Deep Learning and Inverse Problems
Summer 2021
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Problem Set 3
Issued: Wednesday April 28, 2021, Due: Wednesday May 5 2021.

Problem 1 (Compressive sensing)**.** Suppose we are interested in recovering a vector $\tilde{\mathbf{x}} \in \mathbb{R}^n$ from as few measurements as possible. We know that the vector $\tilde{\mathbf{x}}$ consists of no more than k blocks of constant values, but we don't know where those blocks are. An example is the vector $\tilde{\mathbf{x}} = [1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 0, 0, 0]$, consisting of three such blocks.
This is a simplistic model of an image, since an image consists of patches or blocks of color separated by sharp edges, think about the collections of numbers as a shade of gray or block of color.
This problem is a conceptual one, aimed at practicing your understanding of sparse modeling and sparse recovery, your answers should be very brief.

1. Find a dictionary $\mathbf{W}$ in which the vector $\tilde{\mathbf{x}}$ is as sparse as possible.
2. Suppose you wish to recover the vector $\tilde{\mathbf{x}}$ from as few linear measurements obtained as

$$y_i = \langle \mathbf{a}_i, \tilde{\mathbf{x}} \rangle, \quad i = 1, \ldots, m$$

   as possible. How many such linear measurements are necessary to recover $\tilde{\mathbf{x}}$ with any method?
3. Give a choice of measurement vectors $\mathbf{a}_i$ and an efficient algorithm for recovering $\tilde{\mathbf{x}}$, and state how many measurements the algorithm requires for recovering $\tilde{\mathbf{x}}$.

Problem 2 (Coherence and RIP)**.** Use Geshgorin's circle theorem (which you find in the internet or in a linear algebra text book such as Horn & Johnson) to show that a matrix with coherence μ obeys the RIP, and determine the order and constant with which the RIP is satisfied. What is the limitation of using this approach to bound the RIP?