AIRBUS QUANTUM COMPUTING CHALLENGE

Problem Statement n°1

Aircraft Climb Optimization

1. Problem statement

1.1 The climb

The trajectory of an aircraft, also called the "mission" between the departure airport and the destination airport is made of several phases of flight, that are visible on the graph below.

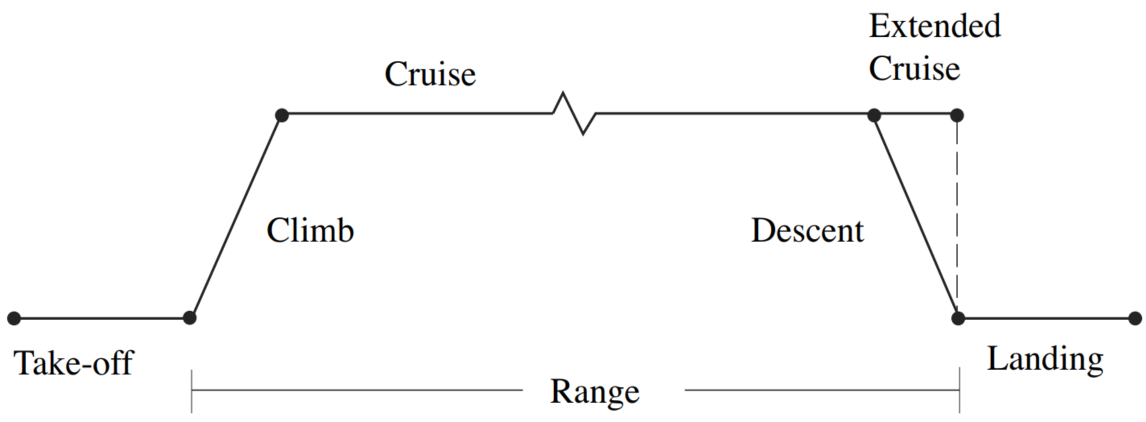

Figure 1: Mission profile

Regarding the fuel and time optimization of the flight, the cruise is often considered as the most important part. However, climb and descent are more critical when it comes to short-haul flights, which tend nowadays to be more and more frequent. For Airlines operating these kind of flights, the optimization of climb and descent is very valuable. Therefore AIRBUS is interested in providing for that topics the best products and associated services. In this problem we will focus on the optimization of the climb trajectory.

1.2 The Cost Index

We look at optimizing speed and thrust laws during climb phase between an initial point I where the aircraft is supposed to equilibrate while climbing at the MCL thrust (Max Climb) and the speed CAS_I, and the first cruise level located at the altitude Zp_F. The optimization criteria is the cost ϕ of the trajectory, which brings into play the time and the fuel consumption through the Cost Index CI:

$$\phi = \text{consumption} + CI \times \text{time}. \quad (1)$$

CI is the Cost-Index: it represents the ratio between the fuel consumed and the flight duration. For instance, the extreme CI values are:

- $CI = 0$: in that case, cost of time is null, meaning that, for instances, the crew has fixed wages, whatever the duration of the flight.
- $CI = CI_{max}$ meaning that fuel price is low compared to flight time cost. In that case, the airline will try to make the trip as fast as possible, whatever the consumption of fuel it will induce.

For instance, a cost index of 30 kg/min means that the cost of one flight minute is the same as the cost of 30 kg of fuel. As an example, the article [1] shows a Cost Index database for a large set of airlines and aircraft. The present document will focus on data applicable to short-haul aircraft such as A320 family.
Supposing that this ratio is constant, it allows to replace a multi-criteria optimization (minimize the fuel consumption and the duration of the flight) by a single-criteria problem, easier to solve.
For the different acceptable solutions to be compared, it is needed that the corresponding trajectories are all in the same final state. For that, we pursue the climb phase with an acceleration at a constant altitude and a constant thrust until reaching cruise Mach number, which value is fixed. We finally complete the trajectory with a cruise segment at constant altitude, constant Mach, and adapted thrust until the total length traveled from the beginning of the climb is equal to L.

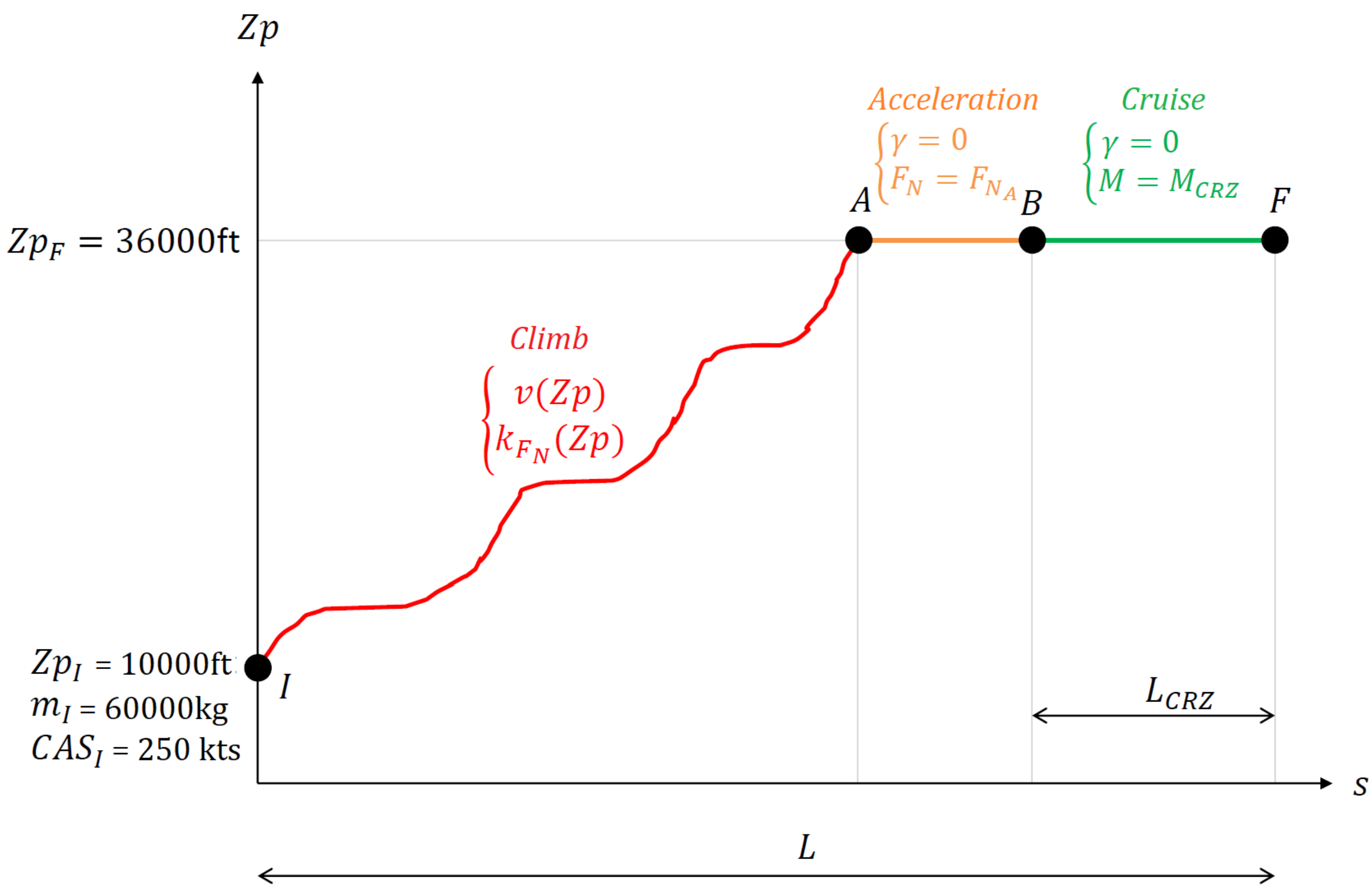

Figure 2: Trajectory profile

The cost of the complete trajectory is therefore equal to the cost of the climb itself, plus the costs of those two additional phases.

2. Problem description

2.1 Mathematic expression

The mathematic problem to solve is the following:

$$\min_{\{v_i,\gamma_i,m_i,t_i,s_i,Cz_i,\lambda_i\}_{1\le i\le N-1}} \phi(v_{N-1},m_{N-1},t_{N-1},s_{N-1},\lambda_{N-1}),$$

s.c.

$$g_{v_i}=\frac{v_{i+1}-v_i}{Zp_{i+1}-Zp_i}-\frac{1}{2}\left(\frac{\lambda_{i+1}F_{N_{MCL_{i+1}}}}{m_{i+1}v_{i+1}\sin\gamma_{i+1}}-\frac{\frac{1}{2}\rho(Zp_{i+1})v_{i+1}S_{REF}\left(Cx_0+kCz_{i+1}^2\right)}{m_{i+1}\sin\gamma_{i+1}}-\frac{g_0}{v_{i+1}}+\frac{\lambda_iF_{N_{MCL_i}}}{m_iv_i\sin\gamma_i}-\frac{\frac{1}{2}\rho(Zp_i)v_iS_{REF}\left(Cx_0+kCz_i^2\right)}{m_i\sin\gamma_i}-\frac{g_0}{v_i}\right)=0\ (0\le i\le N-2),$$

$$g_{\gamma_i}=\frac{\gamma_{i+1}-\gamma_i}{Zp_{i+1}-Zp_i}-\frac{1}{2}\left(\frac{\frac{1}{2}\rho_{i+1}S_{REF}Cz_{i+1}}{m_{i+1}\sin\gamma_{i+1}}-\frac{g_0}{v_{i+1}^2\tan\gamma_{i+1}}+\frac{\frac{1}{2}\rho_iS_{REF}Cz_i}{m_i\sin\gamma_i}-\frac{g_0}{v_i^2\tan\gamma_i}\right)=0\ (0\le i\le N-2),$$

$$g_{m_i}=\frac{m_{i+1}-m_i}{Zp_{i+1}-Zp_i}+\frac{1}{2}\eta\left(\frac{\lambda_{i+1}F_{N_{MCL_{i+1}}}}{v_{i+1}\sin\gamma_{i+1}}+\frac{\lambda_iF_{N_{MCL_i}}}{v_i\sin\gamma_i}\right)=0\ (0\le i\le N-2),$$

$$g_{t_i}=\frac{t_{i+1}-t_i}{Zp_{i+1}-Zp_i}-\frac{1}{2}\left(\frac{1}{v_{i+1}\sin\gamma_{i+1}}+\frac{1}{v_i\sin\gamma_i}\right)=0\ (0\le i\le N-2),$$

$$g_{s_i}=\frac{s_{i+1}-s_i}{Zp_{i+1}-Zp_i}-\frac{1}{2}\left(\frac{1}{\tan\gamma_{i+1}}+\frac{1}{\tan\gamma_i}\right)=0\ (0\le i\le N-2),$$

$$g_{VMO_i}=CAS(v_i,Zp_i)-VMO\le 0\ (1\le i\le N-1),$$

$$g_{MMO_i}=MACH(v_i,Zp_i)-MMO\le 0\ (1\le i\le N-1)$$

$$g_{Vz_i}=Vz_{min}-v_i\sin\gamma_i\le 0\ (1\le i\le N-1),$$

$$Cz_i\le Cz_{max}\ (1\le i\le N-1),$$

$$0\le\lambda_i\le 1\ (1\le i\le N-1),$$

with

$$v_0=TAS(CAS_I,Zp_I),$$

$$\gamma_0=\arcsin\left[\frac{F_{N_{MCL}}(Zp_0)-\frac{1}{2}\rho(Zp_0)v_0^2S_{REF}\left[Cx_0+k\left(\frac{m_0g_0}{\frac{1}{2}\rho(Zp_0)v_0^2S_{REF}}\right)\right]}{m_0g_0}\right],$$

$$m_0=m_I,$$

$$t_0=0,$$

$$s_0=0,$$

$$Cz_0=\frac{m_0g_0}{\frac{1}{2}\rho(Zp_0)v_0^2S_{REF}},$$

$$\lambda_0=1.$$

with :

$$Zp_i = Zp_I + i\frac{Zp_F - Zp_I}{N-1} \qquad (0 \leq \mathrm{i} \leq \mathrm{N}-1),$$

and :

$$\begin{cases} F_{N_{MCL_i}} = F_{N_{MCL}}(Zp_i)(\text{provided by the aircraft performance model}), \\ \rho_i = \rho_0\left(\frac{Ts_0 + L_Z Zp_i}{Ts_0}\right)^{\alpha_0 - 1}, \\ M(v, Zp) = \frac{v}{\sqrt{1.4R(Ts_0 + L_Z Zp)}}, \\ CAS(v, Zp) = \sqrt{7RTs_0\left\{\left\{\left(\frac{Ts_0 + L_Z Zp}{Ts_0}\right)^{\alpha_0} \cdot \left[\left(1 + \frac{v^2}{7R(Ts_0 + L_Z Zp)}\right)^{3.5} - 1\right] + 1\right\}^{\frac{1}{3.5}} - 1\right\}}, \\ TAS(CAS, Zp) = \sqrt{7R(Ts_0 + L_Z Zp)\left\{\left\{\left(\frac{Ts_0 + L_Z Zp}{Ts_0}\right)^{-\alpha_0} \cdot \left[\left(1 + \frac{CAS^2}{7RTs_0}\right)^{3.5} - 1\right] + 1\right\}^{\frac{1}{3.5}} - 1\right\}}, \end{cases}$$

The Cz_i equation is not here for this vers

where:

$$\begin{cases} Ts_0 = 288.15°\mathrm{K}, \\ \rho_0 = 1.225\mathrm{kg.m^{-3}}, \\ L_Z = -0.0065\mathrm{K.m^{-1}}, \\ g_0 = 9.80665.\mathrm{m.s^{-2}}, \\ \alpha_0 = -\frac{g_0}{RL_Z}, \\ R = 287.05287\mathrm{N.m.kg^{-1}.K^{-1}}. \end{cases} \tag{2}$$

The criteria $\phi(\lambda_{N-1}, v_{N-1}, m_{N-1}, s_{N-1}, t_{N-1})$ can be computed as follow:

$$\begin{cases} \rho_F = \rho_0\left(\frac{Ts_0 + L_Z Zp_F}{Ts_0}\right)^{-\left(\frac{g_0}{RL_Z}+1\right)}, \\ v_F = M_{CRZ}\sqrt{1.4R(Ts_0 + L_Z Zp_F)}, \end{cases}$$

then:

$$\begin{cases} A = -\frac{\rho_F S_{REF} Cx_0}{2m_{N-1}} - \frac{6km_{N-1}g_0^2}{\rho_F S_{REF} v_{N-1}^4}, \\ B = \frac{16km_{N-1}g_0^2}{\rho_F S_{REF} v_{N-1}^3}, \\ C = \frac{F_{N_{MCL_{N-1}}}}{m_{N-1}} - \frac{12km_{N-1}g_0^2}{\rho_F S_{REF} v_{N-1}^2}, \\ D = \sqrt{B^2 - 4AC}, \end{cases}$$

and then:

$$
\begin{cases}
t_B = t_{N-1} + \dfrac{2}{D}\left[\tanh^{-1}\left(\dfrac{2Av_{N-1}+B}{D}\right) - \tanh^{-1}\left(\dfrac{2Av_F+B}{D}\right)\right], \\
m_B = m_{N-1} - \eta\lambda_{N-1}F_{N_{MCL_{N-1}}}(t_B - t_{N-1}), \\
s_B = s_{N-1} + \dfrac{1}{A}\log\left(\dfrac{D-2Av_F-B}{D-2Abv_{N-1}-B}\right) - \dfrac{B+D}{2A}(t_B - t_{N-1}).
\end{cases}
$$

b minuscule is not in the formula.Log is natural log = Ln

Finally:

$$
\begin{cases}
m_F = m_B e^{-\frac{2\eta g_0\sqrt{kCx_0}}{v_F}(s_F - s_B)}, \\
t_F = t_B + \dfrac{s_F - s_B}{v_F},
\end{cases}
$$

and

$$
\boxed{\phi(v_{N-1}, m_{N-1}, t_{N-1}, s_{N-1}, \lambda_{N-1}) = -m_F + CI\left(t_B - \frac{s_B}{v_F}\right).}
$$

CHECK

- $Zp_{N-1} = 36000$ ft
- $v_{N-1} \approx 223.61 \text{ m.s}^{-1}$ (=250 kt CAS)
- $v_F \approx 236.15 \text{ m.s}^{-1}$ (Mach=0.8)
- $m_{N-1} = 59042$ kg
- $t_{N-1} = 880.8$ s
- $s_{N-1} = 168717.2$ m
- $\lambda_{N-1} = 1$

give

- $F_{N_{MCL_{N-1}}} = 48920$ N
- $\rho_F \approx 0.3652 \text{ kg.m}^{-3}$
- $A \approx -3.31814e-05 \text{ m}^{-1}$
- $B \approx 0.0166878\ s^{-1}$
- $C \approx -1.970107 \text{ m.s}^{-2}$
- $t_B \approx 992.839$ s
- $m_B \approx 58950.65$ kg
- $s_B \approx 194453.96$ m
- $m_F \approx 58358.27$ kg
- $t_F \approx 1863.24$ s
- $\boxed{\phi \approx -53272.68 \text{ kg}}$

These values are OK

This is not correct if CI is in seconds (which has a value of CI=0.5) and phi= -

2.2 Problem characteristics

The dimension of the vector to optimize (number of unknowns) is equal to $7(\mathrm{N}-1)$ and the number of constraints is equal to $10(\mathrm{N}-1)$. By choosing

$$\boxed{N = 53}$$

(which corresponds to a discretization of one point every 500ft if $Zp_I = 10000\text{ft}$ and $Zp_F = 36000\text{ft}$), we then get a **non-linear problem with 364 unknowns and 520 constraints**.
One should be very careful at the units of all variables, as given in the following section, to keep the problem consistent.

2.3 Parameters

The parameters of this problem are the following:

	Name	Definition	Value	Unit (M=mass, L=length, T=time)
1	Cx_0	2*Aerodynamic polar coefficients	0.014	-
2	k		0.09	
3	Cz_{max}	Maximum lift coefficient	0.7	-
4	S_{REF}	Aerodynamic reference area	120	m^2 $[\mathrm{L}^2]$
5	η	Specific fuel consumption	0.06	$\mathrm{kg(N.h)}^{-1}[\mathrm{L}^{-1}\mathrm{T}]$
6	Zp_I	Initial altitude	10000	ft [L]
7	Zp_F	Final altitude	36000	ft [L]
8	m_I	Initial mass	60000	kg [M]
9	CAS_I	Initial Calibrated Air Speed (CAS)	250	kt $[\mathrm{LT}^{-1}]$
10	VMO	Maximum Calibrated Air Speed (CAS)	350	kt $[\mathrm{LT}^{-1}]$
11	MMO	Maximum Mach number	0.82	-
12	M_{CRZ}	Cruise Mach number	0.80	-
13	$L(= s_F)$	Total length of the trajectory	400	km [L]
14	Vz_{min}	Minimum climb speed	300	$\mathrm{ft.min}^{-1}$ $[\mathrm{LT}^{-1}]$
15	g_0	Acceleration of gravity	9.80665	$\mathrm{m.s}^{-2}$ $[\mathrm{LT}^{-2}]$
16	CI	Cost Index	30	$\mathrm{kg.min}^{-1}$ $[\mathrm{MT}^{-1}]$

We also express $F_{N_{MCL}}(Zp)$ that gives the total Max Climb thrust (sum of the thrust of all engines), which is a function of the altitude of the aircraft :

$$F_{N_{MCL}}(Zp) = 140000 - 2.53\, Zp \qquad [\mathrm{MLT}^{-2}]$$

where Zp is in ft and $F_{N_{MCL}}$ in N.

Finally, we recall that:

- 1ft = 0.3048 m

- $1\text{kt} = \frac{1852}{3600}\,\text{ms}^{-1}$

3. KPI

The following Key Performance Indicators will be used to assess the submitted proposals:

- Provide a Quantum algorithm, or hybrid solution, for this optimization problem
- Provide an estimate of the required Quantum Computing hardware to run the established method: number of qubits, coherence time, connectivity...
- Provide a comparison of the required computation time between a quantum algorithm and the expected implementation of the proposed optimization method on a classical hardware.

References

- Cost-Index Database provided by TOGA Projects and AviationLads
  https://www.scribd.com/document/374519450/Cost-Index-Database-2017 AviationLads.com

Additioanl Information

Please refer to the attached document for more information regarding:

1. Jacobian of constraints
2. Gradient of the criterion

Note: These values could be useful for the optimization algorithm but strictly speaking, they are not necessary in the mathematical formulation of the problem. They are given as information only.