§6.3 Exercícios

1. Calcule $\int_C f ds$, onde

a) $f(x, y) = x + y$ e C é a fronteira do triângulo de vértices (0,0), (1,0) e (0,1).

b) $f(x, y) = x^2 - y^2$ e C é a circunferência $x^2 + y^2 = 4$.

c) $f(x, y) = y^2$ e C tem equações paramétricas $x = t - \operatorname{sen} t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$.

d) $f(x, y, z) = e^{\sqrt{z}}$ e C é definida por $\sigma(t) = (1, 2, t^2)$, $0 \leq t \leq 1$.

e) $f(x, y, z) = yz$ e C é o segmento de reta de extremidades (0,0,0) e (1,3,2).

f) $f(x, y, z) = x + y$ e C é a curva obtida como interseção do semiplano $x = y$, $y \geq 0$, com o parabolóide $z = x^2 + y^2$, $z \leq 2$.

2. Um arame tem a forma da curva obtida como interseção da porção da esfera $x^2 + y^2 + z^2 = 4$, $y \geq 0$, com o plano $x + z = 2$. Sabendo-se que a densidade em cada ponto do arame é dada por $f(x, y, z) = xy$, calcule a massa total do arame.

3. Deseja-se construir uma peça de zinco que tem a forma da superfície do cilindro $x^2 + y^2 = 4$, compreendida entre os planos $z = 0$ e $x + y + z = 2$,

$z \geq 0$. Se o metro quadrado do zinco custa M reais, calcule o preço total da peça.

4. Calcule $\int_C F \cdot dr$, onde

a) $F(x, y) = (x^2 - 2xy, y^2 - 2xy)$ e C é a parábola $y = x^2$ de $(-2, 4)$ a (1,1).

b) $F(x, y) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right)$ e C é a circunferência de centro na origem e raio a, percorrida no sentido anti-horário.

c) $F(x, y) = (y + 3x, 2y - x)$ e C é a elipse $4x^2 + y^2 = 4$, percorrida no sentido anti-horário.

d) $F(x, y) = (x^2 + y^2, x^2 - y^2)$ e C é a curva de equação $y = 1 - |1 - x|$ de (0,0) a (2,0).

e) $F(x, y, z) = (x, y, xz - y)$ e C é o segmento de reta de (0,0,0) a (1,2,4).

f) $F(x, y, z) = (yz, xz, x(y + 1))$ e C é a fronteira do triângulo de vértices (0,0,0), (1,1,1) e $(-1, 1, -1)$, percorrida nesta ordem.

g) $F(x, y, z) = (x^2 - y^2, z^2 - x^2, y^2 - z^2)$ e C é a curva de interseção da esfera $x^2 + y^2 + z^2 = 4$ com o plano $y = 1$, percorrida no sentido anti-horário quando vista da origem.

h) $F(x, y, z) = (xy, x^2 + z, y^2 - x)$ e C é a curva obtida como interseção do cone $x^2 + y^2 = z^2$, $z \geq 0$, com o cilindro $x = y^2$ de (0,0,0) a $(1, 1, \sqrt{2})$.

5. Calcule o trabalho realizado pelo campo de forças $F(x, y) = (x^2 - y^2, 2xy)$ ao mover uma partícula ao longo da fronteira do quadrado limitado pelos eixos coordenados e pelas retas $x = a$ e $y = a$ $(a > 0)$ no sentido anti- horário.

6. Calcule o trabalho realizado pelo campo de forças $F(x, y, z) = (y^2, z^2, x^2)$ ao longo da curva obtida como interseção da esfera $x^2 + y^2 + z^2 = a^2$ com o cilindro $x^2 + y^2 = ax$, onde $z \geq 0$ e $a > 0$. A curva é percorrida no sentido anti-horário quando vista do plano xy.

7. Determine uma função potencial para cada campo gradiente F dado.

a) $F(x, y) = (e^x \operatorname{sen} y, e^x \cos y)$.

b) $F(x, y) = (2xy^2 - y^3, 2x^2y - 3xy^2 + 2)$.

c) $F(x, y) = (3x^2 + 2y - y^2e^x, 2x - 2ye^x)$.

d) $F(x, y, z) = (y + z, x + z, x + y)$.

e) $F(x, y, z) = (e^{y+2z}, xe^{y+2z}, 2xe^{y+2z})$.

f) $F(x, y, z) = (y \operatorname{sen} z, x \operatorname{sen} z, xy \cos z)$.

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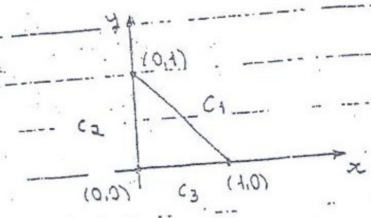
1) a) $f(x,y) = x+y$ — C: triângulo de vértices: $(0,0)$, $(1,0)$ e $(0,1)$



$$\int_C f\,dr = \int_{C_1} f\,dr + \int_{C_2} f\,dr + \int_{C_3} f\,dr$$

$$\int_C f\,dr = \int f(\gamma(t)).\,\|\gamma'(t)\|\,dt$$

C_1: reta $y=-x+1$: $(t,-t+1)$, $0 \le t \le 1$ — $\gamma_1'(t) = (1,-1)$

C_2: reta $x=0$: $(0,t)$, $0 \le t \le 1$ — $\gamma_2'(t) = (0,-1)$

C_3: reta $y=0$: $(t,0)$, $0 \le t \le 1$ — $\gamma_3'(t) = (1,0)$

$$\int_{C_1} f\,dr = \int_0^1 (\cancel{t}-\cancel{t}+1).\sqrt{2}\,dt = \left[\sqrt{2}.t\right]_0^1 = \sqrt{2}$$

$$\int_{C_2} f\,dr = \int_0^1 (0+t).1\,dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$$

$$\int_{C_3} f\,dr = \int_0^1 (t+0).1\,dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$$

$$\int_C f\,dr = \sqrt{2} + \frac{1}{2} + \frac{1}{2} = \boxed{\sqrt{2}+1}$$

b) $f(x,y) = x^2 - y^2$ — C: circunferência: $x^2+y^2=4$

$r(t) = (2\cos t,\ 2\sin t) \qquad 0 \le t \le 2\pi$

$r'(t) = (-2\sin t,\ 2\cos t) \qquad \|r'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t} = \sqrt{4} = 2$

$$\int_C f \cdot dr = \int_0^{2\pi} \underbrace{(4\cos^2 t - 4\sin^2 t)}_{=4(\cos^2 t - \sin^2 t) = 4\cos 2t} \cdot 2\, dt$$

$$= 8 \int_0^{2\pi} \cos 2t \, dt = \qquad \begin{aligned} u &= 2t \\ du &= 2dt \end{aligned}$$

$$= 8 \cdot \frac{1}{2} \left[\sin 2t \right]_0^{2\pi} = \boxed{0}$$

c) $f(x,y) = y^2 \qquad C: \ x = t - \sin t \qquad 0 \le t \le 2\pi$

$\qquad\qquad\qquad\qquad\quad y = 1 - \cos t$

$= (t - \sin t,\ 1 - \cos t) \qquad 0 \le t \le 2\pi$

$r'(t) = (1 - \cos t,\ \sin t)$

$\|r'(t)\| = \sqrt{(1-\cos t)^2 + (\sin t)^2} = \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} =$

$= \sqrt{2(1-\cos t)} = \sqrt{2 \cdot 2\sin^2 (t/2)} = 2 \cdot \sin(t/2)$

(*) $(1 - \cos t) = 2\sin^2(t/2)$

$$\int_C f \cdot dr = \int_0^{2\pi} (1-\cos t)^2 \cdot 2\sin\left(\frac{t}{2}\right) dt =$$

$$= 2 \cdot \int_0^{2\pi} \left(2\sin^2\left(\frac{t}{2}\right)\right)^2 \cdot \sin\left(\frac{t}{2}\right) dt =$$

$$= 8 \cdot 2 \int_0^{2\pi} \sin^4\left(\frac{t}{2}\right) \cdot \underbrace{\frac{1}{2}\sin\left(\frac{t}{2}\right) dt}_{= -2\left(\cos(t/2)\right)} =$$

$$= -16 \int_0^{2\pi} \operatorname{sen}^4\left(\frac{t}{2}\right) \cdot d\left(\cos\left(\frac{t}{2}\right)\right) =$$

$$= -16 \int_0^{2\pi} \left(1-\cos^2(t/2)\right)^2 \cdot d\left(\cos(t/2)\right) =$$

$$= -16 \int_0^{2\pi} 1 - 2\cos^2\left(\frac{t}{2}\right) + \cos^4\left(\frac{t}{2}\right) \; d\left(\cos(t/2)\right) =$$

$$= -16 \left[\cos\left(\frac{t}{2}\right) - \frac{2}{3}\cos^3\left(\frac{t}{2}\right) + \frac{1}{5}\cos^5\left(\frac{t}{2}\right)\right]_0^{2\pi} =$$

$$= -16\left(-1 + \frac{2}{3} - \frac{1}{5} - 1 + \frac{2}{3} - \frac{1}{5}\right) = -16\cdot\left(\frac{-2}{1/15} + \frac{4}{3/5} - \frac{2}{5/3}\right) =$$

$$= -16\cdot\frac{-30+20-6}{15} = -16\cdot\frac{-16}{15} = \boxed{\frac{256}{15}}$$

d) $f(x,y,z) = e^{\sqrt{z}}$ $\qquad$ $C: \sigma(t) = (1, 2, t^2)$, $0 \le t \le 1$

$\sigma'(t) = (0, 0, 2t)$ $\qquad$ $\|\sigma'(t)\| = 2t$

$$\int_C f\, ds = \int_0^1 e^{\sqrt{t^2}} \cdot 2t\, dt = \int_0^1 e^{|t|} \cdot 2t\, dt =$$

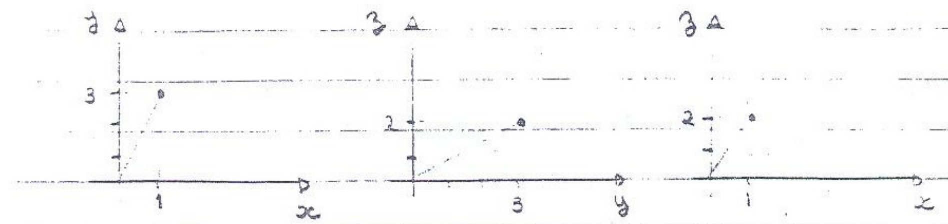
$$= 2\cdot\int_0^1 e^t \cdot t\, dt \qquad u = t \quad du = dt$$

$$dv = e^t dt \quad v = e^t$$

$$= 2\cdot\left(t\cdot e^t - \int_0^1 e^t dt\right) = 2\cdot\left[t\cdot e^t - e^t\right]_0^1 =$$

$$= 2\cdot(\cancel{e} - \cancel{e} + 1) = \boxed{2}$$

e) $f(x, y, z) = yz$ C: segmento de reta c/ extremidades $(0,0,0)$ e $(1,3,2)$



$y = 3x$ $z = \frac{2}{3} y$ $z = 2x$

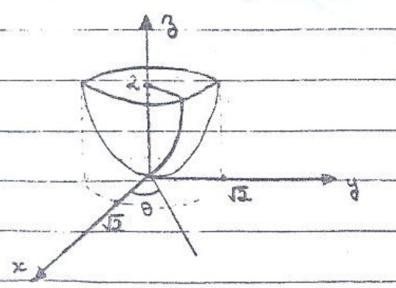
$\sigma(t) = (t, 3t, 2t) \quad 0 \leq t \leq 1$

$\sigma'(t) = (1, 3, 2) \qquad \|\sigma'(t)\| = \sqrt{1+9+4} = \sqrt{14}$

$$\int_C f \cdot ds = \int_0^1 6t^2 \cdot \sqrt{14}\, dt = 6\sqrt{14} \int_0^1 t^2\, dt =$$

$$= 6\sqrt{14} \cdot \left[\frac{t^3}{3}\right]_0^1 = \overset{2}{\cancel{6}}\sqrt{14} \cdot \frac{1}{\cancel{3}} = \boxed{2\sqrt{14}}$$

f) $f(x,y,z) = x+y$ C: interseção: semi-plano $x = y,\ y \geq 0$
parabolóide $z = x^2 + y^2,\ z \leq 2$



$x^2 + y^2 = z \quad \therefore \quad z = 2t^2$

$x = y = t$

$\sigma(t) = (t, t, 2t^2)$

$\sigma'(t) = (1, 1, 4t)$

$\|\sigma'(t)\| = \sqrt{1 + 1 + 16t^2}$
$= \sqrt{2 + 16t^2}$

$\therefore 0 \leq z \leq 2$

$0 \leq 2t^2 \leq 2$

$0 \leq t^2 \leq 1 \quad \therefore \quad 0 \leq t \leq 1$

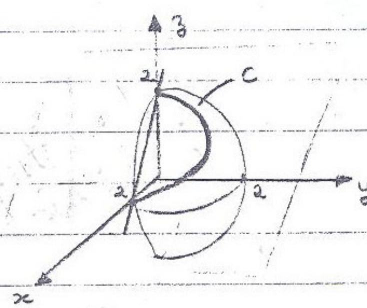
$$\int_C f \cdot dr = \int_0^1 2t \cdot \sqrt{2+16t^2}\, dt \qquad u = 2+16t^2$$
$$du = 32t\, dt$$

$$= \frac{1}{16}\int_0^1 u^{1/2}\, du = \frac{1}{\cancel{16}\,8} \cdot \frac{\cancel{2}}{3}\left[(2+16t^2)^{3/2}\right]_0^1 =$$

$$= \frac{1}{24}\left(\sqrt{18^3} - \sqrt{2^3}\right) = \frac{1}{24}\left(54\sqrt{2} - 2\sqrt{2}\right) = \frac{\overset{13}{\cancel{52}}\sqrt{2}}{\underset{6}{\cancel{24}}} = \boxed{\frac{13\sqrt{2}}{6}}$$

② $f(x,y,z) = xy$ $\qquad$ C: $\overset{\text{intersecção}}{x^2+y^2+z^2=4} \quad y \geq 0$

$x + z = 2 \quad \therefore \quad z = 2 - x$



$x^2+y^2+(2-x)^2 = 4$

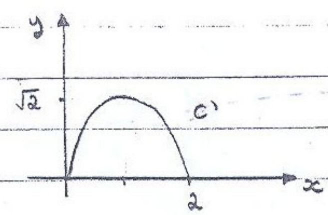
$x^2+y^2+\cancel{4}-4x+x^2 = \cancel{4}$

$2x^2+y^2-4x = 0$

$2(x-1)^2+y^2 = 2$

$(x-1)^2 + \dfrac{y^2}{2} = 1$ $\quad$ C' elipse

↳ projeção de C no plano xy



$\bar{C}: \begin{cases} x-1 = \cos t \\ y = \sqrt{2}\,\text{sen}\, t \end{cases} \qquad 0 \leq t \leq \pi$

$\sigma(t) = \begin{cases} x = 1+\cos t \\ y = \sqrt{2}\,\text{sen}\, t \\ z = 2-(1+\cos t) = 1-\cos t \end{cases} \qquad 0 \leq t \leq \pi$

$\sigma'(t) = \begin{cases} x'(t) = -\text{sen}\, t \\ y'(t) = \sqrt{2}\cos t \\ z'(t) = \text{sen}\, t \end{cases}$ $\qquad \|\sigma'(t)\| = \sqrt{\text{sen}^2 t + 2\cos^2 t + \text{sen}^2 t}$

$= \sqrt{2\underbrace{(\text{sen}^2 t + \cos^2 t)}_{=1}} = \sqrt{2}$

$$M = \int_C f \, dr = \int_0^{\pi} (1+\cos t) \cdot (\sqrt{2} \operatorname{sen} t) \cdot \sqrt{2} \, dt$$

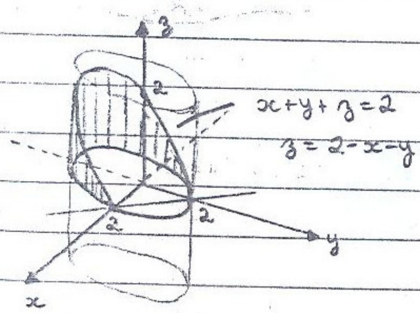
$$M = \int_0^{\pi} (\sqrt{2} \operatorname{sen} t + \sqrt{2} \operatorname{sen} t \cos t) \cdot \sqrt{2} \, dt$$

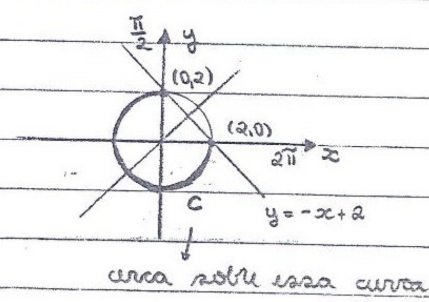
$$M = 2 \int_0^{\pi} \operatorname{sen} t \, dt + 2 \int_0^{\pi} \operatorname{sen} t \underbrace{\cos t \, dt}_{= d(\operatorname{sen} t)}$$

$$M = 2 \cdot \Big[-\cos t \Big]_0^{\pi} + \cancel{2} \cdot \left[\frac{\cancel{\operatorname{sen}^2 t}}{\cancel{2}} \right]_0^{\pi}$$

$$M = 2 \cdot (1+1) = \boxed{4}$$

③ $x^2 + y^2 = 4$ $z = 0$ $x + y + z = 2$ $z \geq 0$





$C: \ \sigma(t) = (2\cos t, 2 \operatorname{sen} t) \quad \pi/2 \leq t \leq 2\pi$

$\sigma'(t) = (-2 \operatorname{sen} t, 2\cos t)$

$\|\sigma'(t)\| = \sqrt{4 \operatorname{sen}^2 t + 4 \cos^2 t} = 2$

área da cerca $= \int_C z \, dS = \int_{\pi/2}^{2\pi} (2 - x - y) \cdot 2 \, dt =$

$$= 2 \cdot \int_{\pi/2}^{2\pi} 2 - 2\cos t - 2\sin t \, dt = 4 \int_{\pi/2}^{2\pi} 1 - \cos t - \sin t \, dt =$$

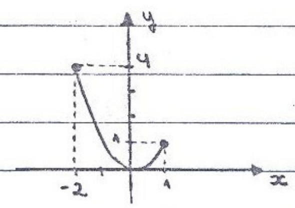
$$= 4 \left[t - \sin t + \cos t \right]_{\pi/2}^{2\pi} = 4 \cdot \left(2\pi - \cancel{0} + 1 - \frac{\pi}{2} + 1 - \cancel{0} \right) =$$

$$= 4 \cdot \left(\frac{3\pi}{2} + 2 \right) = \cancel{4}^{2} \cdot \frac{3\pi + 4}{\cancel{2}_{1}} = (6\pi + 8)\, m^2$$

$1\, m^2 \longrightarrow M$ reais $\therefore$ $\boxed{\text{preço} = (6\pi + 8) \cdot M \text{ reais}}$

(4) a) $F(x,y) = (x^2 - 2xy \, , \, y^2 - 2xy)$

C: parábola $y = x^2$ de $(-2,4)$ a $(1,1)$



$\sigma(t) = (t, t^2) \quad -2 \leq t \leq 1$

$\sigma'(t) = (1, 2t)$

$$\int_C F \cdot dr = \int_{-2}^{1} (t^2 - 2t^3, \; t^4 - 2t^3) \cdot (1, 2t) \, dt =$$

$$= \int_{-2}^{1} (t^2 - 2t^3 + 2t^5 - 4t^4) \, dt = \left[\frac{t^3}{3} - \frac{t^4}{2} + \frac{t^6}{3} - \frac{4t^5}{5} \right]_{-2}^{1} =$$

$$= \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{3} - \frac{4}{5} + \frac{8}{3} + \frac{16}{2} - \frac{64}{3} - \frac{128}{5} \right) =$$

$$= -\frac{54}{3} + \frac{15}{2} - \frac{132}{5} = \frac{-540 + 225 - 792}{30} = -\frac{1107}{30} = \boxed{-\frac{369}{10}}$$

b) $F(x,y) = \left(\dfrac{x}{\sqrt{x^2+y^2}} , \dfrac{y}{\sqrt{x^2+y^2}} \right)$

$C: \ x^2 + y^2 = a^2$ (sentido anti-horário)

$\vec{r}(t) = (a \cos t , a \,\text{sen}\, t) \qquad 0 \leq t \leq 2\pi$

$\vec{r}'(t) = (-a \,\text{sen}\, t , a \cos t)$

$$\int_C F \cdot dr = \int_0^{2\pi} \left(\frac{\cancel{a} \cos t}{\cancel{a}} , \frac{\cancel{a} \,\text{sen}\, t}{\cancel{a}} \right) \cdot (-a \,\text{sen}\, t , a \cos t) \, dt =$$

$$= \int_0^{2\pi} -a \,\cancel{\text{sen}\, t} \cancel{\cos t} + a \,\cancel{\text{sen}\, t} \cos t \, dt = \int_0^{2\pi} 0 \, dt = \boxed{0}$$

c) $F(x,y) = (y + 3x , 2y - x)$

$C: \ 4x^2 + y^2 = 4$ (anti-horário)

$x^2 + \dfrac{y^2}{4} = 1$ elipse

$\vec{r}(t) = (\cos t , 2 \,\text{sen}\, t) \quad 0 \leq t \leq 2\pi$

$\vec{r}'(t) = (-\text{sen}\, t , 2 \cos t)$

$$\int_C F \cdot dr = \int_0^{2\pi} (2 \,\text{sen}\, t + 3 \cos t , 4 \,\text{sen}\, t - \cos t) \cdot (-\text{sen}\, t , 2 \cos t) \, dt =$$

$$= \int_0^{2\pi} -2 \,\text{sen}^2 t - 3 \,\text{sen}\, t \cos t + 8 \,\text{sen}\, t \cos t - 2 \cos^2 t \, dt =$$

$$= \int_0^{2\pi} -2 (\text{sen}^2 t + \cos^2 t) + 5 \,\text{sen}\, t \cos t \, dt =$$

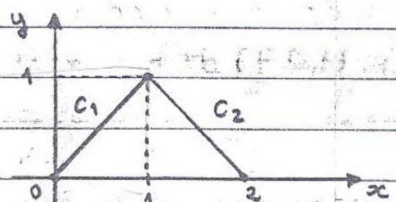
$$= \int_0^{2\pi} -2\,dt + 5.\int_0^{2\pi} \operatorname{sen} t \underbrace{\cos t\,dt}_{d(\operatorname{sen} t)} =$$

$$= \Big[-2t\Big]_0^{2\pi} + 5.\left[\cancel{\frac{\operatorname{sen}^2 t}{2}}^{\,0}\right]_0^{2\pi} = -2\;2\pi = \boxed{-4\pi}$$

d) $F(x,y) = (x^2+y^2,\; x^2-y^2)$

$C: \; y = 1 - |1-x|$ de $(0,0)$ a $(2,0)$

$$C: \begin{cases} y = 1-(1-x) = x & \text{se } x \le 1 \\ y = 1-(x-1) = 2-x & \text{se } x \geq 1 \end{cases}$$



$C_1: \; \sigma_1(t) = (t,t) \quad 0 \le t \le 1$

$\sigma_1'(t) = (1,1)$

$C_2: \; \sigma_2(t) = (t, 2-t) \quad 1 \le t \le$

$\sigma_2'(t) = (1,-1)$

$$\int_{C = C_1 \cup C_2} F \cdot dr = \int_{C_1} F \cdot dr + \int_{C_2} F \cdot dr =$$

$$= \int_0^1 (2t^2, 0)\cdot(1,1)\,dt + \int_1^2 (\underbrace{t^2+(2-t)^2}_{=2t^2-4t+4},\; \underbrace{t^2-(2-t)^2}_{=-4+4t})\cdot(1,-1)\,dt =$$

$$= \int_0^1 2t^2\,dt + \int_1^2 2t^2 - 4t + 4 + 4 - 4t\;dt =$$

$$= \int_0^1 2t^2\,dt + \int_1^2 2t^2 - 8t + 8\,dt =$$

$$= \frac{2}{3}\Big[t^3\Big]_0^1 + \left[\frac{2t^3}{3} - 4t^2 + 8t\right]_1^2 =$$

$$= \frac{2}{3} + \left(\frac{16}{3} - \cancel{16} + \cancel{16} - \frac{2}{3} + 4 - 8 \right) =$$

$$= \frac{2}{3} + \frac{14}{3} - 4 = \frac{16}{3} - 4 = \frac{16-12}{3} = \boxed{\frac{4}{3}}$$

a) $F(x,y,z) = (x, y, xz - y)$

C: segmento de reta de $(0,0,0)$ a $(1,2,4)$

$r(t) = (t, 2t, 4t)$ $\quad r'(t) = (1,2,4)$ $\quad 0 \leq t \leq 1$

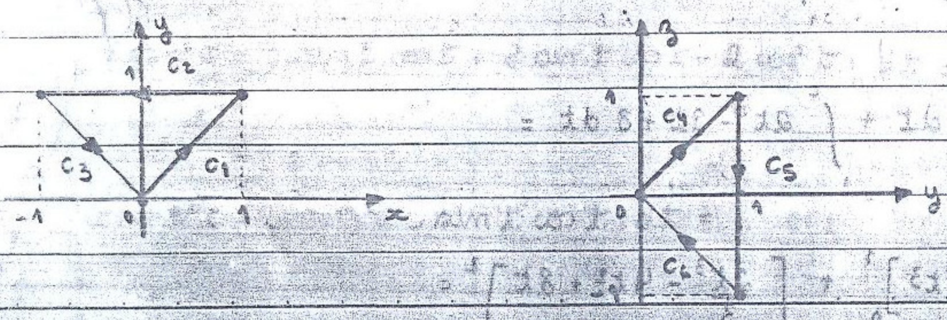
$$\int F \cdot dr = \int_0^1 (t, 2t, 4t^2 - 2t) \cdot (1,2,4)\, dt =$$

$$= \int_0^1 t + 4t + 16t^2 - 8t \, dt = \int_0^1 16t^2 - 3t \, dt =$$

$$= \left[\frac{16t^3}{3} - \frac{3t^2}{2} \right]_0^1 = \frac{16}{3_{/2}} - \frac{3}{2_{/3}} = \frac{32-9}{6} = \boxed{\frac{23}{6}}$$

b) $F(x,y,z) = (yz, xz, x(y+1))$

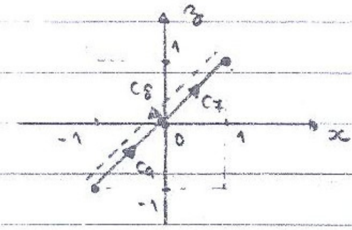
C: triângulo de vértices $(0,0,0)$, $(1,1,1)$ e $(-1,1,-1)$



$C_1: \; y = x$ $\qquad$ $C_4: \; z = y$

$C_2: \; y = 1 \; , \; -1 \leq x \leq 1$ $\qquad$ $C_5: \; y = 1 \; , \; -1 \leq z \leq 1$

$C_3: \; y = -x$ $\qquad$ $C_6: \; z = -y$



$C_7: \; z = x$

$C_8: \; z = x$

$C_9: \; z = x$

$C_a: \; \vec{r}_1 = (t, t, t) \; , \; 0 \leq t \leq 1$ $\qquad$ $\vec{r}_1{}' = (1,1,1)$

$C_b: \; \vec{r}_2 = (-t, 1, -t) \; , \; -1 \leq t \leq 1$ $\qquad$ $\vec{r}_2{}' = (-1,0,-1)$

$C_c: \; \vec{r}_3 = (t, -t, t) \; , \; -1 \leq t \leq 0$ $\qquad$ $\vec{r}_3{}' = (1,-1,1)$

$$\int_{C = C_a \cup C_b \cup C_c} F \cdot dr = \underbrace{\int_{C_a} F \cdot dr}_{\mathrm{I}} + \underbrace{\int_{C_b} F \cdot dr}_{\mathrm{II}} + \underbrace{\int_{C_c} F \cdot dr}_{\mathrm{III}} = \circledast$$

$$\mathrm{I} = \int_0^1 (t^2, t^2, t(t+1)) \cdot (1,1,1) \, dt =$$

$$= \int_0^1 t^2 + t^2 + t^2 + t \; dt = \int_0^1 3t^2 + t \; dt = \left[t^3 + \frac{t^2}{2} \right]_0^1 = \frac{3}{2}$$

$$\mathrm{II} = \int_{-1}^1 (-t, t^2, -2t) \cdot (-1, 0, -1) \, dt =$$

$$= \int_{-1}^1 t + 2t \; dt = \int_{-1}^1 3t \; dt = \left[\frac{3t^2}{2} \right]_{-1}^1 = \frac{3}{2} - \frac{3}{2} = 0$$

$$\mathrm{III} = \int_{-1}^0 (-t^2, t^2, t(-t+1)) \cdot (1, -1, 1) \, dt =$$

$$= \int_{-1}^{0} -t^2 - t^2 - t^2 + t \, dt = \int_{-1}^{0} -3t^2 + t \, dt =$$

$$= \left[-t^3 + \frac{t^2}{2} \right]_{-1}^{0} = -1 - \frac{1}{2} = -\frac{3}{2}$$

$$\circledast \quad \frac{3}{2} + 0 - \frac{3}{2} = \boxed{0_{,,}}$$

g) $F(x,y,z) = (x^2 - y^2,\ z^2 - x^2,\ y^2 - z^2)$

C: $x^2 + y^2 + z^2 = 4 \ \cap\ y = 1$ sentido anti-horário

$= x^2 + z^2 = 4$ (plano // xz)

$\sigma(t) = (2\cos t,\ 1,\ 2 \operatorname{sen} t) \qquad 0 \le t \le 2\pi$

$\sigma'(t) = (-2 \operatorname{sen} t,\ 0,\ 2\cos t)$

$$\int_C F \cdot dr = \int_0^{2\pi} (4\cos^2 t - 1,\ \cancel{4\operatorname{sen}^2 t - 4\cos^2 t},\ 1 - 4\operatorname{sen}^2 t) \cdot (-2\operatorname{sen} t,\ \cancel{0},\ 2\cos t) \, dt =$$

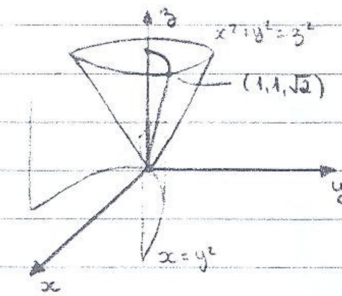
$$= \int_0^{2\pi} -8\operatorname{sen} t \cos^2 t + 2\operatorname{sen} t + 2\cos t - 8\cos t \operatorname{sen}^2 t \, dt =$$

$$= \left[\frac{8\cos^3 t}{3} - 2\cos t + \cancelto{0}{2\operatorname{sen} t} - \cancelto{0}{\frac{8\operatorname{sen}^3 t}{3}} \right]_0^{2\pi} =$$

$$= \cancel{\frac{8}{3}} - \cancel{2} - \cancel{\frac{8}{3}} + \cancel{2} = \boxed{0_{,,}}$$

h) $F(x,y,z) = (xy,\ x^2+z,\ y^2-x)$

$C:\ x^2+y^2=z^2,\ z \geq 0 \ \cap\ x=y^2$ de $(0,0,0)$ a $(1,1,\sqrt{2})$



$y=t \quad x=t^2 \quad z=\sqrt{t^2+t^4}$

$\sigma(t) = (t^2,\ t,\ \sqrt{t^2+t^4})$

$0 \leq t \leq 1$

$\sigma'(t) = \left(2t,\ 1,\ \dfrac{(t^2+t^4)^{-1/2} \cdot (2t+4t^3)}{2}\right)$

$$\int_0^1 (t^3,\ t^4+\sqrt{t^2+t^4},\ \cancel{t^2-t^2}^{\,0}) \cdot \left(2t,1,\ \cancel{\frac{(t^2+t^4)^{-1/2}\cdot(2t+4t^3)}{2}}\right) dt =$$

$$= \int_0^1 2t^4 + t^4 + \sqrt{t^2+t^4}\ dt = \int_0^1 3t^4 + \sqrt{t^2+t^4}\ dt =$$

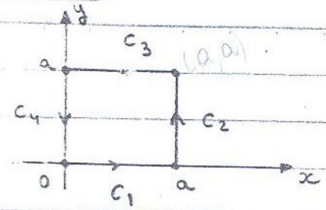
$$= \int_0^1 3t^4\,dt + \int_0^1 t\sqrt{1+t^2}\ dt = \qquad u = 1+t^2 \quad du = 2t\,dt$$

$$= \left[\frac{3t^5}{5}\right]_0^1 + \frac{1}{2}\left[\frac{2}{3}(1+t^2)^{3/2}\right]_0^1 =$$

$$= \frac{3}{5} + \frac{1}{2}\left(\frac{2}{3}\cdot 2\sqrt{2} - \frac{2}{3}\right) = \frac{3}{5_{/3}} + \frac{2\sqrt{2}}{3_{/5}} - \frac{1}{3_{/5}} =$$

$$= \frac{9+10\sqrt{2}-5}{15} = \boxed{\frac{4+10\sqrt{2}}{15}}$$

⑤ $F(x,y) = (x^2-y^2,\ 2xy)$



$C = C_1 \cup C_2 \cup C_3 \cup C_4$

$W = \oint_C F \cdot dr$

$C_1: \sigma_1(t) = (t, 0), \quad 0 \le t \le a \qquad \sigma_1'(t) = (1,0)$

$C_2: \sigma_2(t) = (a, t), \quad 0 \le t \le a \qquad \sigma_2'(t) = (0,1)$

$C_3: \sigma_3(t) = (a-t, a), \quad 0 \le t \le a \qquad \sigma_3'(t) = (-1,0)$

$C_4: \sigma_4(t) = (0, a-t), \quad 0 \le t \le a \qquad \sigma_4'(t) = (0,-1)$

$W = \oint_C F \cdot dr = \int_{C_1} F\,dr + \int_{C_2} F\,dr + \int_{C_3} F\,dr + \int_{C_4} F\,dr$

$= \int_0^a (t^2, 0) \cdot (1, 0)\,dt + \int_0^a (\cancel{*}, 2at) \cdot (\cancel{0}, 1)\,dt + \int_0^a (\underbrace{(a-t)^2 - a^2}_{= t^2 - 2at}, \cancel{*}) \cdot (-1, \cancel{0})\,dt +$

$+ \int_0^a (\cancel{*}, \cancel{0}) \cdot (\cancel{0}, \cancel{-1})\,dt =$

$= \int_0^a \cancel{t^2} + 2at - \cancel{t^2} + 2at\,dt = \int_0^a 4at\,dt = \left[2at^2\right]_0^a = \boxed{2a^3}$

⑥ $F(x,y,z) = (y^2, z^2, x^2)$

$C: \; x^2 + y^2 + z^2 = a^2 \; \cap \; x^2 + y^2 = ax \qquad z \geq 0 \quad a > 0$

intersecção: $\quad x^2 + y^2 = a^2 - z^2 \qquad x^2 + y^2 = ax$

$a^2 - z^2 = ax$

$W = \int_C F \cdot dr$

$z^2 = a^2 - ax$

$z = \pm\sqrt{a^2 - ax}$

Resposta 6: $\frac{\text{pi. } A^3}{4}$

7 a) $F(\cdot\ y) = (e^x \operatorname{sen} y,\ e^x \cos y)$

- $\dfrac{\partial f}{\partial x} = e^x \operatorname{sen} y \longrightarrow f(x,y) = e^x \operatorname{sen} y + A(y)$

- $\dfrac{\partial f}{\partial y} = e^x \cos y \longrightarrow f(x,y) = e^x . \operatorname{sen} y + B(x)$

$$\boxed{f(x,y) = e^x . \operatorname{sen} y_{\,\shortparallel}}$$

b) $F(x,y) = (2xy^2 - y^3,\ 2x^2y - 3xy^2 + 2)$.

- $\dfrac{\partial f}{\partial x} = 2xy^2 - y^3 \longrightarrow f(x,y) = x^2y^2 - xy^3 + A(y)$

- $\dfrac{\partial f}{\partial y} = 2x^2y - 3xy^2 + 2 \longrightarrow f(x,y) = x^2y^2 - xy^3 + 2y + B(x)$

$$\boxed{f(x,y) = x^2y^2 - xy^3 + 2y_{\,\shortparallel}}$$

c) $F(x,y) = (3x^2 + 2y - y^2 . e^x,\ 2x - 2ye^x)$

- $\dfrac{\partial f}{\partial x} = 3x^2 + 2y - y^2 . e^x \longrightarrow f(x,y) = x^3 + 2xy - e^x y^2 + A(y)$

- $\dfrac{\partial f}{\partial y} = 2x - 2ye^x \longrightarrow f(x,y) = 2xy - e^x y^2 + B(x)$

$$\boxed{f(x,y) = x^3 + 2xy - e^x y^2_{\,\shortparallel}}$$

i) $F(x,y,z) = (y+z,\ x+z,\ x+y)$

- $\dfrac{\partial f}{\partial x} = y + z \longrightarrow f(x,y,z) = xy + xz + A(y,z)$

- $\dfrac{\partial f}{\partial y} = x + z \longrightarrow f(x,y,z) = xy + yz + B(x,z)$

- $\dfrac{\partial f}{\partial z} = x + y \longrightarrow f(x,y,z) = xz + yz + C(x,y)$

$$\boxed{f(x,y,z) = xy + xz + yz_{\,\shortparallel}}$$

j) $F(x,y,z) = (e^{y+2z},\ x.e^{y+2z},\ 2x.e^{y+2z})$

- $\dfrac{\partial f}{\partial x} = e^{y+2z} \longrightarrow f(x,y,z) = x.e^{y+2z} + A(y,z)$

- $\dfrac{\partial f}{\partial y} = x.e^{y}.e^{2z} \longrightarrow f(x,y,z) = x.e^{y+2z} + B(x,z)$

- $\dfrac{\partial f}{\partial z} = 2x.e^{y}.e^{2z} \longrightarrow f(x,y,z) = x.e^{y+2z} + C(x,y)$

$$\boxed{f(x,y,z) = x.e^{y+2z}{}_{\,\shortparallel}}$$

k) $F(x,y,z) = (y \operatorname{sen} z,\ x \operatorname{sen} z,\ xy \cos z)$

- $\dfrac{\partial f}{\partial x} = y \operatorname{sen} z \longrightarrow f(x,y,z) = xy \operatorname{sen} z + A(y,z)$

- $\dfrac{\partial f}{\partial y} = x \operatorname{sen} z \longrightarrow f(x,y,z) = x \cdot y \operatorname{sen} z + B(x,z)$

- $\dfrac{\partial f}{\partial z} = xy \cos z \longrightarrow f(x,y,z) = x \cdot y \operatorname{sen} z + C(x,y)$

$$\boxed{f(x,y,z) = xy \operatorname{sen} z}$$