§6.7 Exercícios

1. Considere a integral de linha $\int_C (y^2 - xy)dx + k(x^2 - 4xy)dy$.

a) Determine a constante k para que esta integral seja independente do caminho.

b) Calcule o valor da integral de $A = (0,0)$ a $B = (1,1)$ para o valor de k encontrado em a).

2. Verifique que as seguintes integrais independem do caminho e calcule seus valores.

a) $\displaystyle\int_{(1,-2)}^{(3,4)} \frac{ydx - xdy}{x^2}$.

b) $\displaystyle\int_{(0,2)}^{(1,3)} \frac{3x^2}{y}dx - \frac{x^3}{y^2}dy.$

c) $\displaystyle\int_{(1,1)}^{(x_0,y_0)} 2xydx + (x^2 - y^2)dy.$

d) $\displaystyle\int_{(0,0)}^{(x_0,y_0)} \operatorname{sen} ydx + x\cos ydy.$

3. a) Caso exista, encontre uma função potencial para $V(x,y) =$
$= (2xy^3 - y^2\cos x, 1 - 2y \operatorname{sen} x + 3x^2y^2)$.

b) Calcule $\int_C (2xy^3 - y^2\cos x)dx + (1 - 2y\operatorname{sen} x + 3x^2y^2)dy$, onde C é o arco da parábola $2x = \pi y^2$, de $P_1 = (0,0)$ a $P_2 = \left(\frac{\pi}{2}, 1\right)$.

4. Calcule $\displaystyle\oint_C \frac{yx^2dx - x^3dy}{(x^2+y^2)^2}$, onde C é a curva dada pela equação $\frac{x^2}{4} + \left(y - \frac{1}{3}\right)$
1, percorrida no sentido anti-horário.

5. Encontre todos os possíveis valores de $\displaystyle\int_C \frac{(x+y)dx + (y-x)dy}{x^2+y^2}$, onde C é uma curva fechada qualquer que não passa pela origem.

6. Mostre que as integrais $\oint_C F_1(x,y)dx + F_2(x,y)dy$ são nulas, quaisquer que sejam os contornos fechados C contidos no domínio das funções F_1 e F_2, onde:

a) $F_1(x,y) = \operatorname{sen} x + 4xy$ e $F_2(x,y) = 2x^2 - \cos y$.

b) $F_1(x,y) = \dfrac{y}{x^2+y^2}$ e $F_2(x,y) = \dfrac{-x}{x^2+y^2}$, e C **não envolve** a origem.

7. Sejam F_1 e F_2 funções com derivadas parciais contínuas no plano xy tais que $\dfrac{\partial F_2}{\partial x} = \dfrac{\partial F_1}{\partial y}$ em $\mathbb{R}^2$, exceto nos pontos (4,0), (0,0) e $(-4,0)$. Indique por C_1, C_2, C_3 e C_4 as circunferências de equações:

$$(x-2)^2+y^2=9\ ,\ (x+2)^2+y^2=9\ ,\ x^2+y^2=25 \text{ e } x^2+y^2=1,$$

respectivamente, orientadas no sentido anti-horário. Sabendo que

$$\oint_{C_1} F_1dx+F_2dy=11,\ \oint_{C_2} F_1dx+F_2dy=9 \text{ e } \oint_{C_3} F_1dx+F_2dy=13,$$

calcule $\oint_{C_4} F_1dx+F_2dy$.

8. Sejam $F_1(x,y)$ e $F_2(x,y)$ funções reais de classe C^1 em $U=\mathbb{R}^2-\{A,B\}$ (A e B como na figura 6.18), tais que $\dfrac{\partial F_1}{\partial y}=\dfrac{\partial F_2}{\partial x}$ em U. Sendo C_1, C_2, C_3 as curvas dadas na figura 6.18, calcule $\oint_{C_3} F_1dx+F_2dy$, supondo que

$$\oint_{C_1} F_1dx+F_2dy=12 \text{ e } \oint_{C_2} F_1dx+F_2dy=15.$$

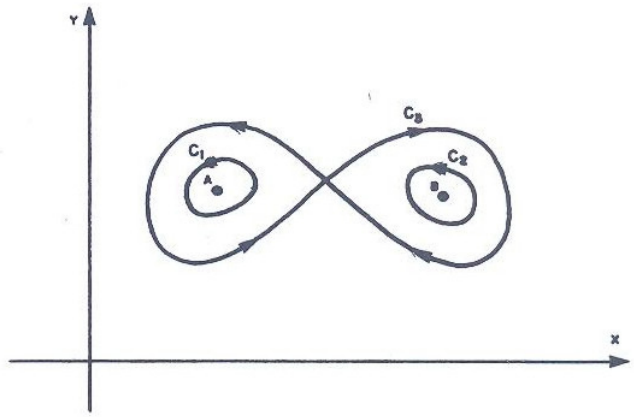

Figura 6.18

9. Seja D a região do plano xy limitada pelas circunferências C_1 e C_2 de equações $x^2+y^2=1$ e $x^2+y^2=25$,respectivamente.Se $F_1(x,y)$ e $F_2(x,y)$ são de classe C^1 em D e $\dfrac{\partial F_2}{\partial x}=\dfrac{\partial F_1}{\partial y}$ em D, quais os possíveis valores da integral $\oint_C F_1dx+F_2dy$, onde C é qualquer curva fechada contida em D, C^1 por partes, sabendo-se que $\oint_{C_1} F_1dx+F_2dy=\oint_{C_2} F_1dx+F_2dy=2\pi$, quando C_1 e C_2 estão orientadas no sentido anti-horário. Justifique sua resposta.

10. Calcule $\oint_C \dfrac{-y}{x^2+y^2}\,dx+\dfrac{x}{x^2+y^2}\,dy$, onde C é a curva definida por $y^2=2(x+2)$, $-2\leq x\leq 2$, orientada no sentido decrescente de y.

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1) a) $$\int_C \underbrace{(y^2 - xy)}_{F_1} dx + \underbrace{K(x^2 - 4xy)}_{F_2} dy$$

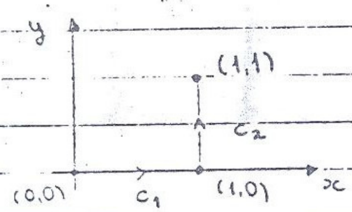
$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

$$K(2x - 4y) = 2y - x$$

$$2K(x - 2y) = -x + 2y$$

$$K = \frac{-x + 2y}{2(x-2y)} = \frac{-1\cancel{(x-2y)}}{2\cancel{(x-2y)}} = \boxed{-\frac{1}{2}}$$

ii) $$\int_{(0,0)}^{(1,1)} (y^2 - xy)\, dx - \frac{1}{2}(x^2 - 4xy)\, dy \quad ⊛$$



$C_1: \sigma_1(t) = (t, 0) \quad 0 \leq t \leq 1$

$\sigma_1'(t) = (1, 0)$

$C_2: \sigma_2(t) = (1, t) \quad 0 \leq t \leq 1$

$\sigma_2'(t) = (0, 1)$

$$⊛ = \int_0^1 \overbrace{(0, -1/2\, t^2)}^{F(\sigma_1(t))} \cdot \overbrace{(1,0)}^{\sigma_1'(t)}\, dt + \int_0^1 \overbrace{\left(t^2 - t, -\frac{1}{2}(1 - 4t)\right)}^{F(\sigma_2(t))} \cdot \overbrace{(0,1)}^{\sigma_2'(t)}\, dt$$

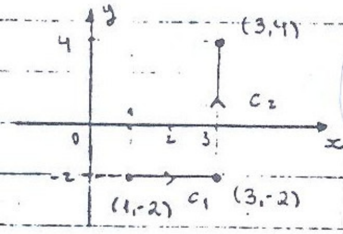
$$= \int_0^1 \overbrace{0\, dt}^{0} - \frac{1}{2}\int_0^1 1 - 4t\, dt =$$

$$= -\frac{1}{2}\left[t - 2t^2\right]_0^1 = -\frac{1}{2}(1 - 2) =$$

$$= -\frac{1}{2}(-1) = \boxed{\frac{1}{2}}$$

② a) $\displaystyle\int_{(1,-2)}^{(3,4)} \underbrace{\frac{y}{x^2}}_{F_1}\, dx - \underbrace{\frac{1}{x}}_{F_2}\, dy$ ⊛

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \quad \therefore \quad \frac{1}{x^2} = \frac{1}{x^2} \quad \text{OK}$$



$\begin{pmatrix}3\\-2\end{pmatrix} + t\begin{pmatrix}0\\6\end{pmatrix} =$

$\begin{pmatrix}1\\-2\end{pmatrix} + t\begin{pmatrix}2\\0\end{pmatrix}$

$C_1: \quad \sigma_1(t) = (t, -2) \quad 1 \leq t \leq 3$

$\sigma_1'(t) = (1, 0)$

$C_2: \quad \sigma_2(t) = (3, t) \quad -2 \leq t \leq 4$

$\sigma_2'(t) = (0, 1)$

$$⊛ = \int_1^3 \left(\frac{-2}{t^2}, \frac{-1}{t}\right) \cdot (1,0)\, dt + \int_{-2}^4 \left(\frac{t}{9}, -\frac{1}{3}\right) \cdot (0,1)\, dt$$

$$= \int_1^3 -\frac{2}{t^2}\, dt + \int_{-2}^4 -\frac{1}{3}\, dt =$$

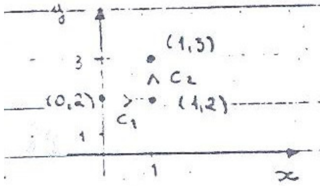
$$= -2\int_1^3 t^{-2}\, dt - \frac{1}{3}\int_{-2}^4 dt =$$

$$= -2\left[-\frac{1}{t}\right]_1^3 - \frac{1}{3}\,[t]_{-2}^4 =$$

$$= -2\left(-\frac{1}{3} + 1\right) - \frac{1}{\cancel{3}_1} \cdot \cancel{6}^2 = -\frac{4}{3} - 2 = \boxed{-\frac{10}{3}}$$

b) $\displaystyle\int_{(0,2)}^{(1,3)} \frac{3x^2}{y}\, dx - \frac{x^3}{y^2}\, dy$ ⊛

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \quad \therefore \quad -\frac{3x^2}{y^2} = -\frac{3x^2}{y^2} \quad \text{OK}$$



$C_1: \sigma_1(t) = (t, 2) \quad 0 \le t \le 1$

$\sigma_1'(t) = (1,0)$

$C_2: \sigma_2(t) = (1, t) \quad 2 \le t \le 3$

$(0,1)$ $(1, 2+t)$

$$⊛ \int_0^1 \left(\frac{3t^2}{2}, -\frac{t^3}{4}\right) \cdot (1,0)\, dt + \int_2^3 \left(\frac{3}{t}, -\frac{1}{t^2}\right) \cdot (0,1)\, dt$$

$$= \int_0^1 \frac{3t^2}{2}\, dt + \int_2^3 -\frac{1}{t^2}\, dt =$$

$$= \frac{3}{2}\left[\frac{t^3}{3}\right]_0^1 + \left[\frac{1}{t}\right]_2^3 =$$

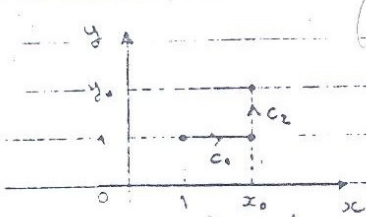
$$= \frac{\cancel{3}}{2} \cdot \frac{1}{\cancel{3}} + \left(\frac{1}{3} - \frac{1}{2}\right) = \cancel{\frac{1}{2}} + \frac{1}{3} - \cancel{\frac{1}{2}} = \boxed{\frac{1}{3}}$$

c) $$\int_{(1,1)}^{(x_0, y_0)} 2xy\, dx + (x^2 - y^2)\, dy \quad ⊛$$

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \quad \therefore \quad 2x = 2x \quad \text{OK}$$

$\begin{pmatrix} x_0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ y_0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} x_0 - 1 \\ \cancel{t}\ 1 \end{pmatrix}$



$C_1: \sigma_1(t) = (t, 1) \quad 1 \le t \le x_0$

$\sigma_1'(t) = (1,0)$

$C_2: \sigma_2(t) = (x_0, t) \quad 1 \le t \le y_0$

$\sigma_2'(t) = (0,1)$

$(1,t)$ $(x_0, 1)$

$$⊛ = \int_1^{x_0} (2t, t^2 - 1)(1,0)\, dt + \int_1^{y_0} (2x_0 t, x_0^2 - t^2)(0,1)\, dt$$

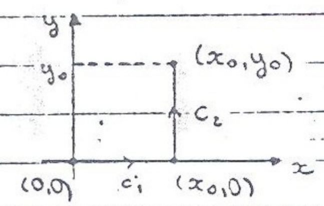
$$= \int_1^{x_0} 2t\,dt + \int_1^{y_0} x_0^2 - t^2\,dt =$$

$$= \left[t^2 \right]_1^{x_0} + \left[x_0^2 t - \frac{t^3}{3} \right]_1^{y_0} =$$

$$= \cancel{x_0^2} - 1 + x_0^2 \cdot y_0 - \frac{y_0^3}{3} - \cancel{x_0^2} + \frac{1}{3} = \boxed{x_0^2 \cdot y_0 - \frac{y_0^3}{3} - \frac{2}{3}}$$

l) $\displaystyle\int_{(0,0)}^{(x_0,y_0)} \sin y\,dx + x\cos y\,dy$ ⊛

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \therefore \cos y = \cos y \quad \text{OK}$$



$C_1: \sigma_1(t) = (t, 0) \quad 0 \le t \le x_0$

$\sigma_1'(t) = (1, 0)$

$C_2: \sigma_2(t) = (x_0, t) \quad 0 \le t \le y_0$

$\sigma_2'(t) = (0, 1)$

$$⊛ = \int_0^{x_0} (0, t.1) \cdot (1,0)\,dt + \int_0^{y_0} (\sin t, x_0 \cos t) \cdot (0,1)\,dt =$$

$$= \int_0^{x_0} \cancel{0\,dt}^{\,0} + \int_0^{y_0} x_0 \cos t\,dt =$$

$$= \left[x_0 . \sin t \right]_0^{y_0} = x_0 . \sin y_0 - x_0 . \cancel{\sin 0}^{\,0} =$$

$$= \boxed{x_0 . \sin y_0}$$

(3) a) $V(x,y) = (2xy^3 - y^2\cos x,\ 1-2y\,\text{sen}\,x + 3x^2y^2)$ 177

integrando

$$\frac{\partial f}{\partial x} = 2xy^3 - y^2\cos x \Rightarrow f(x,y) = x^2y^3 - y^2\,\text{sen}\,x + A(y)$$

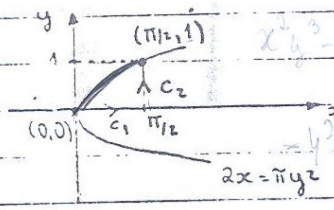
$$\frac{\partial f}{\partial y} = 1 - 2y\,\text{sen}\,x + 3x^2y^2 \Rightarrow f(x,y) = y - y^2\,\text{sen}\,x + x^2y^3 + B(x)$$

$A(y) = y \qquad B(x) = 0$

$$\boxed{f(x,y) = y - y^2\,\text{sen}\,x + x^2y^3}$$

b) $$\int_{(0,0)}^{(\pi/2,1)} (2xy^3 - y^2\cos x)\,dx + (1 - 2y\,\text{sen}\,x + 3x^2y^2)\,dy \quad ⊛$$

onde C é arco da parábola $2x = \pi y^2 \therefore x = y^2 \cdot \frac{\pi}{2}$



$C_1: \sigma_1(t) = (t, 0) \quad 0 \le t \le \pi/2$

$\sigma_1'(t) = (1, 0)$

$C_2: \sigma_2(t) = (\pi/2, t) \quad 0 \le t \le 1$

$\sigma_2'(t) = (0, 1)$

$$\left.\begin{aligned} \frac{\partial F_2}{\partial x} &= -2y\cos x + 6xy^2 \\ \frac{\partial F_1}{\partial y} &= 6xy^2 - 2y\cos x \end{aligned}\right\}$$ logo, a integral independe do caminho

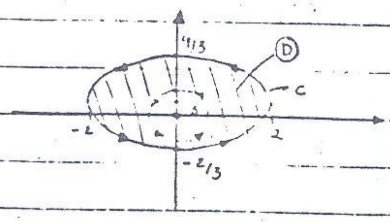
$$⊛ = \int_0^{\pi/2} (0,1)\cdot(1,0)\,dt + \int_0^1 \left(\pi\cdot t^3,\ 1 - 2t + \frac{3\pi^2}{4}t^2\right)\cdot(0,1)\,dt$$

$$= \int_0^{\pi/2} \cancel{0\,dt}^{\,0} + \int_0^1 1 - 2t + \frac{3\pi^2}{4}\cdot t^2\,dt$$

$$= \left[\cancel{t} - t^2 + \frac{2\pi^2}{4} \cdot \frac{t^3}{3} \right]_0^1 = \cancel{1} - \cancel{1} + \frac{\pi^2}{4} = \boxed{\frac{\pi^2}{4}}$$

(4) $$\oint_C \frac{4x^2}{(x^2+y^2)^2}\,dx - \frac{x^3}{(x^2+y^2)^2}\,dy \qquad C: \frac{x^2}{4} + \left(y - \frac{1}{3}\right)^2 = 1$$

sentido anti-horário



$\gamma: x^2 + y^2 = a^2$

$\sigma(\theta) = (a\cos\theta, a\,\text{sen}\,\theta)$

$0 \le \theta \le 2\pi$

$\sigma'(\theta) = (-a\,\text{sen}\,\theta, a\cos\theta)$

$$\oint_{C \cup \gamma^-} F.dr = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy$$

$$\frac{\partial F_2}{\partial x} = \frac{-3x^2(x^2+y^2)^{\cancel{2}} + 2\cancel{(x^2+y^2)}.2x.x^3}{(x^2+y^2)^{\cancel{4}\,3}} = \frac{x^2[\overbrace{-3(x^2+y^2)+4x^2}^{= -3x^2-3y^2+4x^2 \,=\, x^2-3y^2}]}{(x^2+y^2)^3}$$

$$\frac{\partial F_1}{\partial y} = \frac{x^2(x^2+y^2)^{\cancel{2}} - 2\cancel{(x^2+y^2)}.2y.4x^2}{(x^2+y^2)^{\cancel{4}\,3}} = \frac{x^2[\overbrace{(x^2+y^2)-4y^2}^{= x^2-3y^2}]}{(x^2+y^2)^3}$$

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \Rightarrow \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy = 0$$

$$\oint_{C\cup\gamma^-} F.dr = 0 \quad \therefore \quad \oint_C F dr + \oint_{\gamma^-} F dr = 0$$

$$\oint_C F.dr = \oint_\gamma F dr =$$

$$= \int_0^{2\pi} \left(\frac{a^3\,\text{sen}\,\theta \cos^2\theta}{a^4}, -\frac{a^3 \cos^3\theta}{a^4} \right) . (-a\,\text{sen}\,\theta, a\cos\theta)\, d\theta =$$

$$= \int_0^{2\pi} - \frac{\cancel{a^4} \operatorname{sen}^2\theta \cos^2\theta}{\cancel{a^4}} - \frac{\cancel{a^4}\cos^4\theta}{\cancel{a^4}} \, d\theta$$

$$= \int_0^{2\pi} - \cos^2\theta \, (\underbrace{\operatorname{sen}^2\theta + \cos^2\theta}_{=1}) \, d\theta$$

$$= - \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta =$$

$$= - \left[\frac{\theta}{2} + \frac{\operatorname{sen} 2\theta}{4} \right]_0^{2\pi} = - \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{-\pi}$$

(5) $\displaystyle\int_C \frac{x+y}{x^2+y^2} \, dx + \frac{y-x}{x^2+y^2} \, dy$ C: curva fechada qualquer que não passa pela origem

$$\frac{\partial F_2}{\partial x} = \frac{-1(x^2+y^2) - 2x(y-x)}{(x^2+y^2)^2} = \frac{x^2 - y^2 - 2xy}{(x^2+y^2)^2}$$

$$\frac{\partial F_1}{\partial y} = \frac{1(x^2+y^2) - 2y(x+y)}{(x^2+y^2)^2} = \frac{x^2 - y^2 - 2xy}{(x^2+y^2)^2}$$

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

- se o domínio simplesmente conexo que contém C não contém a origem:

$$\oint_C F \cdot dr = \boxed{0}$$

• se o domínio simplesmente conexo que contém C também contém a origem:

1º) $\oint_{C \cup \gamma^-} F \cdot dr = 0$ onde $\gamma: x^2 + y^2 = a^2$

$\sigma(\theta) = (a\cos\theta, a \operatorname{sen}\theta) \quad 0 \leq \theta \leq 2\pi$

C: sentido anti-horário $\qquad \sigma'(\theta) = (-a \operatorname{sen}\theta, a\cos\theta)$

$$\oint_C F \cdot dr = \oint_\gamma F \cdot dr$$

$$= \int_0^{2\pi} \left(\frac{a\cos\theta + a\operatorname{sen}\theta}{a^2}, \frac{a\operatorname{sen}\theta - a\cos\theta}{a^2} \right) \cdot (-a\operatorname{sen}\theta, a\cos\theta)\, d\theta$$

$$= \int_0^{2\pi} \frac{-a^2 \cancel{\operatorname{sen}\theta\cos\theta} - a^2\operatorname{sen}^2\theta + a^2 \cancel{\operatorname{sen}\theta\cos\theta} - a^2\cos^2\theta}{a^2}\, d\theta =$$

$$= \int_0^{2\pi} \frac{-\cancel{a^2}\overbrace{(\operatorname{sen}^2\theta + \cos^2\theta)}^{=1}}{\cancel{a^2}}\, d\theta =$$

$$= \int_0^{2\pi} -1\, d\theta = -[\theta]_0^{2\pi} = \boxed{-2\pi_{\,\shortparallel}}$$

2º) $\oint_{C \cup \gamma^-} F \cdot dr = 0$

C: sentido horário

$$\oint_{C^-} F \cdot dr = -\oint_\gamma F \cdot dr$$

$$\oint_{C^-} F \cdot dr = \boxed{2\pi_{\,\shortparallel}}$$

⑥ i) $F_1(x,y) = \operatorname{sen} x + 4xy$ $F_2(x,y) = 2x^2 - \cos y$

$\frac{\partial F_2}{\partial x} = 4x$, $\frac{\partial F_1}{\partial y} = 4x$ $\therefore \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$

$$\oint_C F_1\,dx + F_2\,dy = \iint_D \underbrace{\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)}_{=0} dx\,dy = \boxed{0}_{//}$$

j) $F_1(x,y) = \frac{y}{x^2+y^2}$ $F_2(x,y) = \frac{-x}{x^2+y^2}$

C não envolve a origem

$\frac{\partial F_2}{\partial x} = \frac{-1(x^2+y^2) + 2x.x}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$

$\frac{\partial F_1}{\partial y} = \frac{1(x^2+y^2) - 2y.y}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$

$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$

$$\oint_C F_1\,dx + F_2\,dy = \iint_D \underbrace{\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)}_{=0} dx\,dy = \boxed{0}_{//}$$

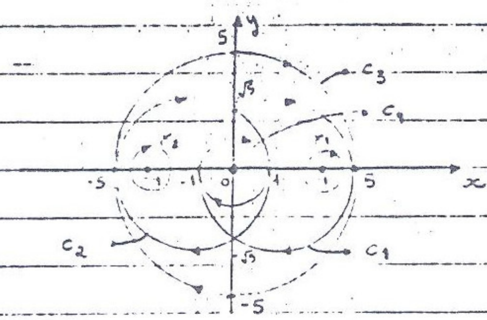
⑦ $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$ singularidades: $(4,0)$, $(0,0)$, $(-4,0)$

C_1: $(x-2)^2 + y^2 = 9$ anti-h $\oint_{C_1} = 11$

C_2: $(x+2)^2 + y^2 = 9$ anti-h $\oint_{C_2} = 9$

C_3: $x^2 + y^2 = 25$ anti-h $\oint_{C_3} = 13$

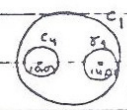
C_4: $x^2 + y^2 = 1$ anti-h $\oint_{C_4} = ?$



• $$\oint_{C_1 \cup C_4^- \cup \gamma_1^-} F\cdot dr = \iint_D \overbrace{\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)}^{=0} dx\,dy = 0$$

$$\oint_{C_1 \cup C_4^- \cup \gamma_1^-} F\cdot dr = \underbrace{\oint_{C_1} F\,dr}_{=11} - \oint_{C_4} F\,dr - \oint_{\gamma_1} F\cdot dr = 0$$

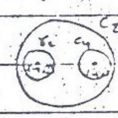
$$\boxed{\oint_{C_4} F\,dr + \oint_{\gamma_1} F\,dr = 11}$$ (1)



• $$\oint_{C_2 \cup C_4^- \cup \gamma_2^-} F\cdot dr = \iint_D \overbrace{\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)}^{=0} dx\,dy = 0$$

$$\oint_{C_2 \cup C_4^- \cup \gamma_2^-} F\,dr = \underbrace{\oint_{C_2} F\,dr}_{=9} - \oint_{C_4} F\,dr - \oint_{\gamma_2} F\,dr = 0$$

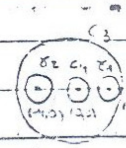
$$\boxed{\oint_{C_4} F\,dr + \oint_{\gamma_2} F\,dr = 9}$$ (2)



• $$\oint_{C_3 \cup C_4^- \cup \gamma_1^- \cup \gamma_2^-} F\cdot dr = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy = 0$$

$$\oint_{C_3 \cup C_4^- \cup \gamma_1^- \cup \gamma_2^-} F\cdot dr = \underbrace{\oint_{C_3} F\,dr}_{=13} - \oint_{C_4} F\,dr - \oint_{\gamma_1} F\,dr - \oint_{\gamma_2} F\,dr = 0$$

$$\boxed{\oint_{C_4} F\,dr + \oint_{\gamma_1} F\,dr + \oint_{\gamma_2} F\,dr = 13}$$ (3)



de ①, ② e ③ temos:

$$\begin{cases} \oint_{C_4} + \oint_{\gamma_1} = 11 \\ \oint_{C_4} + \oint_{\gamma_2} = 9 \implies \oint_{C_4} = 9 - \oint_{\gamma_2} = 9 - 2 = \boxed{7} \\ \underbrace{\oint_{C_4} + \oint_{\gamma_1}}_{=11} + \oint_{\gamma_2} = 13 \implies \oint_{\gamma_2} = 13 - 11 = 2 \end{cases}$$

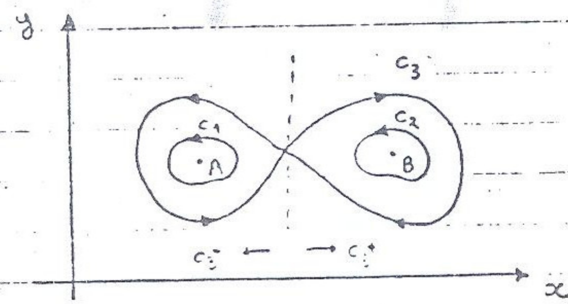
$$\boxed{\oint_{C_4} F_1\,dx + F_2\,dy = 7}$$

⑦ $\dfrac{\partial F_2}{\partial x} = \dfrac{\partial F_1}{\partial y}$

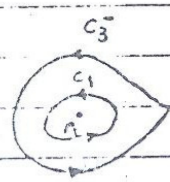
C_1 e C_2: sentido horário

C_3: sentido anti-horário

$\oint_{C_1} F_1\,dx + F_2\,dy = 12$ $\qquad$ $\oint_{C_2} F_1\,dx + F_2\,dy = 15$



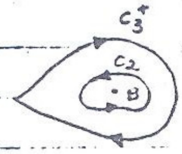
$$\oint_{C_3} = \oint_{C_3^-} + \oint_{C_3^+}$$



$$\oint_{C_3^- \cup C_1^-} F\,dr = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy = 0$$

$$\oint_{C_3^- \cup C_1^-} F\,dr = \oint_{C_3^-} F\,dr - \oint_{C_1} F\,dr = 0$$

$$\oint_{C_3^-} F\,dr = 12$$



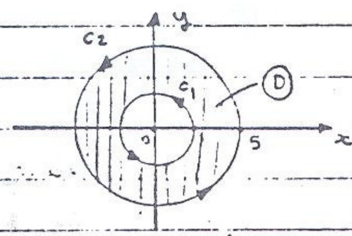
$$\oint_{C_3^+ \cup C_2} F\,dr = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy = 0$$

$$\oint_{C_3^+ \cup C_2} F\,dr = \oint_{C_3^+} F\,dr + \oint_{C_2} F\,dr = 0$$

$$\oint_{C_3^+} F\,dr = -15$$

$$\Rightarrow \oint_{C_3} F\,dr = \oint_{C_3^-} F\,dr + \oint_{C_3^+} F\,dr = 12 - 15 = \boxed{-3}$$

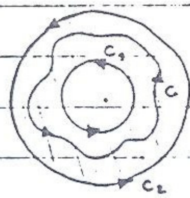
9



$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

$$\oint_{C_1} F\,dr = \oint_{C_2} F\,dr = 2\pi$$

C: qualquer curva fechada contida em D

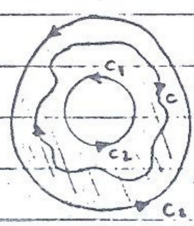


C: sentido anti-horário

$$\oint_{C \cup C_1^-} F \cdot dr = 0$$

$$\oint_C F\,dr - \oint_{C_1} F\,dr = 0$$

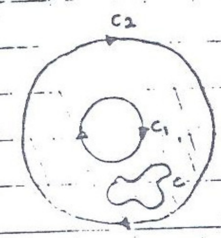
$$\oint_C F \cdot dr = \oint_{C_1} F\,dr = \boxed{2\pi}$$



C: sentido horário

$$\oint_{C \cup C_1} F \cdot dr = 0$$

$$\oint_C F\,dr + \oint_{C_1} F\,dr = 0 \;\therefore\; \oint_C F\,dr = \boxed{-2\pi}$$



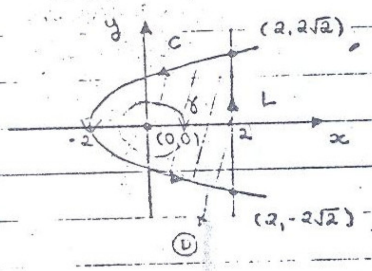
C : sentido horário ou anti-horário

$$\oint_C F\, dr = \iint_D \underbrace{\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)}_{=0} dx\,dy$$

$$\oint_C F\cdot dr = \boxed{0}_{\shortparallel}$$

(ic) $\displaystyle\oint_C \frac{-y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy$ C: $y^2 = 2(x+2)$, $-2 \le x \le 2$

sentido crescente de y



=> O campo está definido num domínio que NÃO é simplesmente conexo (pq, na origem, o campo vetorial não é contínuo). Logo, o campo NÃO é conservativo, apesar de $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$.

$$\frac{\partial F_2}{\partial x} = \frac{1(x^2+y^2) - 2x^2}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$F(x,y)$ só é de classe C^1 para $(x,y) \neq (0,0)$

$$\frac{\partial F_1}{\partial y} = \frac{-1(x^2+y^2) + 2y^2}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$ se $(x,y) \neq (0,0)$

=> A curva C envolve a origem. E todo domínio simplesmente conexo que contém C também contém a origem, logo, o teorema 6.4 não pode ser aplicado. Temos que usar o Teorema de GREEN.

Sendo $\tilde{C} = C \cup L$; γ = circunferência $x^2 + y^2 = a^2$

$$\bullet \oint_{\tilde{C} \cup \gamma^-} F \cdot dr = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy = 0$$

$$\oint_{\tilde{C}} F \cdot dr + \oint_{\gamma^-} F\,dr = 0 \quad \therefore \oint_{\tilde{C}} F \cdot dr = -\oint_{\gamma^-} F\,dr = \oint_{\gamma} F\,dr$$

γ: $\sigma(\theta) = (a\cos\theta, a\,\text{sen}\,\theta)$ $\quad 0 \le \theta \le 2\pi$

$\sigma'(\theta) = (-a\,\text{sen}\,\theta, a\cos\theta)$

$$\oint_{\gamma} F \cdot dr = \int_0^{2\pi} F(\sigma(\theta)) \cdot \sigma'(\theta)\,d\theta =$$

$$= \int_0^{2\pi} \left(-\frac{a\,\text{sen}\,\theta}{a^2}, \frac{a\cos\theta}{a^2} \right) \cdot (-a\,\text{sen}\,\theta, a\cos\theta)\,d\theta$$

$$= \int_0^{2\pi} \frac{a^2\,\text{sen}^2\theta}{a^2} + \frac{a^2\cos^2\theta}{a^2}\,d\theta = \int_0^{2\pi} 1\,d\theta = 2\pi = \oint_{\tilde{C}} F\,dr$$

$$\bullet \oint_{\tilde{C}} F\,dr = \oint_{C \cup L} F\,dr = \oint_C F \cdot dr + \oint_L F\,dr = 2\pi$$

$$\oint_C F\,dr = 2\pi - \oint_L F \cdot dr \quad (*)$$

L: $\sigma_1(t) = (2, t)$ $\quad -2\sqrt{2} \le t \le 2\sqrt{2}$

$\sigma_1'(t) = (0, 1)$

$$\oint_L F \cdot dr = \int_{-2\sqrt{2}}^{2\sqrt{2}} \underbrace{\left(\frac{-t}{4+t^2}, \frac{2}{4+t^2} \right)}_{F(\sigma_1(t))} \cdot \underbrace{(0,1)}_{\sigma_1'(t)}\,dt =$$

$$= \int_{-2\sqrt{2}}^{2\sqrt{2}} \frac{2}{4+t^2}\, dt = \frac{2}{4} \int_{-2\sqrt{2}}^{2\sqrt{2}} \frac{1}{1+(t/2)^2}\, dt \qquad u = t/2, \quad du = 1/2\, dt$$

$$= \frac{1}{2} \cdot 2 \left[\mathrm{arc\,tg} \left(\frac{t}{2} \right) \right]_{-2\sqrt{2}}^{2\sqrt{2}} = \mathrm{arc\,tg}\, \sqrt{2} - \underbrace{\mathrm{arc\,tg}\,(-\sqrt{2})}_{= -(-\mathrm{arc\,tg}\,\sqrt{2})}$$

$$= \boxed{2\, \mathrm{arc\,tg}\, \sqrt{2}}$$

Voltando em (*):

$$\oint_C F \cdot dr = 2\pi - \oint_L F \cdot dr = \boxed{2\pi - 2\, \mathrm{arc\,tg}\, \sqrt{2}}$$