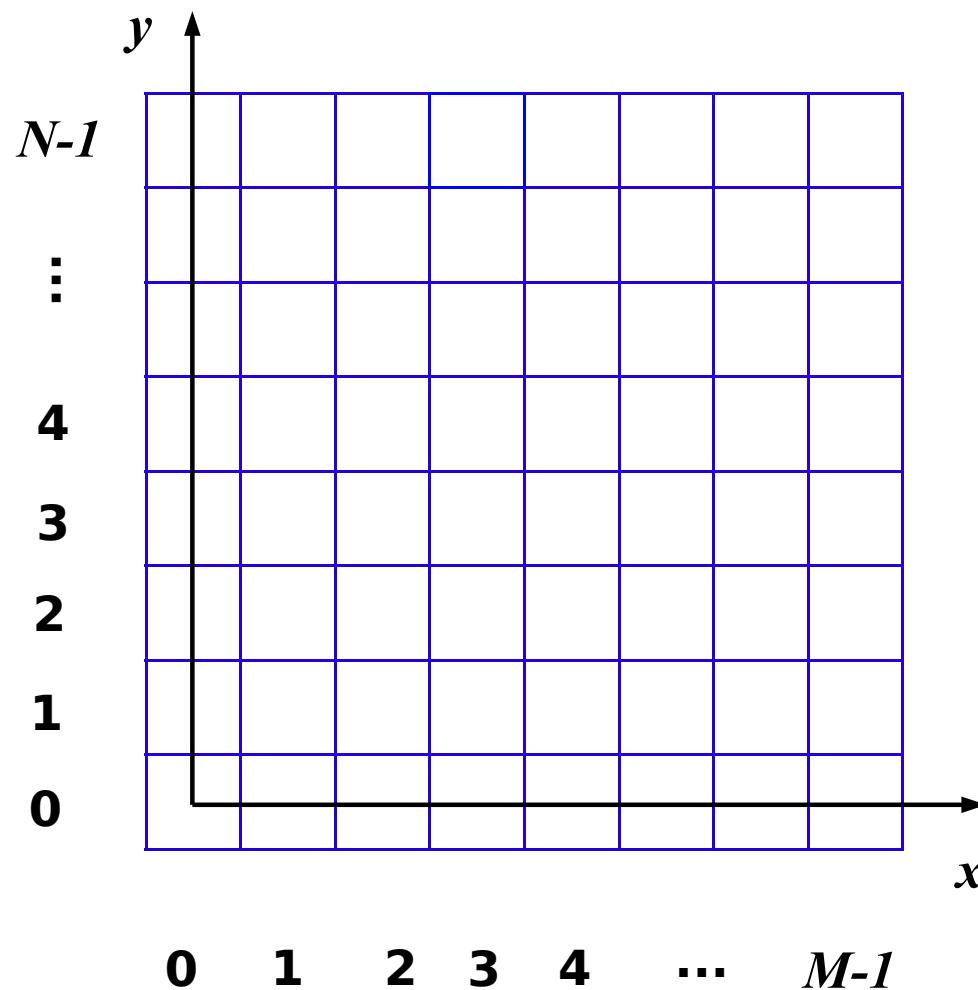
$$f_k(x, y) = m_{ij}$$

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float Imagem [M] [N] [3] ;
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Sistemas de Coordenadas de Imagens Digitais

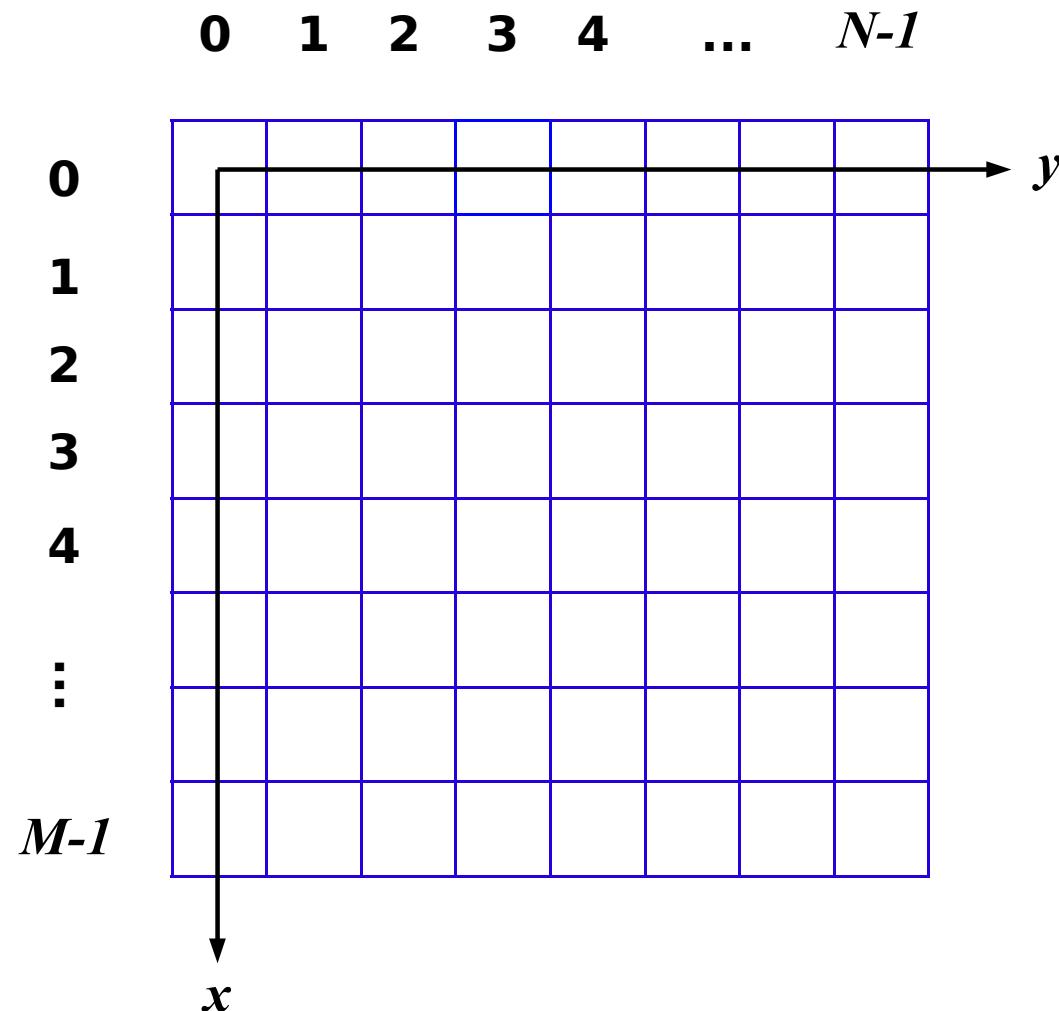
Sistema de coordenadas da imagem digital

Computação Gráfica



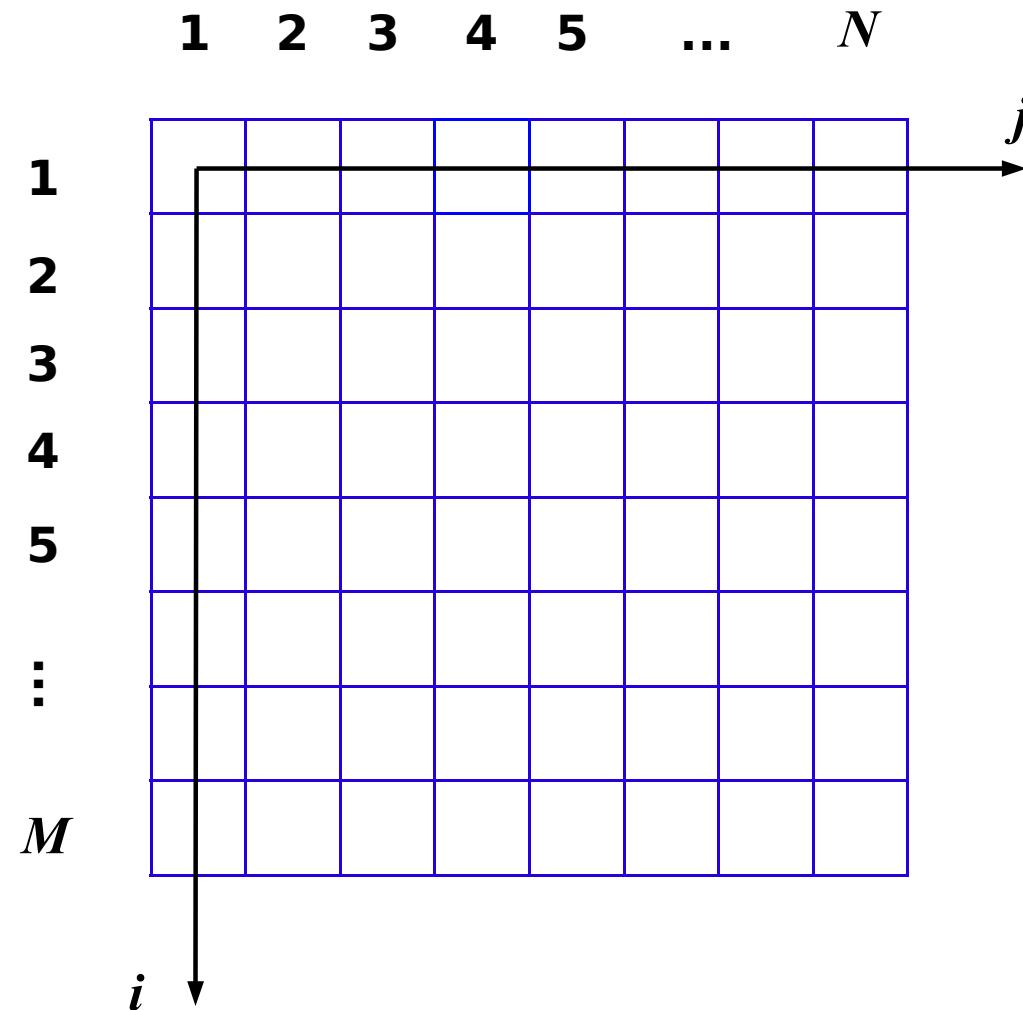
Sistema de coordenadas da imagem digital

Processamento Digital de Imagens



Sistema de coordenadas da imagem digital

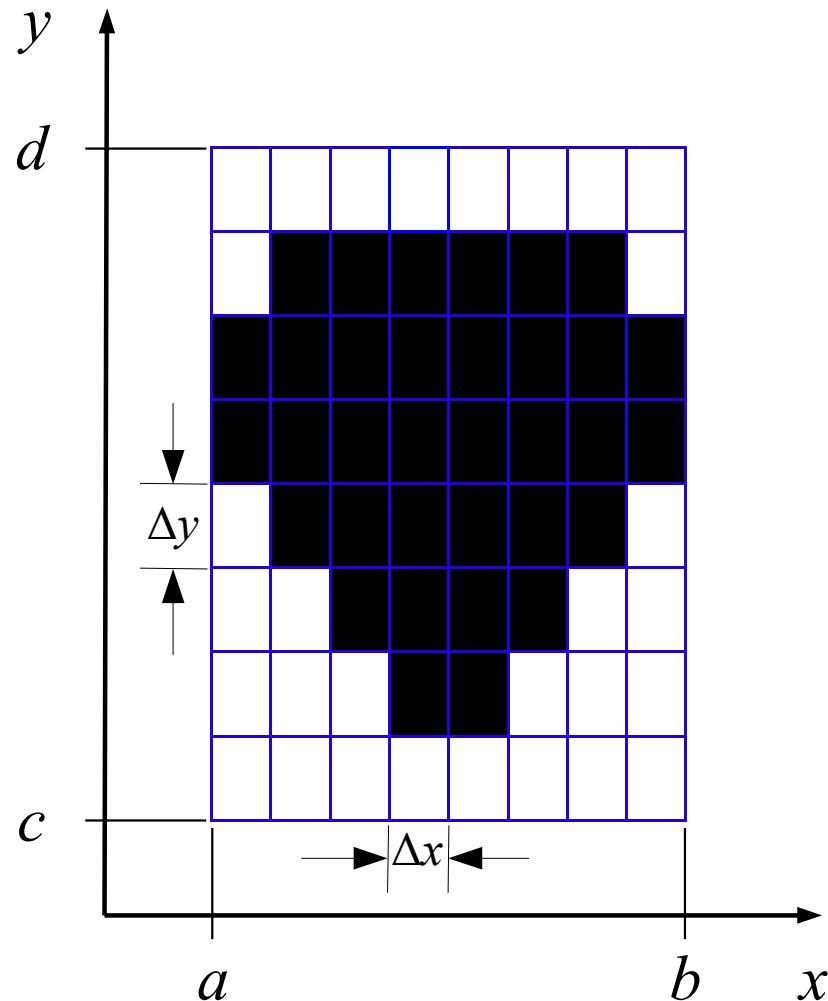
Ambientes matriciais (exemplo Octave)



Representação de uma Imagem

Representação Espacial

Imagen discreta



$$U = [a, b] \times [c, d] = (x, y) \in \mathbb{R}^2; a \leq x \leq b \text{ e } c \leq y \leq d$$

Para $a = c = 0$

$P\Delta = (x_j, y_k) \in \mathbb{R}^2$ (*reticulado bidimensional*)

onde,

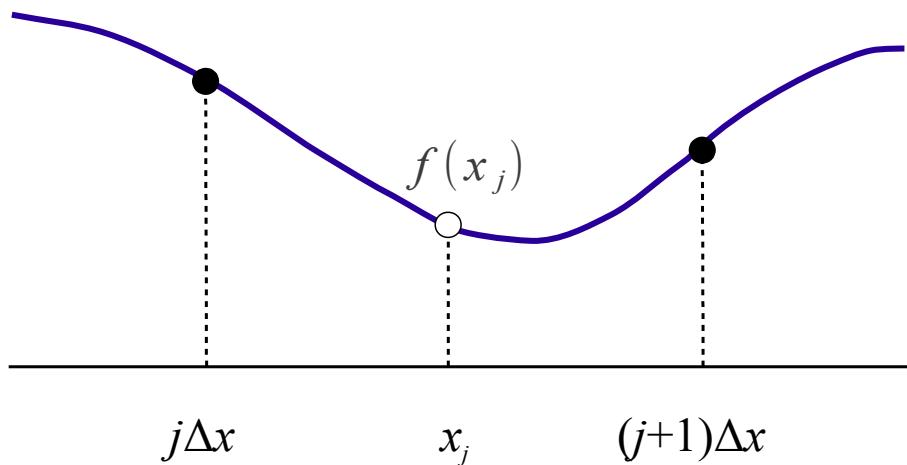
$$x_j = j \Delta x; j = 0, 1, \dots, m-1; \Delta x = b/m$$

$$y_k = k \Delta y; k = 0, 1, \dots, n-1; \Delta y = d/n$$

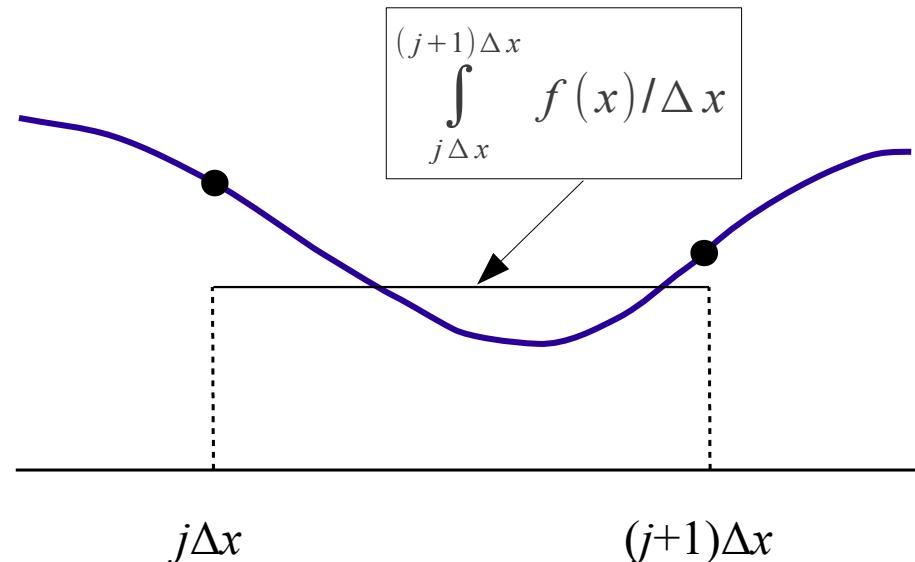
$$c_{jk} = [j \Delta x, (j+1) \Delta x] \times [k \Delta y, (k+1) \Delta y] \subset P\Delta$$
$$j = 0, \dots, m-1; k = 0, \dots, n-1$$

Discretização

Amostragem pontual



Amostragem por área



Amostragem Pontual

Imagen continua

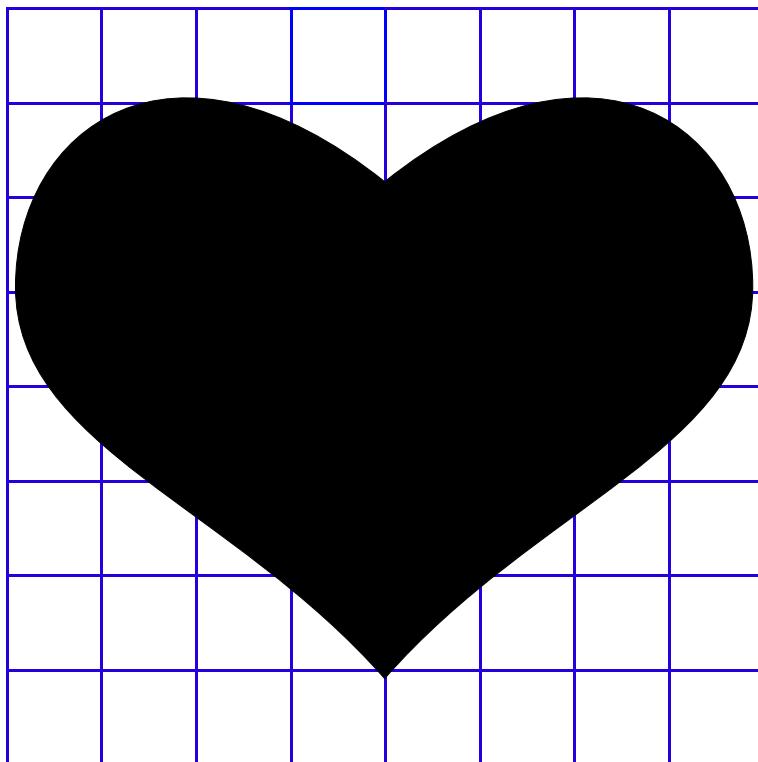
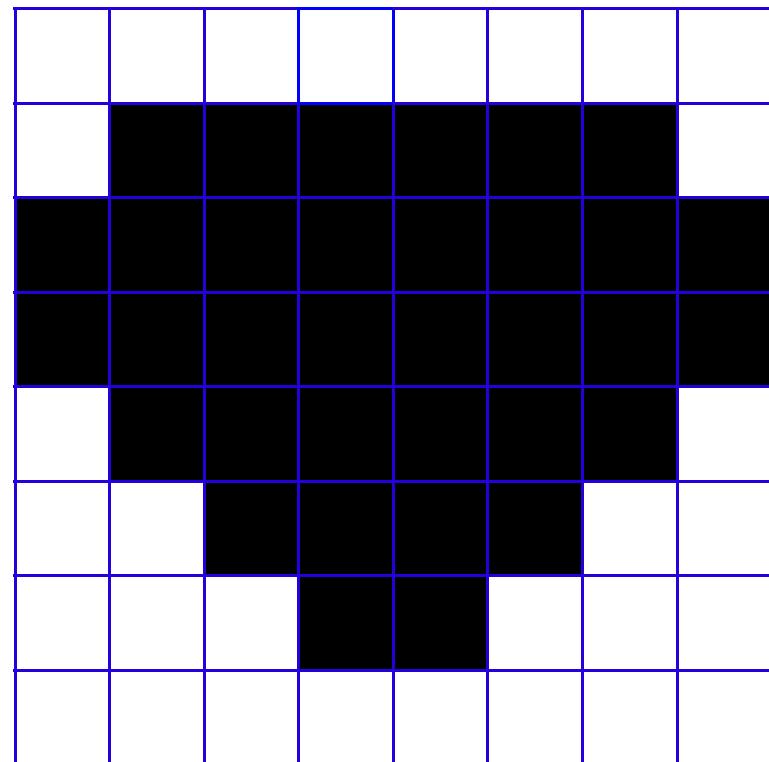


Imagen discreta



Amostragem por Área

Imagen continua

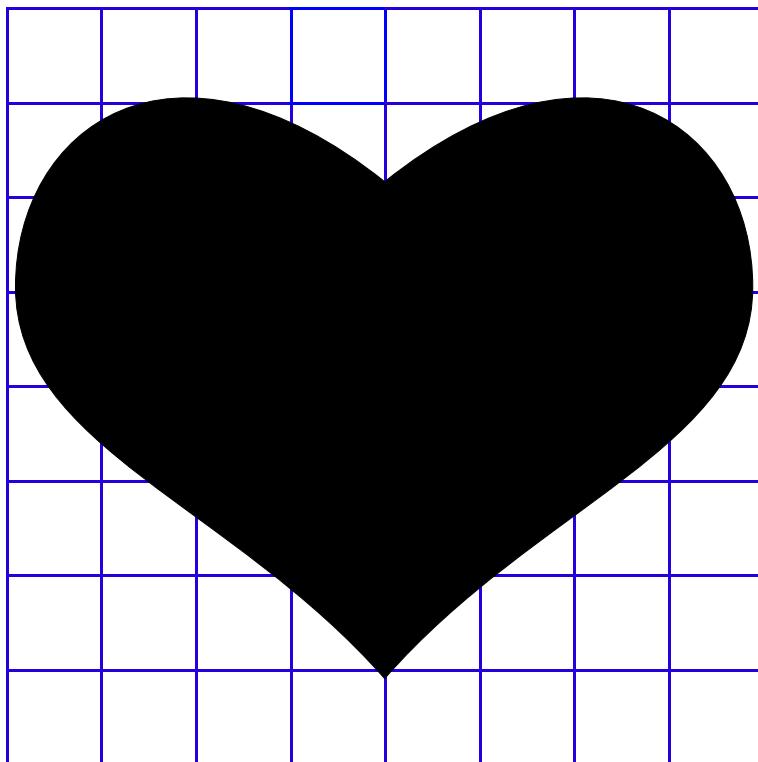
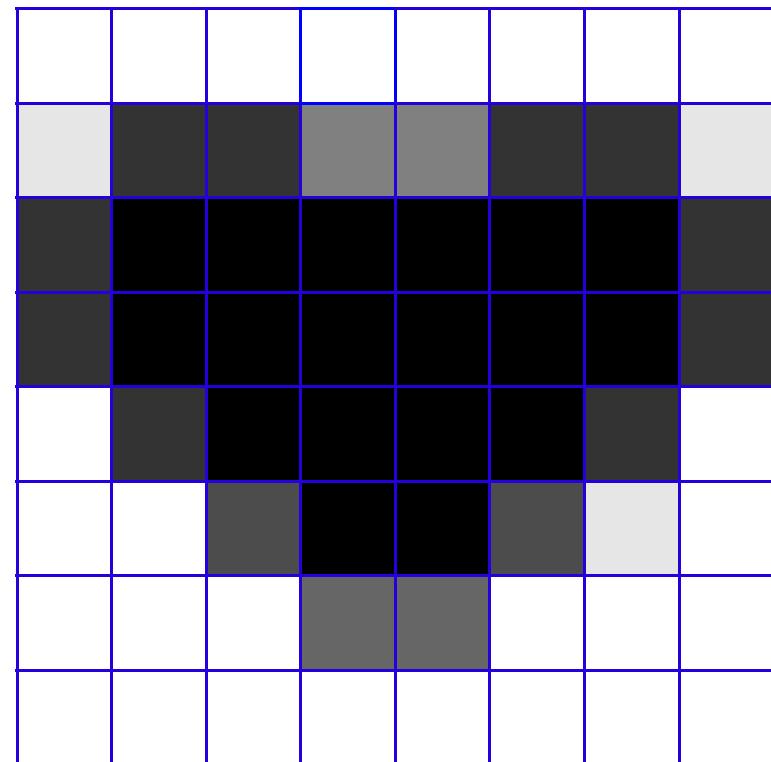
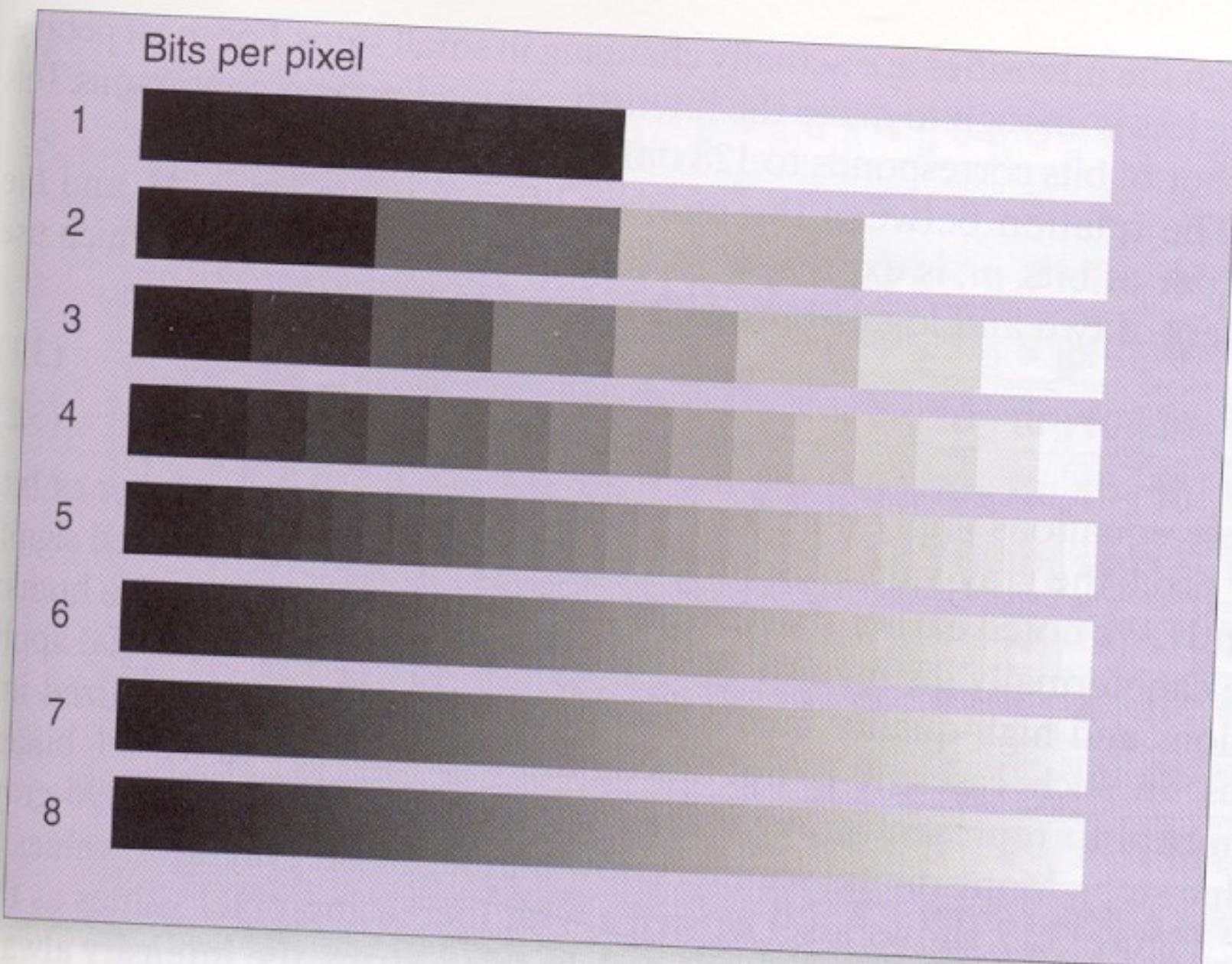


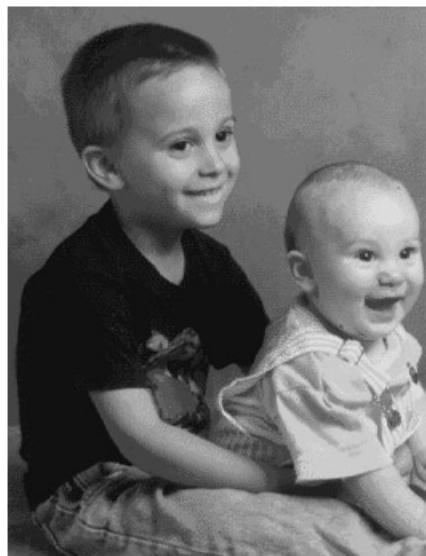
Imagen discreta



Quantização - Bits por pixel



Quantização - Bits por pixel



16 tons de cinza



8 tons de cinza



4 tons de cinza



2 tons de cinza

Quantização - Bits por pixel



24 bits (8 bits por canal/banda)

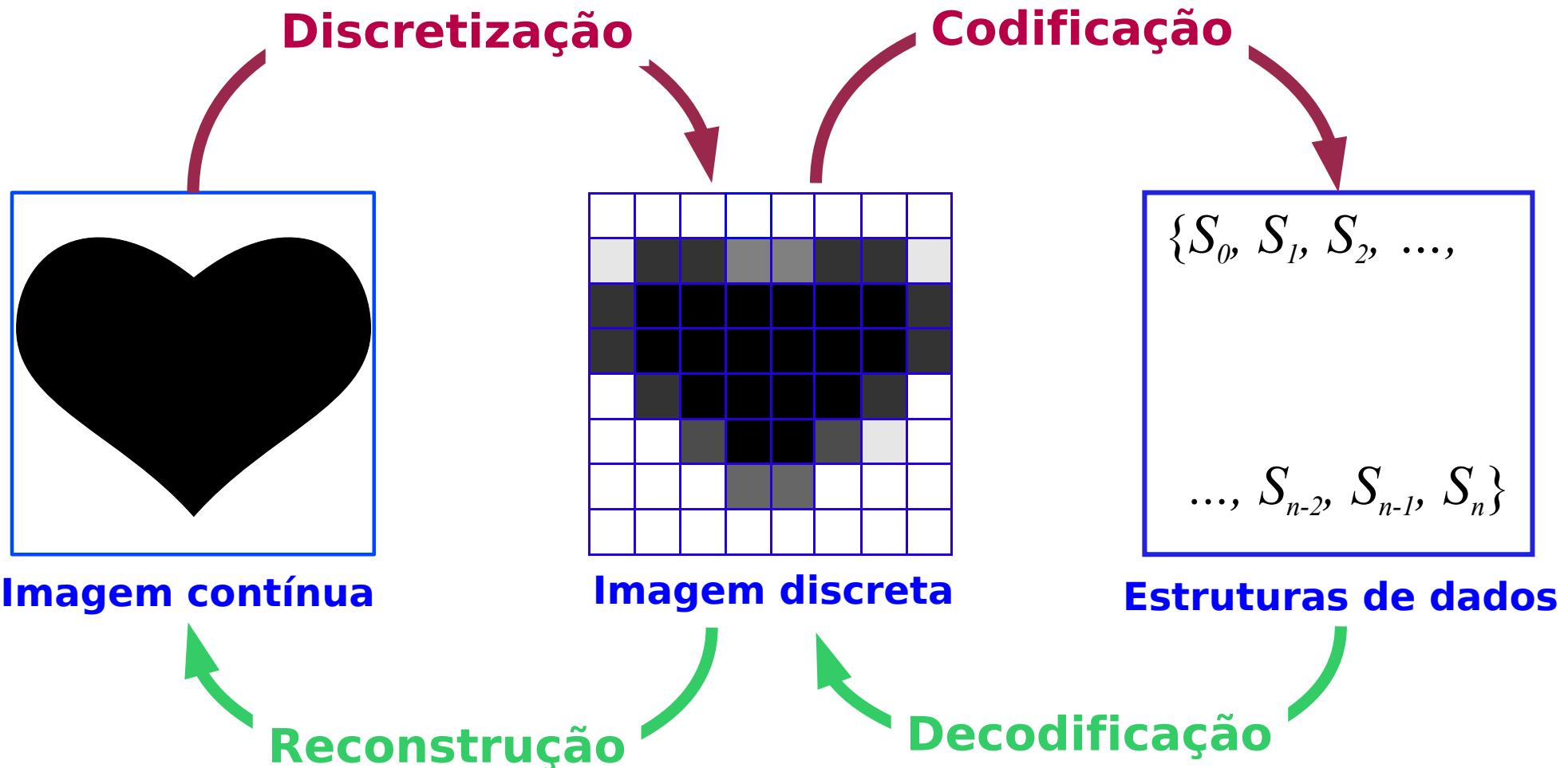
Quantização - Mapa de Cores



256 cores

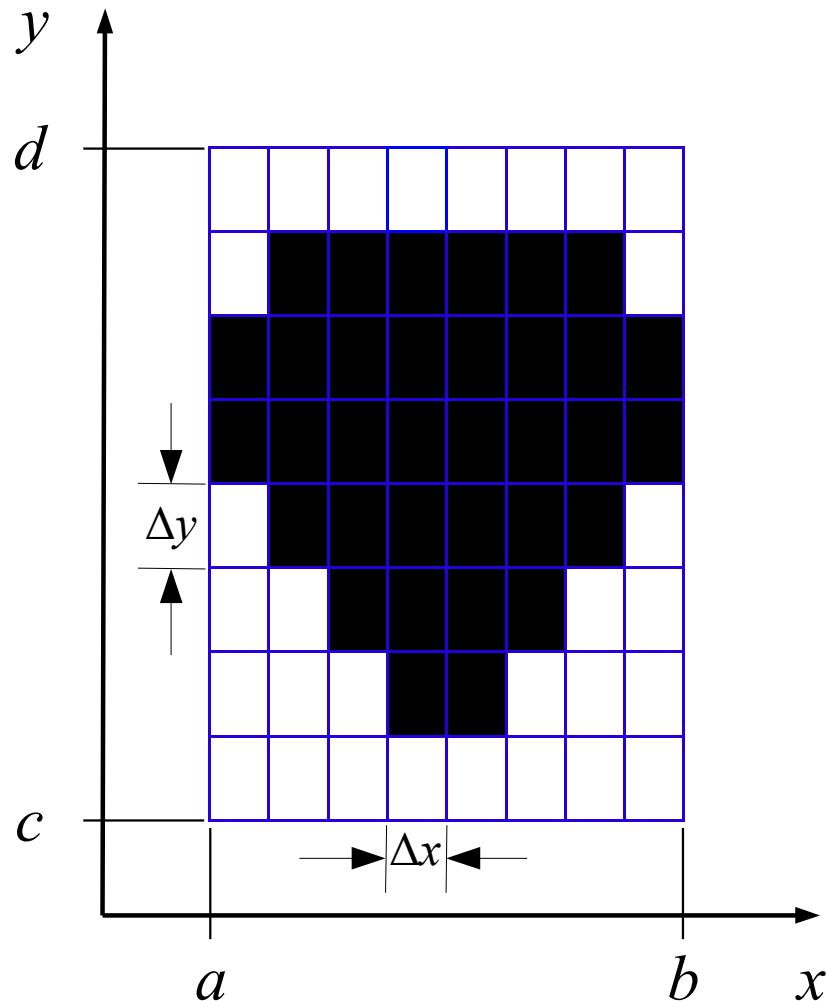
Reconstrução de Imagens Digitais

Níveis de Abstração na Representação de Imagens



Representação Espacial

Imagen discreta



$$U = [a, b] \times [c, d] = (x, y) \in \mathbb{R}^2; a \leq x \leq b \text{ e } c \leq y \leq d$$

Para $a = c = 0$

$P\Delta = (x_j, y_k) \in \mathbb{R}^2$ (*reticulado bidimensional*)

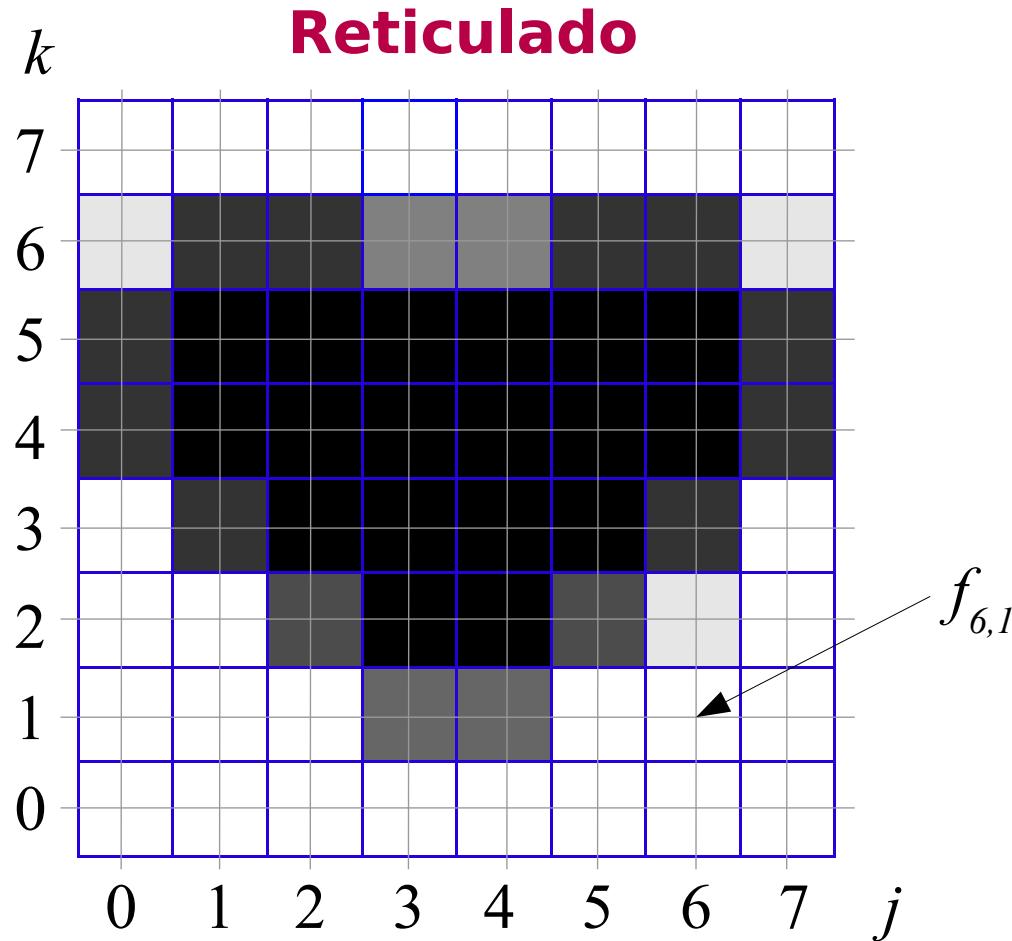
onde,

$$x_j = j \Delta x; j = 0, 1, \dots, m-1; \Delta x = b/m$$

$$y_k = k \Delta y; k = 0, 1, \dots, n-1; \Delta y = d/n$$

$$c_{jk} = [j \Delta x, (j+1) \Delta x] \times [k \Delta y, (k+1) \Delta y] \subset P\Delta$$
$$j = 0, \dots, m-1; k = 0, \dots, n-1$$

Núcleos de Reconstrução 1D



Posição de uma amostra: p_j, p_k
Valor da amostra em p_j, p_k : $f_{j,k}$

Cálculo da Função Reconstruída

Núcleo ϕ é posicionado no centro da amostra

x é a distância à posição p_j

y é a distância à posição p_k

1D

$$f_r(x) = \sum_j f_j \phi(x - p_j)$$

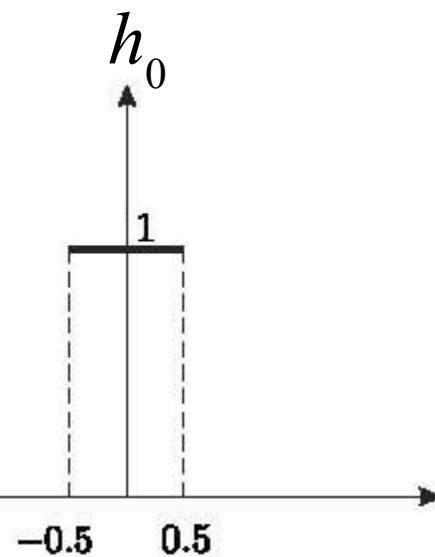
2D

$$f_r(x, y) = \sum_j \sum_k f_{jk} \phi(x - p_j, y - p_k)$$

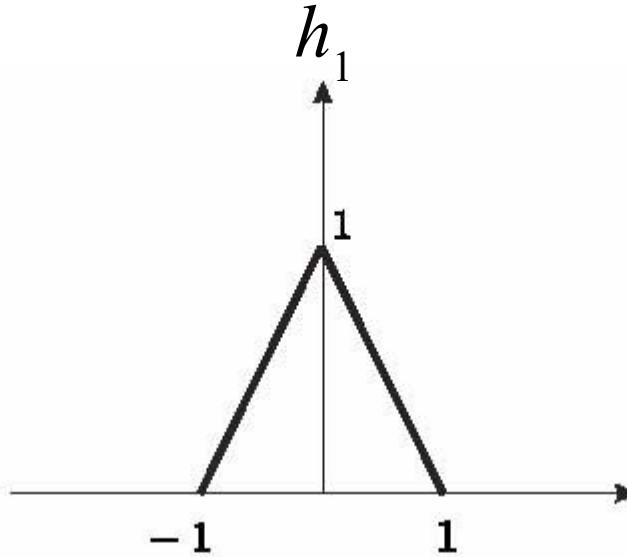
Núcleos de Reconstrução 1D

Núcleo ϕ é posicionado no centro da amostra

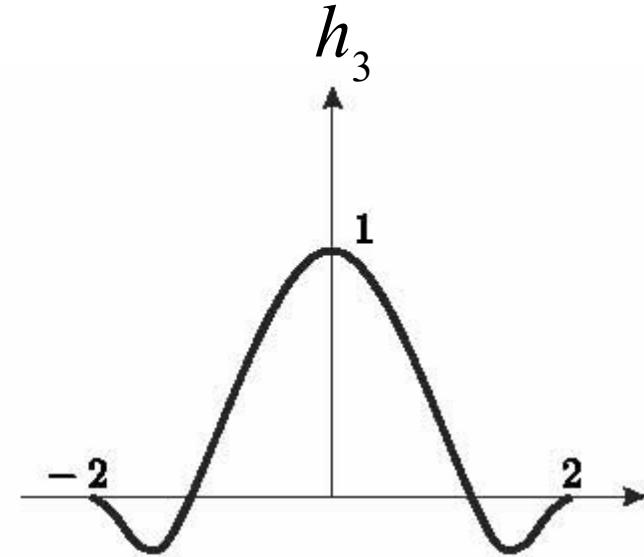
Constante



Triangular



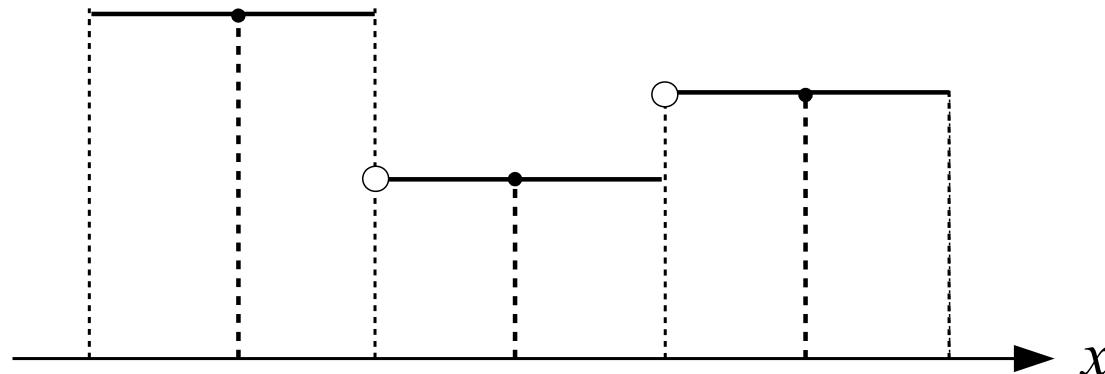
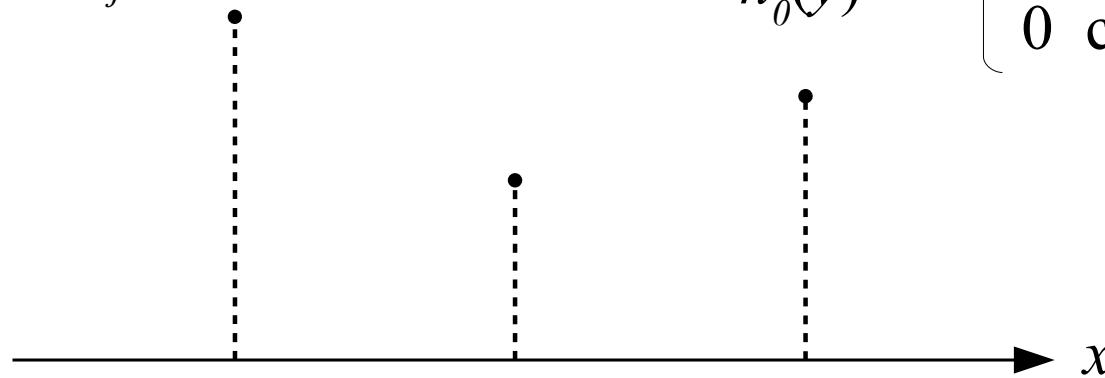
Cúbico



Reconstrução 1D - Núcleo Constante

$$f_r(x) = \sum_j f_j h_0(x - p_j)$$

$$h_0(y) = \begin{cases} 1 & \text{se } -0,5 < y \leq 0,5 \\ 0 & \text{caso contrário} \end{cases}$$



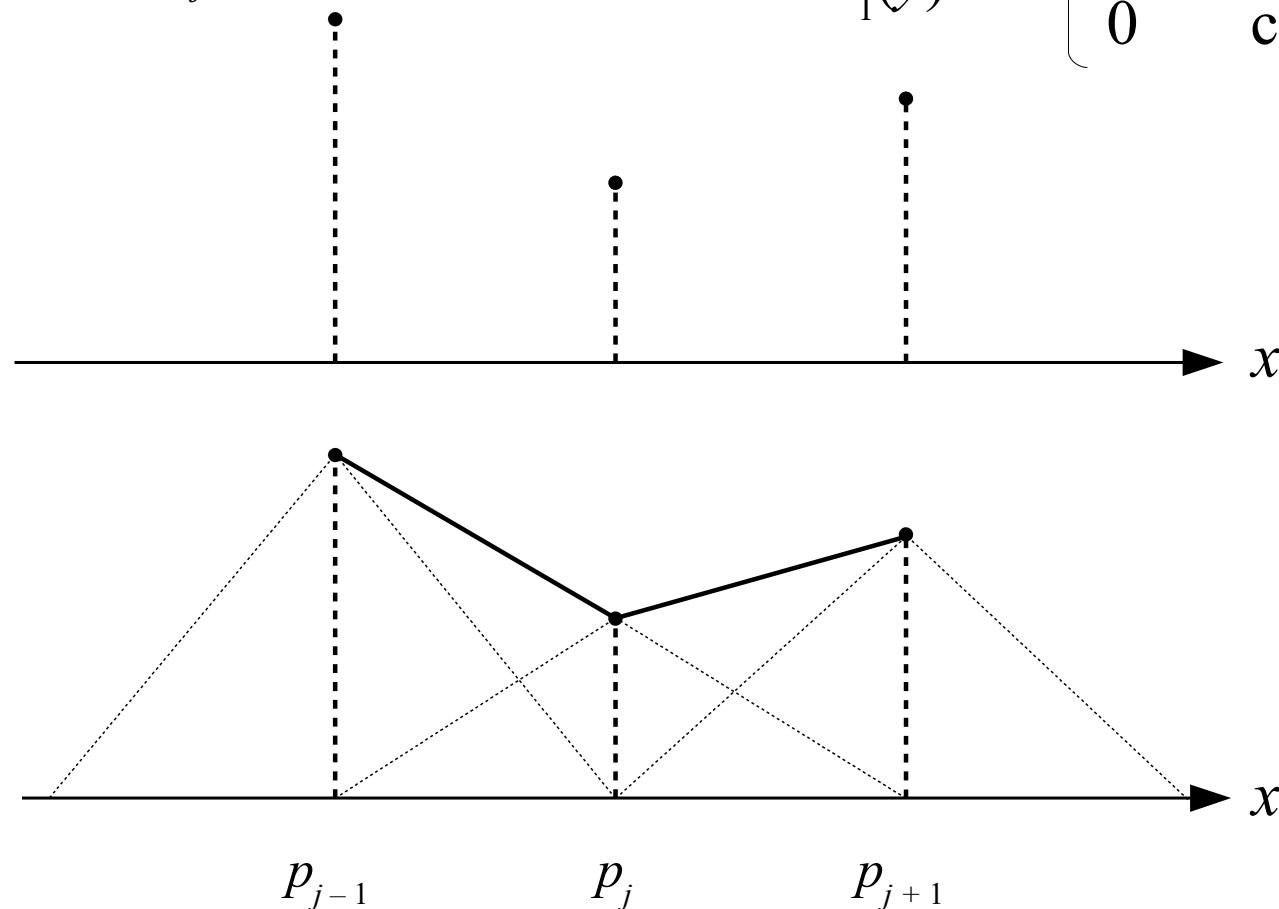
$$p_{j-1} \quad p_j \quad p_{j+1}$$

Valor contínuo corresponde ao valor discreto do vizinho mais próximo.

Reconstrução 1D - Núcleo Triangular

$$f_r(x) = \sum_j f_j h_1(x - p_j)$$

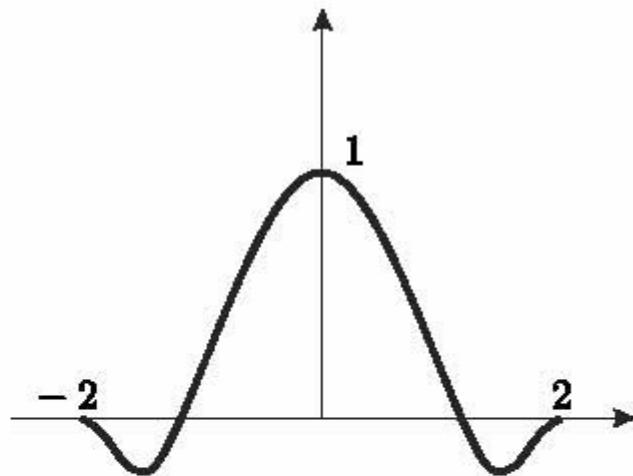
$$h_1(y) = \begin{cases} 1-|y| & \text{se } |y| \leq 1 \\ 0 & \text{caso contrário} \end{cases}$$



Valor contínuo corresponde à combinação de duas funções lineares, uma para cada vizinho discreto

Reconstrução 1D - Núcleo Cúbico

$$f_r(x) = \sum_j f_j h_3(x - p_j)$$



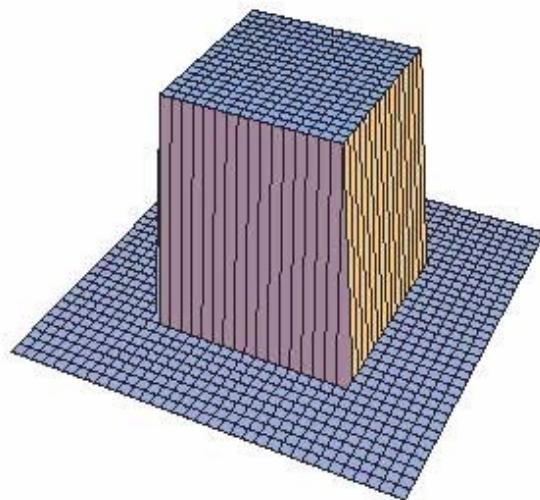
$$h_3(y) = \begin{cases} 1 - 2|y|^2 + |y|^3 & \text{se } 0 < |y| \leq 1 \\ 4 - 8|y| + 5|y|^2 - |y|^3 & \text{se } 1 \leq |y| \leq 2 \\ 0 & \text{caso contrário} \end{cases}$$

Valor contínuo corresponde à combinação de quatro funções, para os quatro vizinhos mais próximos

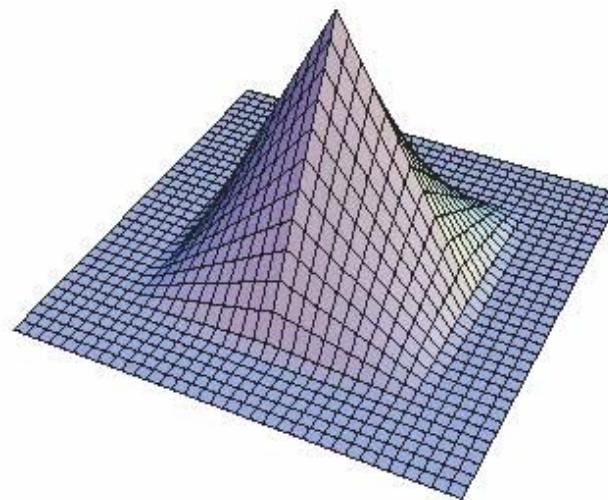
Núcleos de Reconstrução 2D

$$f_r(x, y) = \sum_j \sum_k f_{jk} h_{nx}(x - p_j) h_{ny}(y - p_k)$$

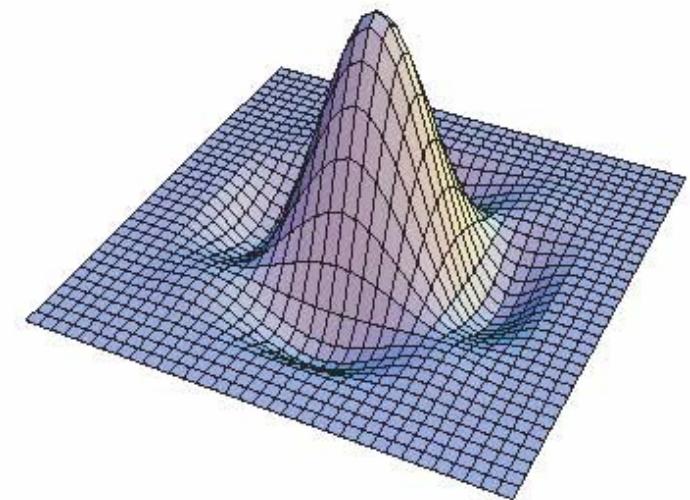
Constante



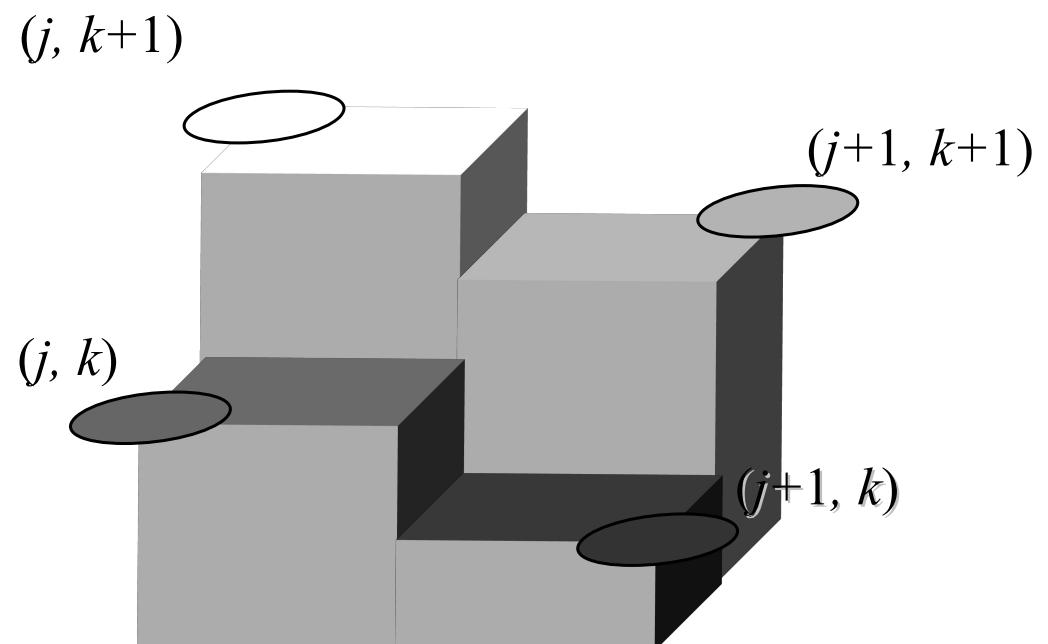
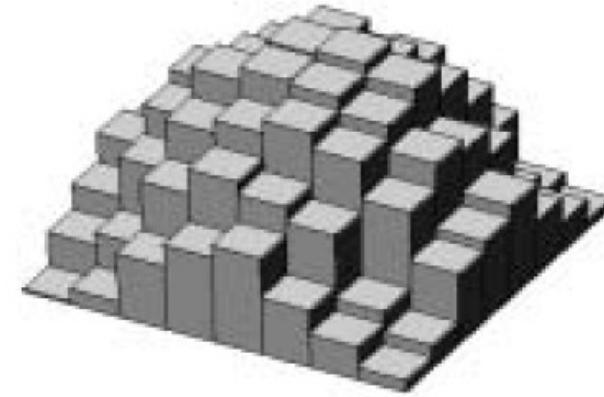
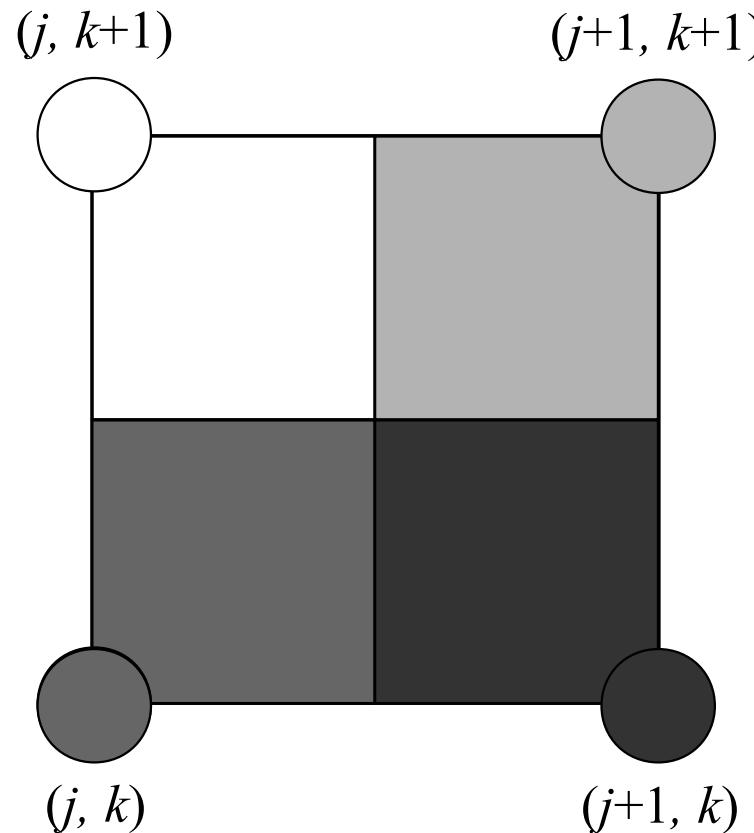
Triangular



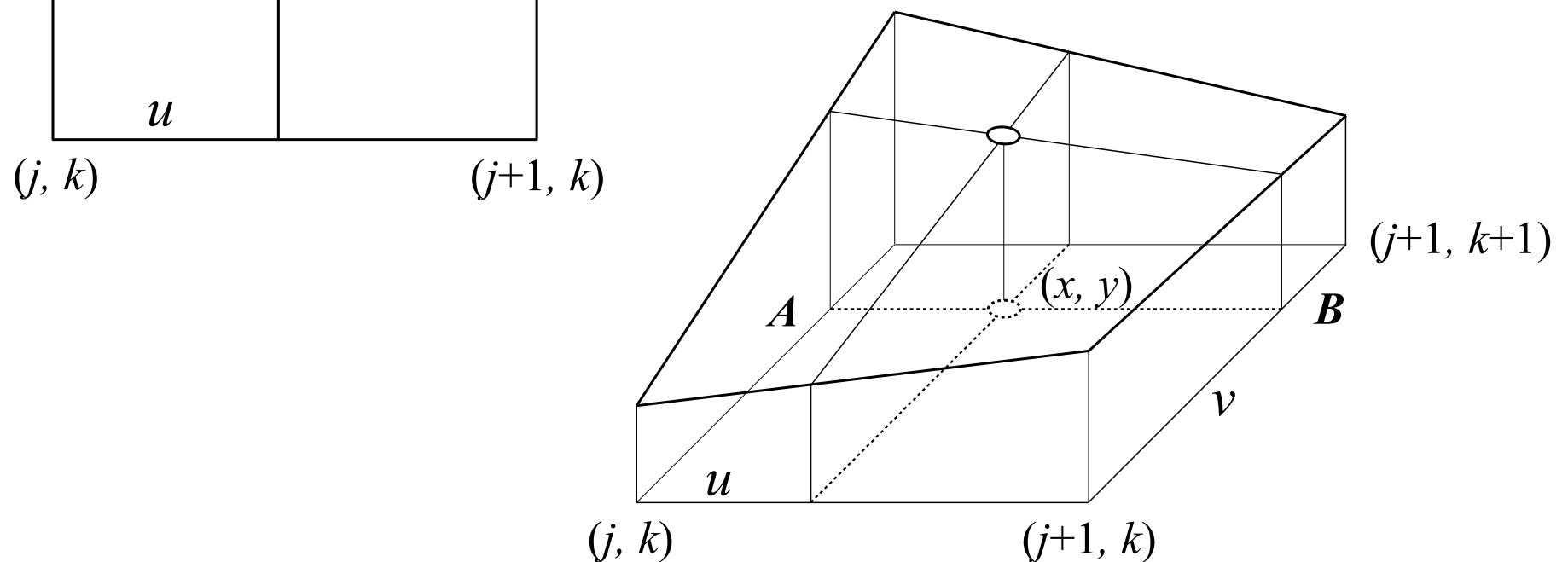
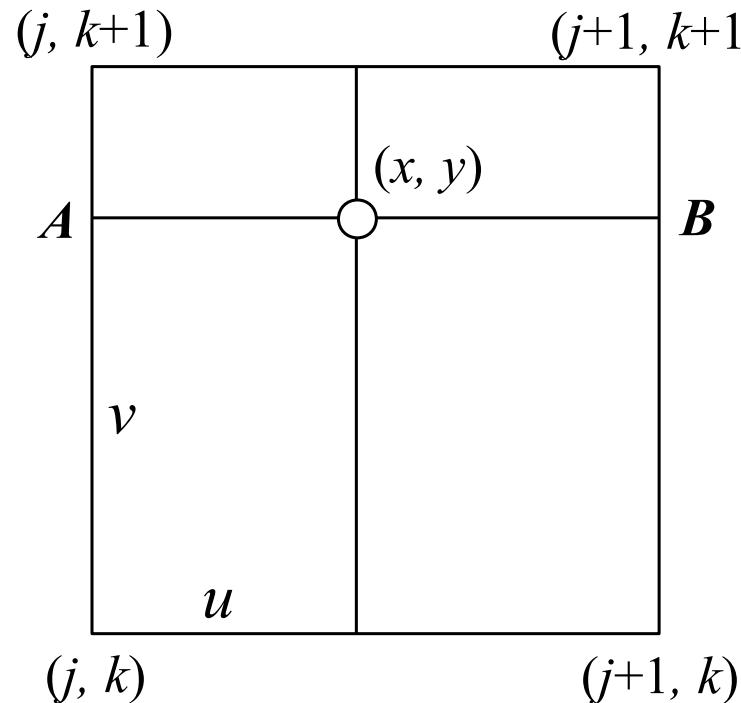
Cúbico



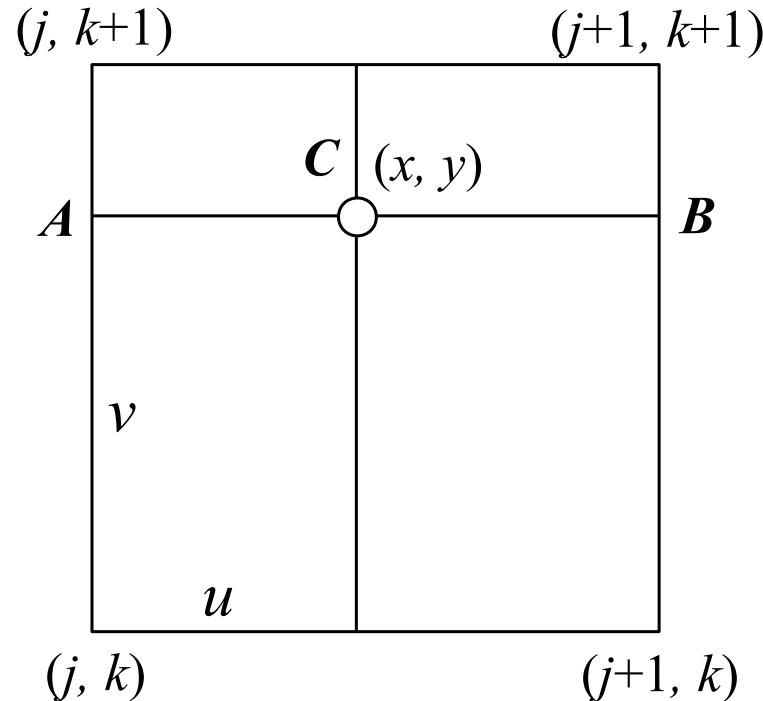
Reconstrução 2D - Núcleo Constante



Reconstrução 2D - Núcleo Triangular



Reconstrução 2D - Núcleo Triangular



$$f_A = (1-v)f_{j,k} + vf_{j,k+1}$$

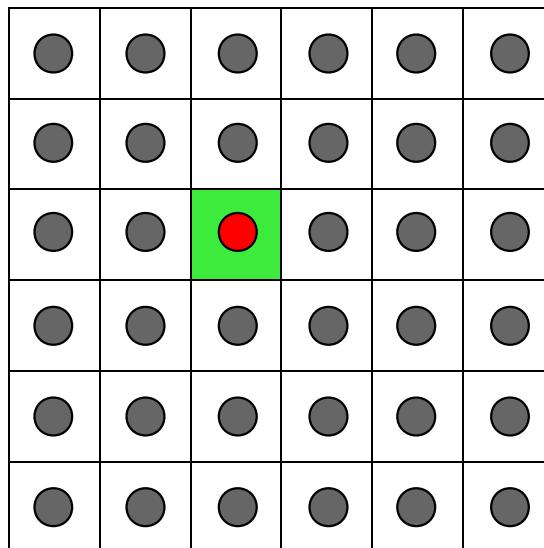
$$f_B = (1-v)f_{j+1,k} + vf_{j+1,k+1}$$

$$f_C = (1-u)f_A + uf_B$$

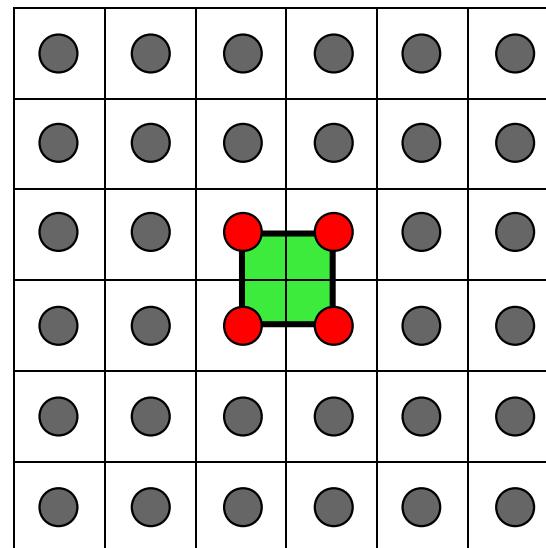
$$f_C = (1-u) [(1-v)f_{j,k} + vf_{j,k+1}] + u [(1-v)f_{j+1,k} + vf_{j+1,k+1}]$$

Reconstrução 2D: Área de influência

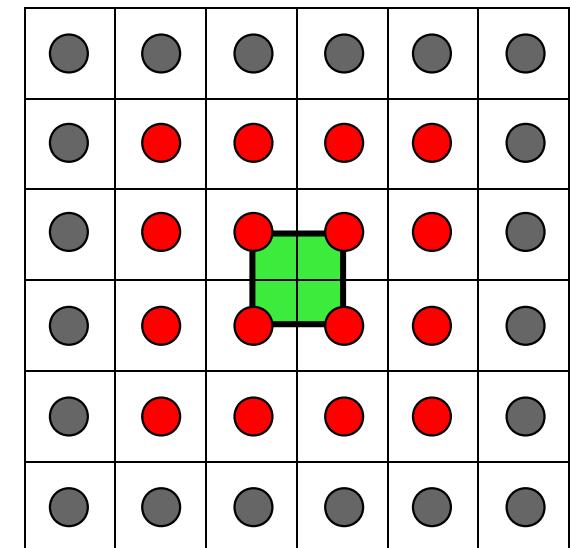
Constante



Triangular



Cúbico



Resultado de Reconstrução 2D



Constante



Triangular



Cúbico

Conceitos de Imagens Digitais

Resolução Espacial



460 x 296 pixels



115 x 74 pixels

Resolução Espacial 512 x 512



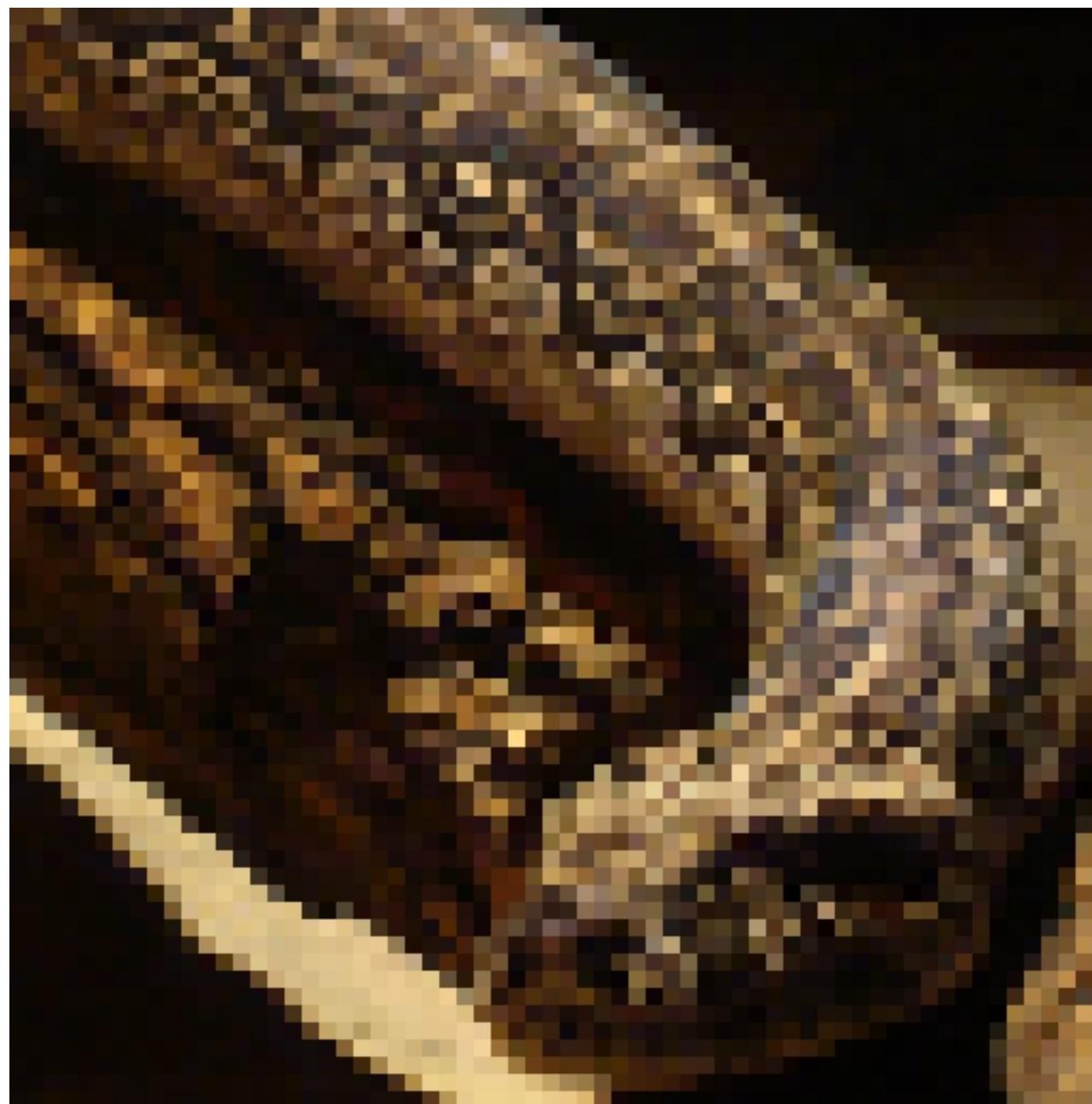
Resolução Espacial 256 x 256



Resolução Espacial 128 x 128



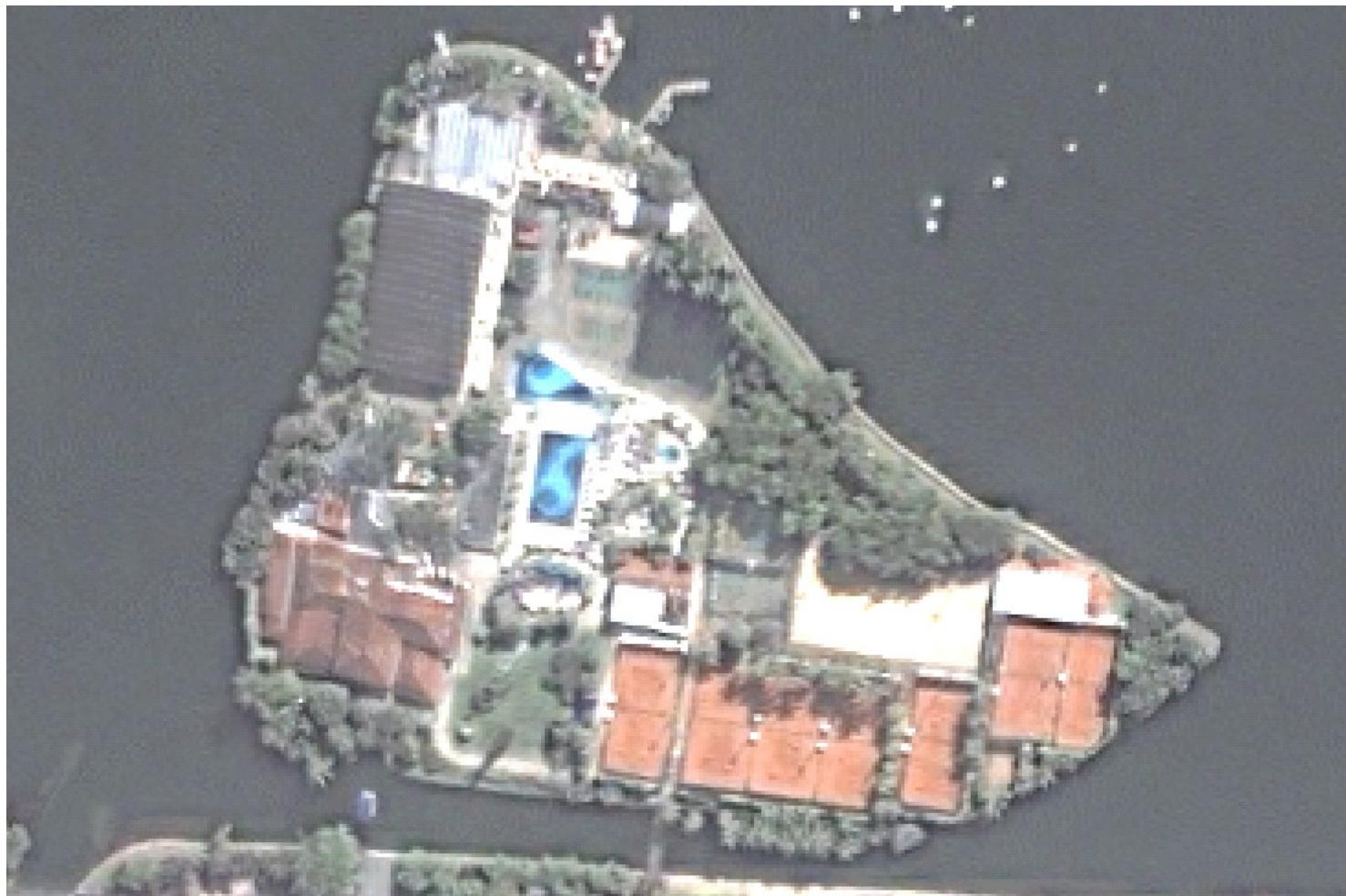
Resolução Espacial 64 x 64



Resolução Espacial 30 x 30 m



Resolução Espacial 1 x 1 m



Resolução Espacial

Resolução absoluta:
colunas x linhas

Pixels por unidade de distância:
dots per inch (dpi)
pixels per inch (ppi)

Sensoriamento Remoto:
ground sampling distance (GSD)
resolução do pixel, e.g., 1x1m

Histograma de Imagenes

Histograma de Imagens Digitais

O histograma de uma imagem digital com tons de cinza entre [0, L-1] é uma função discreta:

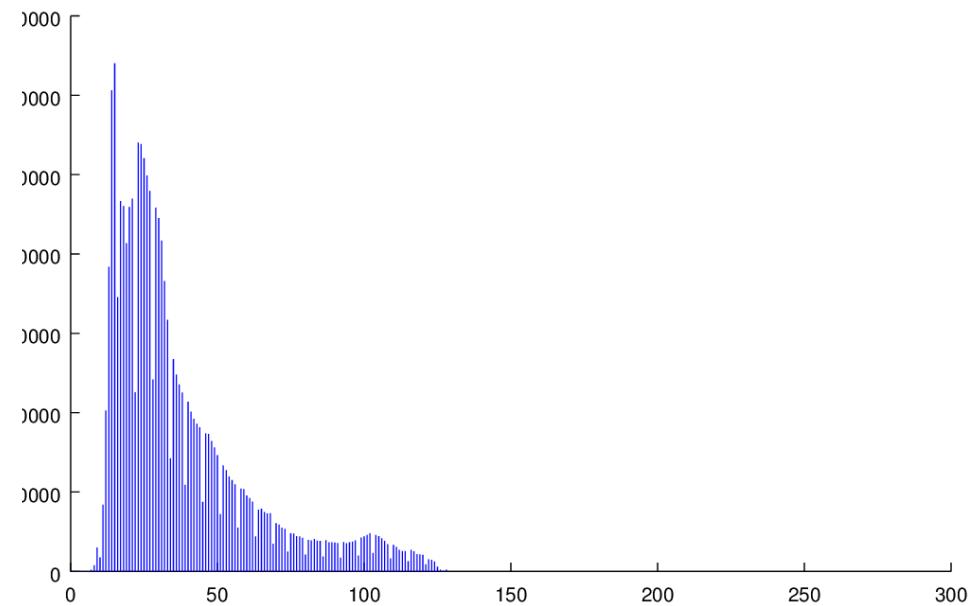
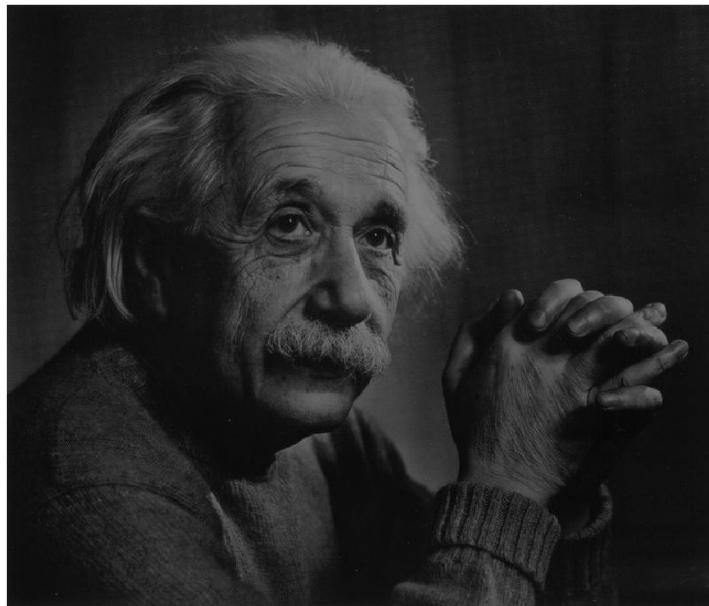
$$h(r_k) = n_k$$

r_k é o k-ésimo tom de cinza,

n_k é o número de pixels com o tom de cinza r_k

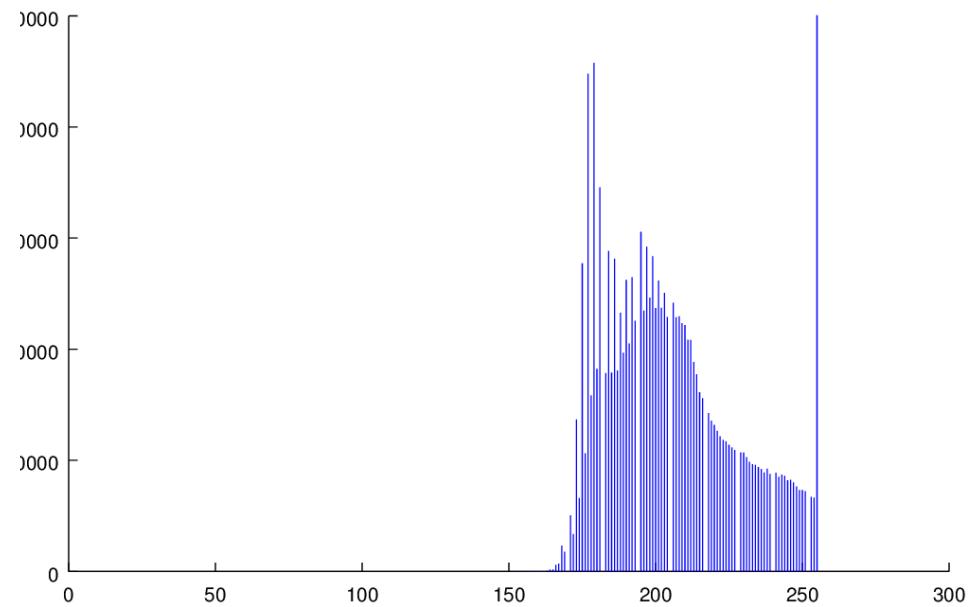
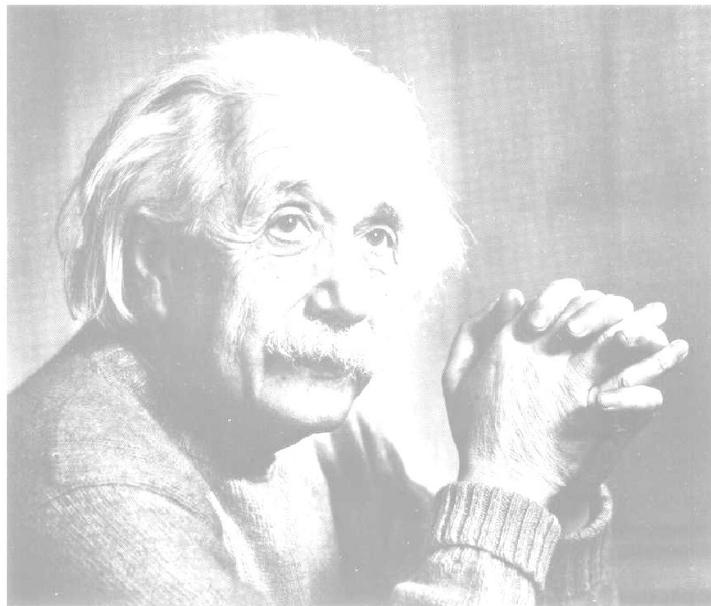
Histograma de Imagens Digitais

Imagen escura



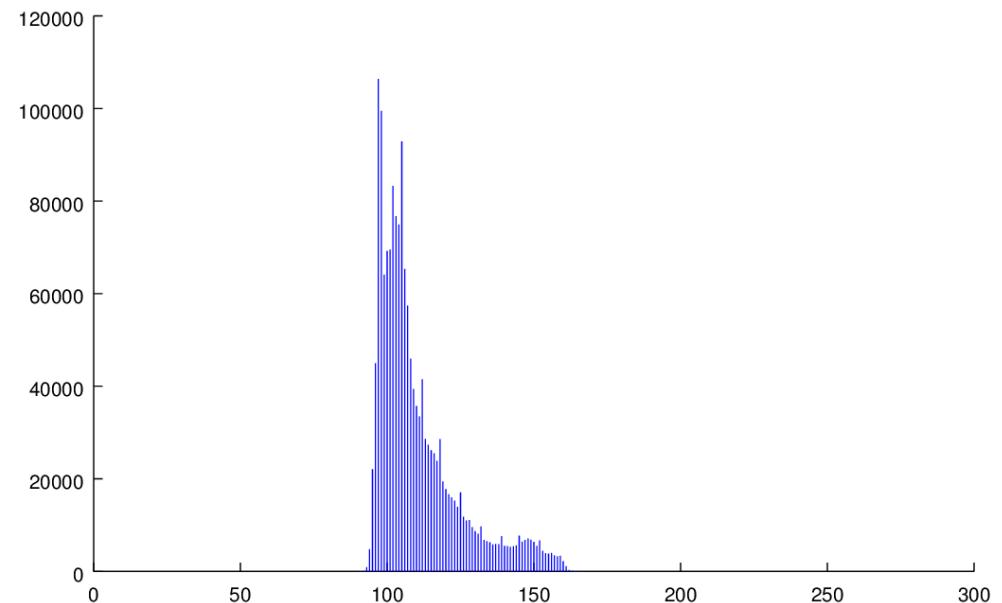
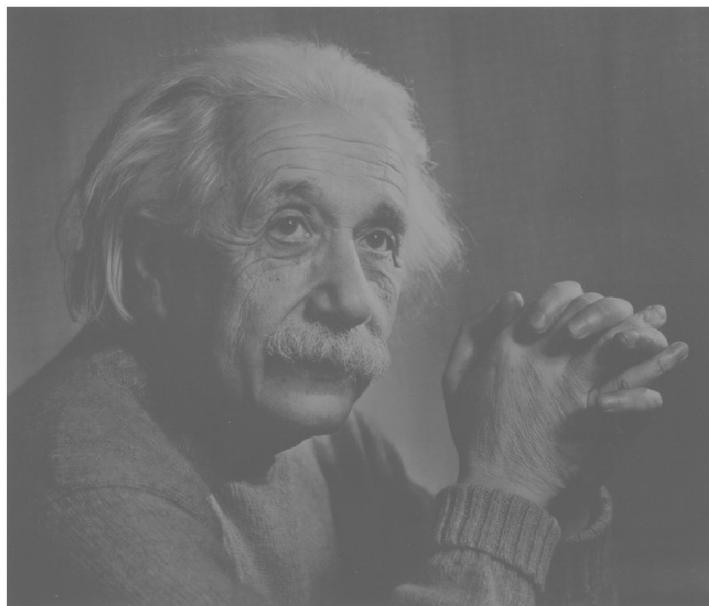
Histograma de Imagens Digitais

Imagen clara



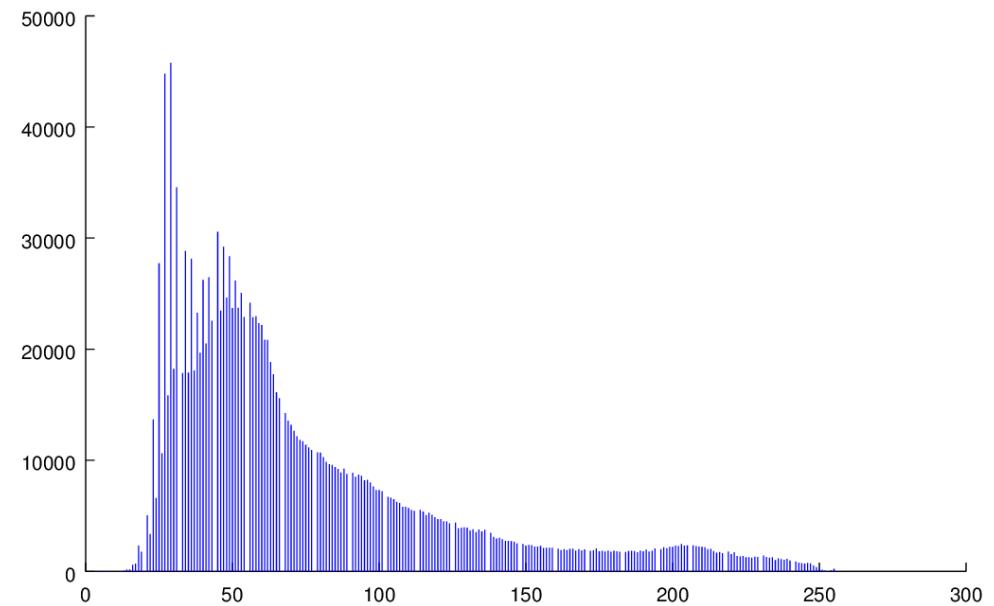
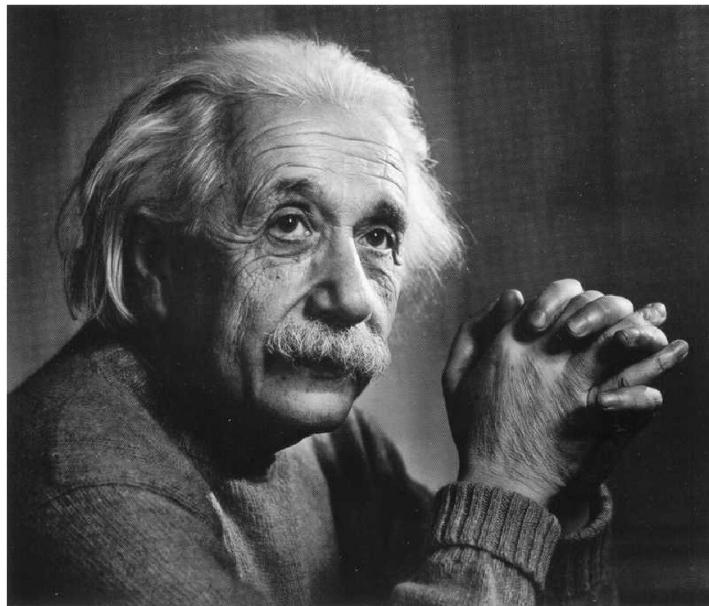
Histograma de Imagens Digitais

Imagen com baixo contraste



Histograma de Imagens Digitais

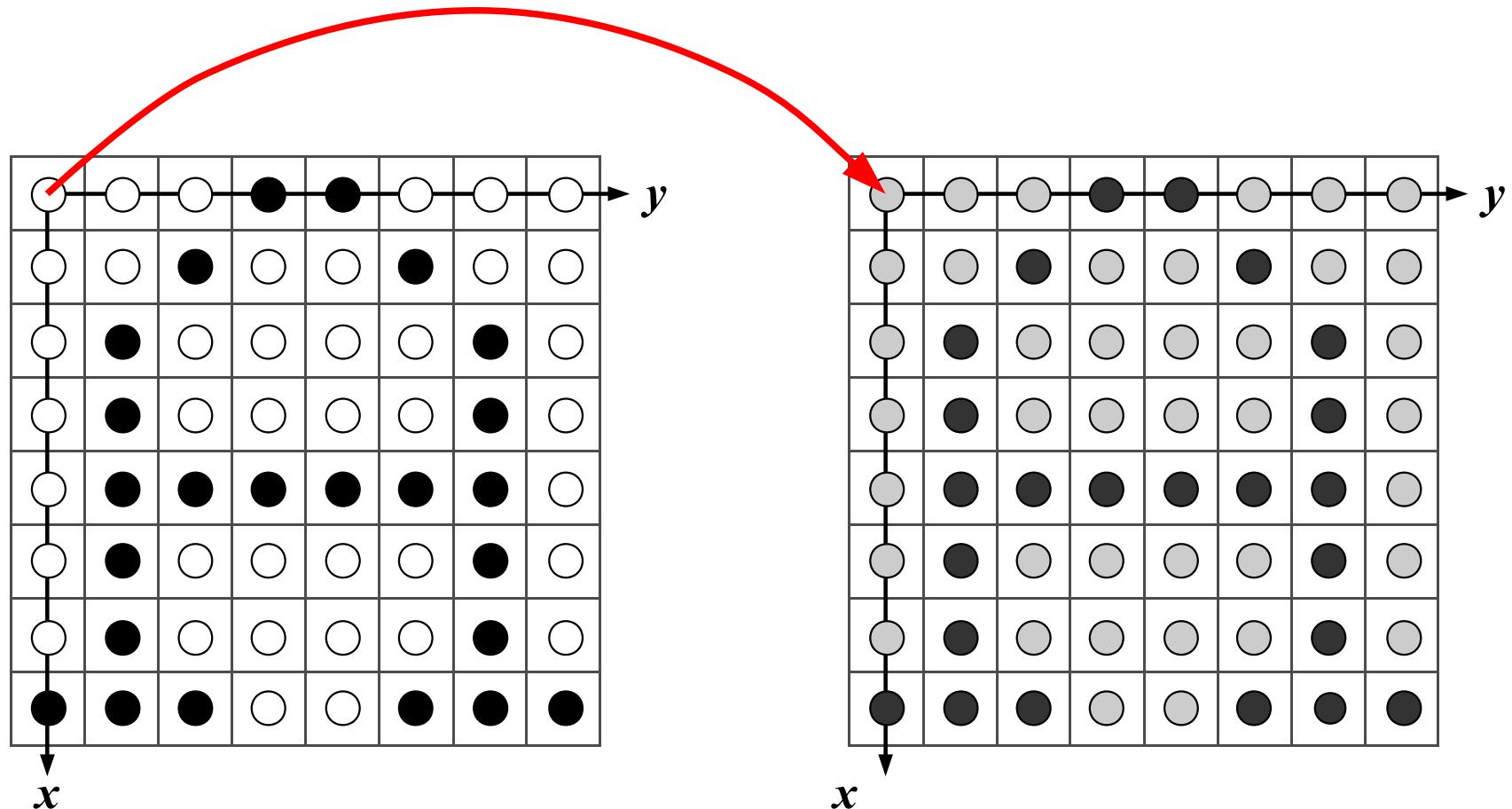
Imagen com alto contraste



Operações Pontuais

Operações Pontuais: $c'_i = g(c_i)$

$g : c_i \rightarrow c_i'$ (função de transformação)



$$f : \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C \quad f' : \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C'$$

$$g(f) \Leftrightarrow f'$$

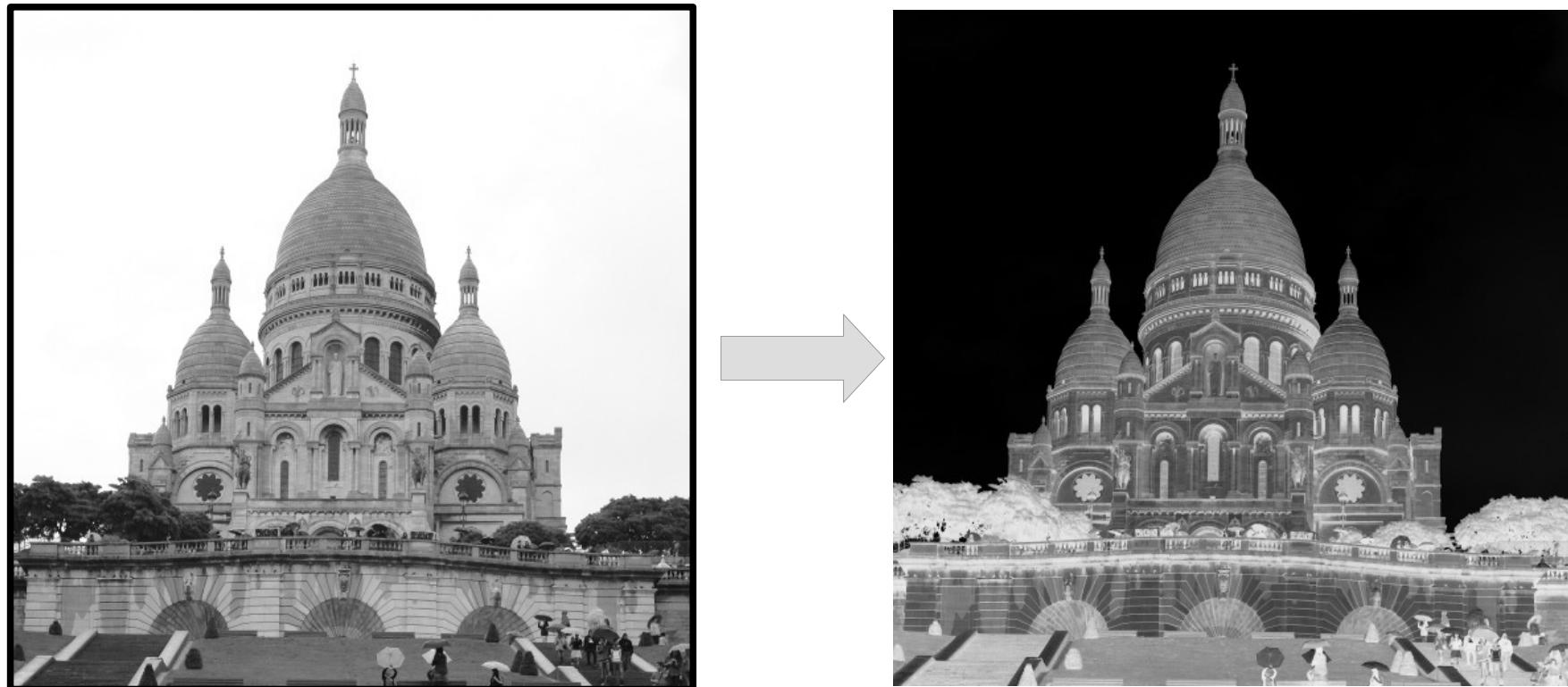
Operações Pontuais: $c_i' = g(c_i)$



$$f: \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C \quad f': \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C'$$

$$g: c_i \rightarrow c_i' \quad g(f) \Leftrightarrow f'$$

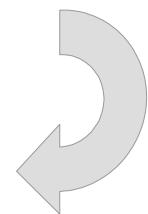
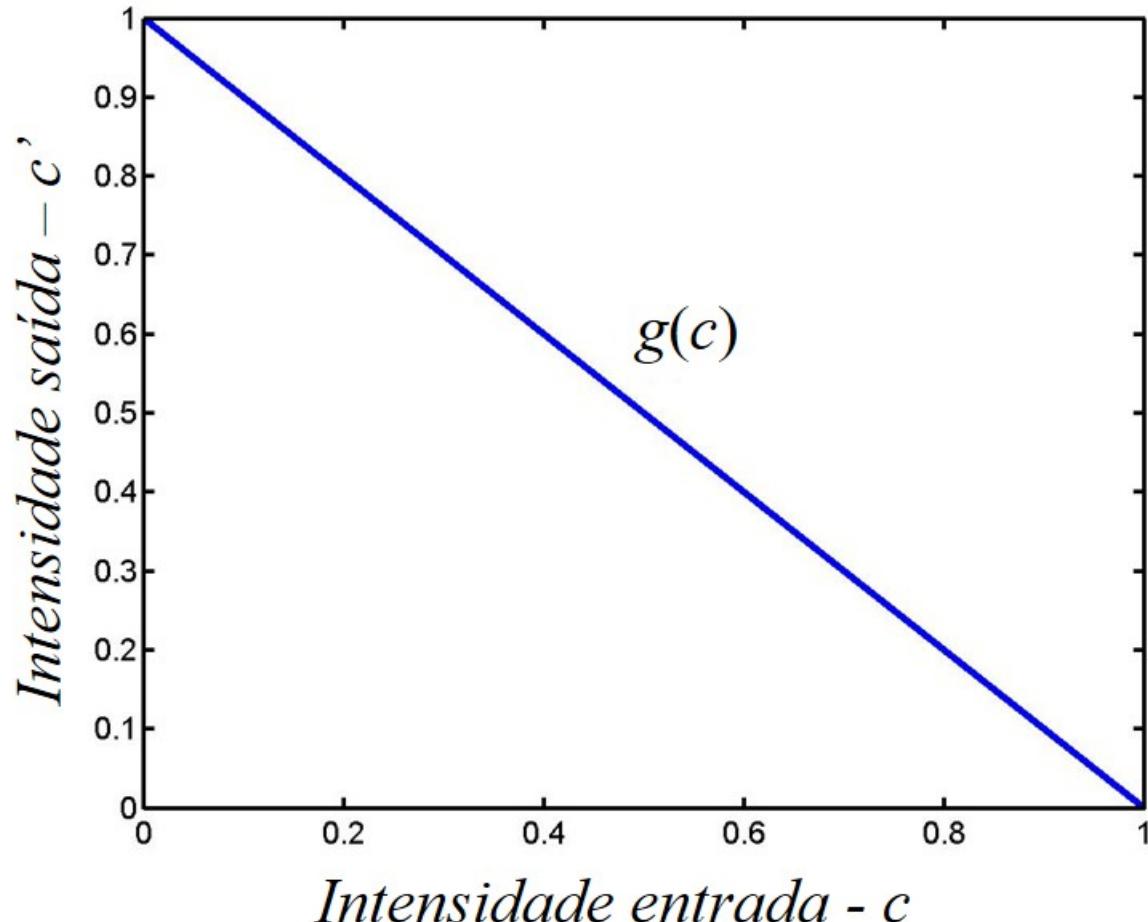
Operações Pontuais: Negativo



$$f: \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C \quad f': \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C'$$

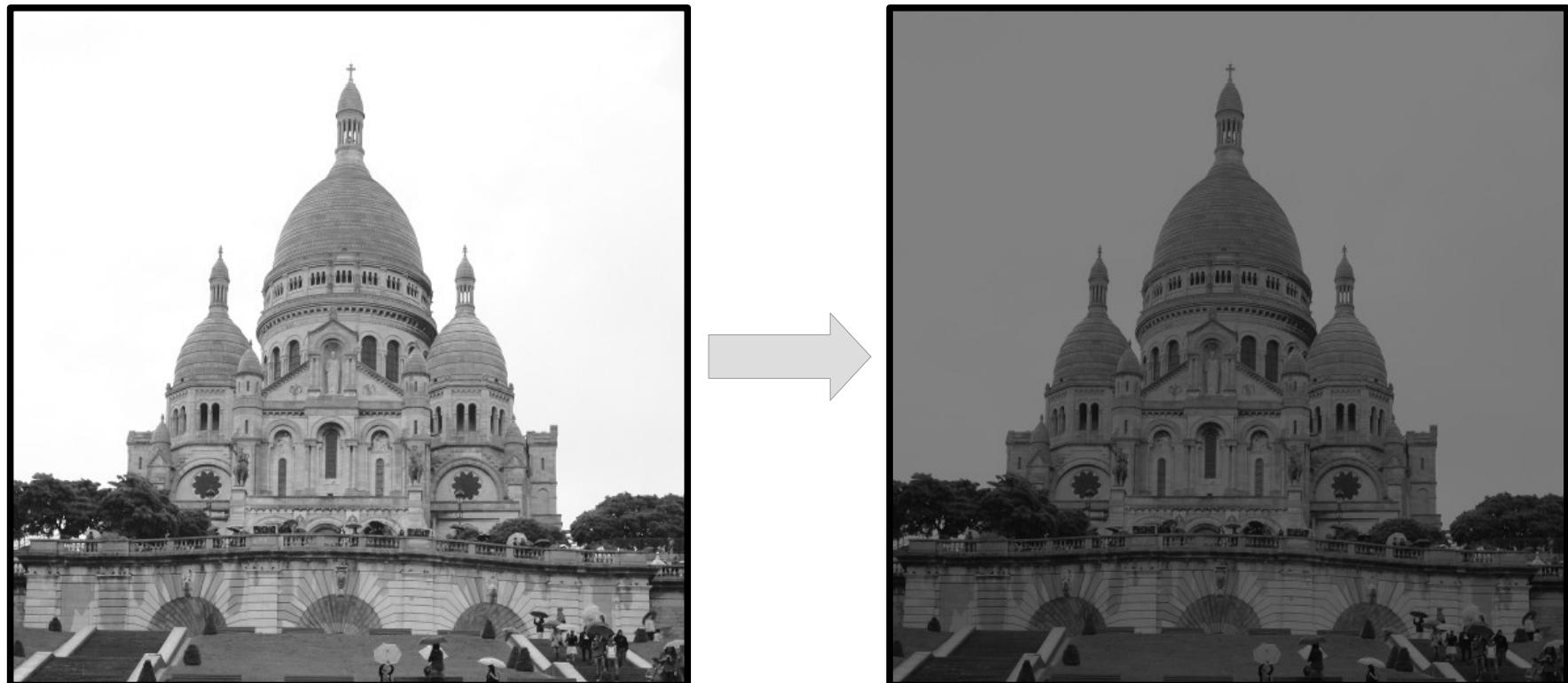
$$g: c_i \rightarrow c_i'; \quad c_i' = 1 - c_i$$

Operações Pontuais: Negativo



$$g: c_i \rightarrow c_i'; \quad c_i' = 1 - c_i$$

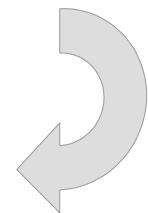
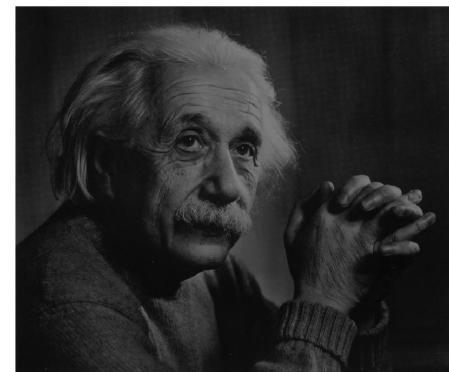
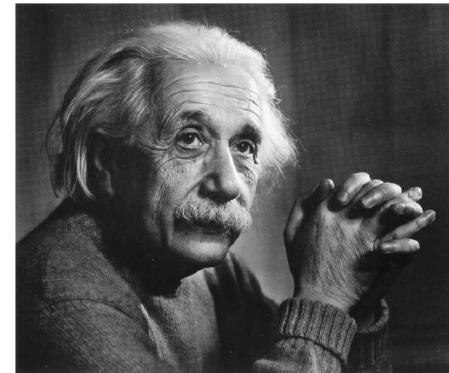
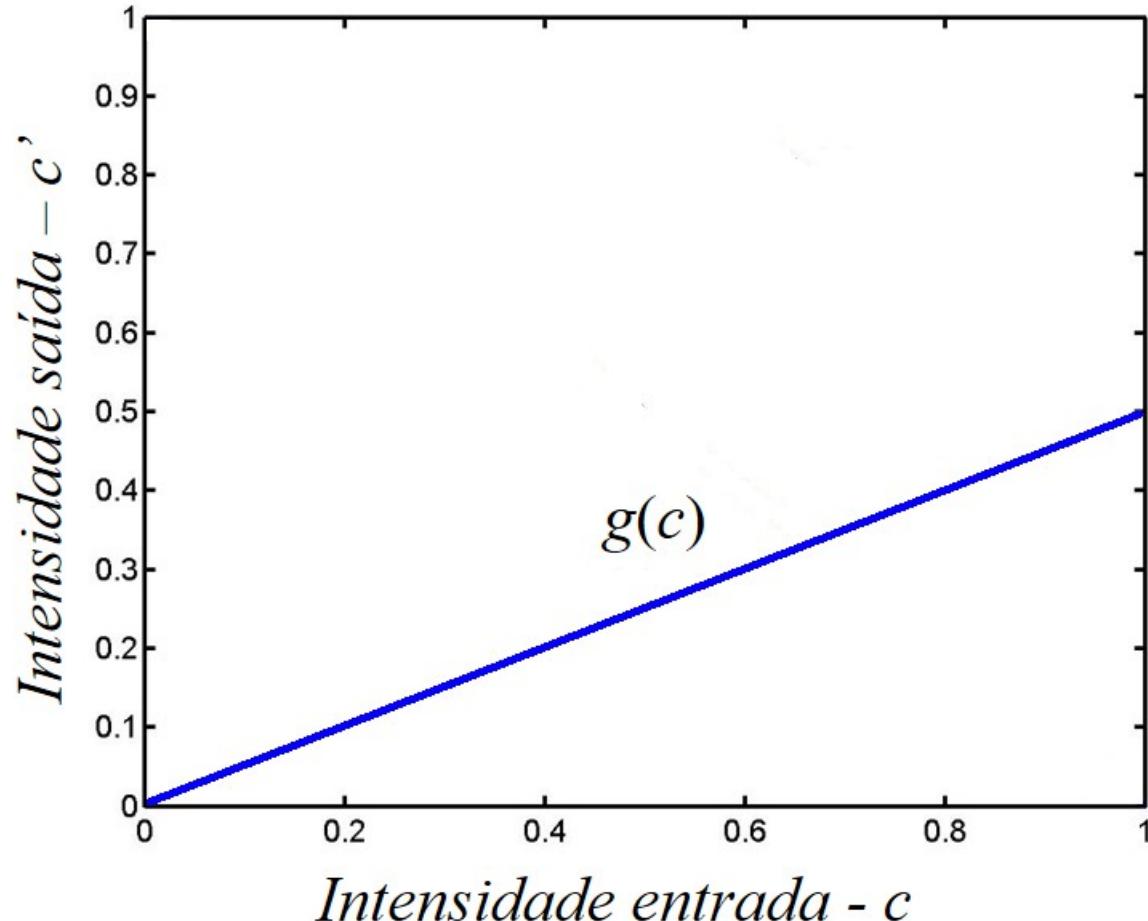
Operações Pontuais: $c'_i = c_i/2$



$$f: \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C \quad f': \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C'$$

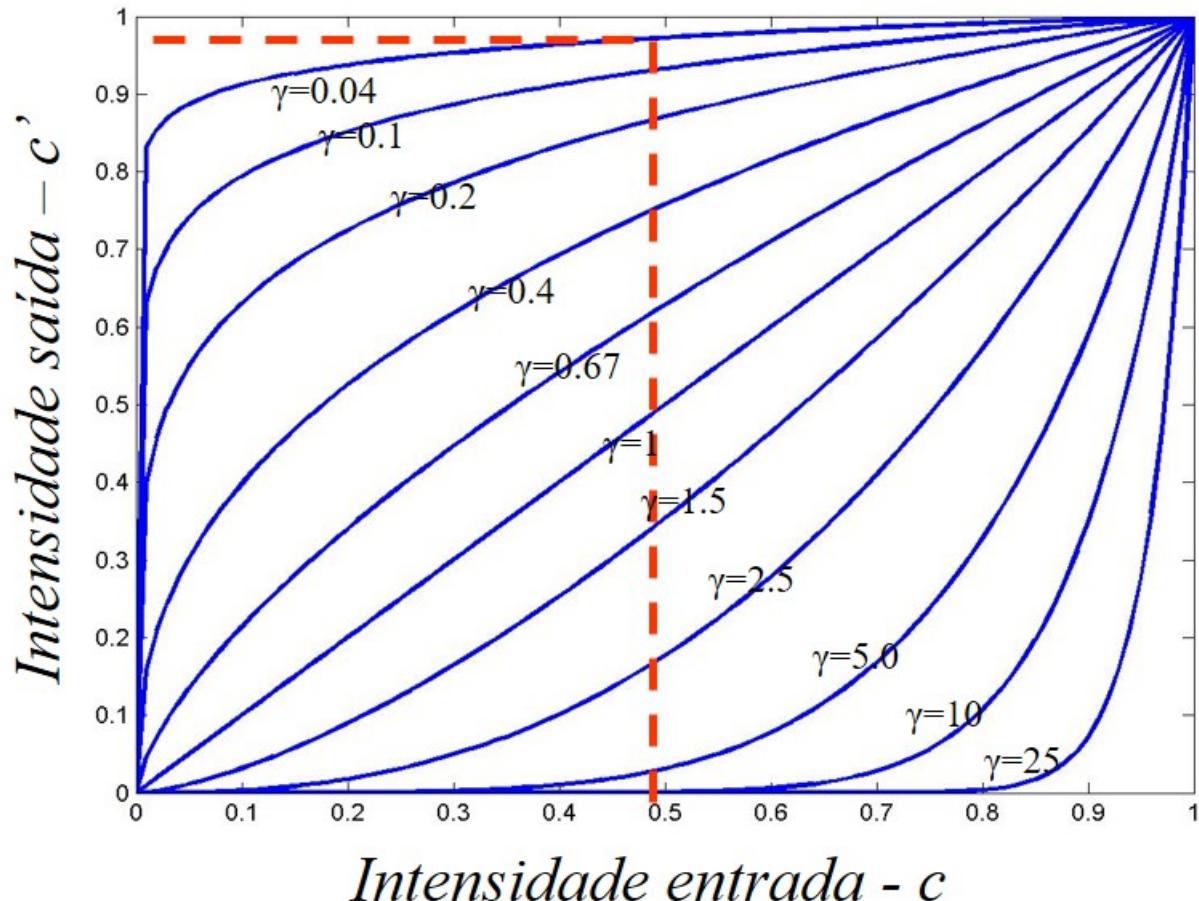
$$g: c_i \rightarrow c'_i; \quad c'_i = \frac{c_i}{2}$$

Operações Pontuais: $c'_i = c_i/2$



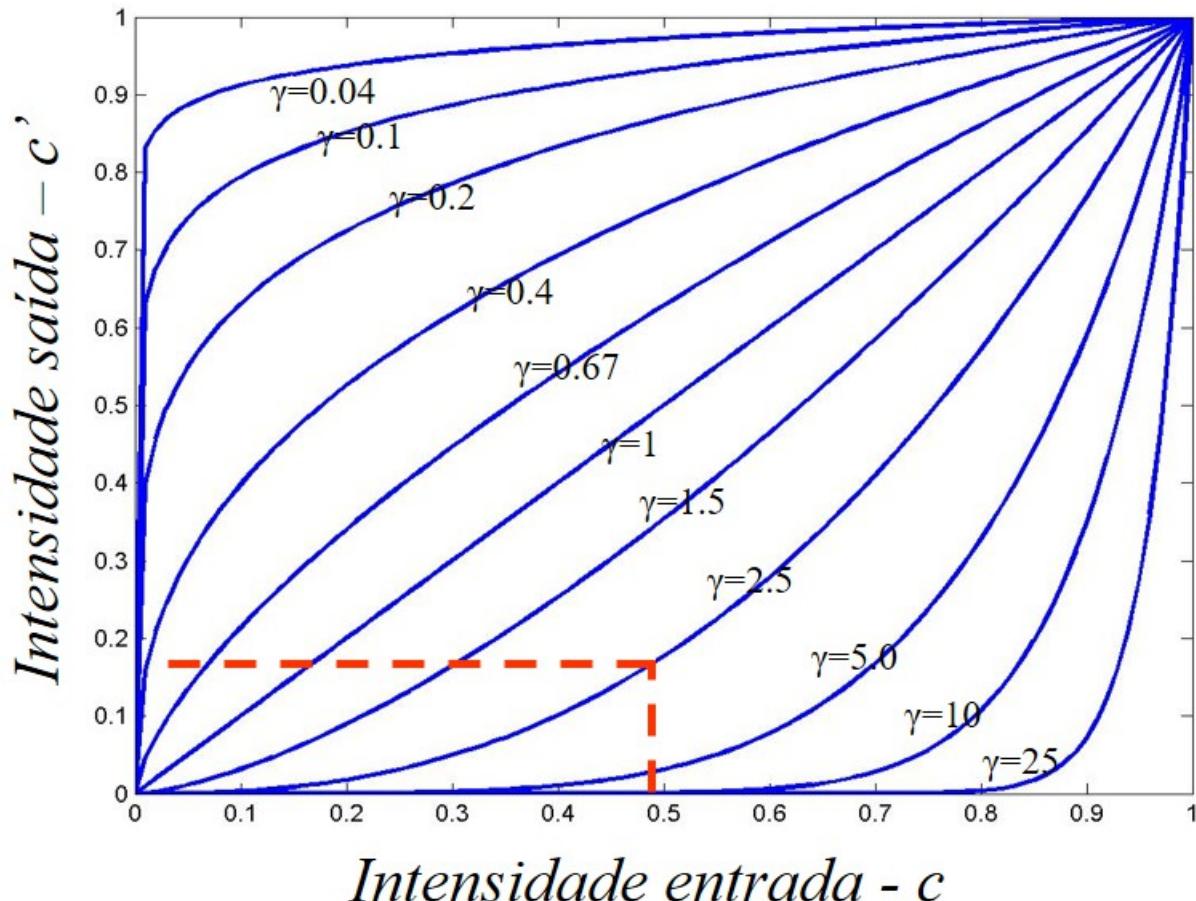
$$g: c_i \rightarrow c'_i; \quad c'_i = \frac{c_i}{2}$$

Operações Pontuais: Correção Gamma



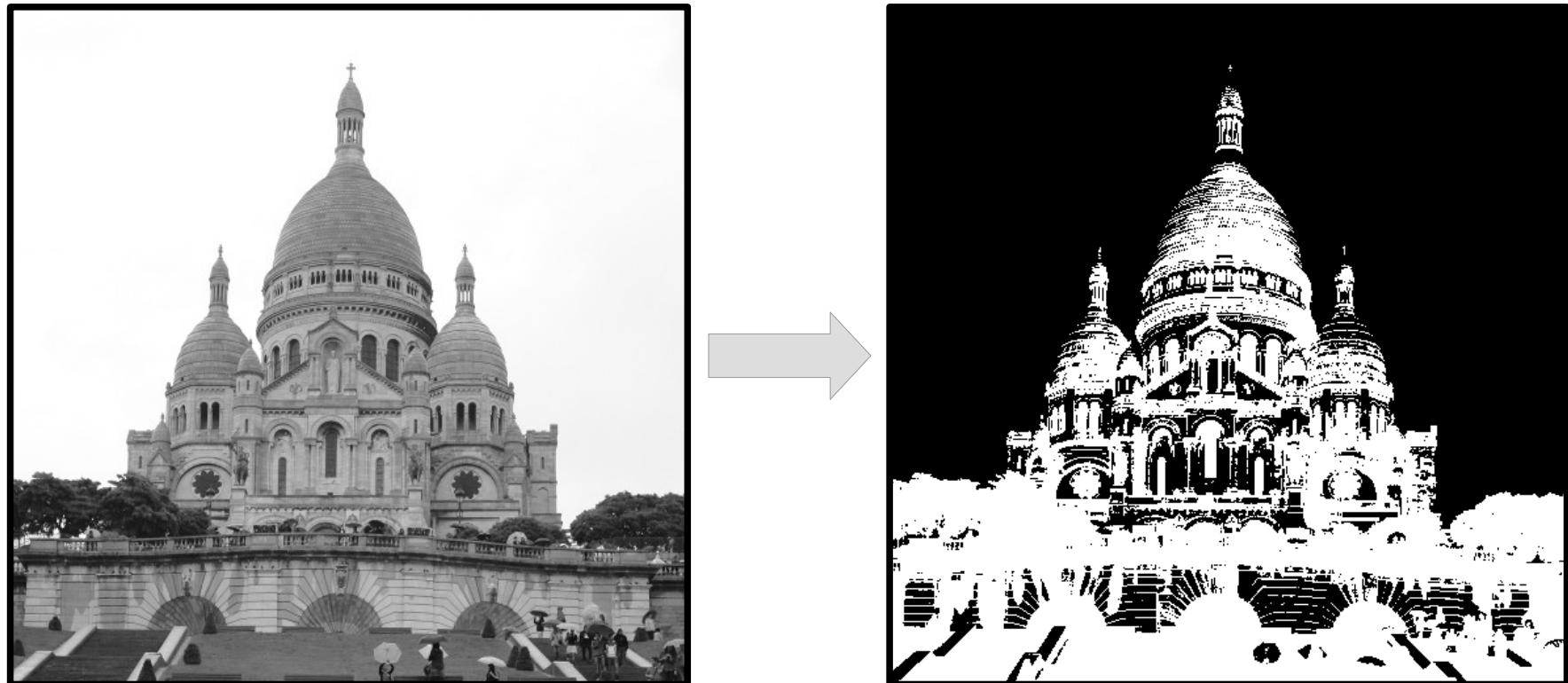
$$g: c_i \rightarrow c_i' ; \quad c_i' = c_i^\gamma \text{ para } \gamma = 0,04$$

Operações Pontuais: Correção Gamma



$$g: c_i \rightarrow c'_i; \quad c'_i = c_i^\gamma \text{ para } \gamma = 2,5$$

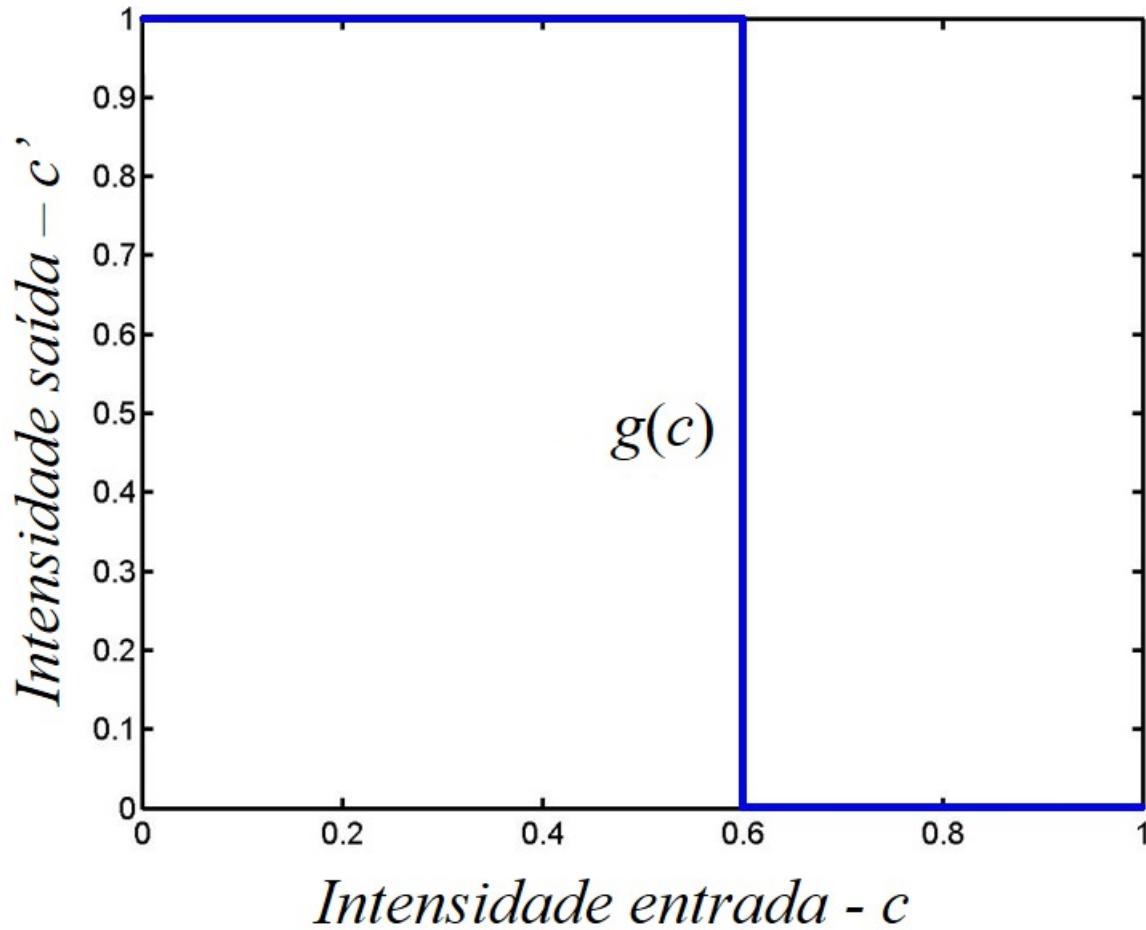
Operações Pontuais: Limiarização



$$f: \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C \quad f': \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C'$$

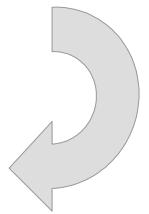
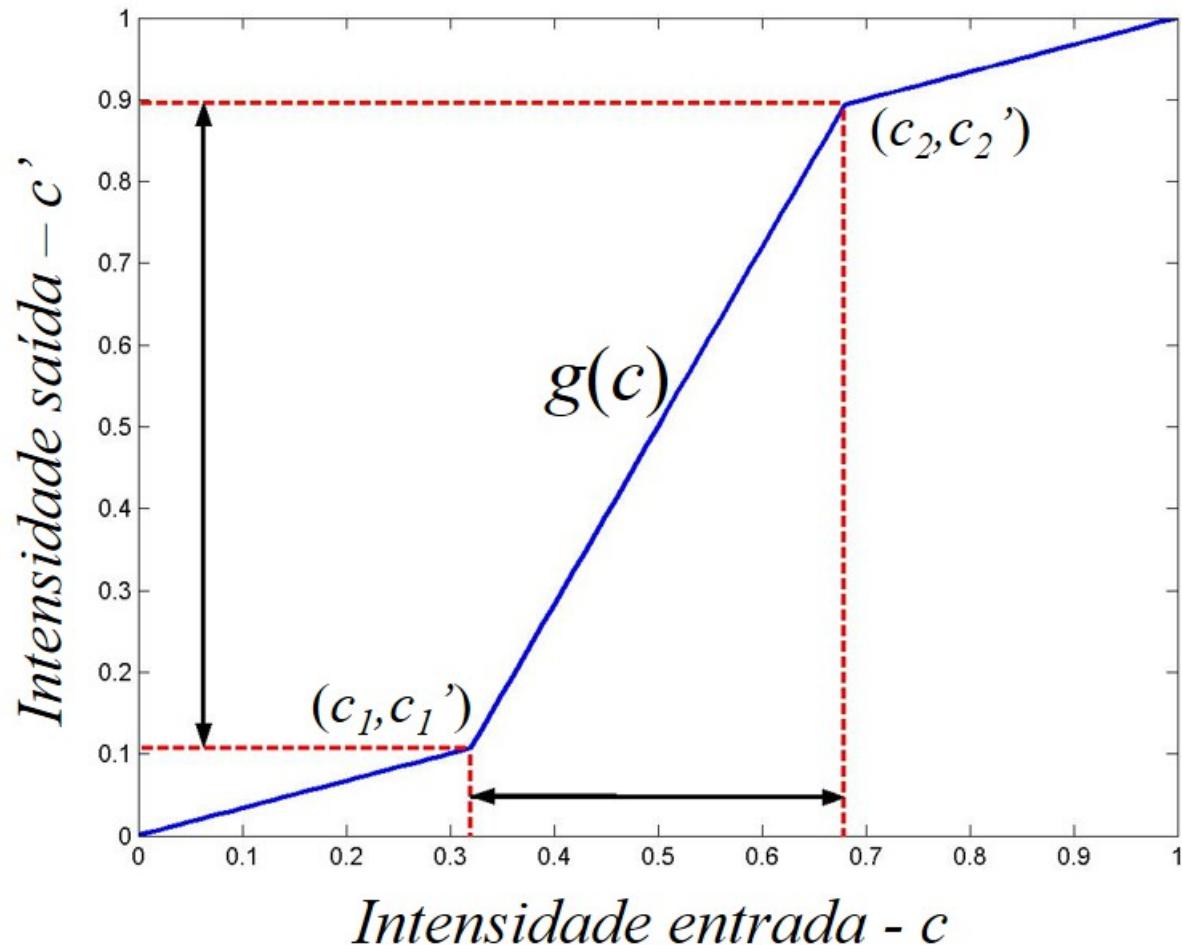
$$g: c_i \rightarrow c_i'; \quad c_i' = c_i < 0.6$$

Operações Pontuais: Limiarização

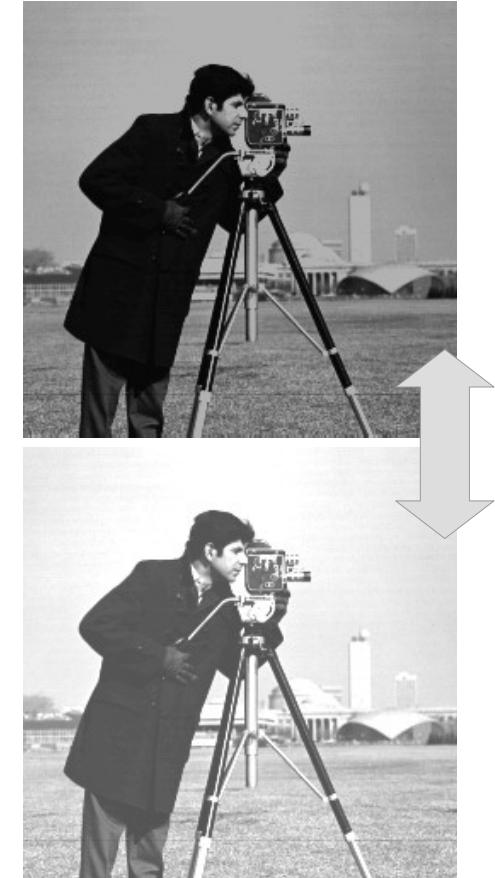
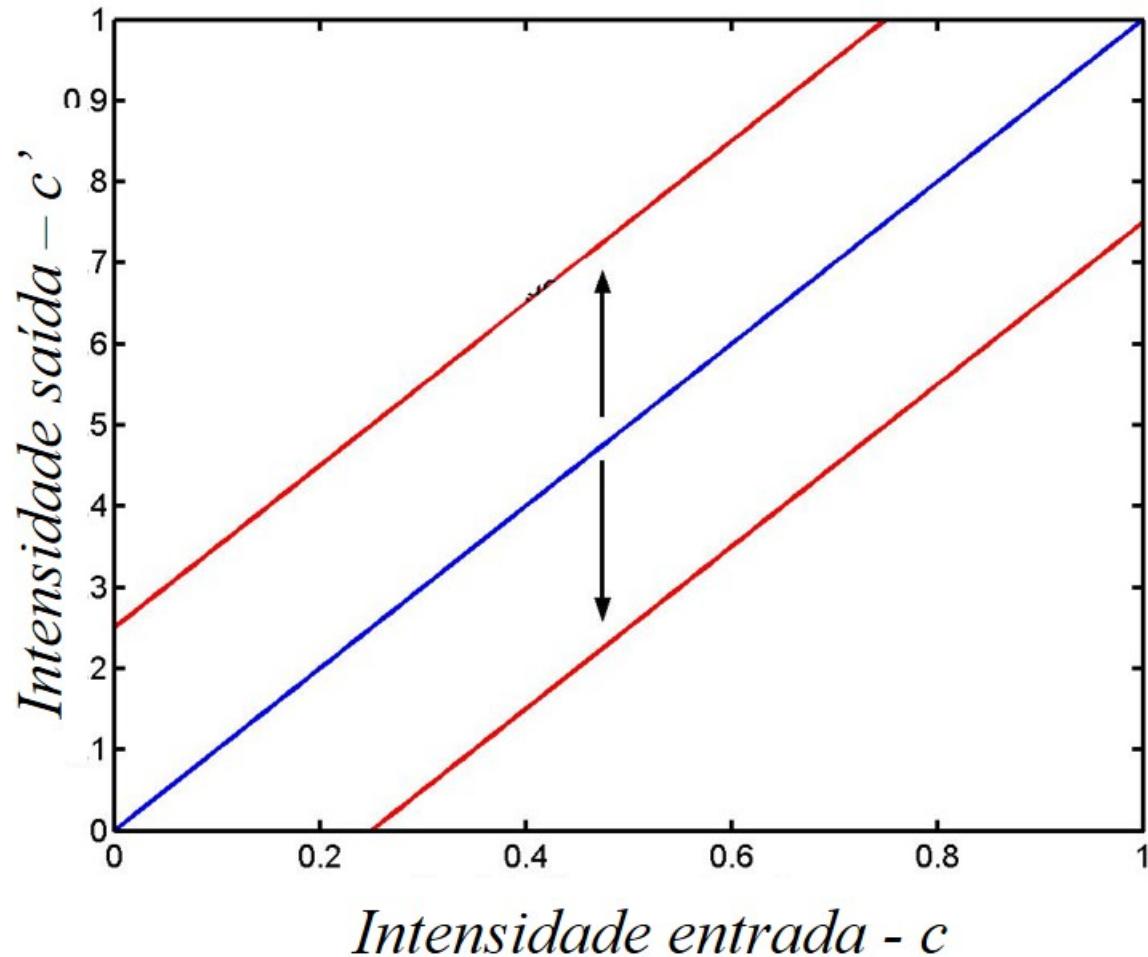


$$g: c_i \rightarrow c_i'; c_i' = c_i < 0.6$$

Operações Pontuais: Alargamento de Contraste



Operações Pontuais: Ajuste de Brilho



$$g: c_i \rightarrow c_i'; \quad c_i' = c_i \pm b$$

Filtros Espaciais Lineares

Filtro Espacial Linear

Uma máscara é posicionada sobre cada pixel da imagem de entrada:

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Intensidade do pixel na imagem de saída é:

$$f'(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

Filtros Espaciais Lineares

Exemplo: filtro de média (suavização)

36	36	36	36	36
36	36	45	45	54
36	36	45	54	54
36	45	54	54	63
36	45	54	63	63

$f(x,y)$



1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$w(s,t)$

?	?	?	?	?
?	38	41	45	?
?	41	46	52	?
?	43	50	56	?
?	?	?	?	?

$f'(x,y)$

$$f'(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

Filtros Espaciais Lineares (Suavização)

Filtros de suavização: borram a imagem

A soma dos elementos da máscara é igual a 1

1/9

1	1	1
1	1	1
1	1	1

Filtro de média 3×3

1/88

1	7	1
7	54	7
1	7	1

Filtro gaussiano 3×3

Filtros Espaciais Lineares (Suavização)

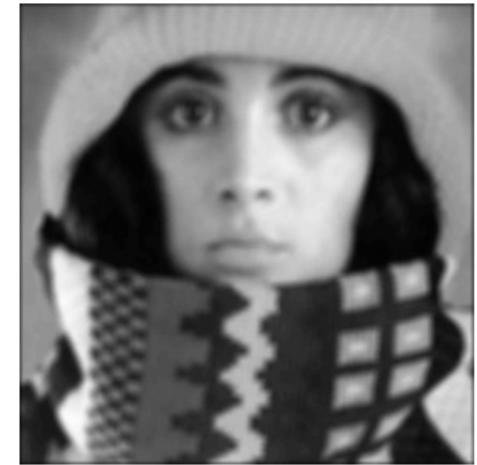
Exemplos: filtros de média



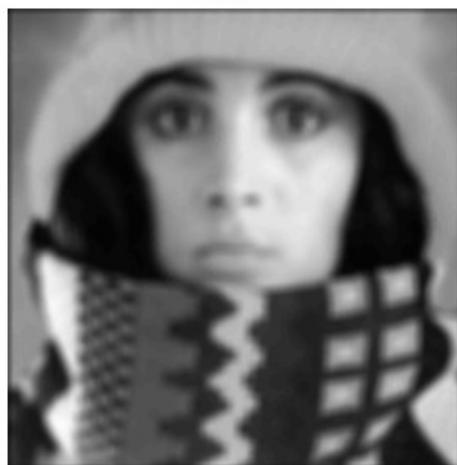
original



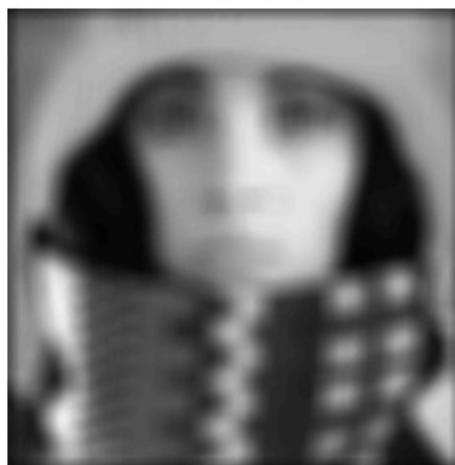
3×3



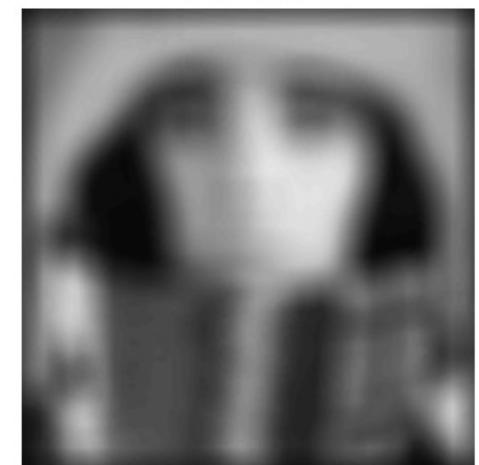
5×5



7×7



15×15



25×25

Filtros Espaciais Lineares (Aguçamento)

Filtros de aguçamento: tornam as bordas dos objetos na imagem mais claras

Exemplo: filtros Laplacianos

0	$-C$	0
$-C$	$4C+1$	$-C$
0	$-C$	0

$-C$	$-C$	$-C$
$-C$	$8C+1$	$-C$
$-C$	$-C$	$-C$

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

Filtros Espaciais Lineares (Aguçamento)



Filtros Espaciais Lineares (Aguçamento)



Filtros Espaciais Lineares (Aguçamento)



Filtros Espaciais Lineares (Aguçamento)



Filtros Espaciais Lineares (Aguçamento)

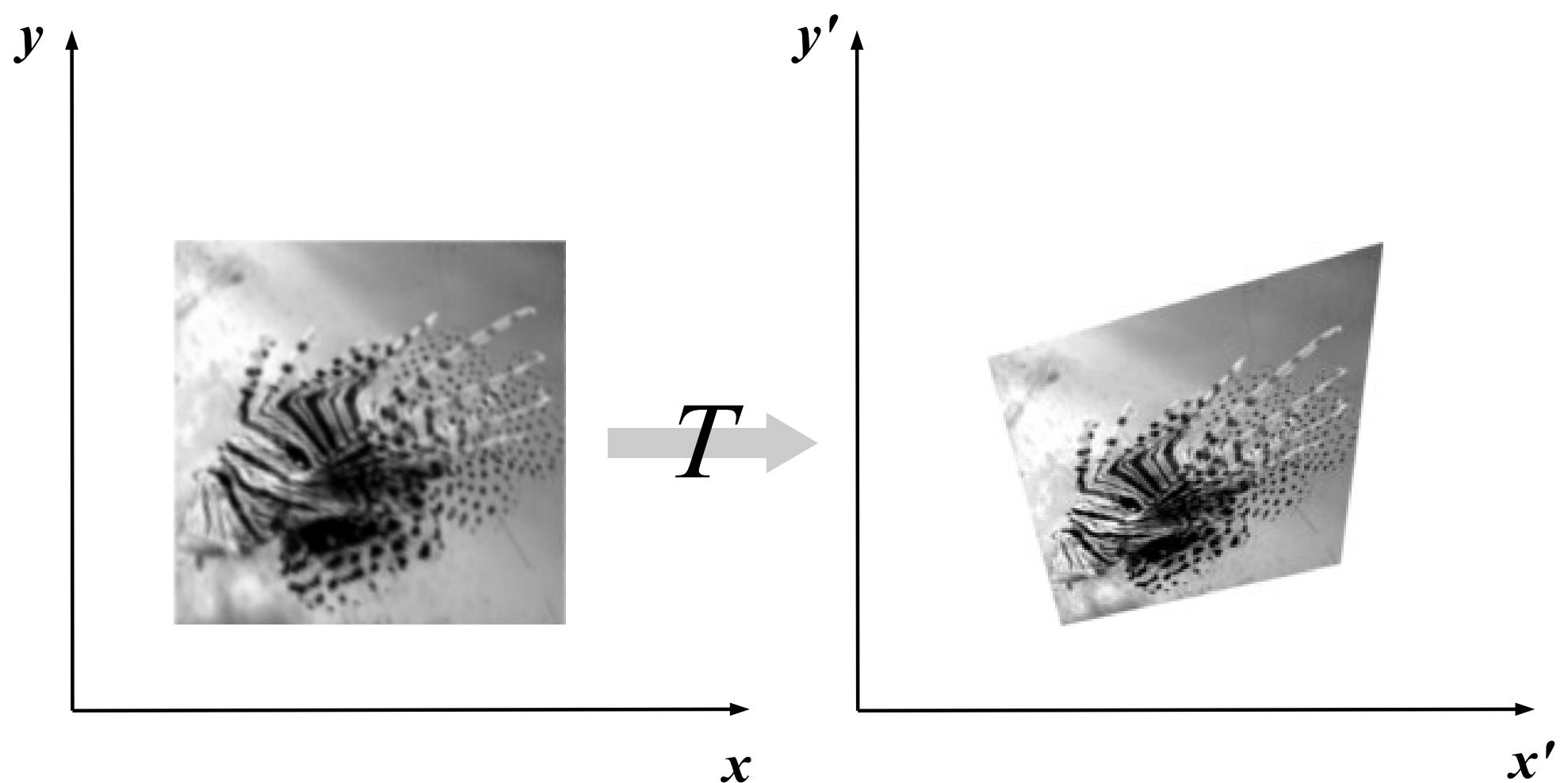


Filtros Espaciais Lineares (Aguçamento)

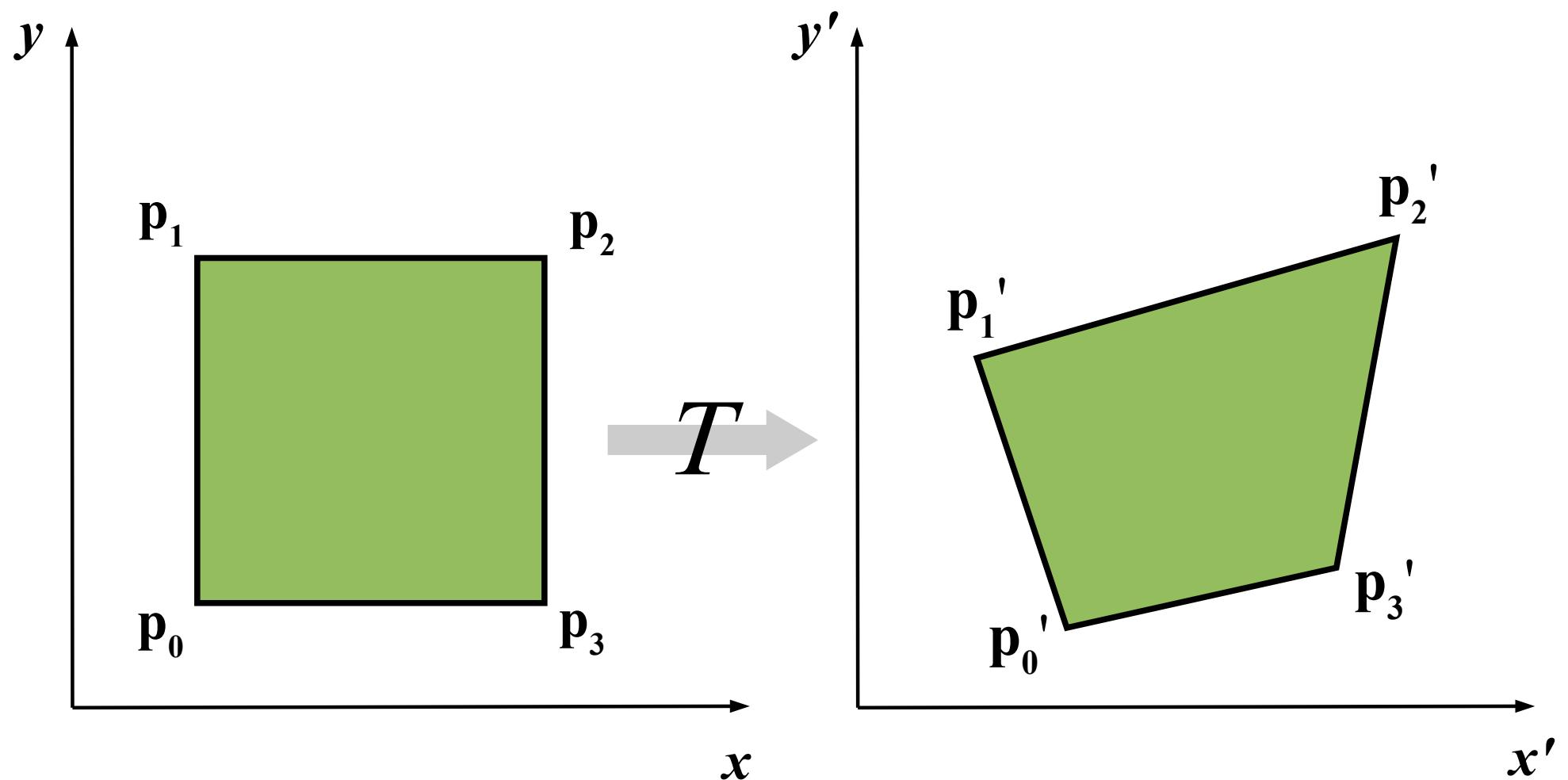


Transformações Geométricas

Transformação Geométrica de uma Imagem



Transformação Geométrica de uma Imagem



Transformação Geométrica de uma Imagem

- Sejam:

$$f: \{(x, y) \in U \subset \mathbb{R}^2\} \rightarrow C$$

$$f': \{(x', y') \in U' \subset \mathbb{R}^2\} \rightarrow C$$

$$\mathbf{T}: \{(x, y) \in \mathbb{R}^2\} \rightarrow \{(x', y') \in \mathbb{R}^2\}$$

$$\mathbf{T}^{-1}: \{(x', y') \in \mathbb{R}^2\} \rightarrow \{(x, y) \in \mathbb{R}^2\}$$

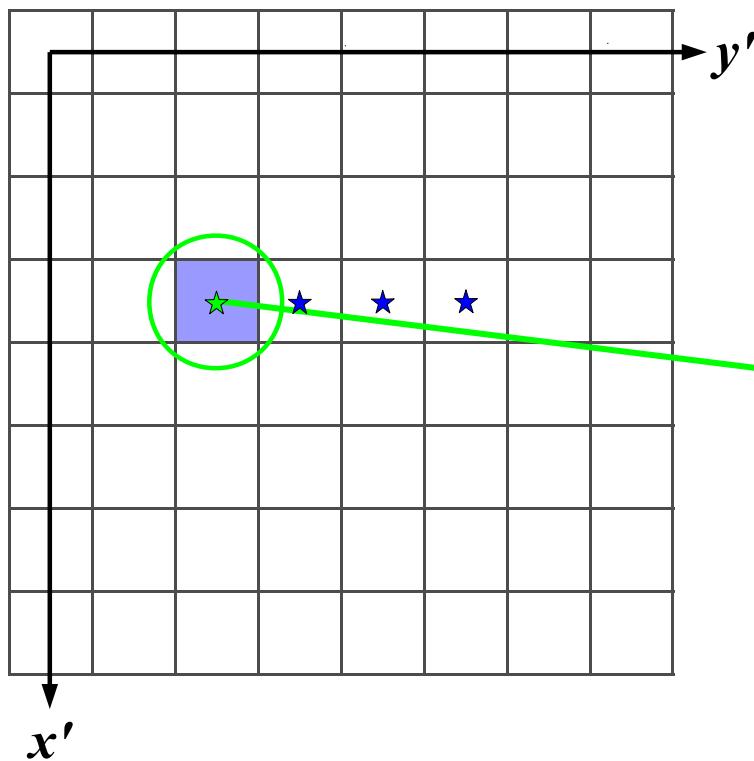
- Algoritmo de Transformação

$$\forall (x_i', y_i') \in U': \mathbf{T}^{-1}(x_i', y_i') \Rightarrow (x_i, y_i)$$

$$f_r = \sum_j \sum_k f_{jk} \phi(x - j, y - k)$$

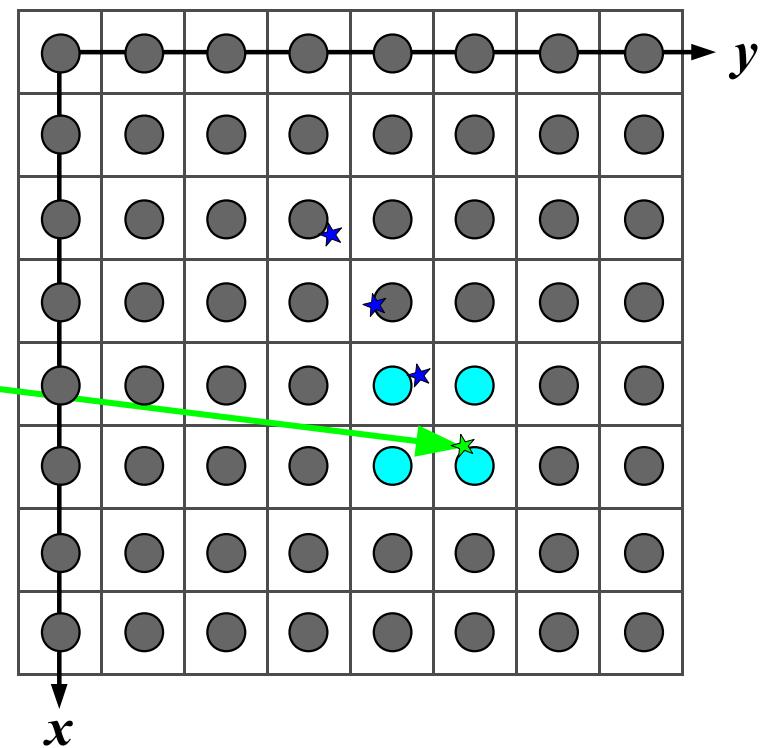
Algoritmo Básico de Transformação Geométrica

Suporte da imagem transformada

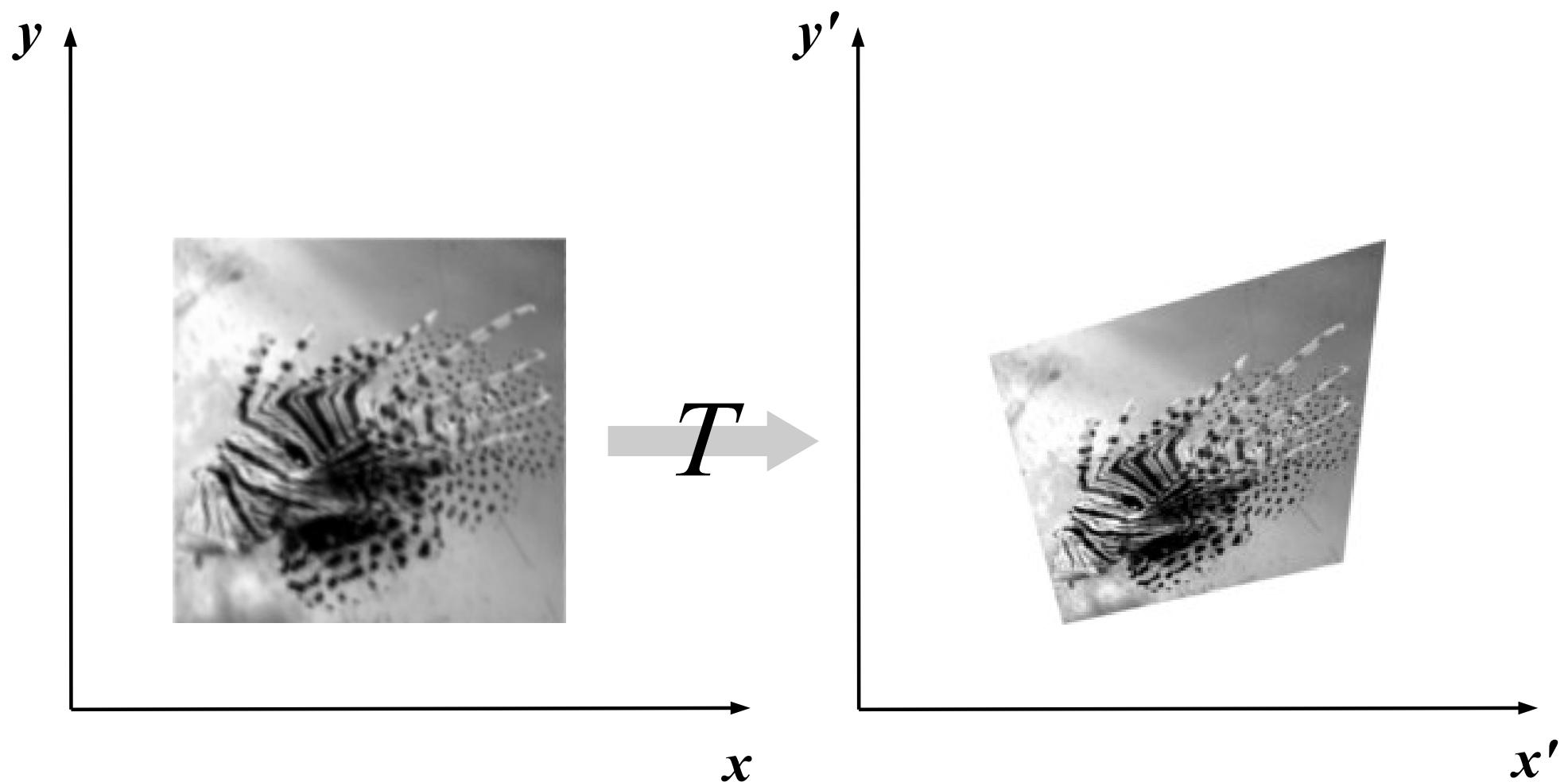


$$T^{-1}$$

Suporte da imagem original

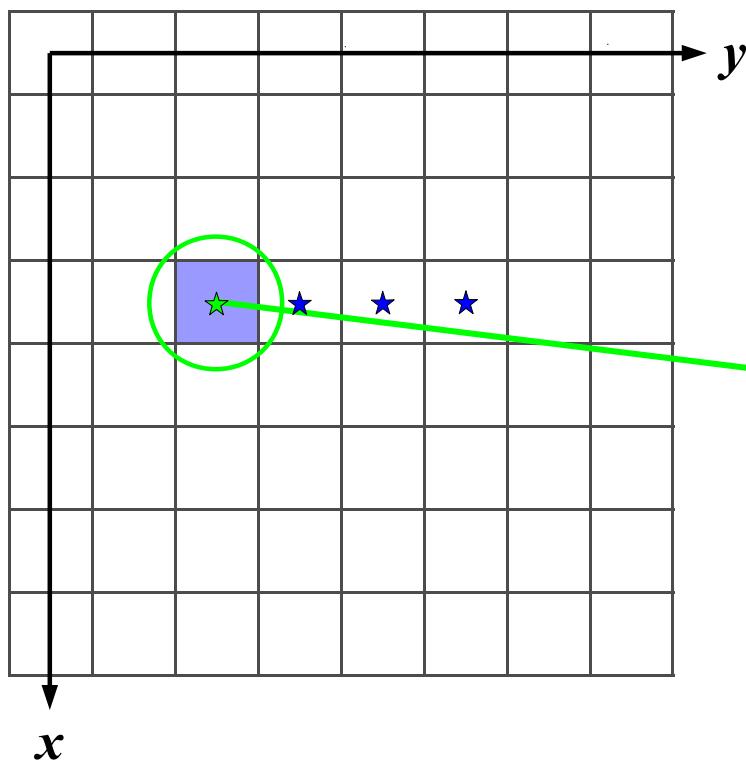


Reconstrução da Imagem Transformada



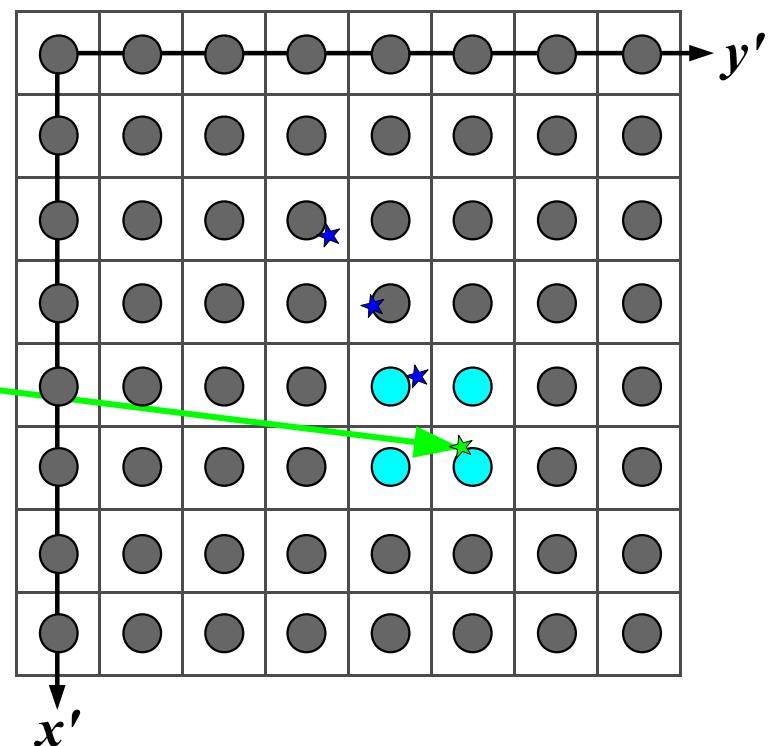
Reconstrução da Imagem Transformada

Suporte da imagem
original



T

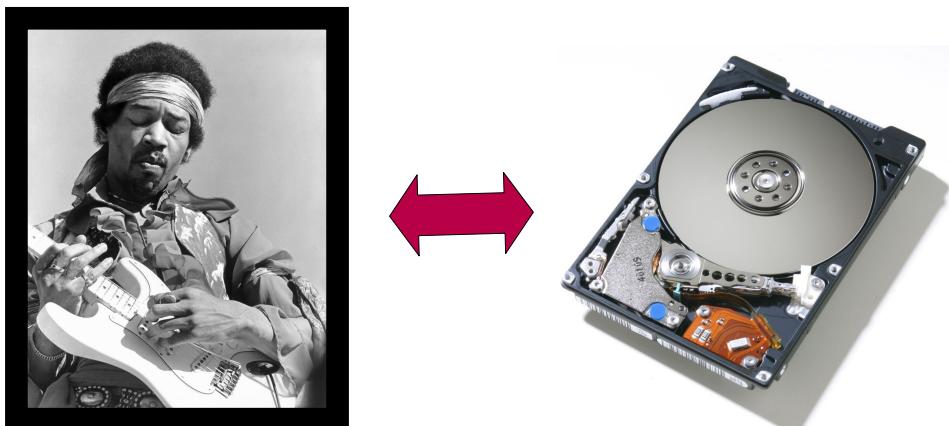
Suporte da imagem
Transformada



Persistência de Imagens Digitais

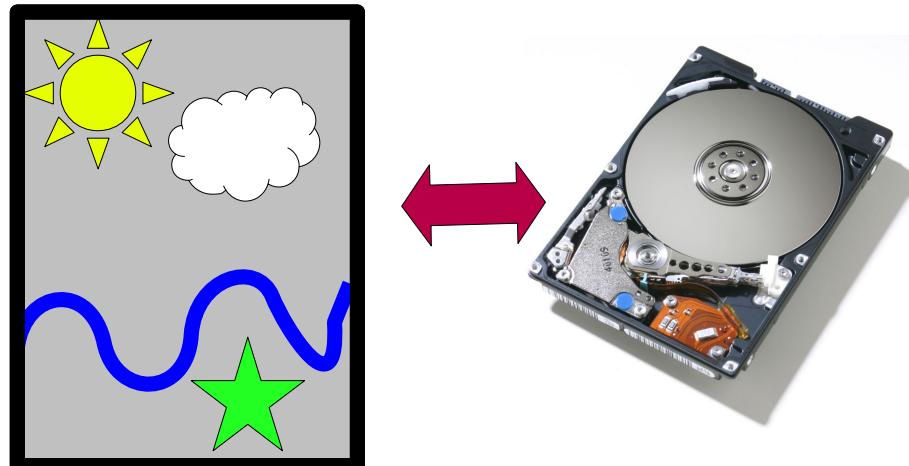
Formatos raster de arquivos de imagem

- Não comprimido
 - BMP (Windows bitmap)
 - Família PNM
(Portable Any Map)
 - PBM (binário)
 - PGM (tons de cinza)
 - PPM (pixelmap)
- Comprimido
 - Com perda de informação
 - JPEG (Joint Photographic Experts Group)
 - Sem perda de informação
 - TIFF (Tagged Image File Format) Compressão LZW opcional
 - GIF (Graphics Interchange Format)
 - PNG (Portable Network Graphic) Sucessor open source do GIF



Formatos vetoriais de arquivos de imagem

- SVG (Scalable Vector Graphics) – Padrão aberto criado e mantido pelo W3C (World Wide Web Consortium)
- PDF (Portable Document File)
- CDR – Formato proprietário do Corel Draw não existe documento público de descrição deste formato
- EPS (Encapsulated PostScript)



Fim