UERJ, Cálculo dif e int 2 , 2013/1

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Lista 08 Regra da cadeia 14/06

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1. Se $z = \cos(xy) + y\cos(x), onde\ x = u^2 + v\ e\ y = u - v^2, utilize\ a\ Regra$ $da\ Cadeia$ para determinar $\frac{\partial z}{\partial u}\ e\ \frac{\partial z}{\partial v}$

2. Sejam $z = f(x,y) = xy^2 - x\ e\ x = g1(u,v) = u - 3v\ e\ y = g2(u,v) = u^2v - 1$
Determine $\frac{\partial z}{\partial u}\ e\ \frac{\partial z}{\partial v}$.
   a) Usando a regra da cadeia
   b) Usando função composta

3. Determinar $\frac{dz}{dt}$ usando a regra da cadeia.
   a) $z = tg(x^2 + y), x = 2t, y = t^2.$
   b) $z = xcos(y), x = sen(t), y = t.$
   c) $z = e^x(\cos(x) + \cos(y)), x = t^3, y = t^2.$

4. Determinar as derivadas parciais $\frac{dz}{dx}\ e\ \frac{dz}{dy}$
   a) $z = \frac{r^2+s}{s}, r = 1 + x, s = x + y$
   b) $z = uv^2 + vln(u), u = 2x - y, v = 2x + y$

5. Se $z = f(u,v), onde\ u = xy, v = \frac{y}{x} e\ f\ \text{têm}\ derivadas\ parciais\ de\ segunda$ $ordem\ \text{contínuas}. mostre\ que$

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = -4uv\frac{\partial^2 z}{\partial u \partial v} + 2v\frac{\partial z}{\partial v}$$

6. A equação de Laplace para $u(x,y,z)$ é dada por:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

   a) Mostre que em coordenadas cilíndricas ela escrita como:

$$x = rcos(\theta); y = rsen(\theta); z = z$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

   b) Mostre que, quando a equação de Laplace é escrita em coordenadas esféricas, ela fica:

$$x = \rho cos(\theta)sen(\varphi); \quad y = \rho sen(\theta)sen(\varphi); \quad z = \rho\cos(\varphi)$$

$$\frac{\partial^2 u}{\partial \rho^2}+\frac{2}{\rho}\frac{\partial u}{\partial \rho}+\frac{cotan(\varphi)}{\rho^2}\frac{\partial u}{\partial \varphi}+\frac{1}{\rho^2}\frac{\partial^2 u}{\partial \varphi^2}+\frac{1}{\rho^2 sen^2(\varphi)}\frac{\partial^2 u}{\partial \theta^2}=0$$

7. Uma partícula de massa m se move sobre uma superfície $z = f(x, y)$. Sejam $x = x(t), y = y(t) as\ coordenadas\ x\ e\ y\ da\ partícula\ no\ instante\ t.$
   a) Determina o vetor velocidade $\vec{v} = \frac{df}{dt}$ e a energia cinética $K = \frac{1}{2}m|\vec{v}|^2\ da\ partícula.$
   b) Estabeleça o vetor aceleração $\vec{a} = \frac{d^2f}{dt^2}$.
   c) Seja $z = x^2 + y^2\ e\ x(t) = tcos(t), y(t) = tsen(t). Determine\ o\ vetor\ velocidade, a\ energia\ cinética\ e\ o\ vetor\ aceleração.$

Gabarito revisado

1. $\frac{\partial z}{\partial u}\ e\ \frac{\partial z}{\partial v}$

$$-3\sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)u^2+2\sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)u\,v^2 - \sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)v+\cos\left(u^2+v\right)-2\sin\left(u^2+v\right)u^2 + 2\sin\left(u^2+v\right)u\,v^2$$

e

$$-\sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)u+3\sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)v^2 + 2\sin\left(\left(u^2+v\right)\left(u-v^2\right)\right)v\,u^2-2\,v\cos\left(u^2+v\right)-\sin\left(u^2+v\right)u + \sin\left(u^2+v\right)v^2$$

2. $\frac{\partial z}{\partial u}\ e\ \frac{\partial z}{\partial v}$, mesmo valor nos dois casos

$$\left(u^2 v-1\right)^2+4\,(u-3\,v)\left(u^2 v-1\right)u\,v-1$$

e

$$-3\left(u^2 v-1\right)^2+2\,(u-3\,v)\left(u^2 v-1\right)u^2+3$$

3. $\frac{dz}{dt}$
   a) $10\left(1+\tan\left(5\,t^2\right)^2\right)t$ $==10tsec^2(5t^2)$
   b) $\cos(t)^2-\sin(t)^2$ retificado
   c) $e^{t^3}\,t\left(3\,t\cos\left(t^3\right)+3\,t\cos\left(t^2\right)-3\sin\left(t^3\right)t-2\sin\left(t^2\right)\right)$

4. $\frac{dz}{dx}$ e $\frac{dz}{dy}$

a)

$$\frac{2y + x^2 + 2xy - 1}{(x+y)^2}$$

e

$$-\frac{1 + 2x + x^2}{(x+y)^2}$$

b)

$$2(2x+y)^2 + 4(2x-y)(2x+y) + 2\ln(2x-y) + \frac{2(2x+y)}{2x-y}$$

e

$$-(2x+y)^2 + 2(2x-y)(2x+y) + \ln(2x-y) - \frac{2x+y}{2x-y}$$

7.a

a) $\vec{v} = \hat{\imath}\frac{\partial f}{\partial x}\frac{dx}{dt} + \hat{\jmath}\frac{\partial f}{\partial y}\frac{dy}{dt}$

$$|\vec{v}| = \sqrt{\left(\frac{\partial f}{\partial x}\frac{dx}{dt}\right)^2 + \left(\frac{\partial f}{\partial y}\frac{dy}{dt}\right)^2}$$

$$K = \frac{1}{2}m\left(\left(\frac{\partial f}{\partial x}\frac{dx}{dt}\right)^2 + \left(\frac{\partial f}{\partial y}\frac{dy}{dt}\right)^2\right)$$

b) $\vec{a} = \hat{\imath}\left[\left(\frac{\partial^2 f}{\partial x^2}\frac{dx}{dt}\frac{dx}{dt} + \frac{\partial^2 f}{\partial y \partial x}\frac{dy}{dt}\frac{dx}{dt} + \frac{\partial f}{\partial x}\frac{d^2 x}{dt^2}\right)\right] +$

$$\hat{\jmath}\left[\left(\frac{\partial^2 f}{\partial x \partial y}\frac{dx}{dt}\frac{dy}{dt} + \frac{\partial^2 f}{\partial y^2}\frac{dy}{dt}\frac{dy}{dt} + \frac{\partial f}{\partial y}\frac{d^2 y}{dt^2}\right)\right]$$

c) Calcule as derivadas necessárias dos itens a) e b) e substitua nas fórmulas encontradas.

$$\frac{\partial f}{\partial x} = 2x \ e \ \frac{\partial^2 f}{\partial x^2} = 2; \ \frac{\partial f}{\partial y} = 2y \ e \ \frac{\partial^2 f}{\partial y^2} = 2; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{dx}{dt} = \cos(t) - t\sin(t) \ , \frac{d^2x}{dt^2} = -2\sin(t) - t\cos(t) \ ; \frac{dy}{dt} = \sin(t) + t\cos(t) \ , \frac{d^2y}{dt^2} = 2\cos(t) - t\sin(t)$$